Muon Bundle Multiplicities in Deep Underground Detectors as a Probe of Pion-Air Interactions

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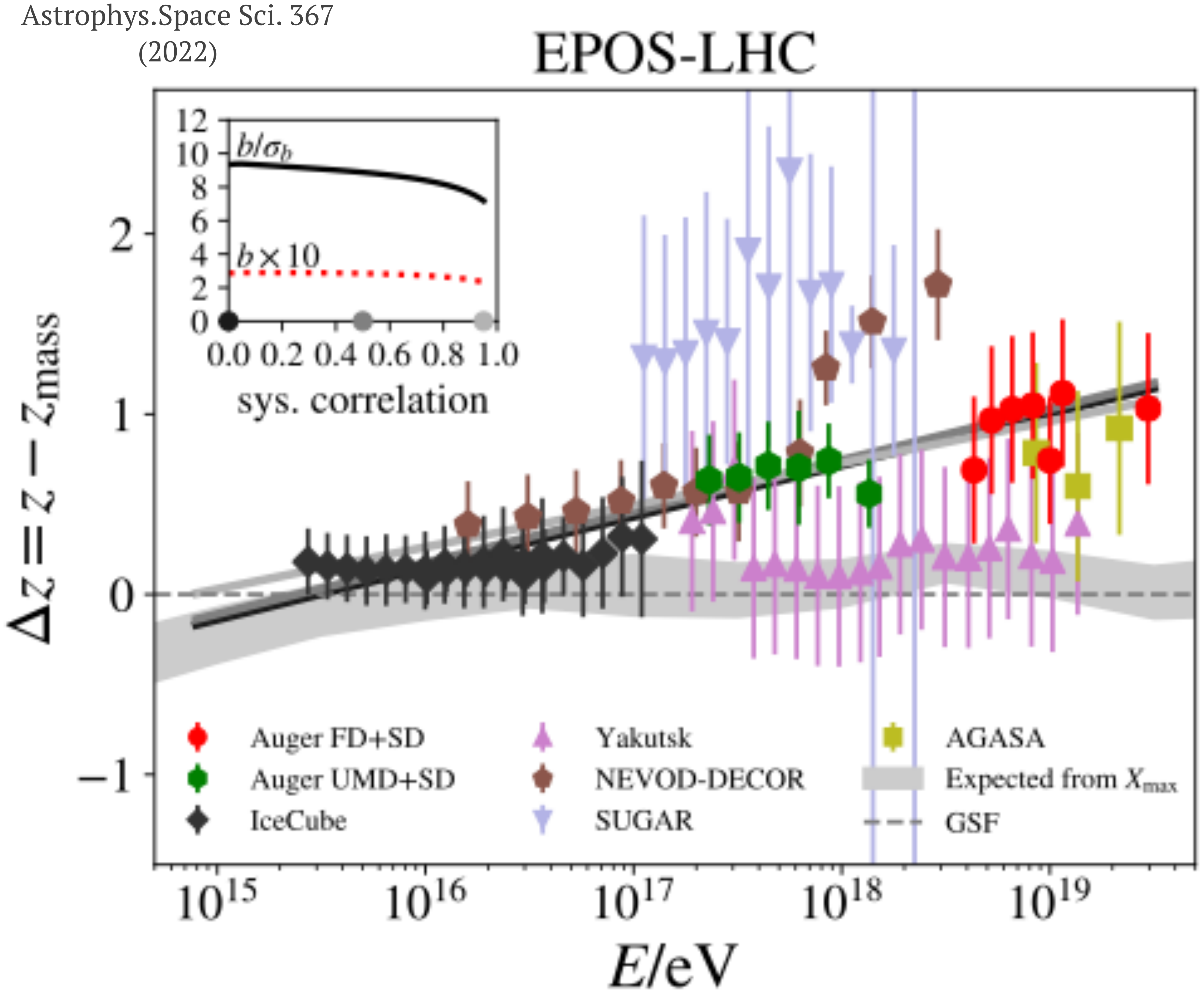
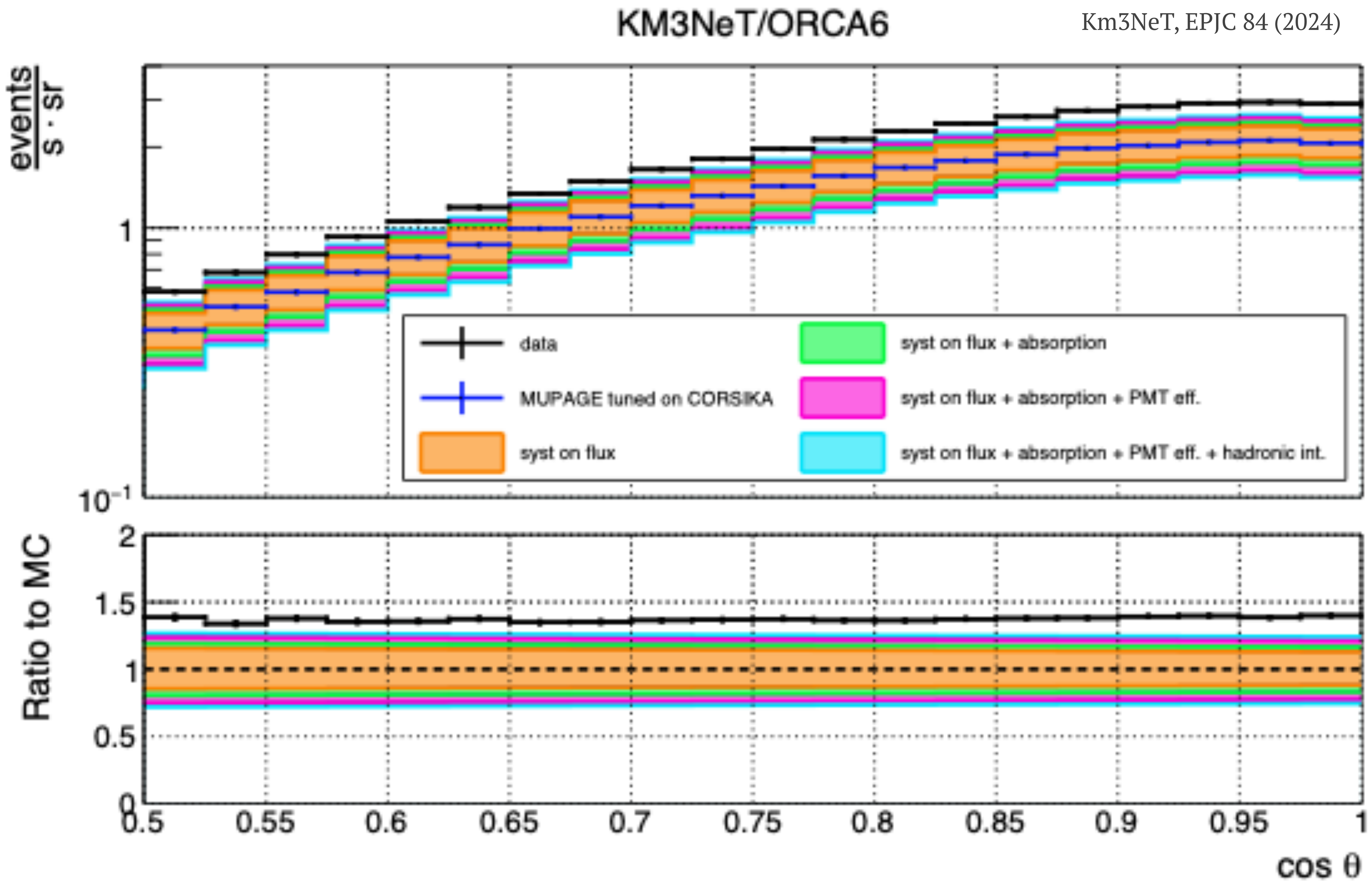
Muon number discrepancies

Abrecht et al.,
Astrophys.Space Sci. 367
(2022)



Mismatch between data & simulation at EeV energies at cosmic ray air shower experiments

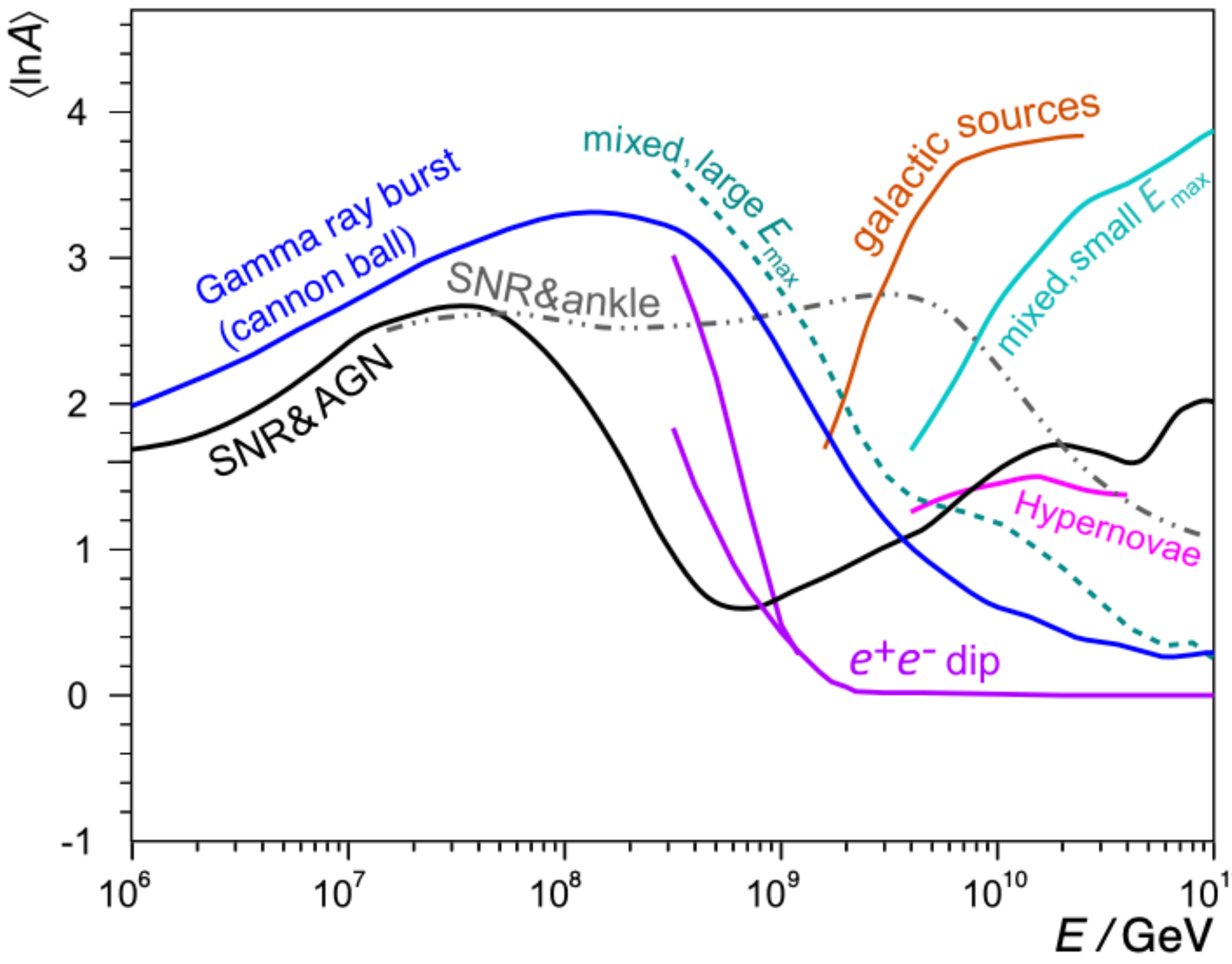
Km3NeT, EPJC 84 (2024)

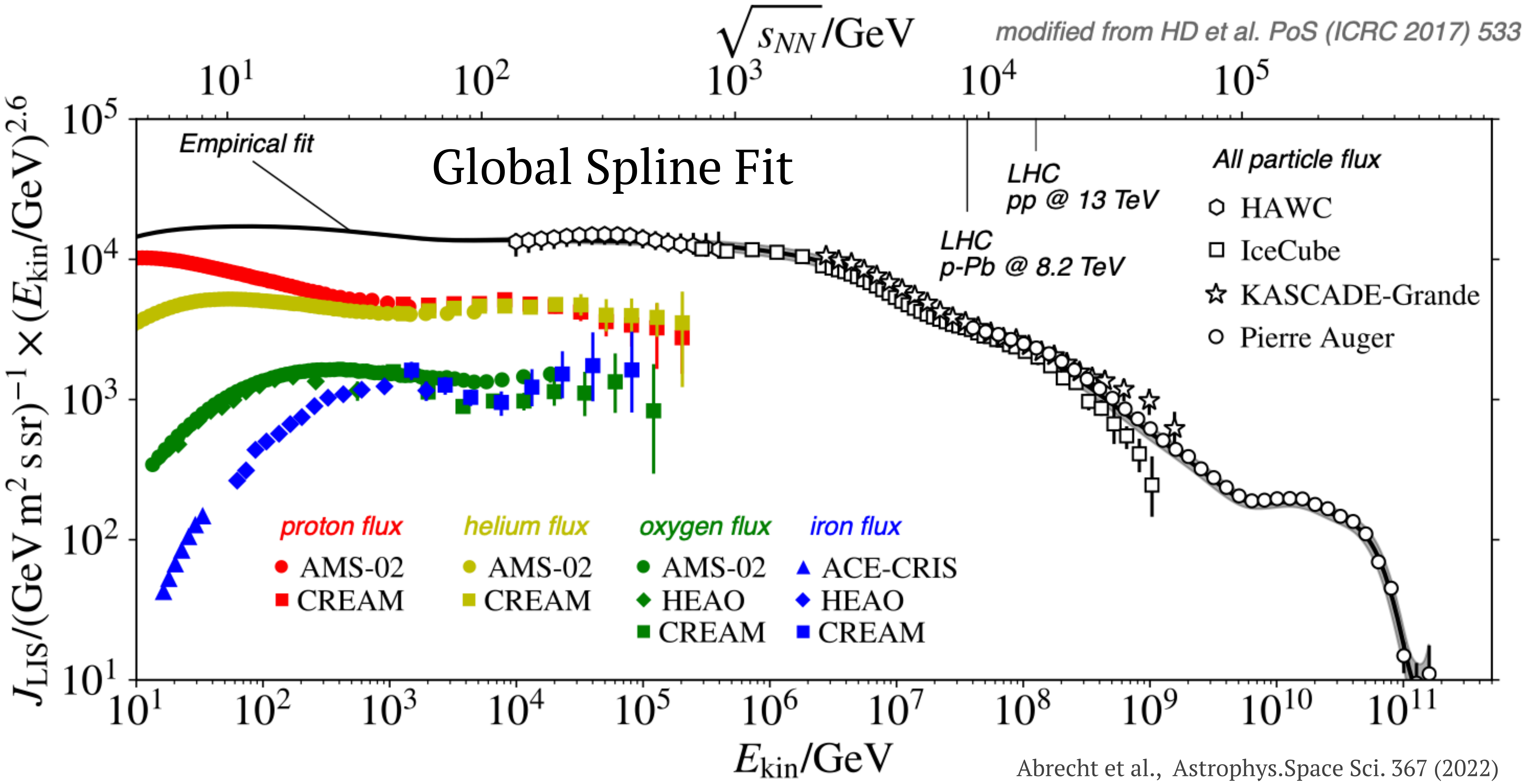


40% disagreement between data and MC in TeV range measured by the underground KM3Net neutrino telescope (SIBYLL2.3D & GSF)

Cosmic Rays

Updated GSF 2024 by Fujisue et al.



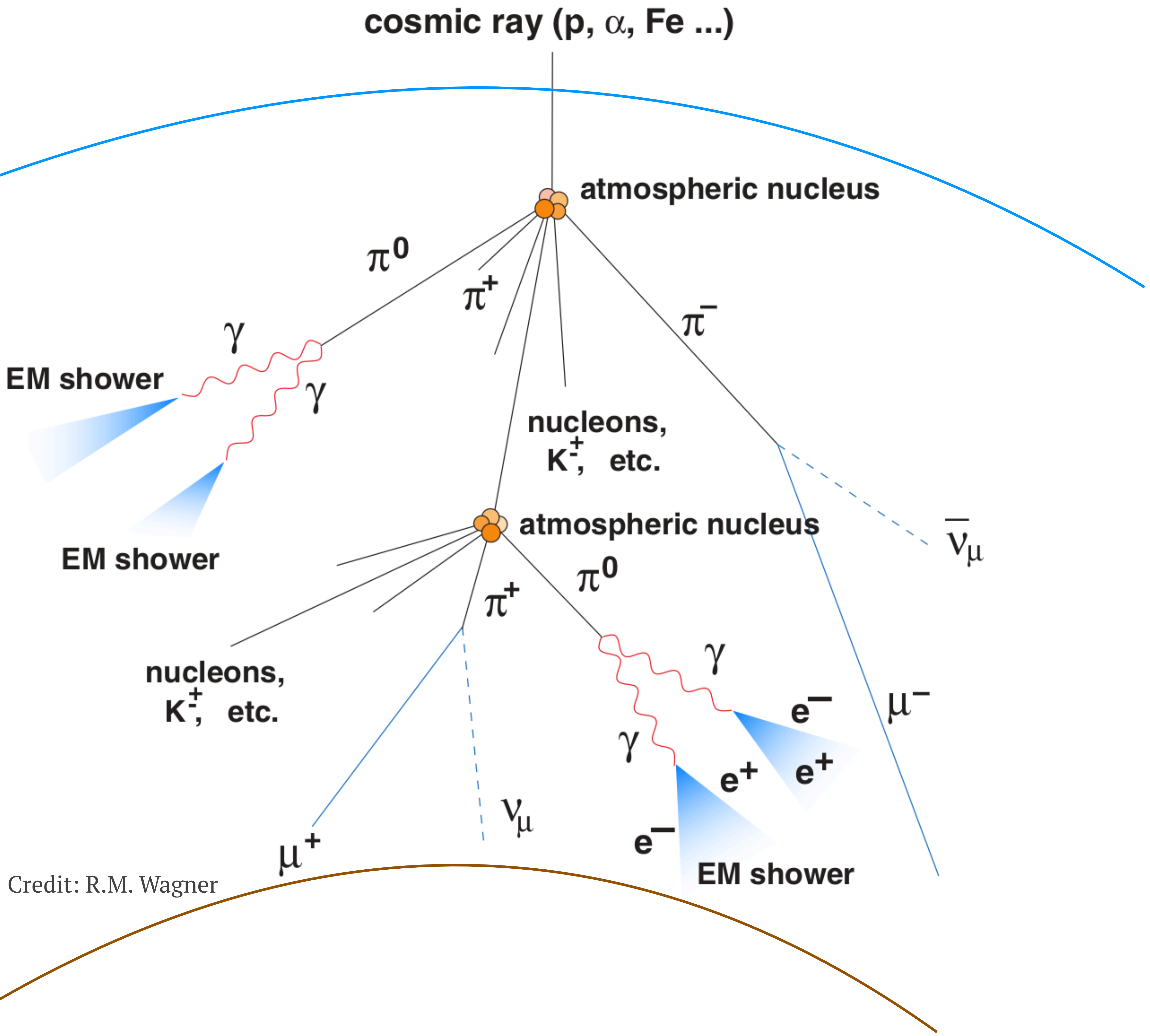


modified from HD et al. PoS (ICRC 2017) 533

Abrecht et al., Astrophys.Space Sci. 367 (2022)

Charged nuclei accelerated to energies beyond 100 EeV, but large uncertainties in mass composition

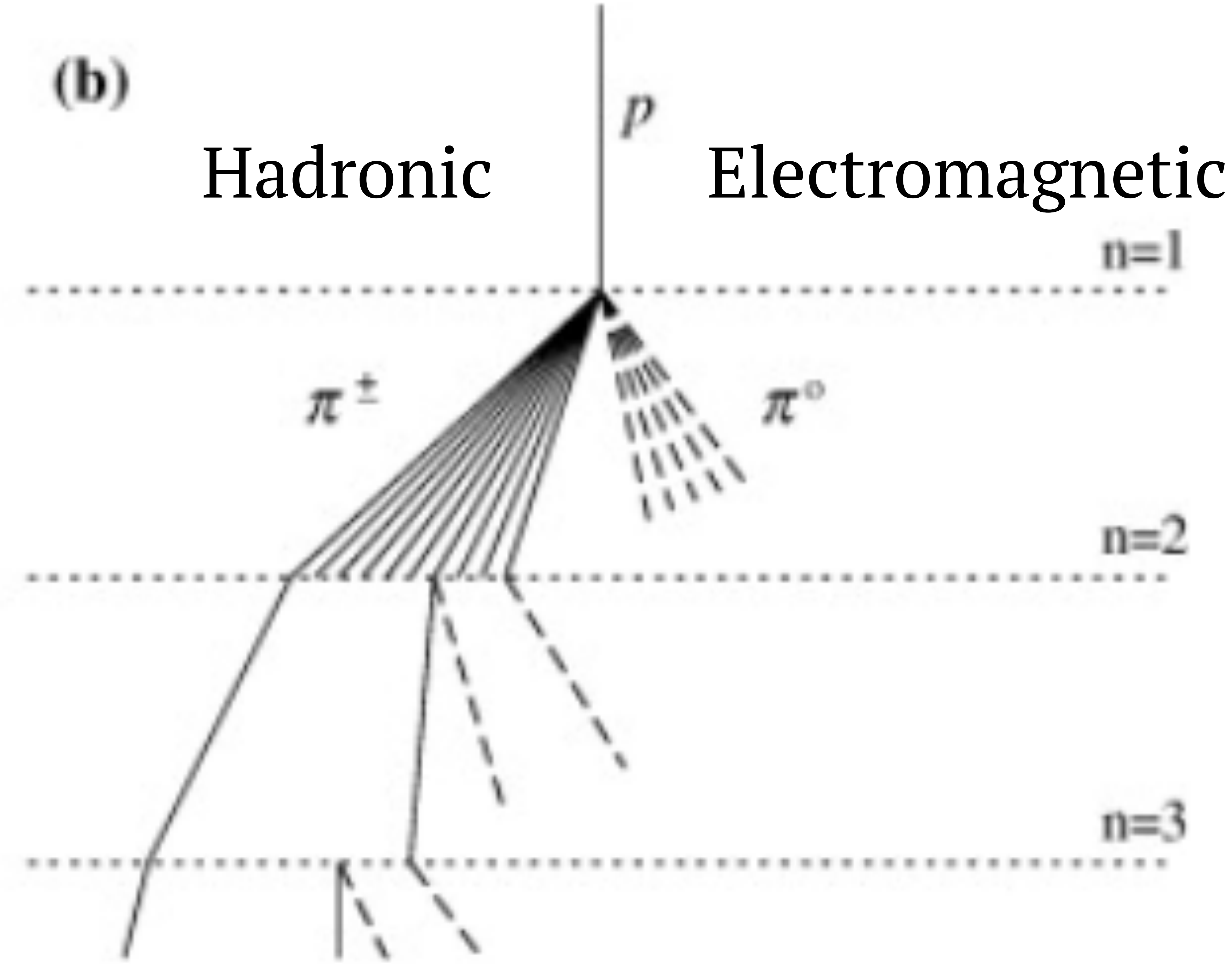
Cosmic Ray Air Showers



Credit: R.M. Wagner

Secondaries reveal information on primary particle

Simplified view: Heitler model



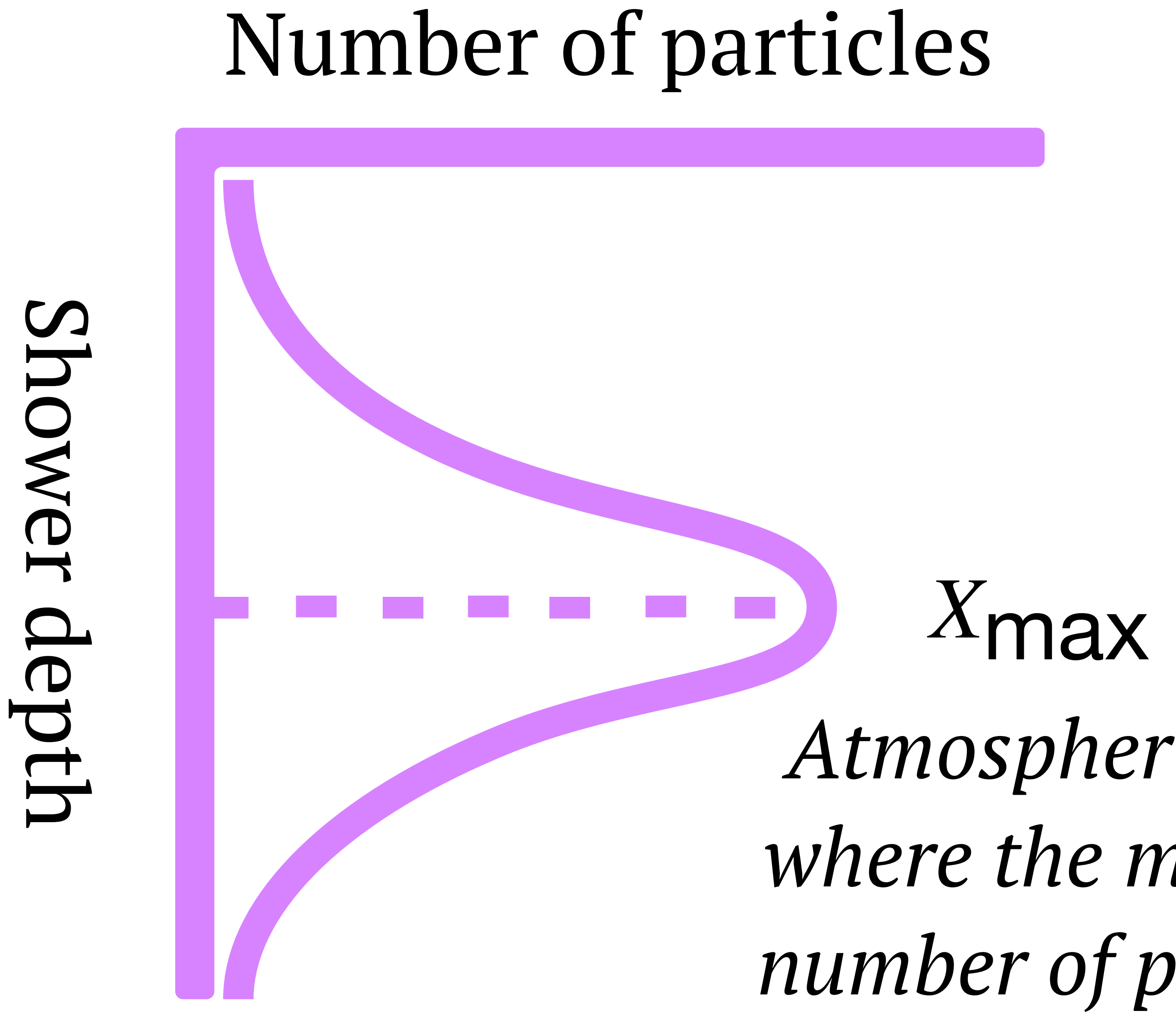
Matthews, Astroparticle Physics (2005)

Superposition model for nuclear primaries

Cosmic Ray Air Showers

Atmospheric slant depth along the shower axis

$$X = \int \rho(l) \mathrm{d}l$$



Atmospheric depth where the maximum number of particle is reached

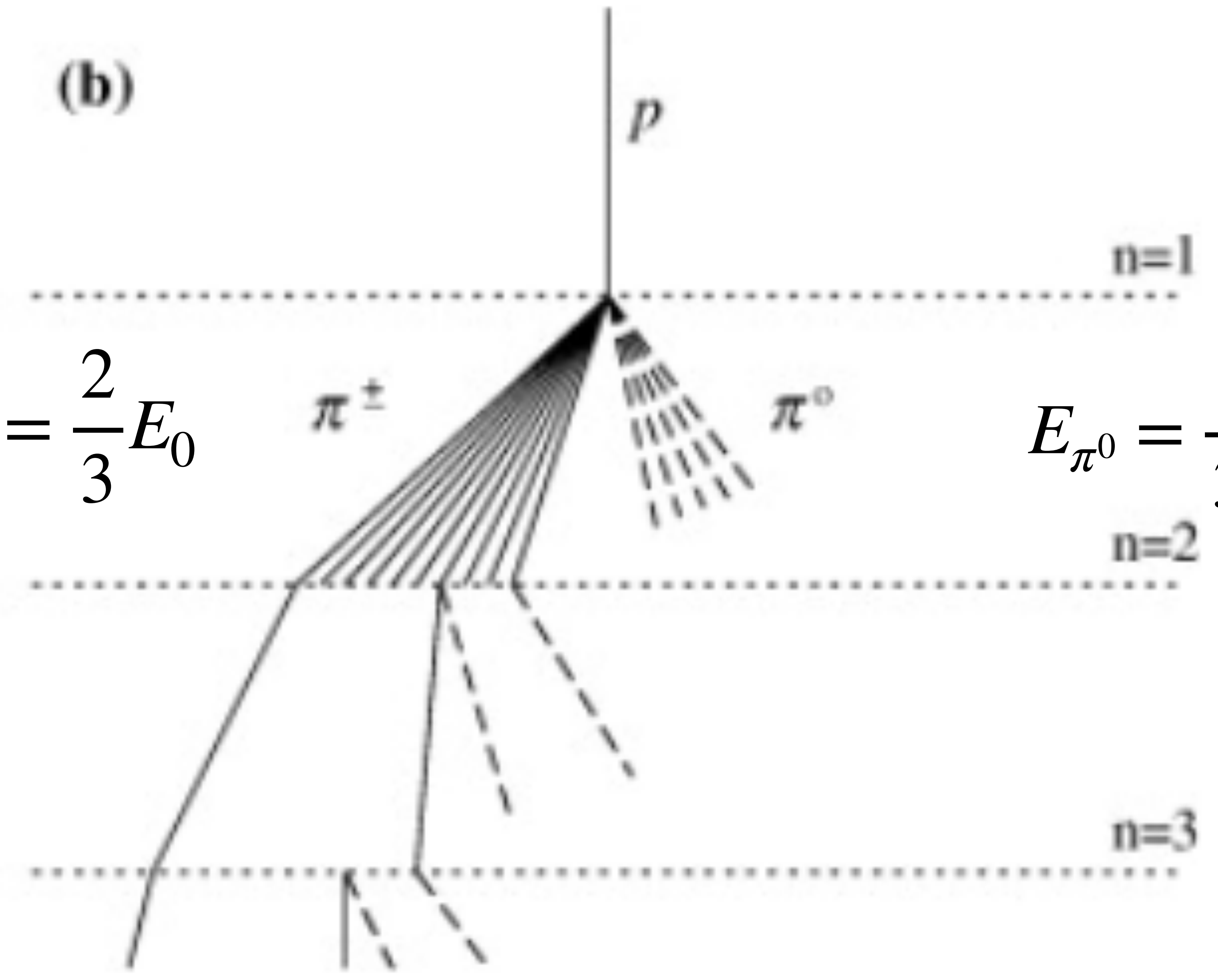
Number of muons given by

$$N_\mu = A \left(\frac{E}{AE_0} \right)^\beta$$

with $\beta \sim 0.9$

Cascade stops when interaction length > decay length at critical energy

$$E_{\mathrm{hadr}} = \frac{2}{3} E_0$$



$$E_{\pi^0} = \frac{1}{3} E_0$$

$$E_{\mathrm{hadr}} = \left(\frac{2}{3} \right)^n E_0$$ after n interactions

Muon production in cosmic ray air showers

p(decay) = p(interaction)

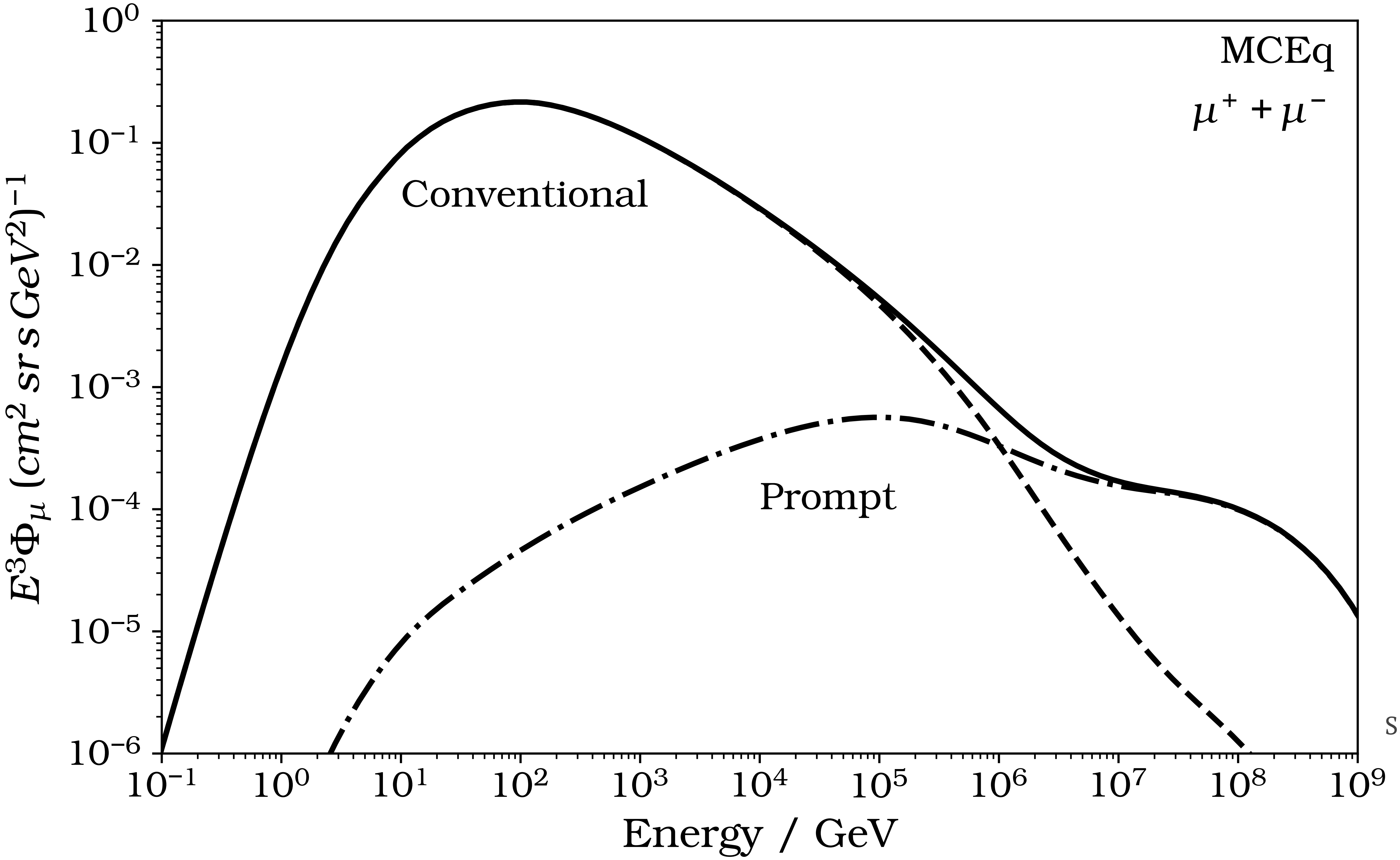
$$\varepsilon_i = T(X) \cdot \frac{R}{Mg} \frac{c \cdot m_i}{\tau_i}$$

Conventional muons

$$\pi^{\pm}, K^{\pm} \rightarrow \mu^{\pm} \nu_{\mu} (\bar{\nu}_{\mu})$$

Prompt muons/neutrinos

$$p, A + \mathrm{air} \rightarrow D, \Lambda_C \rightarrow \nu_{\mu}, \nu_e, \mu$$

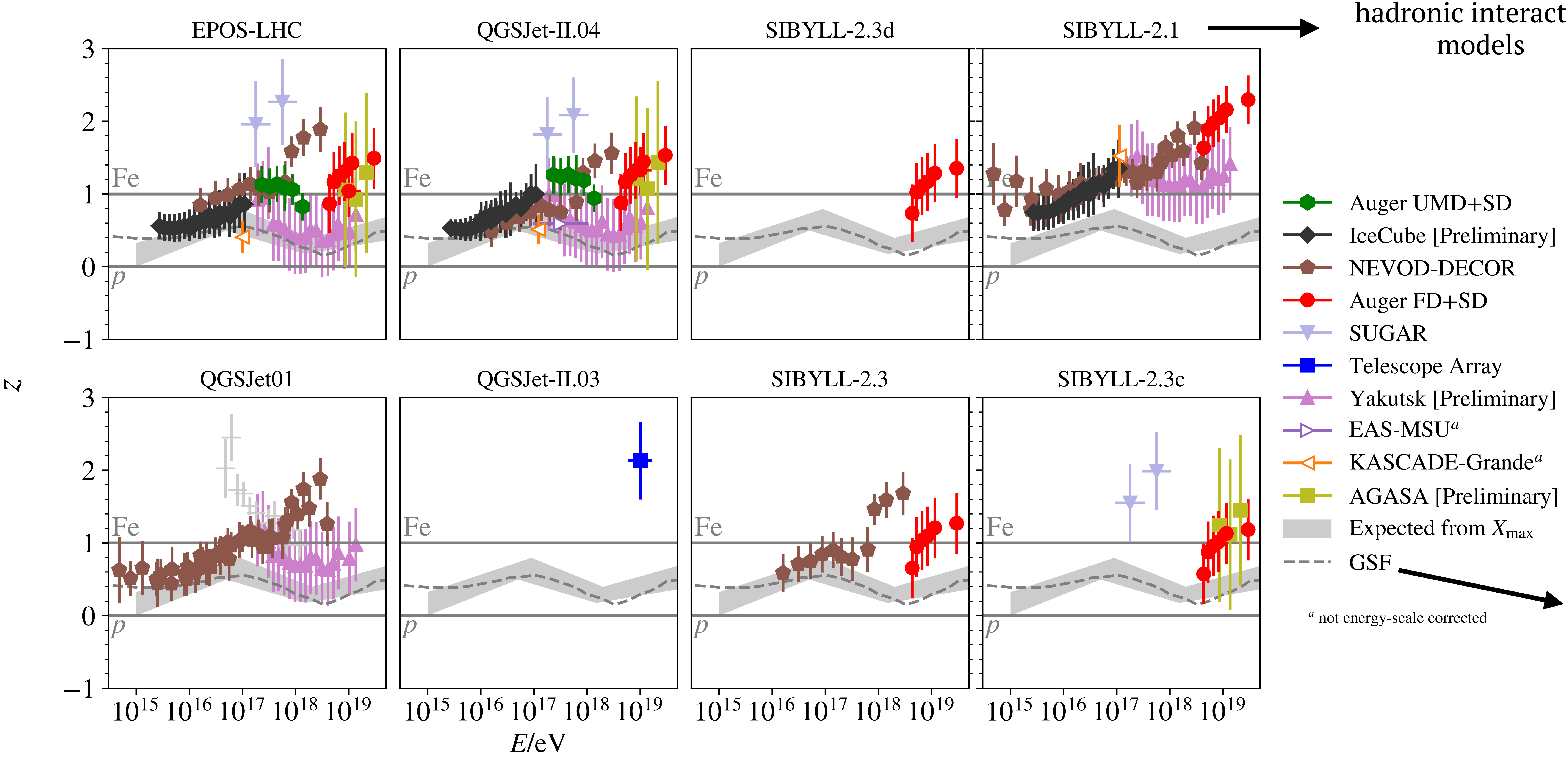


S. Fröse

Muon Puzzle

Muon deficit in cosmic ray air shower experiments with respect to simulation = *muon puzzle*

Abrecht et al., Astrophys.Space Sci. 367 (2022)



hadronic interaction models

cosmic-ray flux model

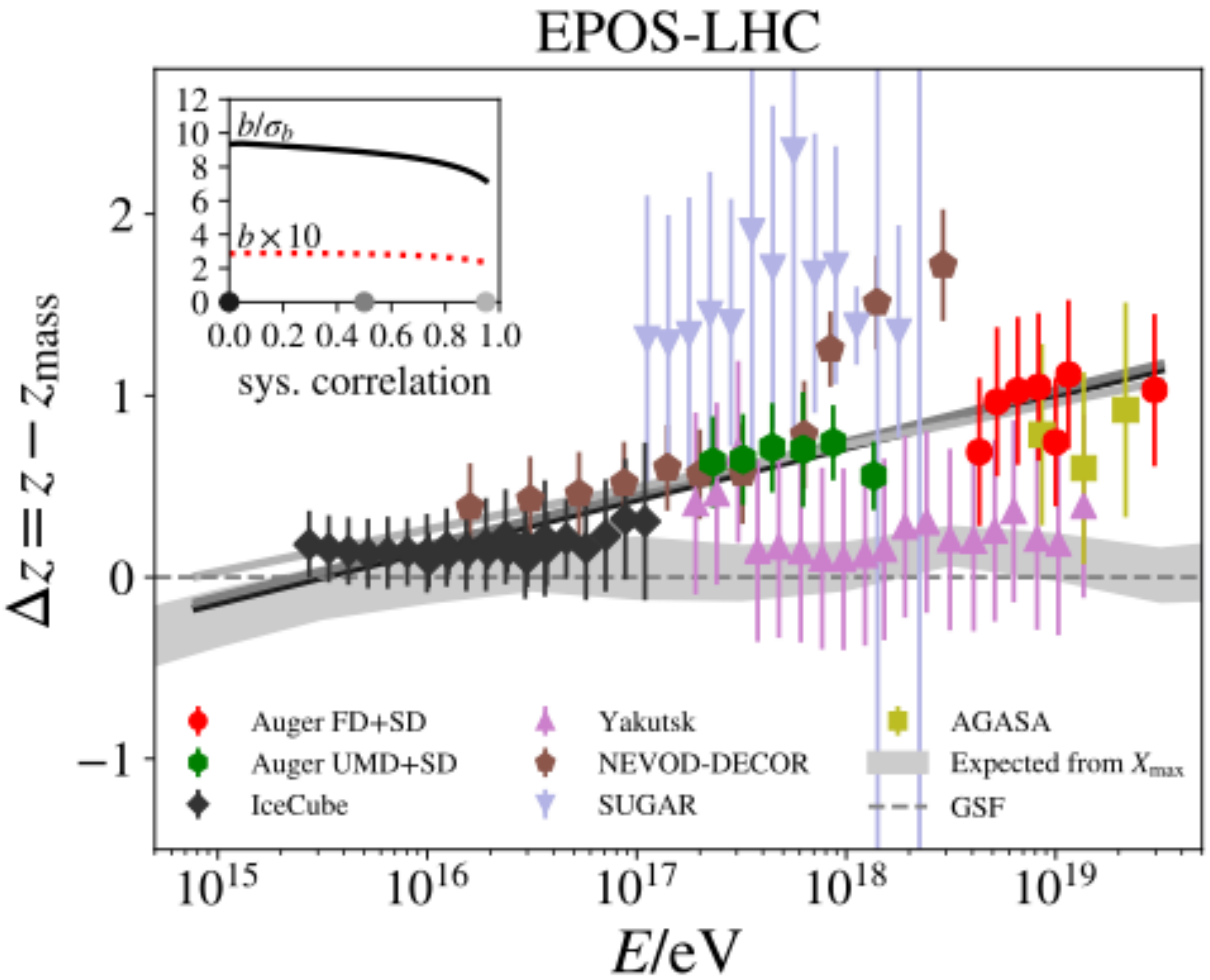
z-scale

$$z = \frac{\ln\langle N_\mu\rangle - \ln\langle N_\mu\rangle_p}{\ln\langle N_\mu\rangle_{\mathrm{Fe}} - \ln\langle N_\mu\rangle_p}$$

Muon puzzle persists across hadronic interaction models —> hadronic origin?

Muon production by hadronic interactions

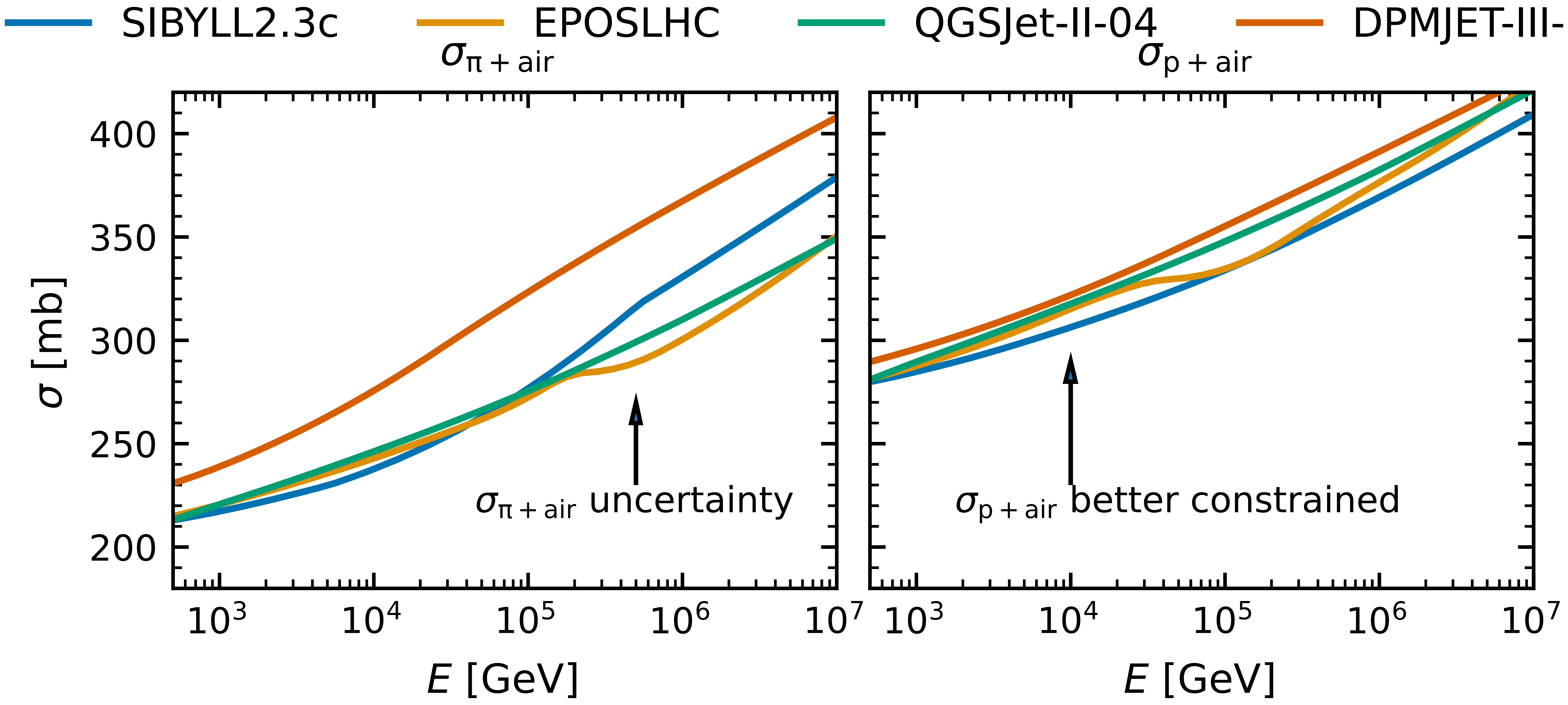
- Muon puzzle between data & simulation increases linear with energy for some experiments
- Muon production in air showers governed by **nucleon-air**, **K-air** and **π-air** inelastic interactions
- Degeneracy between uncertainties in cosmic-ray flux composition & hadronic interaction models complicate muon flux modeling



Abrecht et al.,
Astrophys.Space Sci. 367
(2022)

Muon production by hadronic interactions

Inelastic cross sections contributing to muon production, selected hadronic model landscape

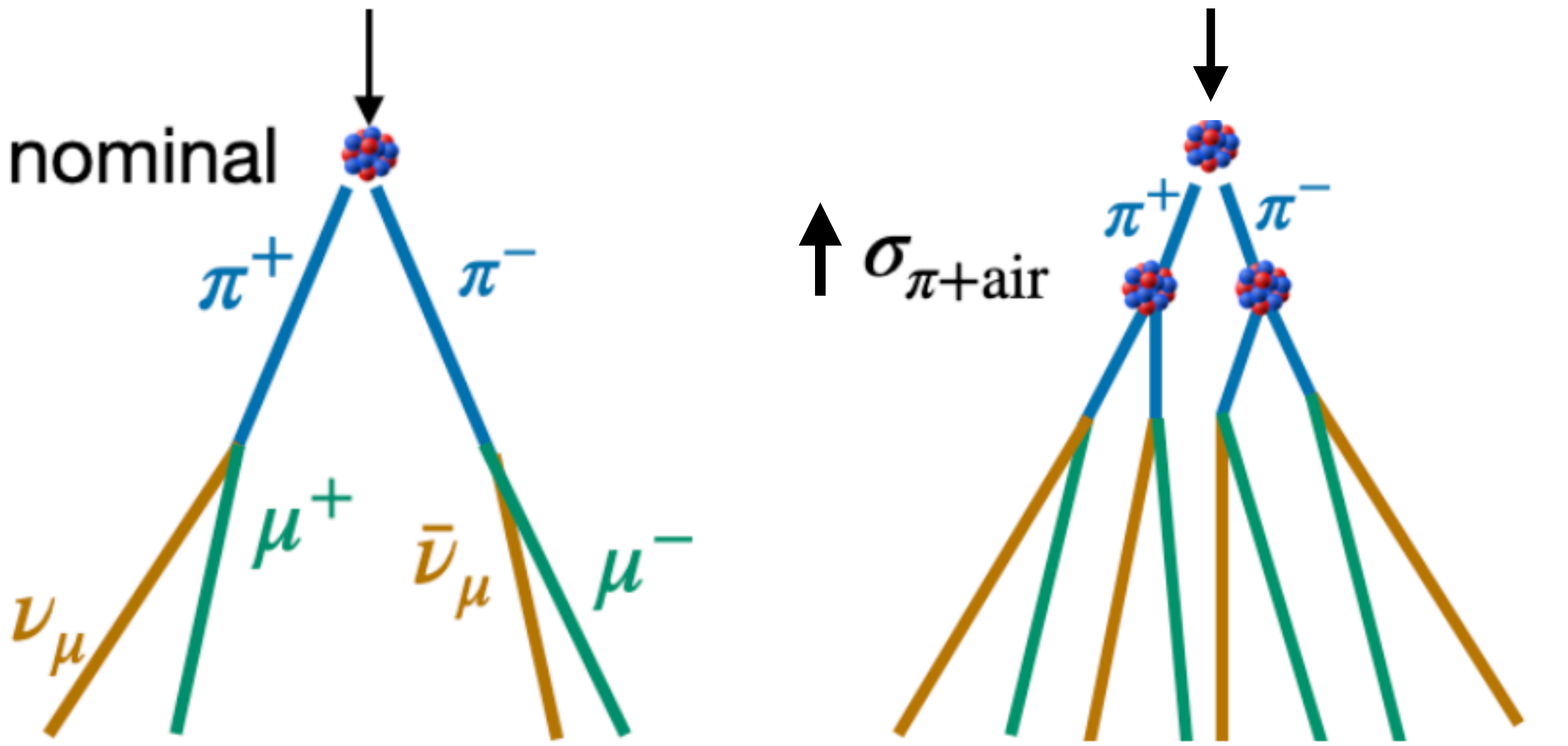


How do the cross section of the first and secondary interactions impact the muon number?

Hadronic cross sections & muon bundle multiplicities

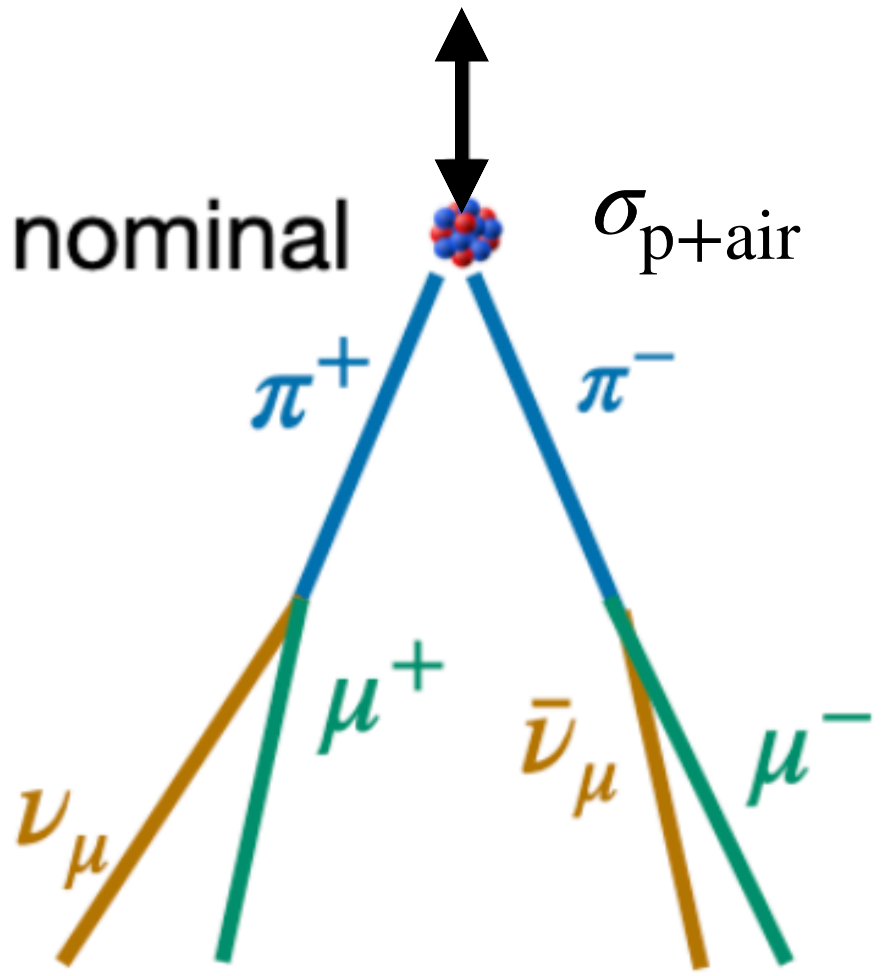
Above critical energy (~ 100 - 200 GeV) :

$\lambda_{\mathrm{decay}} \ll \lambda_{\mathrm{int}}$



Suppression higher-energy secondary particle fluxes —> more lower-energy tertiary particles

Change of interaction height



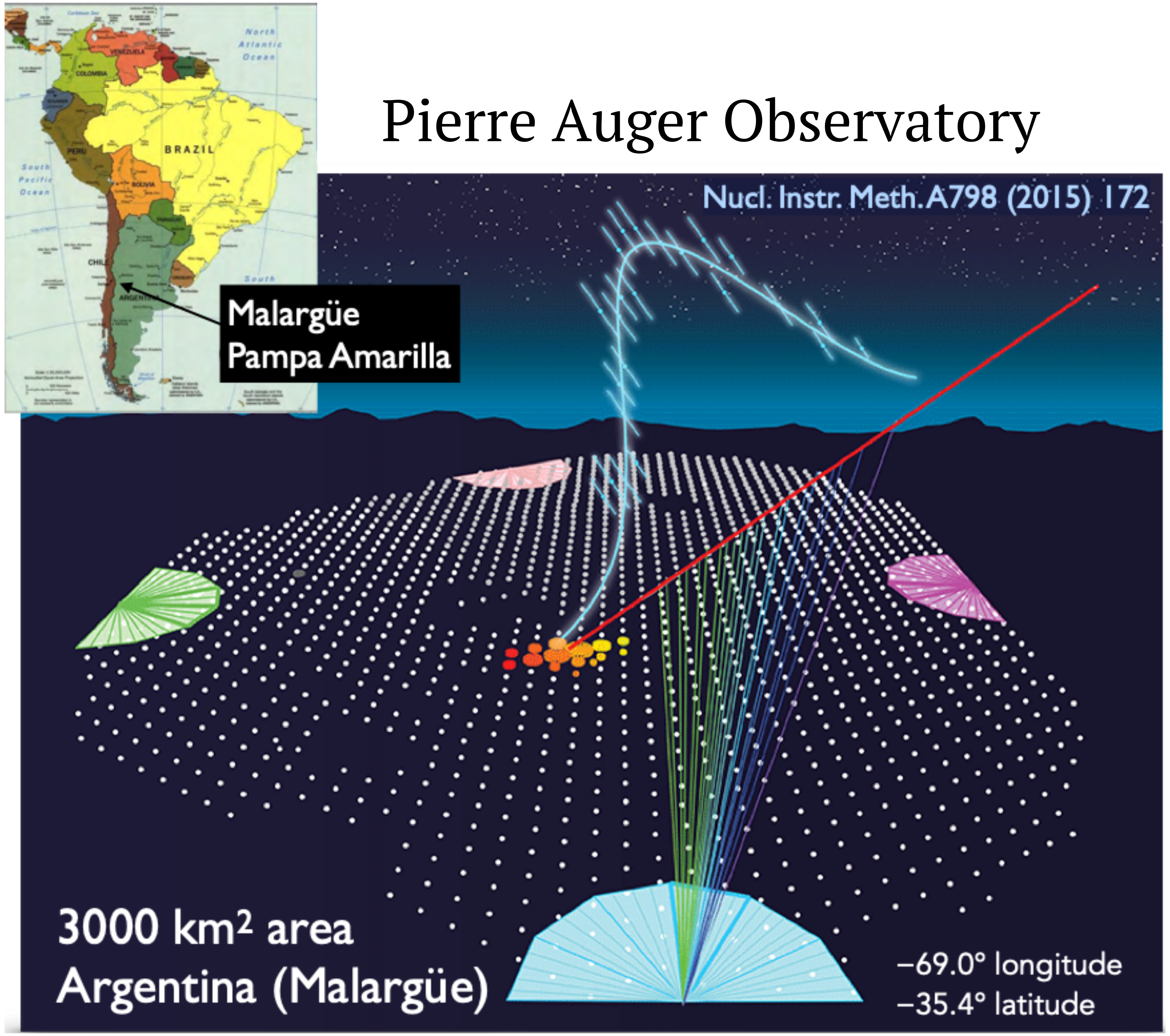
Impact on energy & longitudinal profile

Does 1st or 2nd interactions yield muon puzzle?

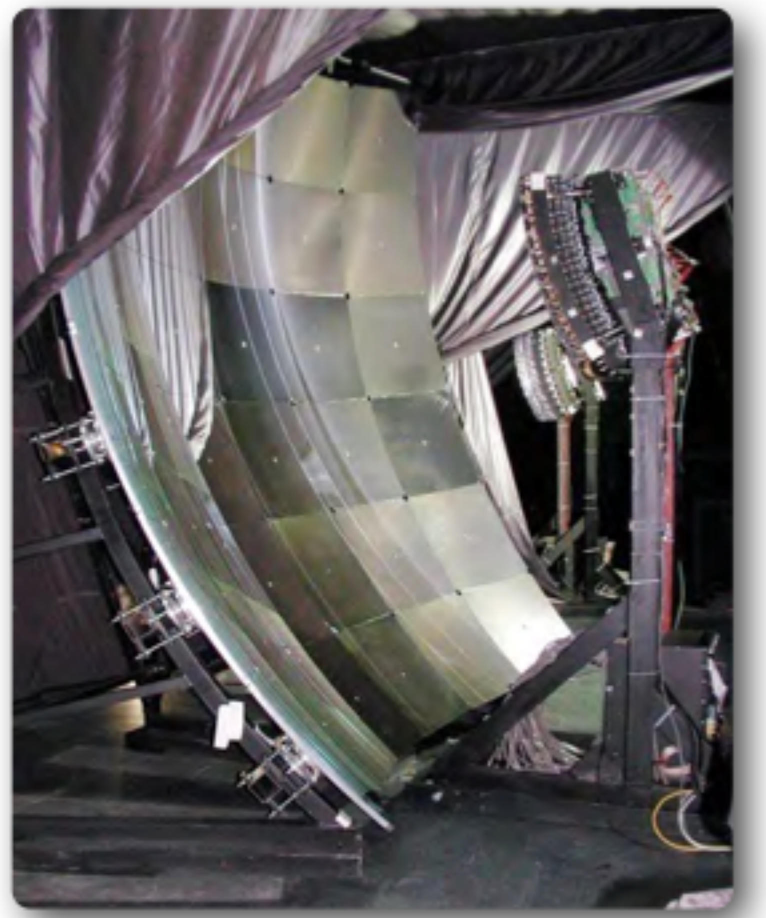
Indirect cosmic ray measurements

Pierre Auger Observatory

Telescope Array
smaller
counterpart in
the Northern
hemisphere



K-H. Kampert



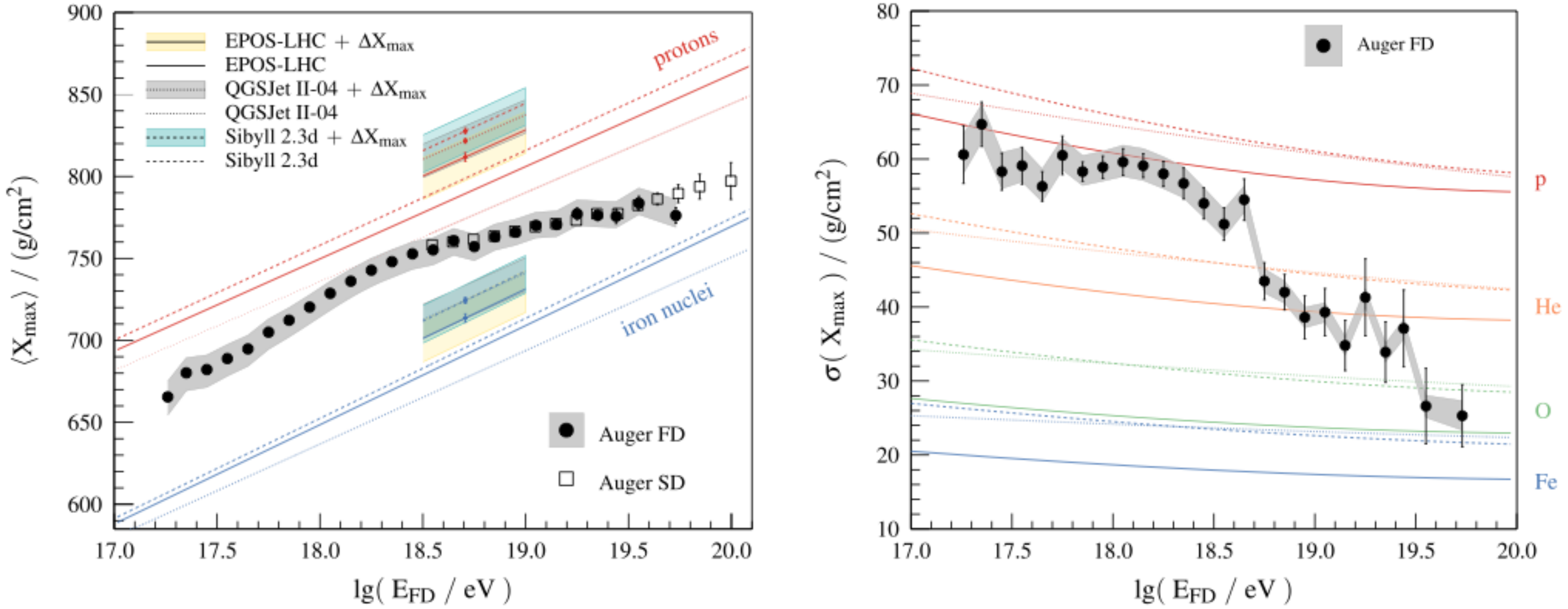
FD

SD

Xmax measured by Auger

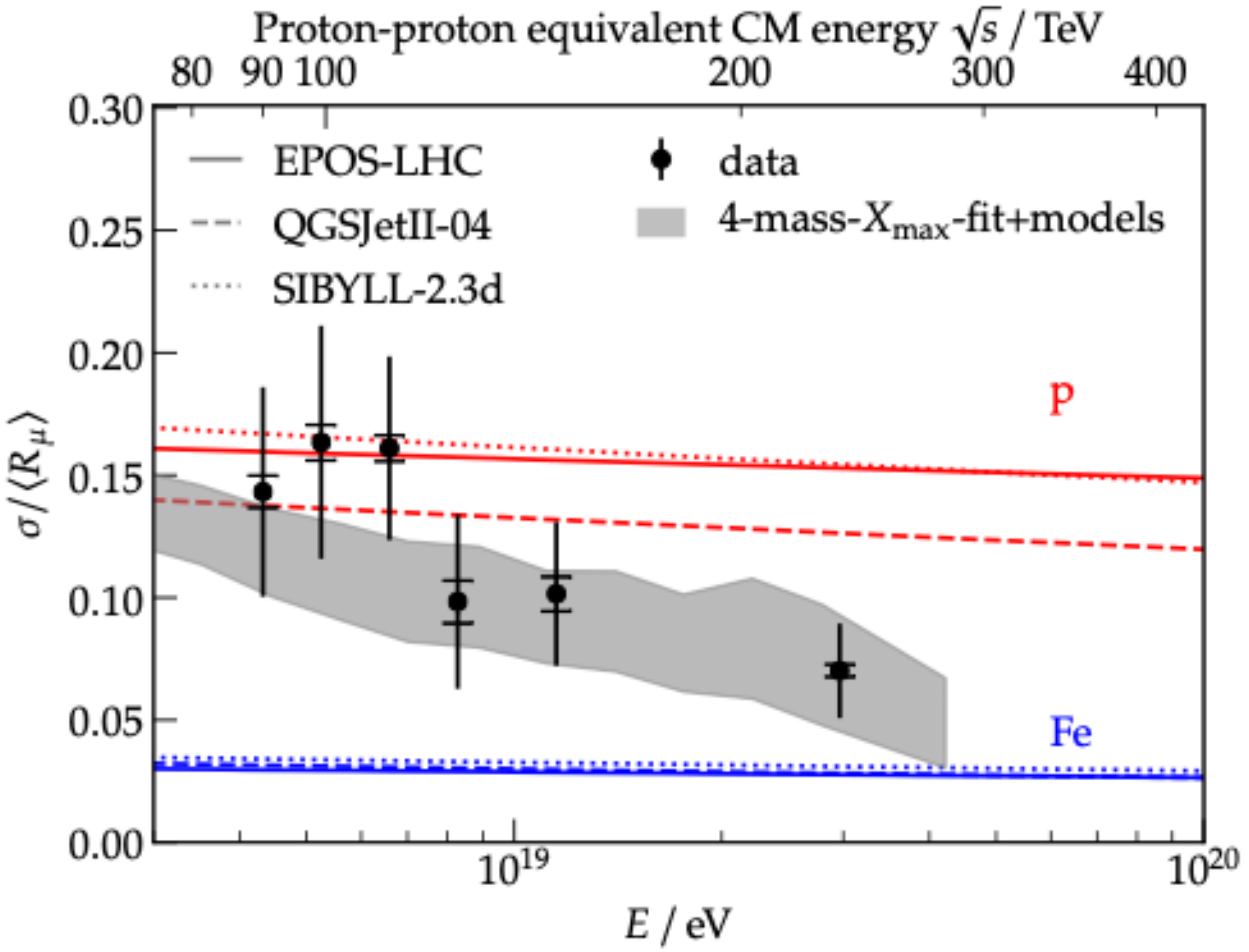
If 1st interaction is the origin, Xmax distribution would also show a mismatch between data & simulation

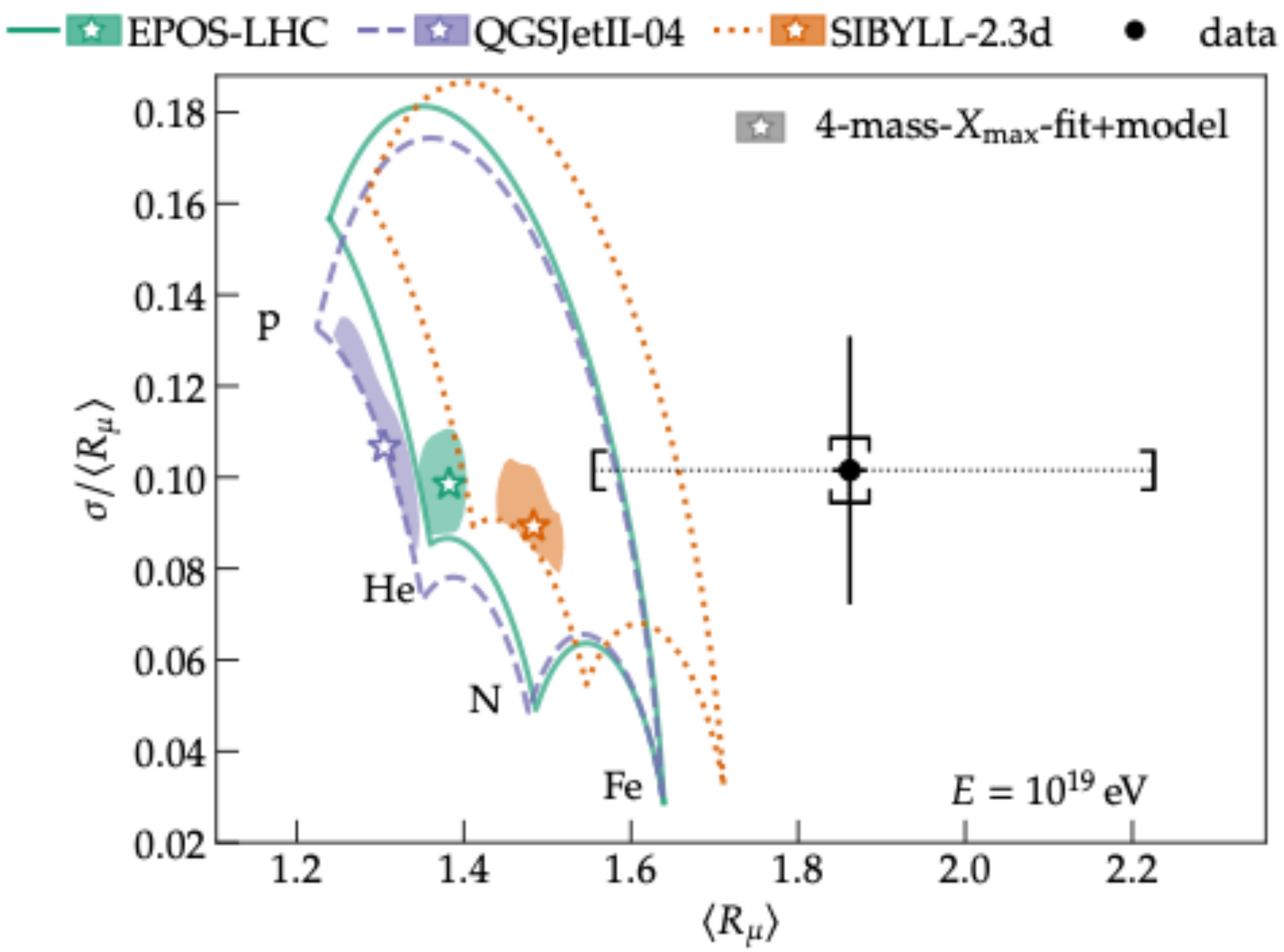
Auger, PRD109 (2024)



Mean and fluctuations as expected from the models —> unlikely that 1st interaction causes muon puzzle

Muon number measured by Auger





Auger, PRL 126 (2021)

Muon number fluctuation remains within expectation, but total number of muons doesn’t match models

➡ Discrepancies which propagate though multiple interactions through the cascade

Can we measure the pion-air cross section?

Colliders

Cosmic-ray Experiments

Underground muon detectors

… and can we constrain it from measurements?

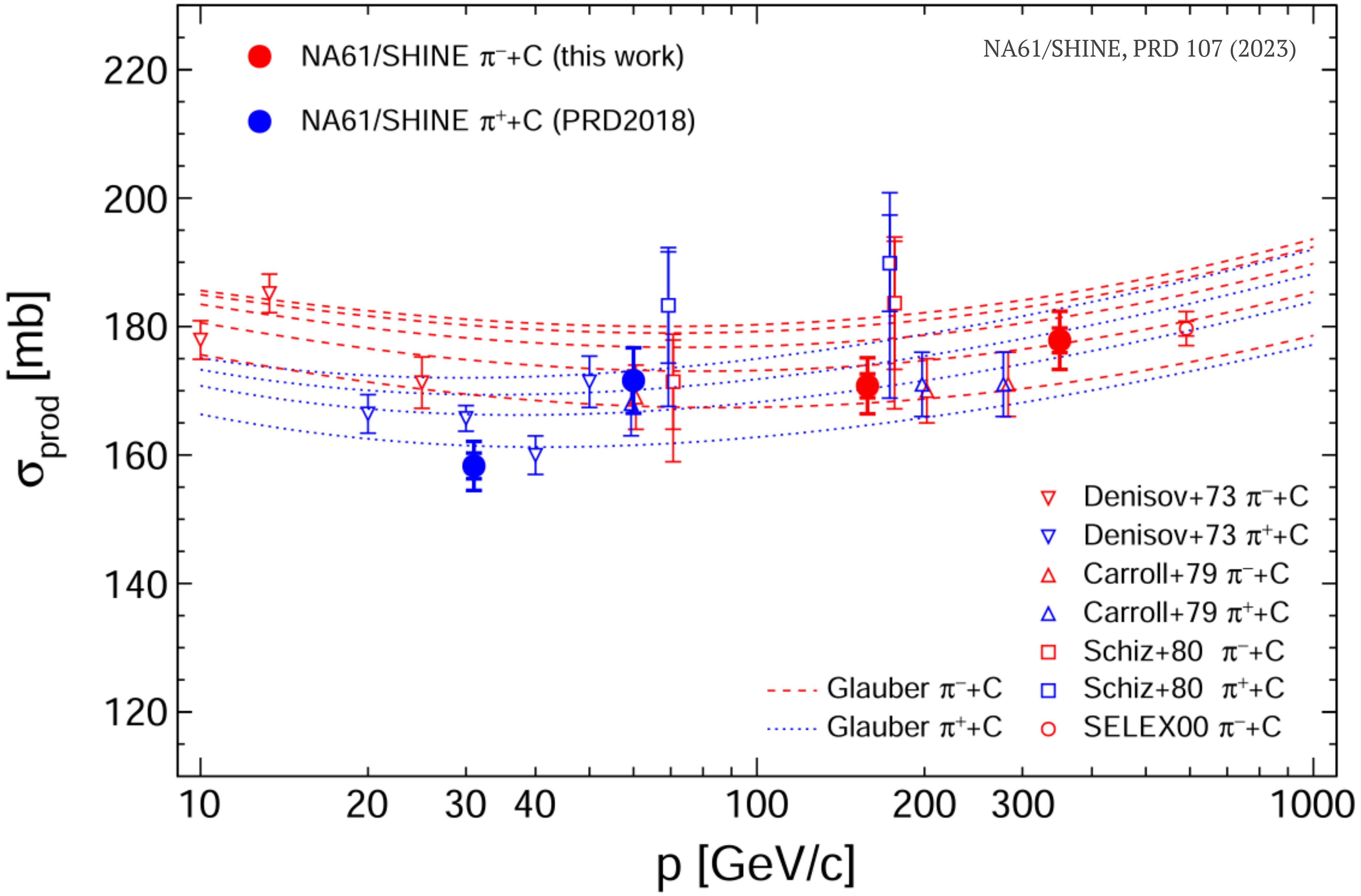
… how does the p-air cross section contribute?

Colliders

Pion-air cross section at colliders

We need: 10^4–10^7 GeV equivalent CMS energy hitting N/O target —> forward fragmentation region, large rapidity

NA61/SHINE: pion beam + Carbon target



Forward facilities: LHCf, FASER

—> constraints through forward spectra from pp, not pion interactions

LHC fixed-target: SMOG2/LHCb

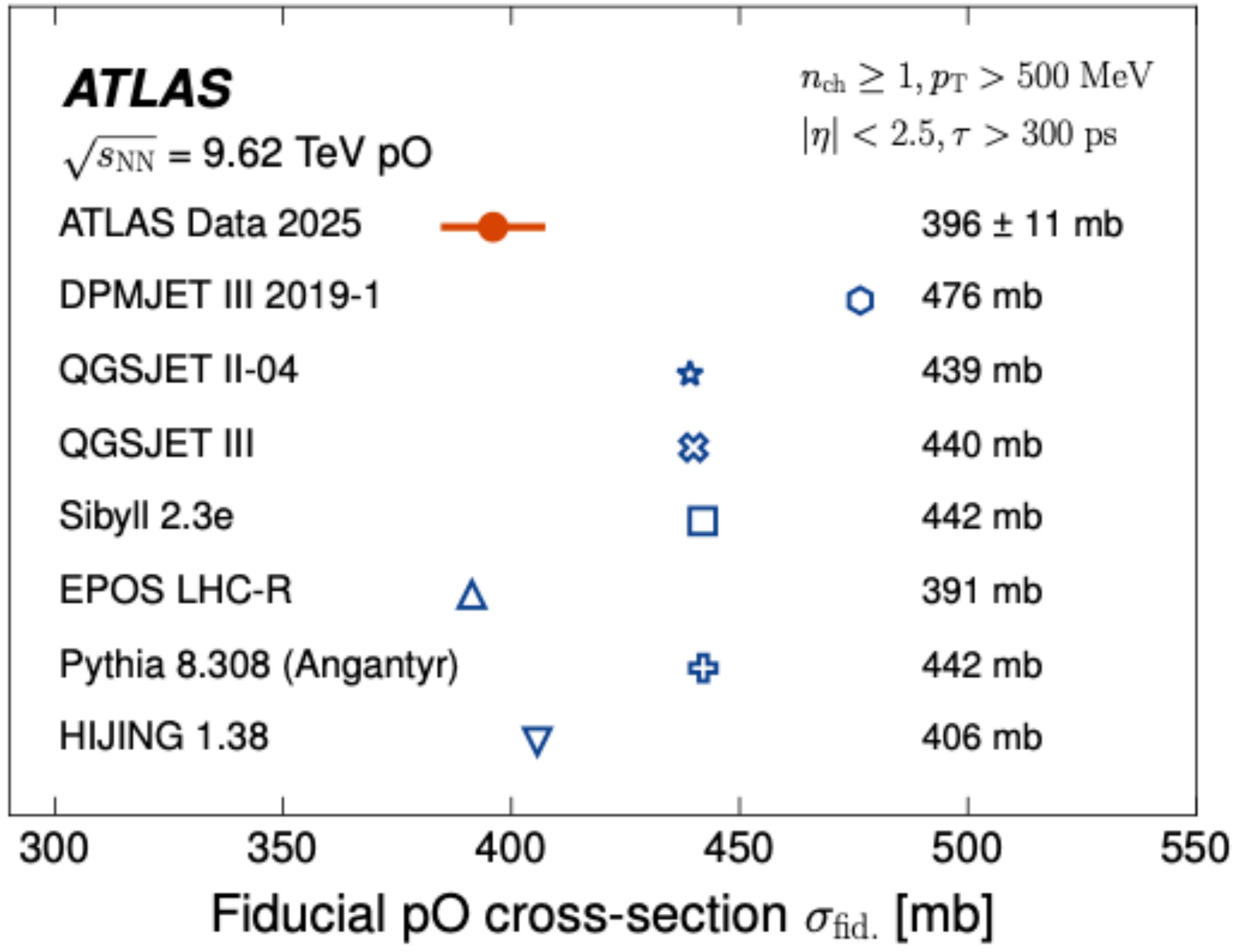
—> no pion beam, but constraints through multiple nuclear targets (pO)

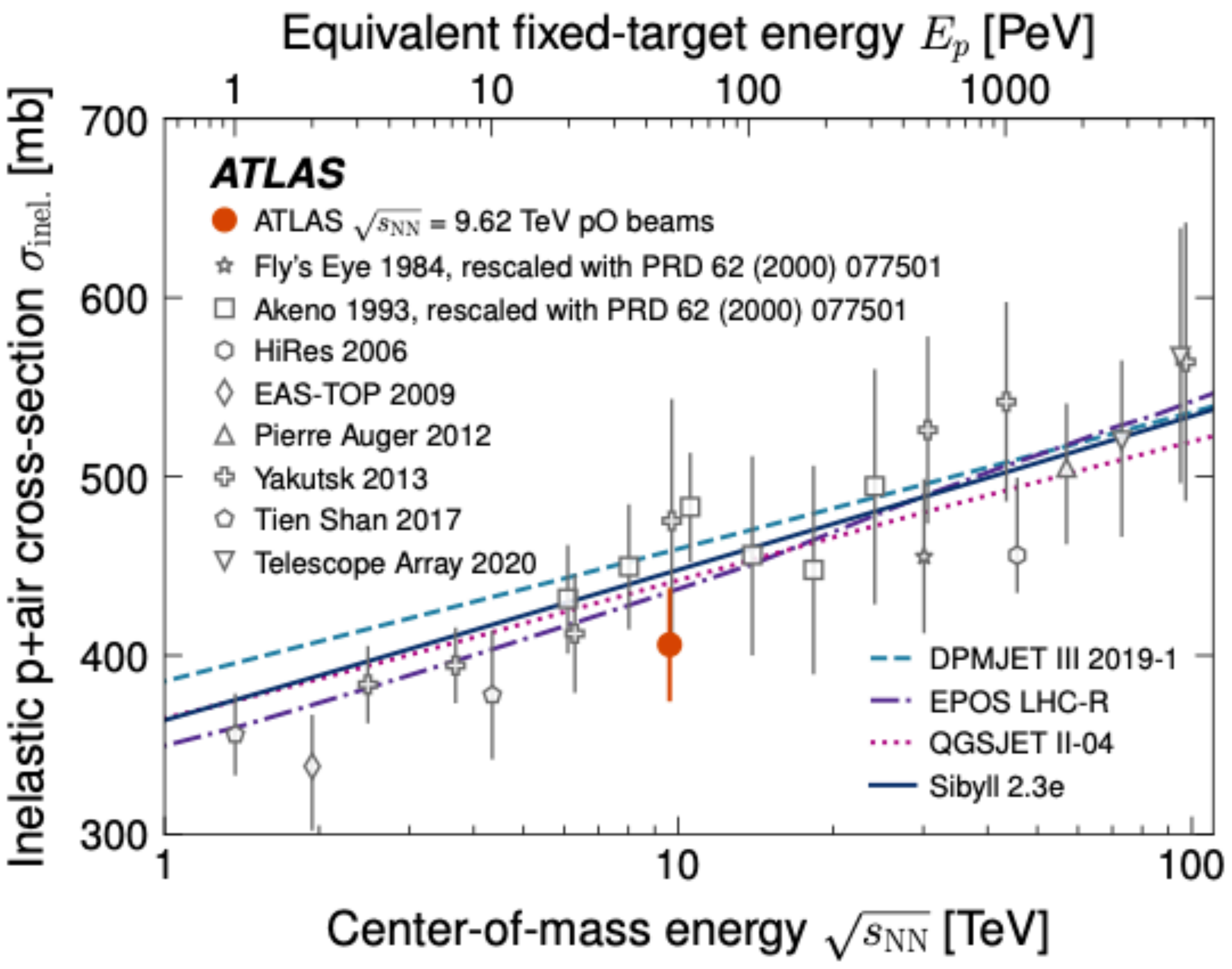
P-air cross section at colliders

First measurement of pO @LHC

Extrapolation to $\sigma^{p+\text{air}}_{\text{inel}} = f_{\text{air}} \cdot \sigma^{p+O}_{\text{inel}}$

ATLAS Collab., arXiv:2604.05512 (2025)



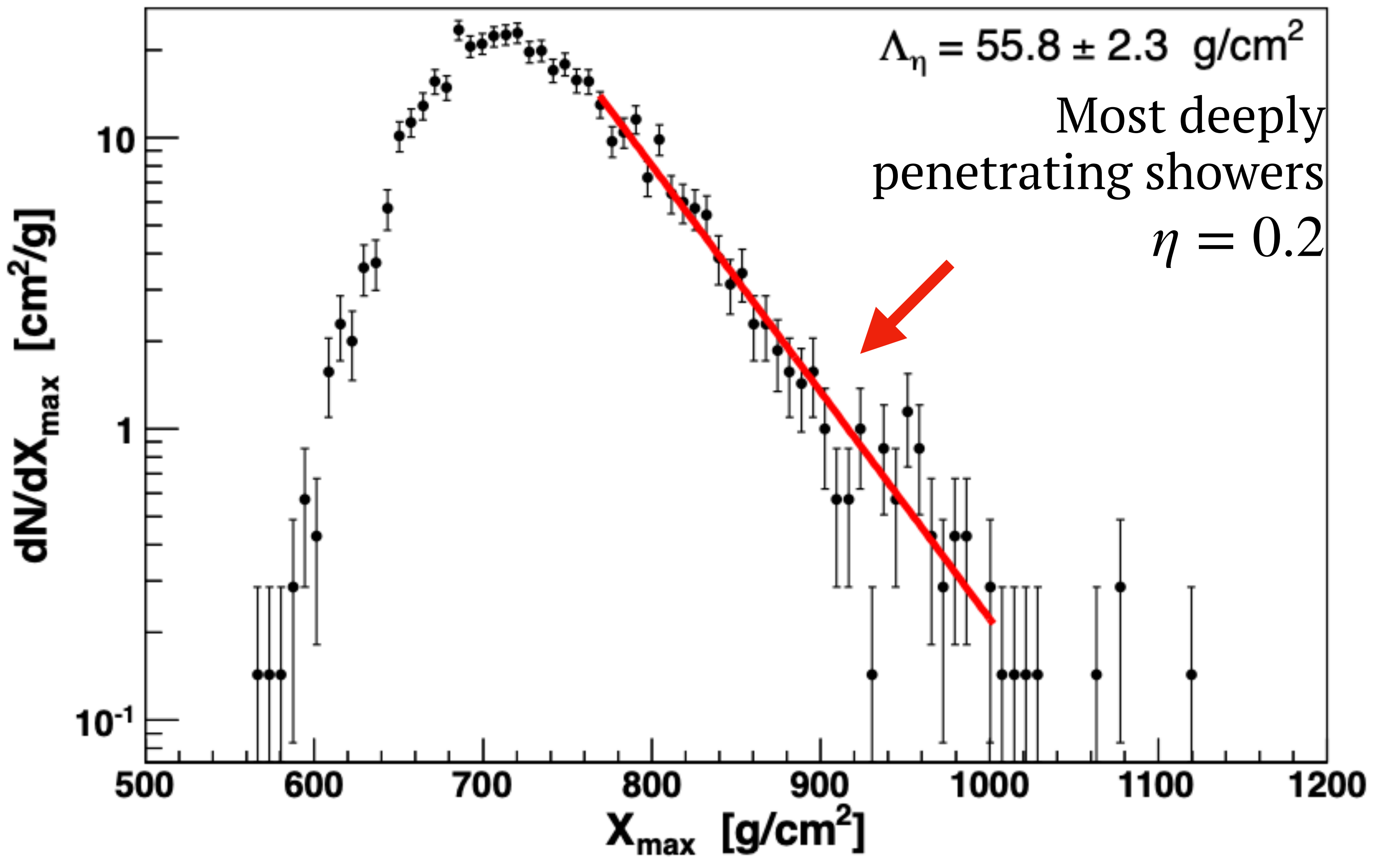


No collider experiment can measure $\sigma_{\pi+\text{air}}$ in the energy range relevant to the muon puzzle in the foreseeable future

P-air cross section measurement with Auger

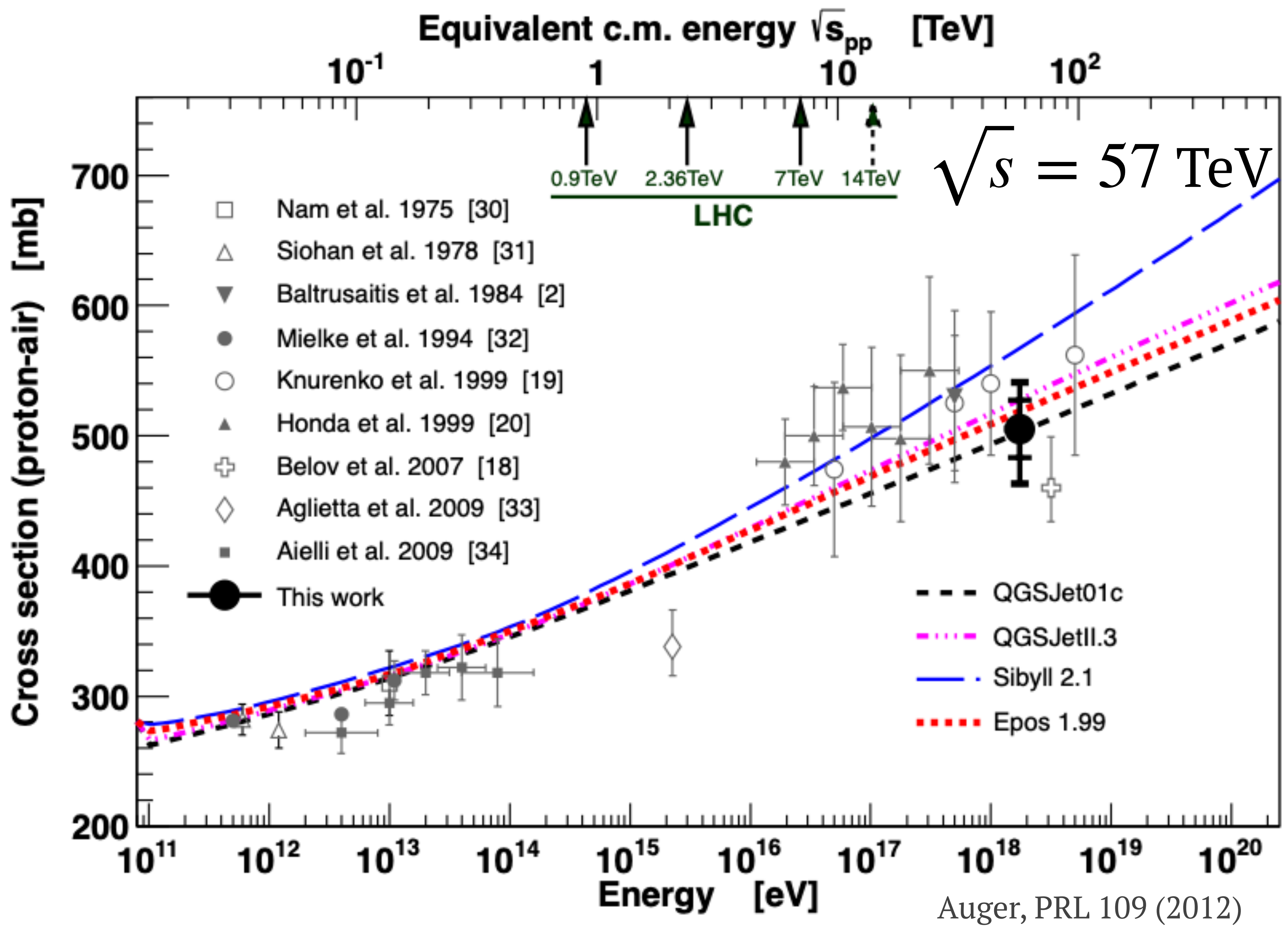
Cosmic-ray Experiments

- Tail of X_max distribution $\frac{\mathrm{d}N}{\mathrm{d}X_\mathrm{max}} \propto \exp\left(-\frac{X_\mathrm{max}}{\Lambda_\eta}\right)$ sensitive to $\Lambda_\eta \propto \sigma_{\mathrm{p+air,\ prod}}$
- $\Lambda_\eta^\mathrm{measured}$ matched to scaled hadronic interaction models —> systematic uncertainty represents model differences



Most deeply penetrating showers

$\eta = 0.2$



$\sqrt{s} = 57$ TeV

Auger, PRL 109 (2012)

Same measurement by Telescope Array at $\sqrt{s} = 73$ TeV, but no possibility to measure $\sigma_{\pi+\mathrm{air}}$

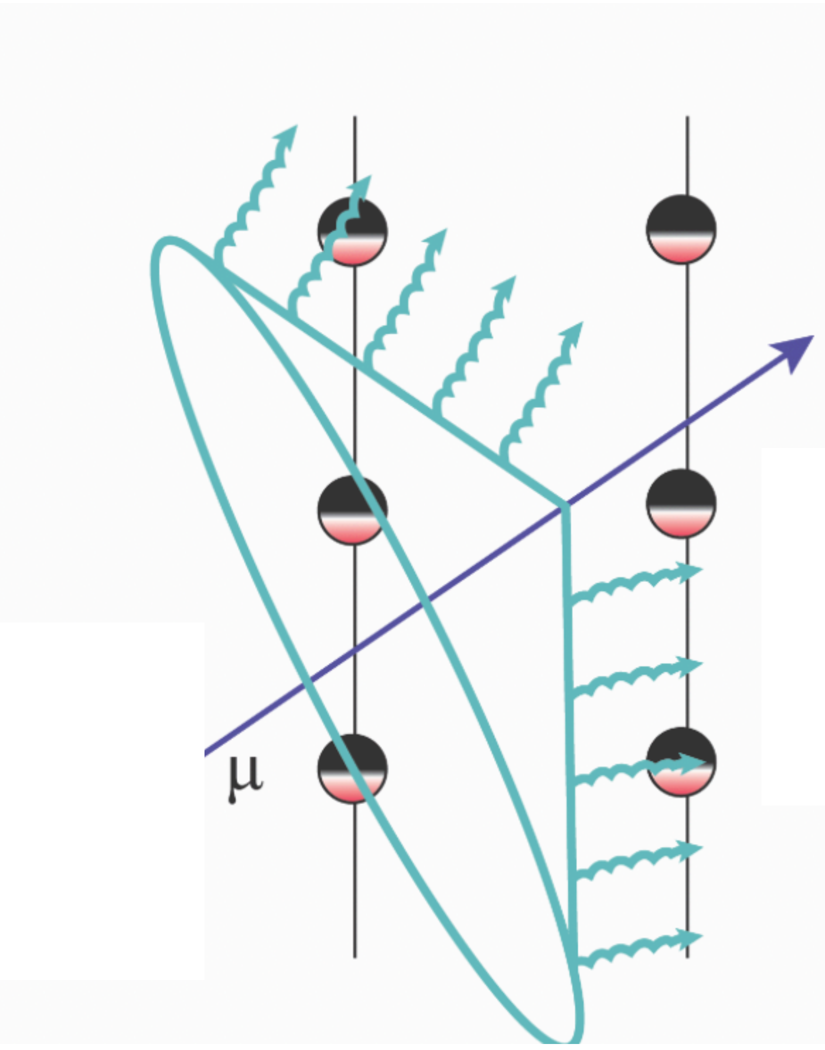
TA, PRD 102 (2020)

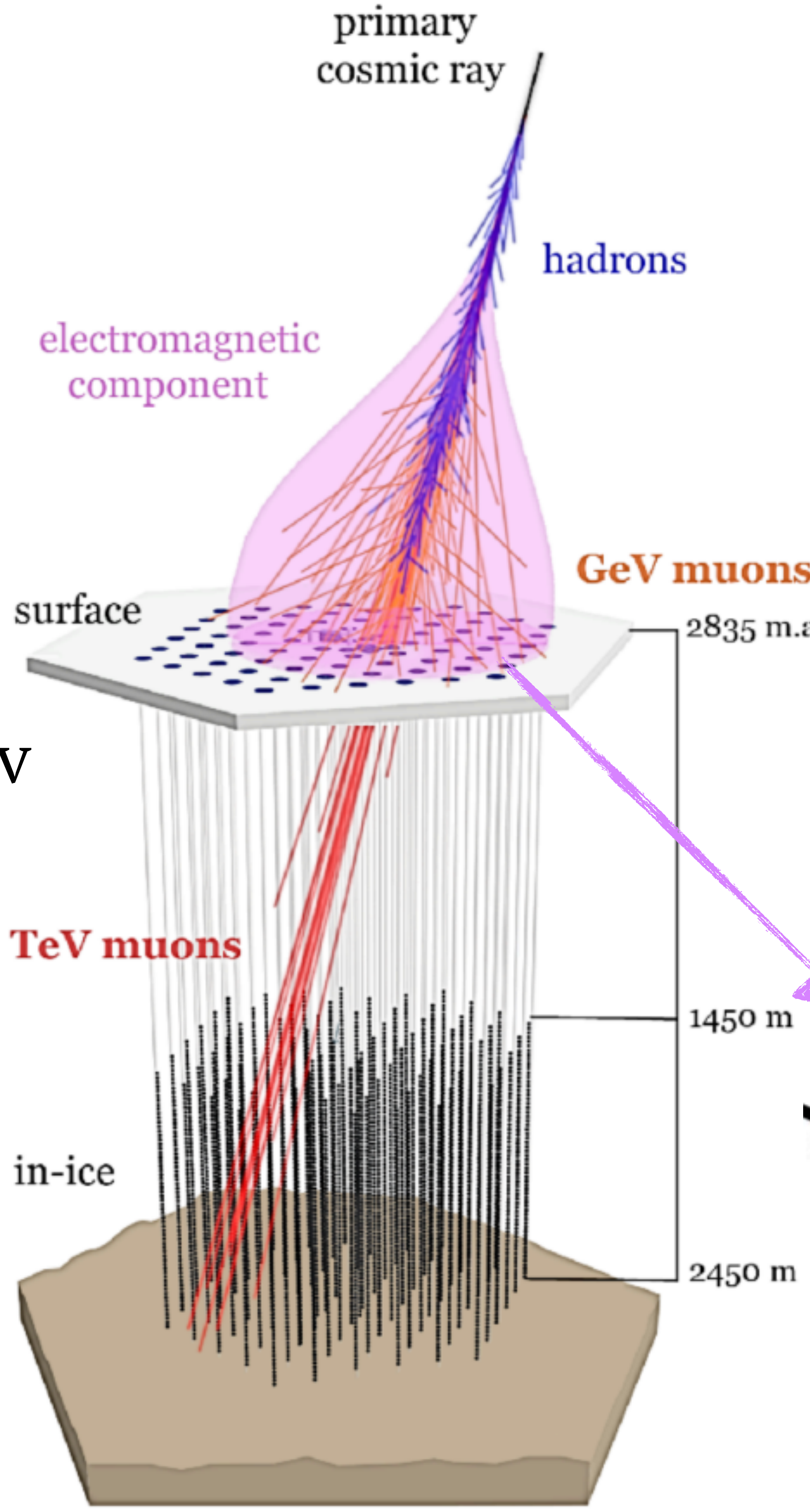
Can we measure the pion-air cross section?

ICECUBE
COMPLETED
2011

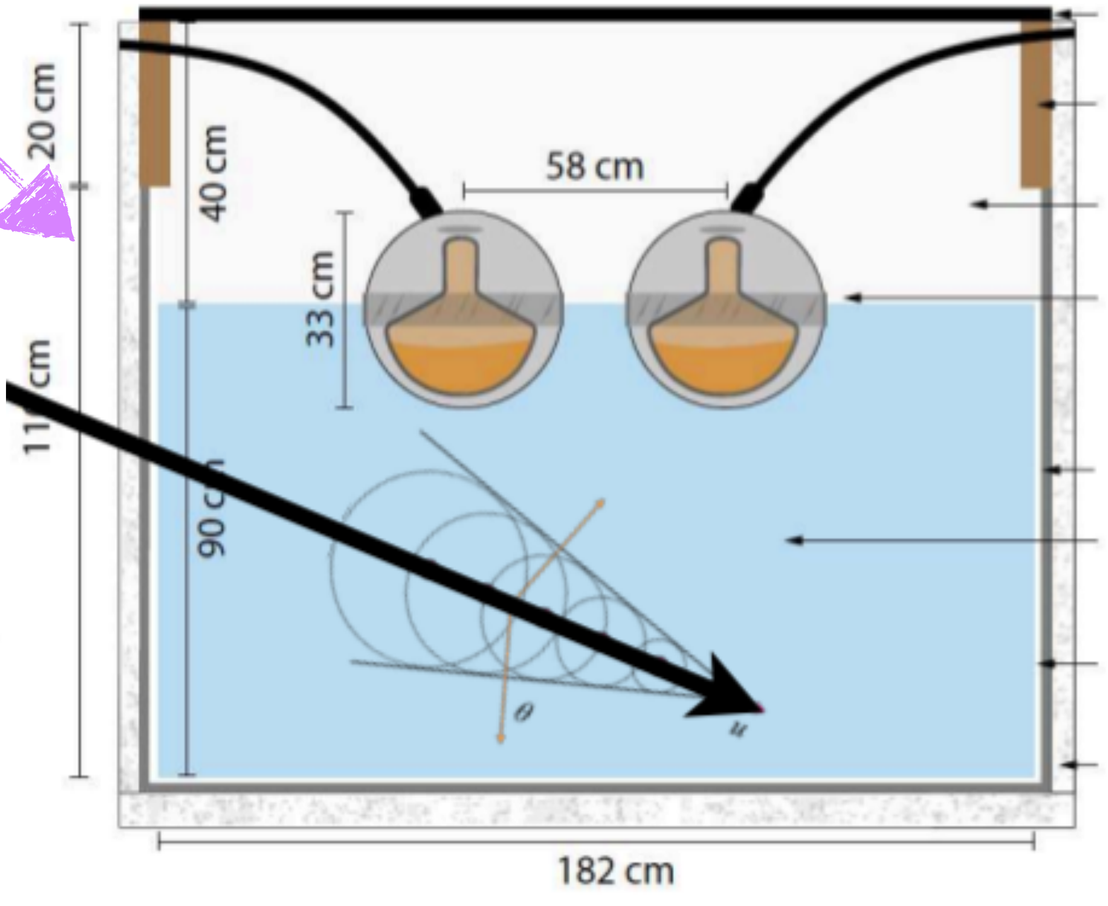
Underground muon detectors

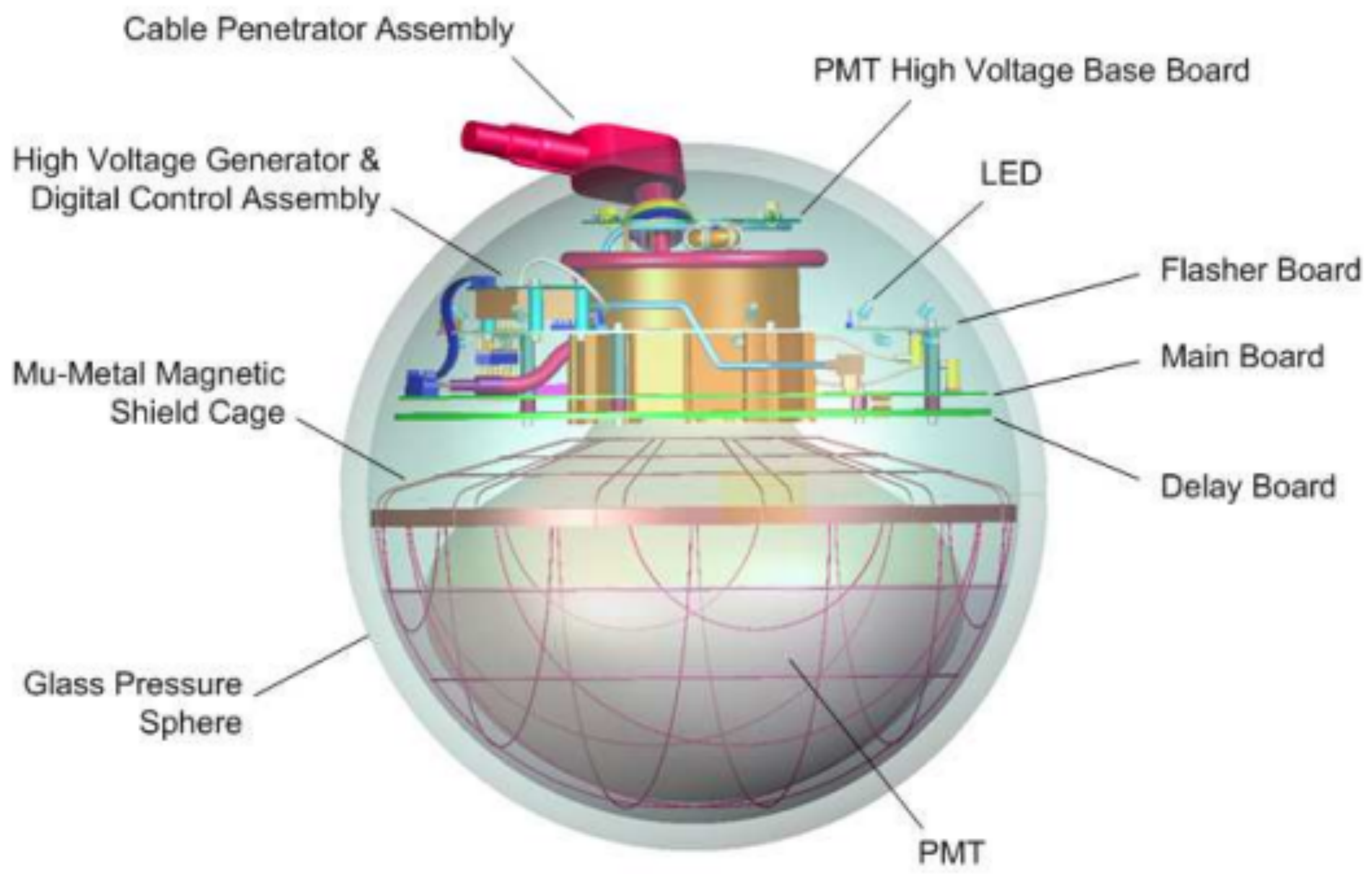
Detection via Cherenkov light in dense medium





Currently the largest neutrino telescope with 15 years of data

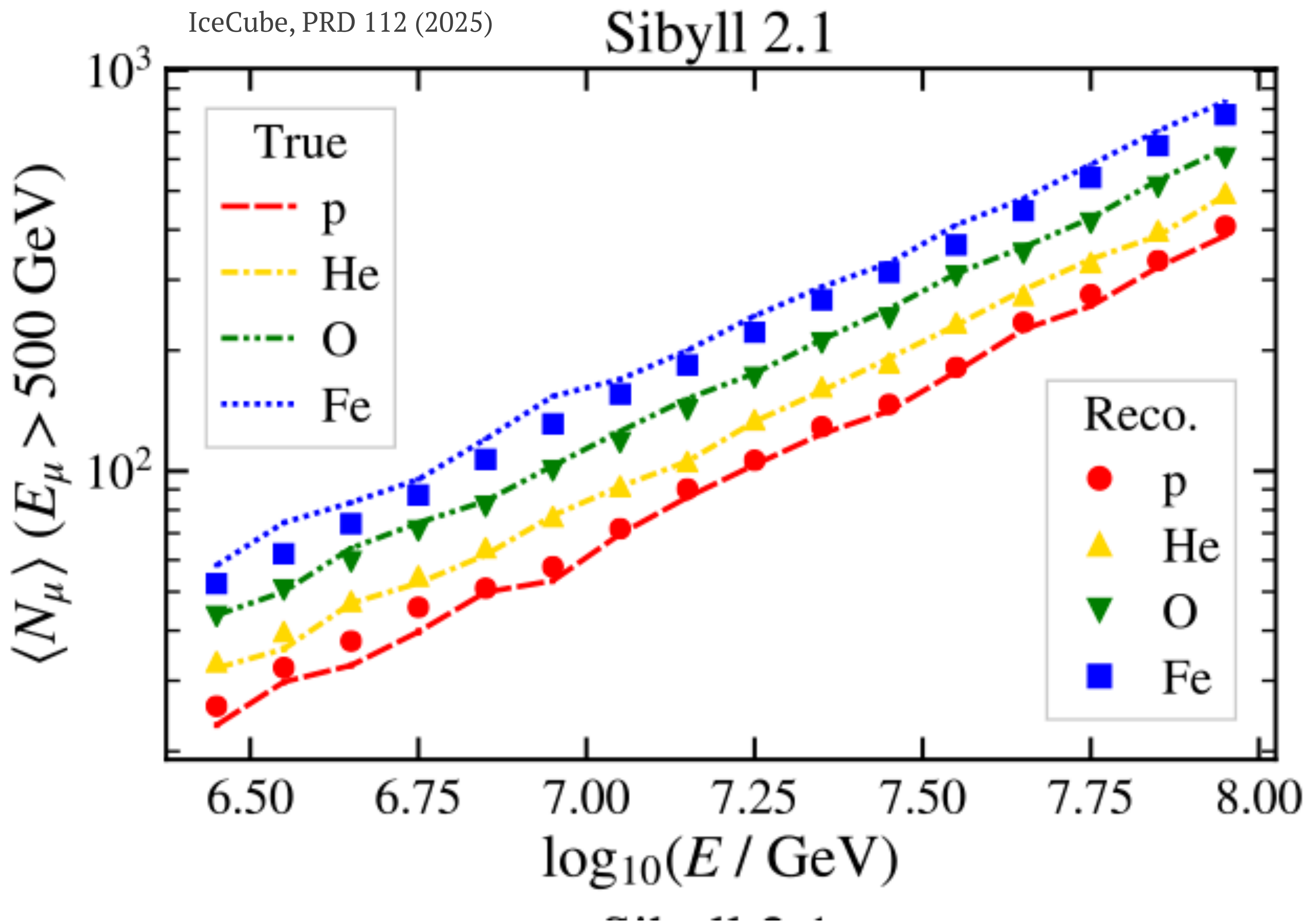


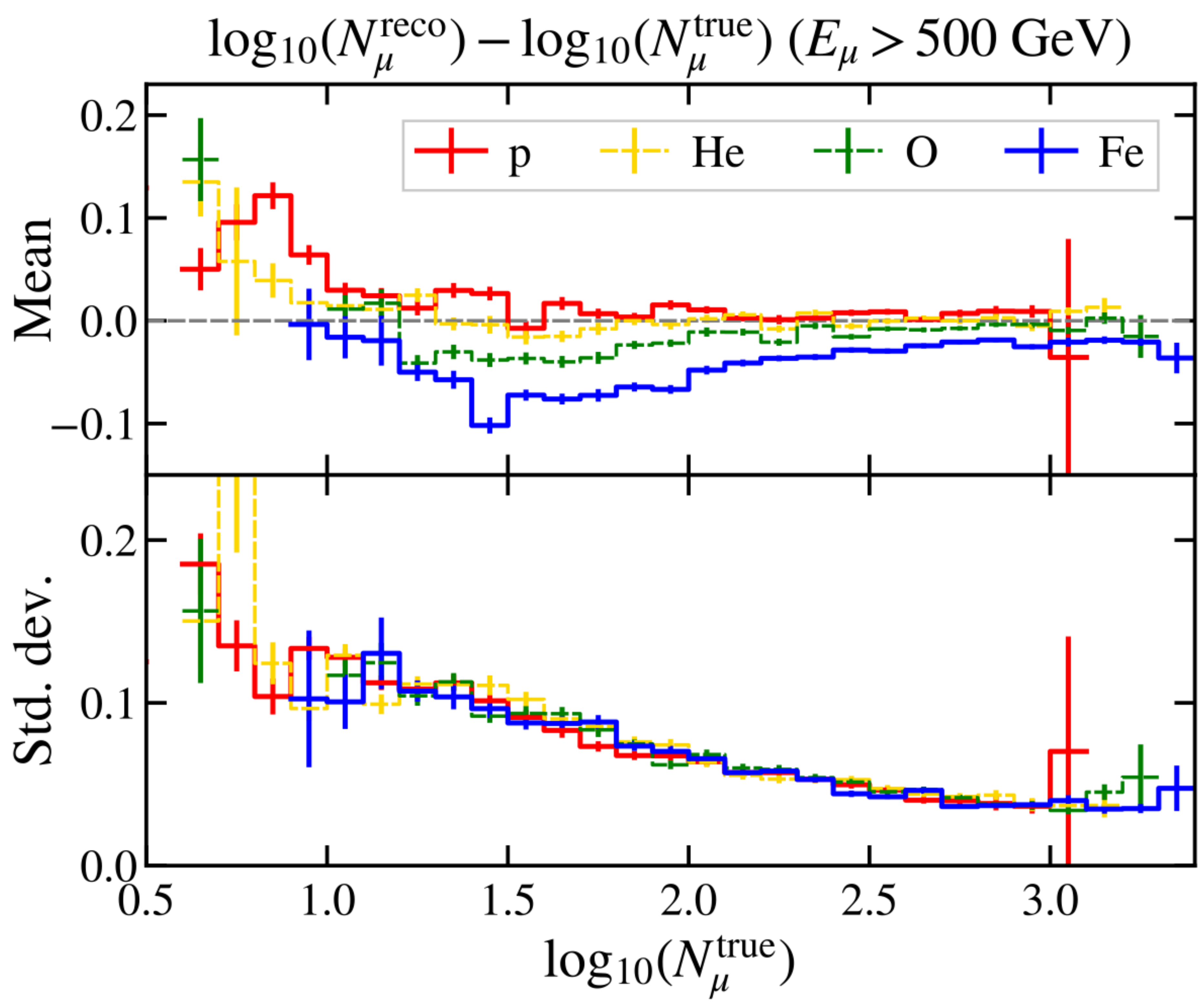


Only neutrino telescope with joint cosmic-ray detection array!

Muon measurements in IceCube

Muon number estimation through neural network for joint measurement with IceTop & in-ice


IceCube, PRD 112 (2025)

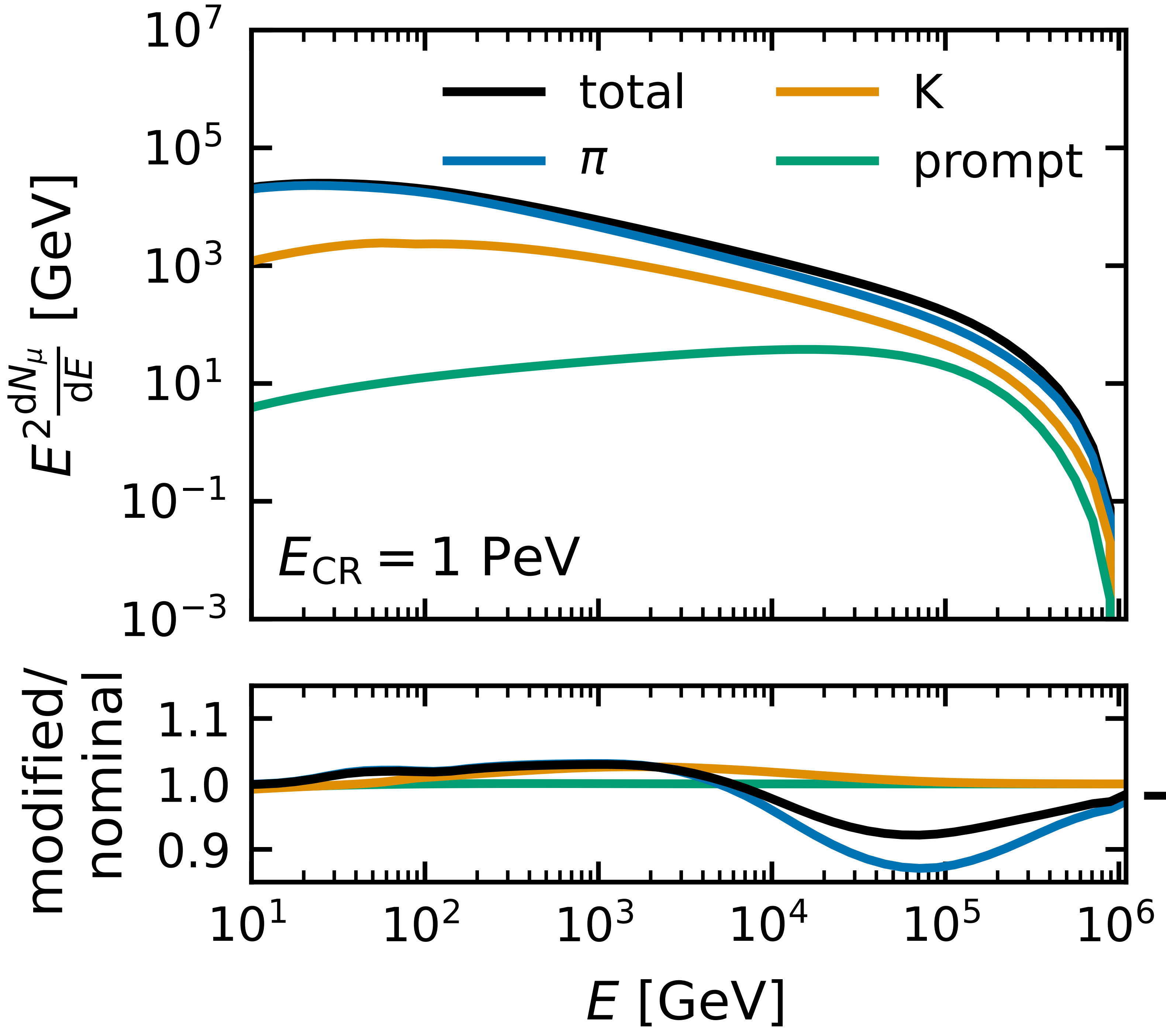


Which tools and muon number measurements are available to model the muon number?

Pion-air cross section at underground detectors?

Muon yield

$$\langle N_\mu(E_{\mathrm{CR}})\rangle = \int_{E_{\mathrm{threshold}}}^{\infty} \mathrm{d}E_\mu \; \mathscr{Y}(E_{\mathrm{CR}})$$



Changing the cross section impacts the number of muons detected underground

$\longrightarrow$ $\sigma_{\pi+\mathrm{air}}(E \sim \mathrm{TeV}) + 50\,\%$

Which tools and muon number measurements are available to study the detection ability?

Modeling atmospheric lepton fluxes: MCEQ

Numerical solver of the cascade equations

https://github.com/mceq-project/MCEq

Fedynitch, Fröse

$$\frac{\mathrm{d}\Phi_h(E,X)}{\mathrm{d}X} = -\frac{\Phi_h(E,X)}{\lambda_{int,h}} - \frac{\Phi_h(E,X)}{\lambda_{dec,h}}$$

Sink term

$$-\frac{\partial}{\partial E}\big(\mu(E)\,\Phi_h(E,X)\big) \quad + \sum_k \int_E^\infty \mathrm{d}E_k\, \frac{\mathrm{d}N_{k(E_k)\to h(E)}}{\mathrm{d}E}\,\frac{\Phi_k(E_k,X)}{\lambda_{int,l}(E_k)} + \sum_k \int_E^\infty \mathrm{d}E_k\, \frac{\mathrm{d}N^{dec}_{k(E_k)\to h(E)}}{\mathrm{d}E}\,\frac{\Phi_k(E_k,X)}{\lambda_{dec,l}(E_k)}$$

Radiative losses

Source terms

Cosmic Ray Flux Model

+

Hadronic Interaction Model

+

Atmospheric Model

New version v1.4 available

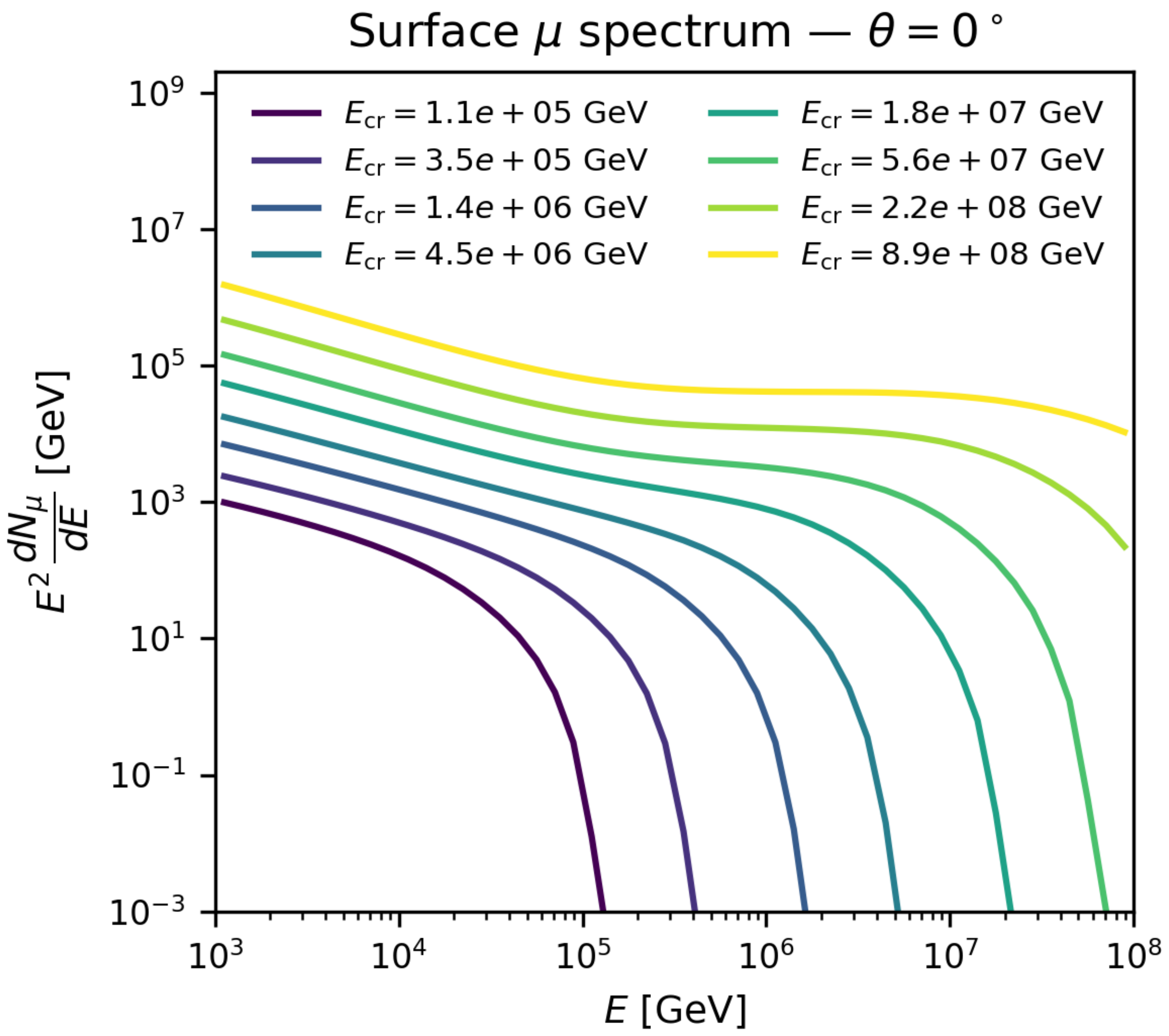
Underground multiplicities: MUTE

Woodley, Fedynitch, Piro, PRD 110 (2024)

Muon flux propagation to deep underground laboratories

https://github.com/wjwoodley/mute

MCEq or **Daemonflux**



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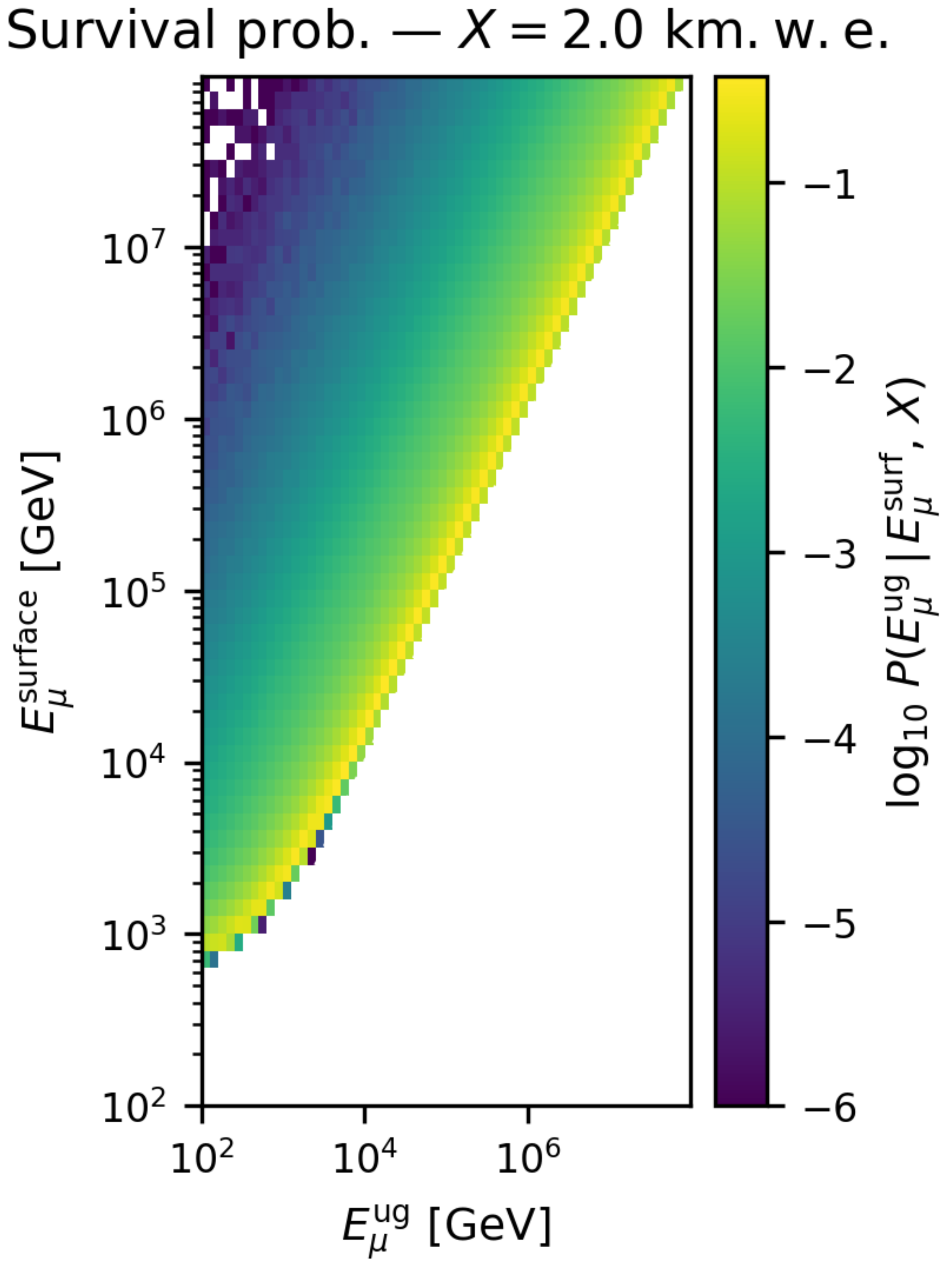
PROPOSAL



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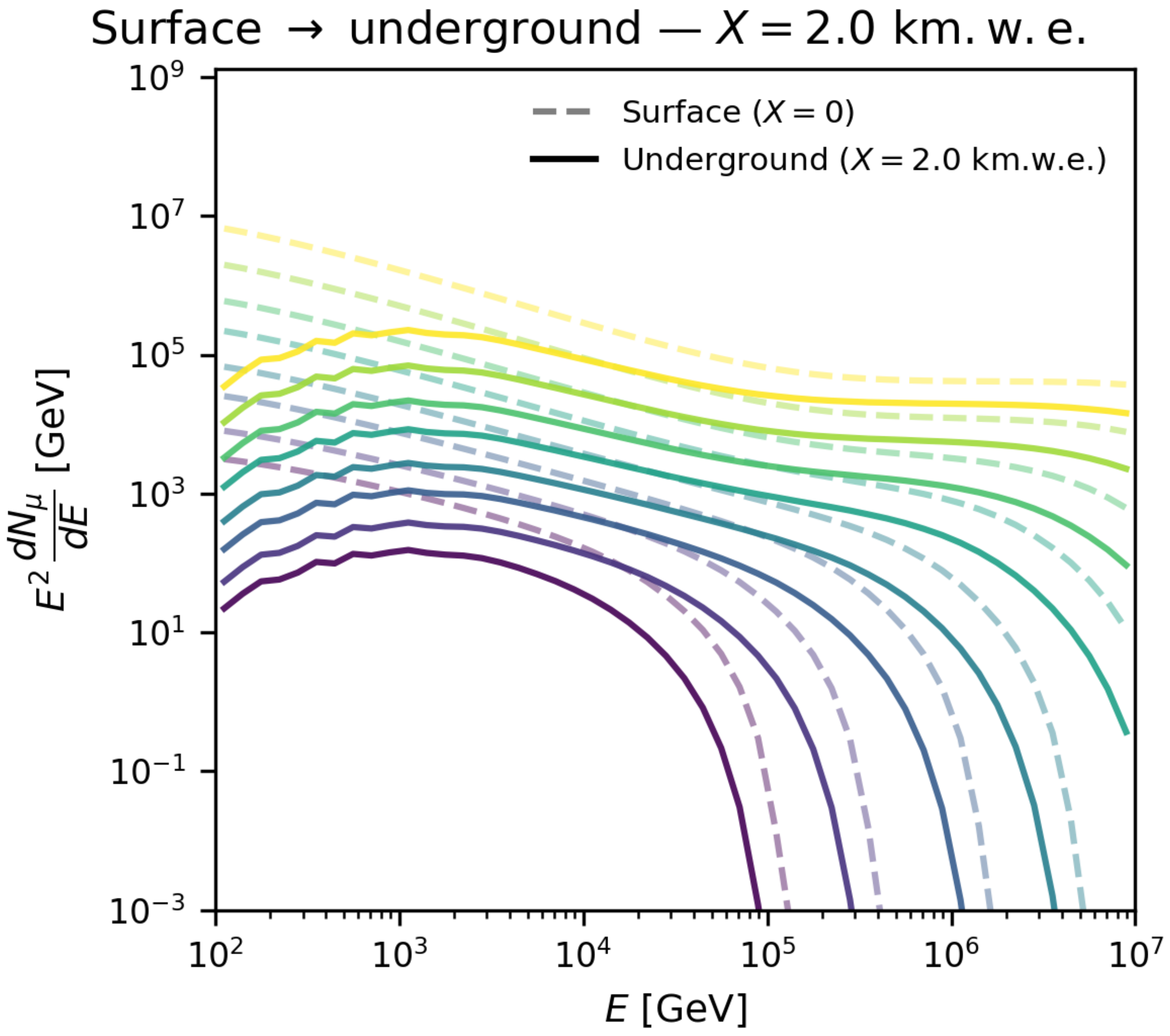
MUTE v3

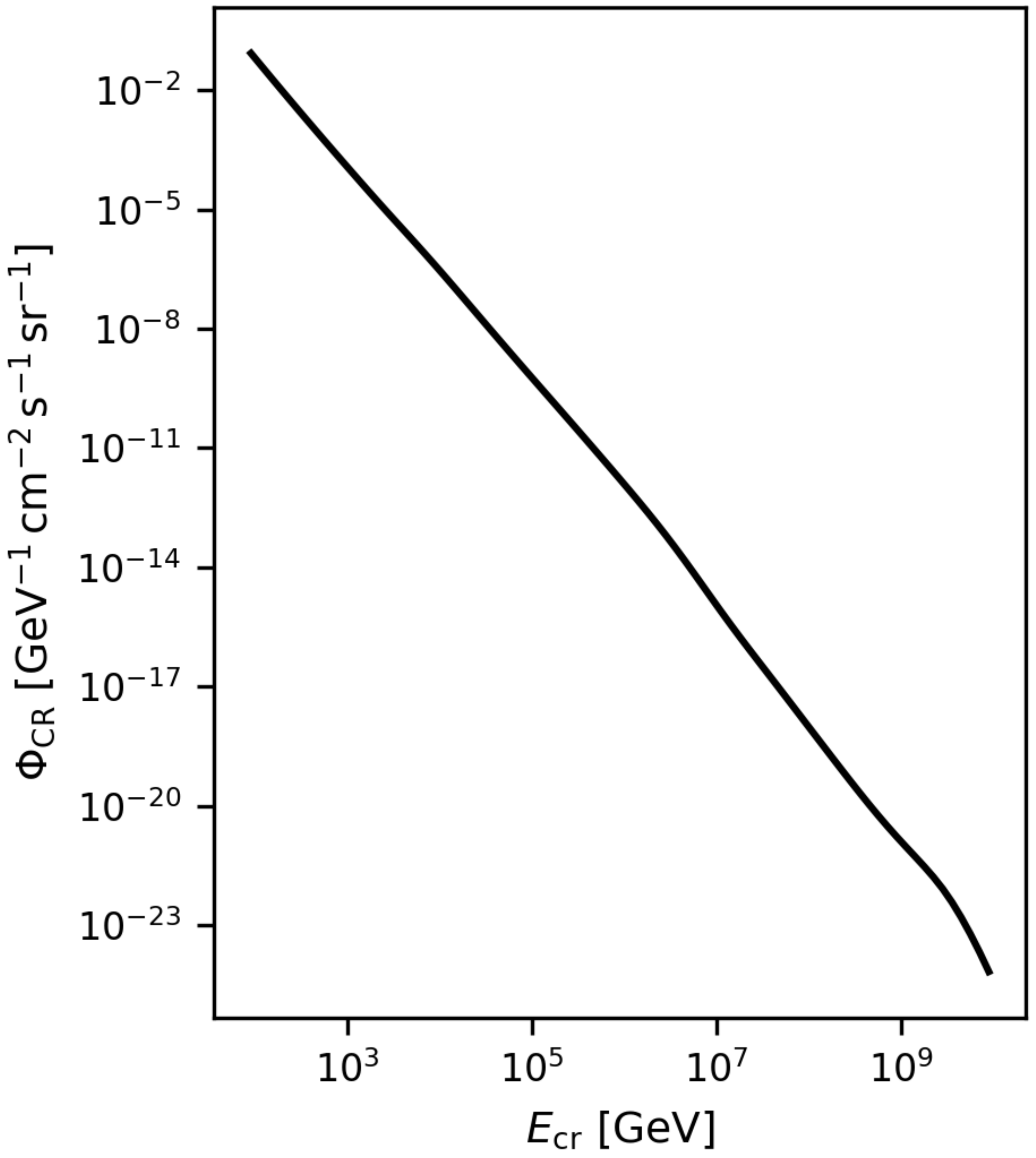


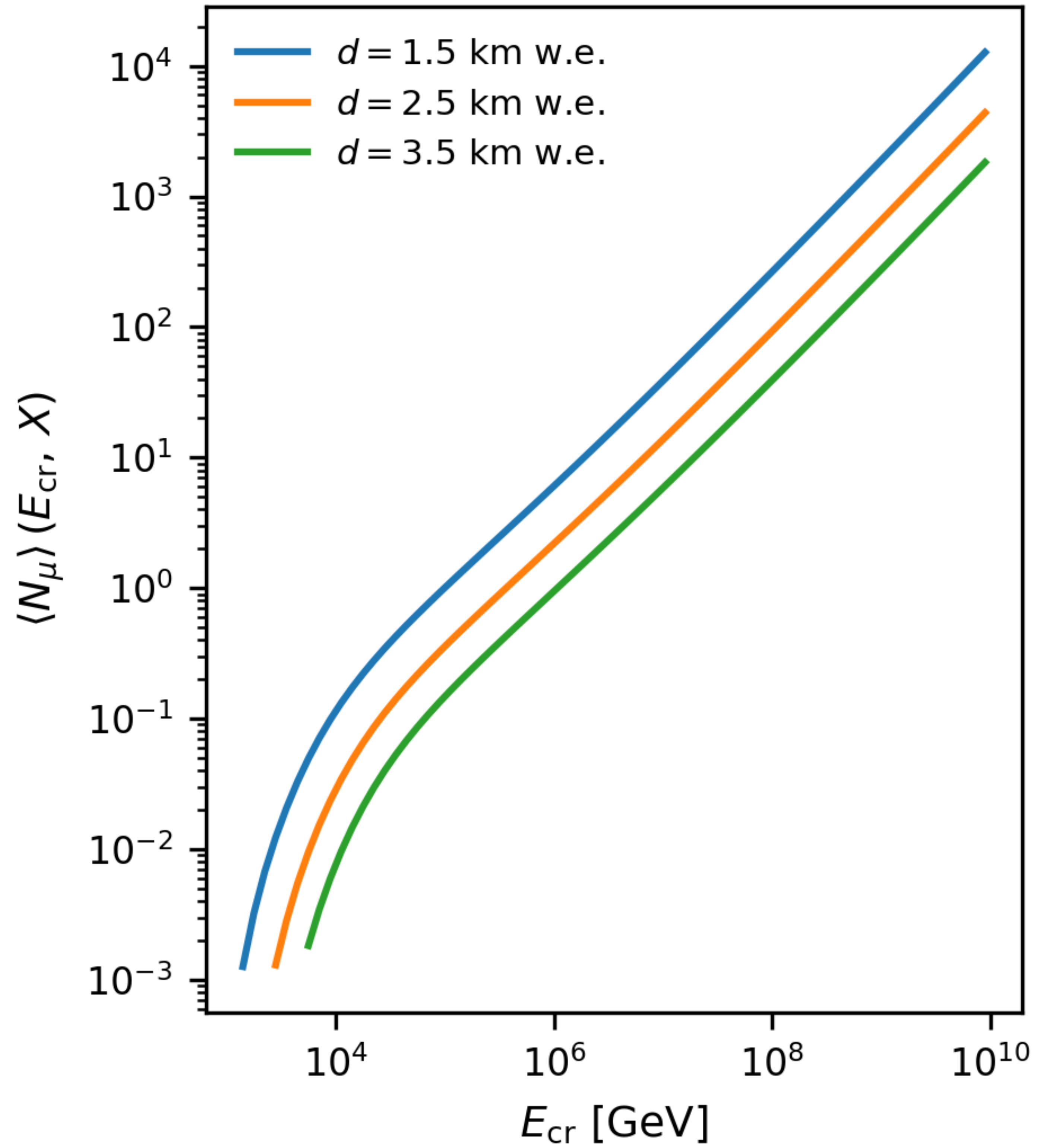
Here: single particle mode

Underground Muon Bundle Flux

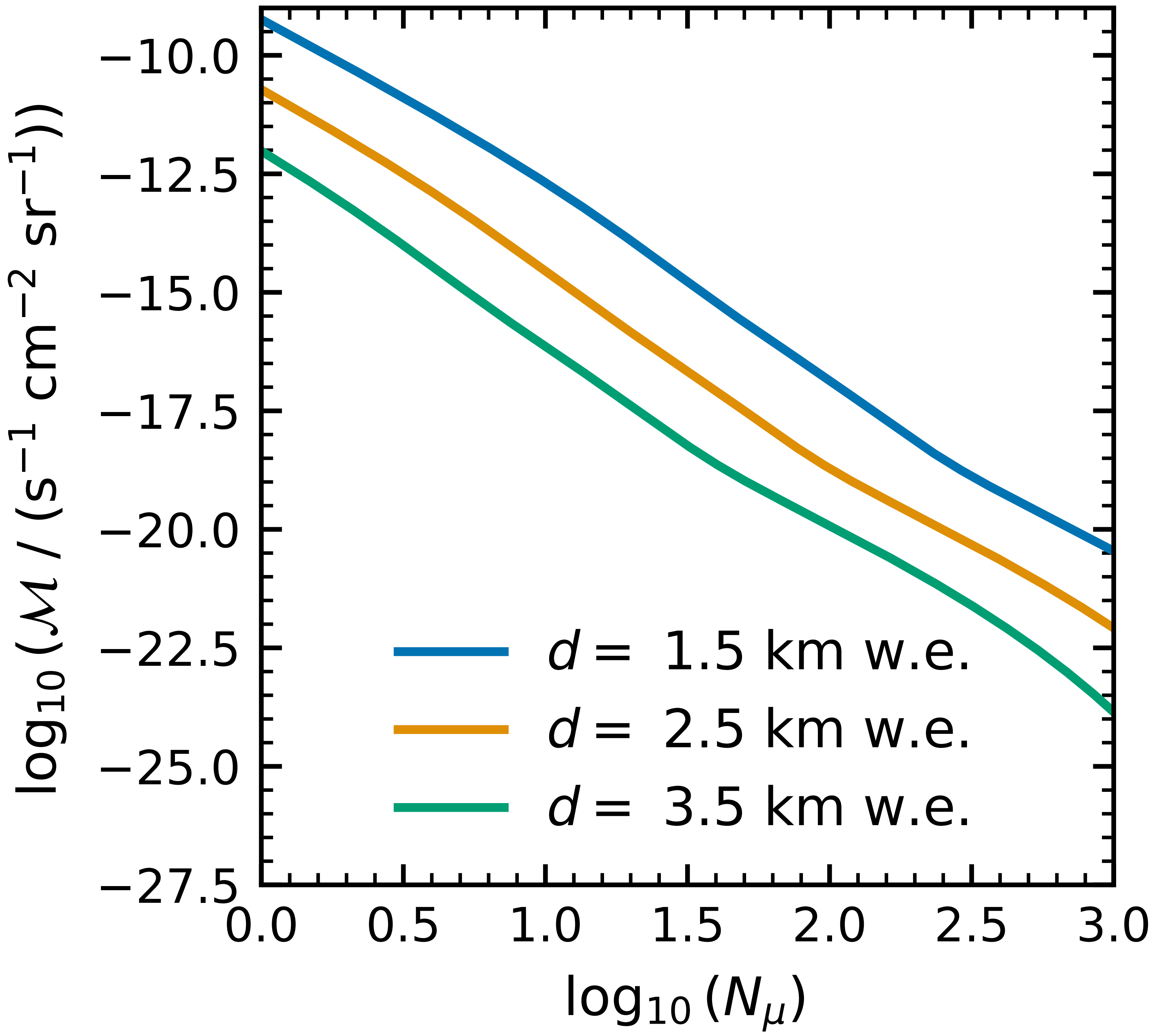
$$\mathscr{M}(N_\mu) := \frac{\mathrm{d}N}{\mathrm{d}N_\mu} = \Phi_{\mathrm{CR}}(E_{\mathrm{CR}}(N_\mu))\frac{\mathrm{d}E_{\mathrm{CR}}}{\mathrm{d}N_\mu}$$

inclusive muon bundle flux as new observable



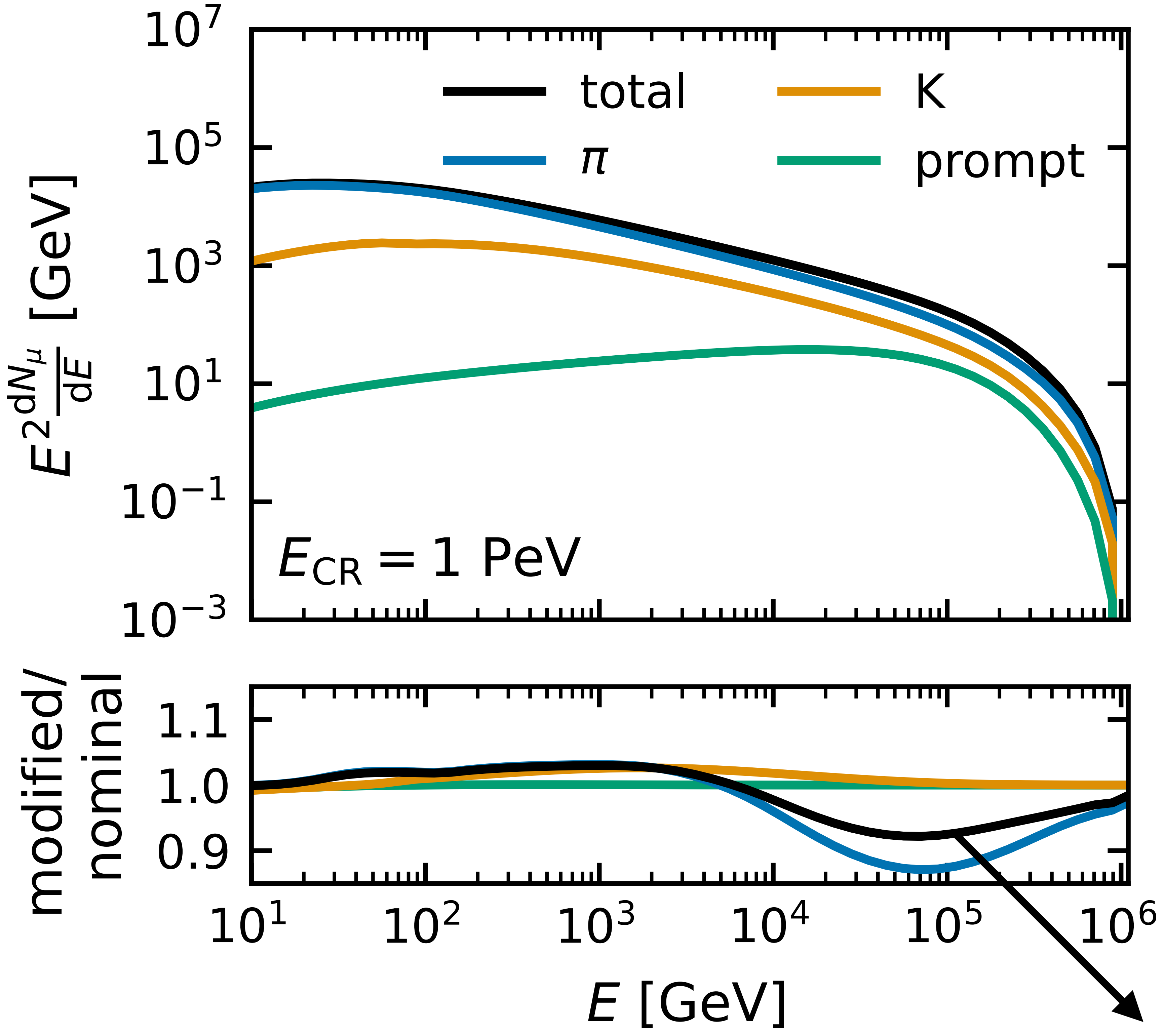


KH AF, PoS 293 ICRC 2025



$$\langle N_\mu(E_{\mathrm{CR}})\rangle = \int_{E_{\mathrm{threshold}}}^{\infty} \mathrm{d}E_\mu \, \boxed{\mathscr{Y}(E_{\mathrm{CR}})}$$ Muon yield

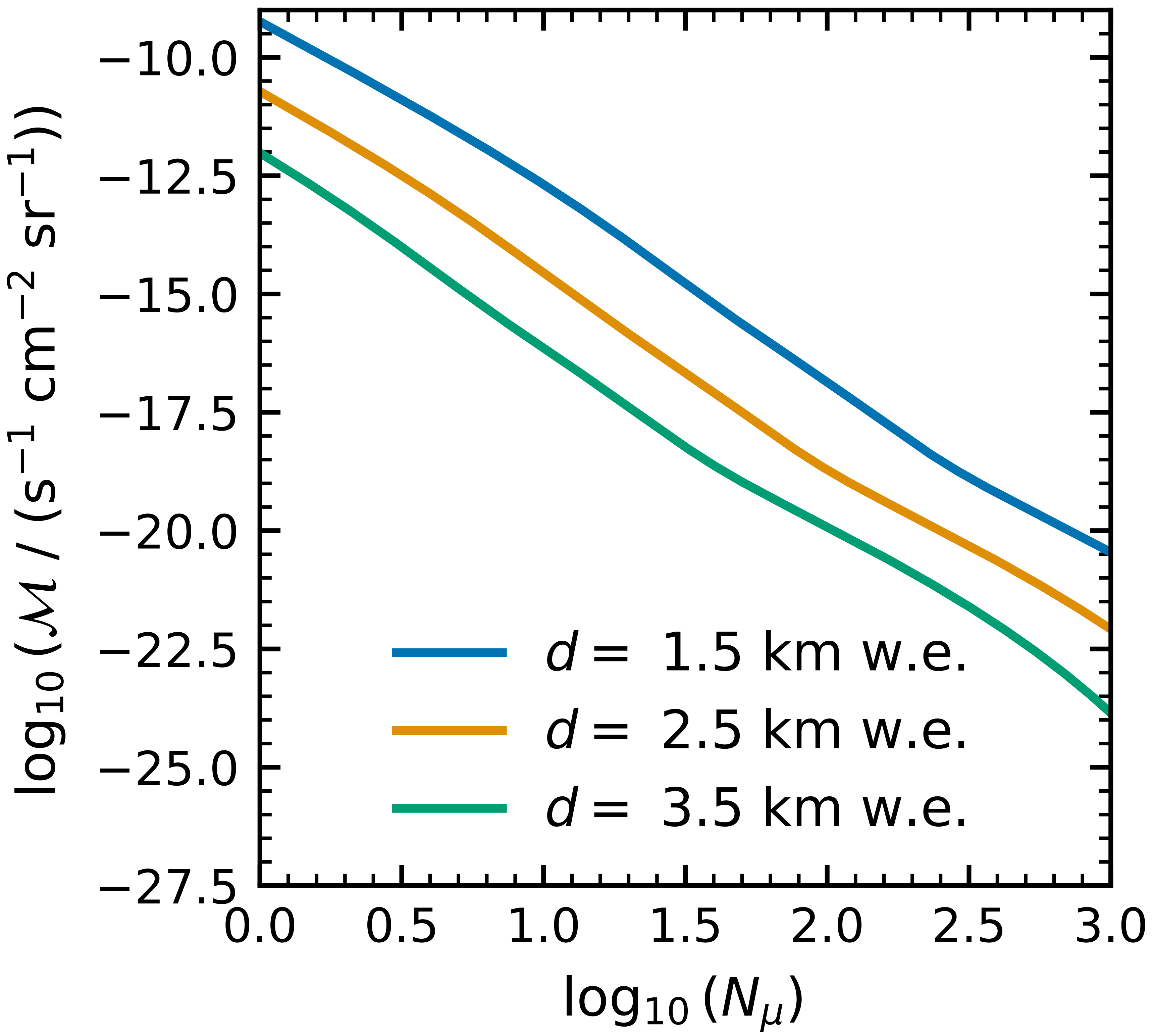
How does the cross section impact the muon bundle flux?



$\sigma_{\pi+\mathrm{air}}(E \sim \mathrm{TeV}) + 50\,\%$

?

KH AF, PoS 293 ICRC 2025

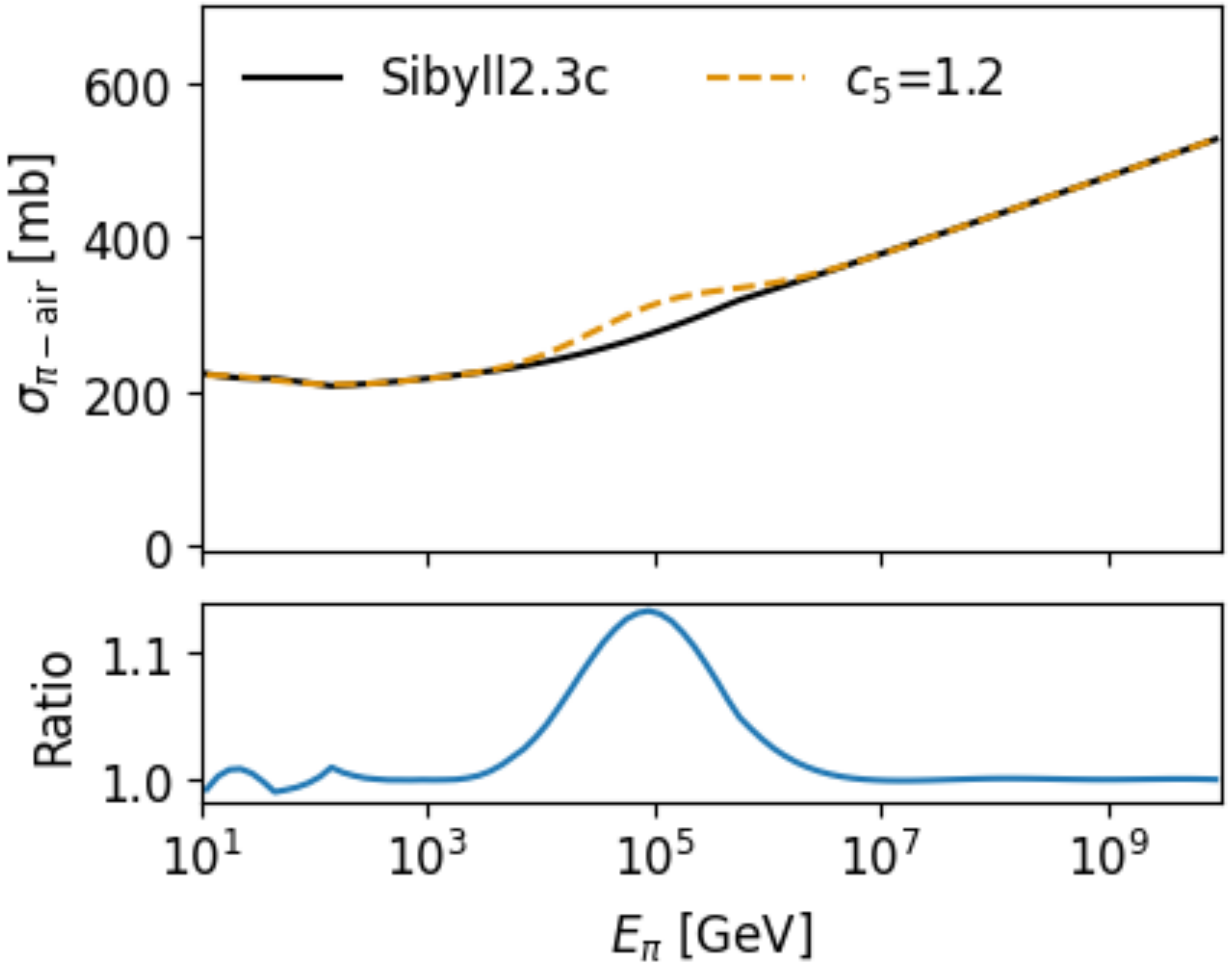


Here: **MCEq v1.3** (SIBYLL2.3c + GSF)

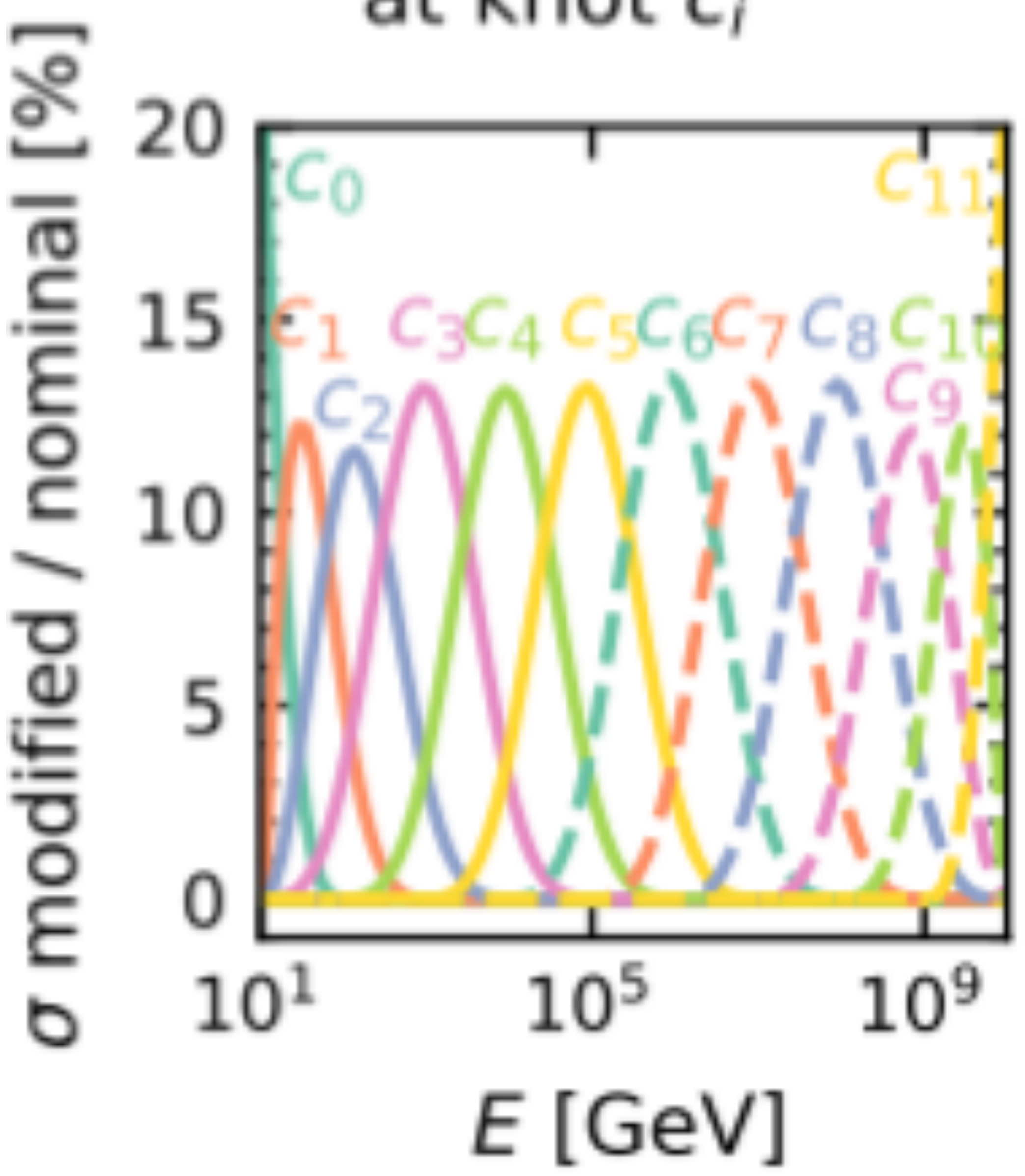
Determine cross section sensitivity for underground detectors

➔ Parametrization of **SIBYLL2.3c** cross section by cubic spline (10 knots —>12 coefficients c_i— 10 to 10^{10} GeV) & calculation of perturbation gradients

$$\boxed{\frac{\partial \mathscr{M}}{\partial \hat{c}_i}} = \frac{\mathscr{M}\left((1+\delta)\hat{c}_i\right) - \mathscr{M}\left((1-\delta)\hat{c}_i\right)}{2\delta}$$



Cross section modification at knot c_i



Cross Section Sensitivity Fit

- Parametrization of **SIBYLL2.3c** cross section by cubic spline (10 knots —>12 coefficients c_i— 10 to 10^{10} GeV) & calculation of perturbation gradients

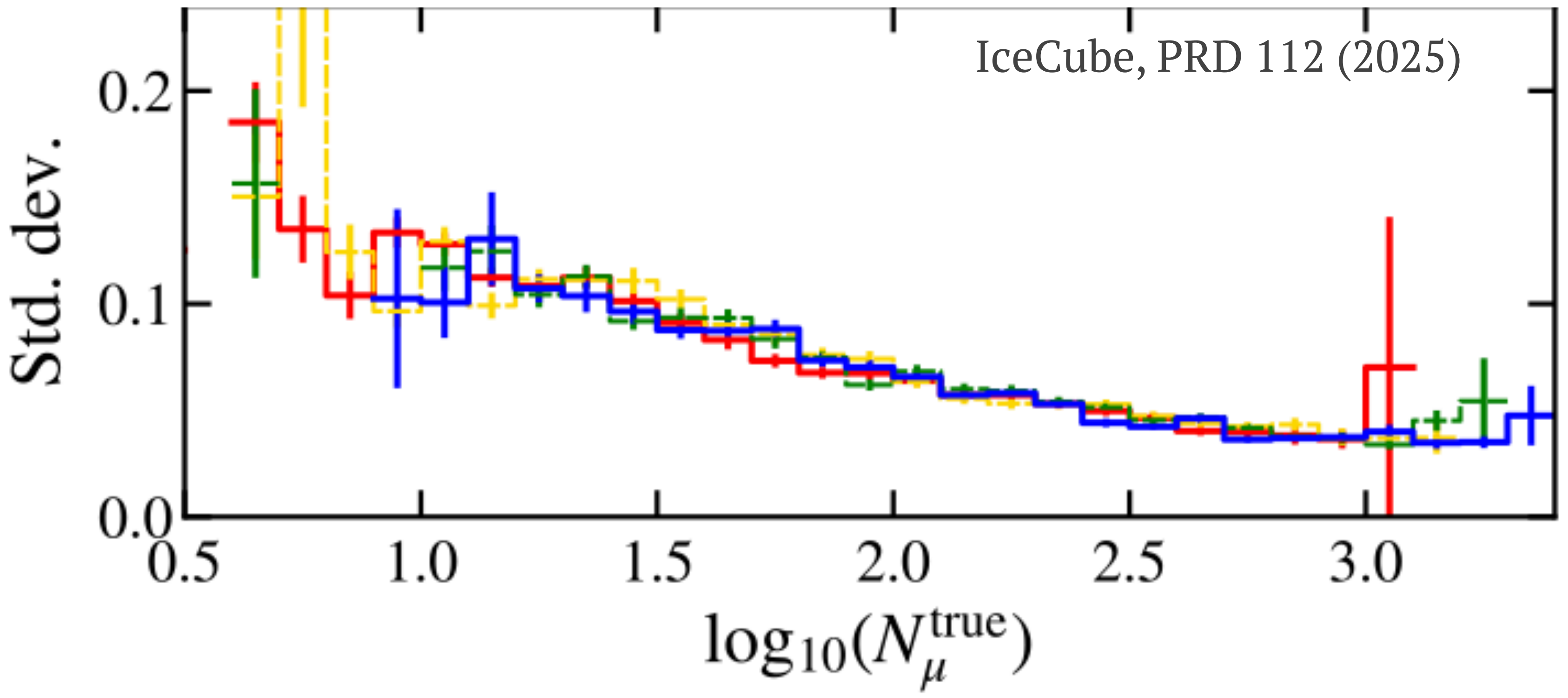
$$\boxed{\frac{\partial \mathscr{M}}{\partial \hat{c}_i}} = \frac{\mathscr{M}\left((1+\delta)\hat{c}_i\right) - \mathscr{M}\left((1-\delta)\hat{c}_i\right)}{2\delta}$$

- Total modification

$$\mathscr{M}_{\text{model}}(N_\mu, d, \theta, c_1, \ldots, c_n) = \mathscr{M}_{\text{nominal}} + \sum_i \boxed{\frac{\partial \mathscr{M}}{\partial \hat{c}_i} c_i}$$

- Assumption of **measurement uncertainty** (e.g. IceCube)

$$\boxed{\log_{10}(\Delta\mathscr{M}/\mathscr{M}) = 0.1}$$

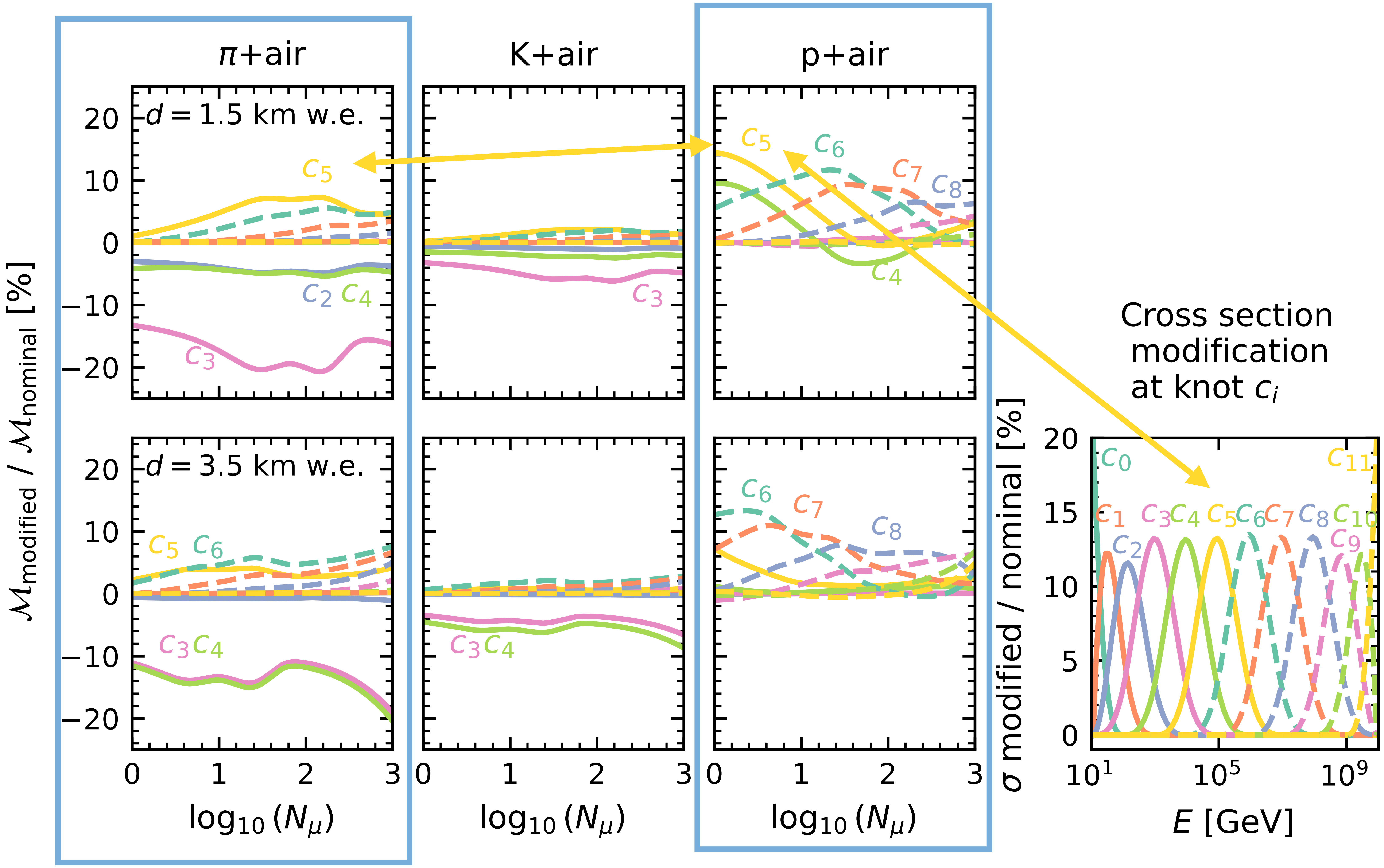


IceCube, PRD 112 (2025)

- Penalized χ^2-fit

$$\chi^2 = \sum_n \left(\frac{\log_{10}(\mathscr{M}^{\text{nominal}}) - \log_{10}(\mathscr{M}^{\text{model}}(\vec{c}))}{\boxed{0.1}}\right)^2 + \sum_i \left(\frac{c_i - c_i^{\text{prior}}}{\delta_{c_i}}\right)^2$$

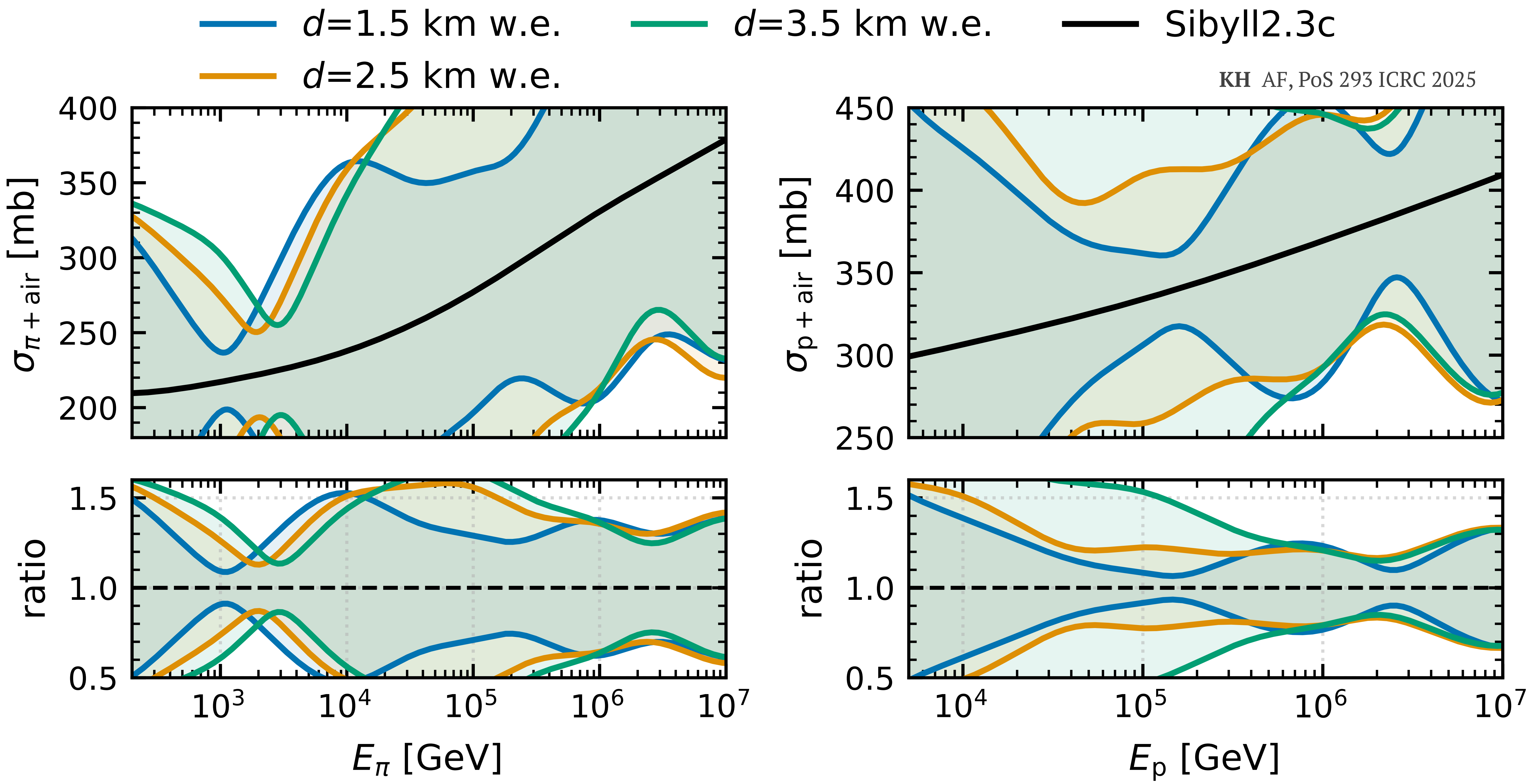
Impact of Cross Section Modification on Bundle Flux



- Kaon contribution negligible —> neutrino production
- Each coefficient corresponds to energy range

$\sigma_{\pi+\mathrm{air}}$ & $\sigma_{\mathrm{p+air}}$ sensitivity region overlapping in multi 100 TeV to PeV region

Projected Sensitivity



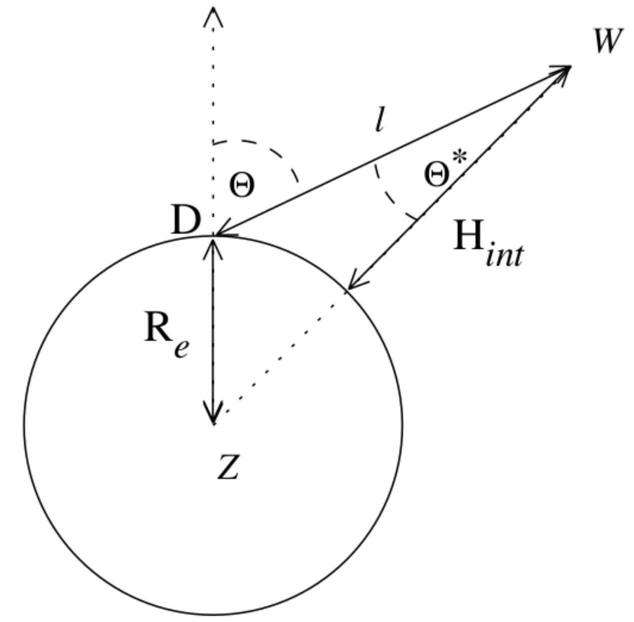
KH AF, PoS 293 ICRC 2025

Propagation of covariance matrix of fitted spline coefficients to cross section

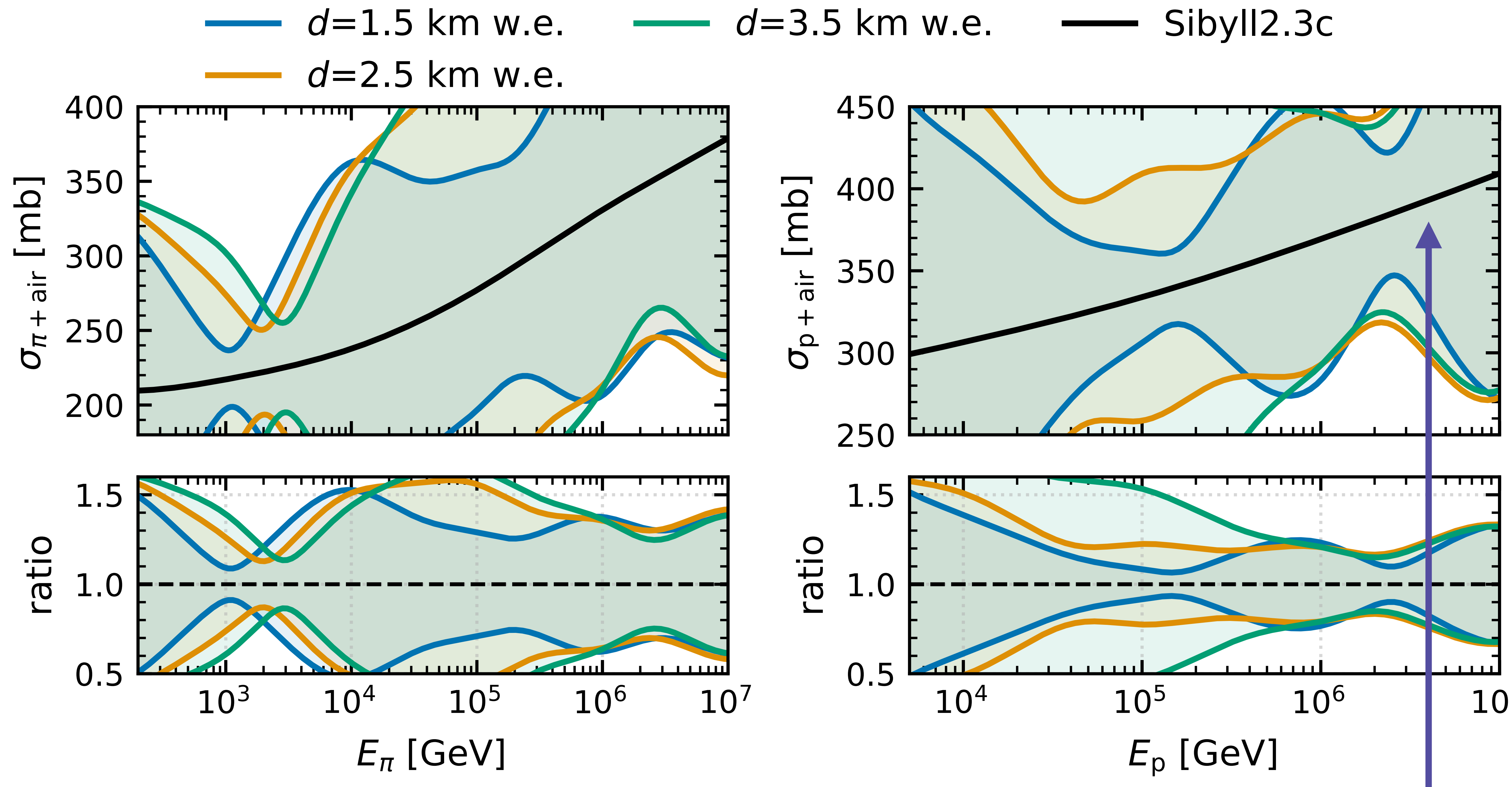
$$\mathrm{Var}[\sigma_{\pi+\mathrm{air}}(E)] = \mathbf{J}^T \mathbf{\Sigma}_c \mathbf{J}$$

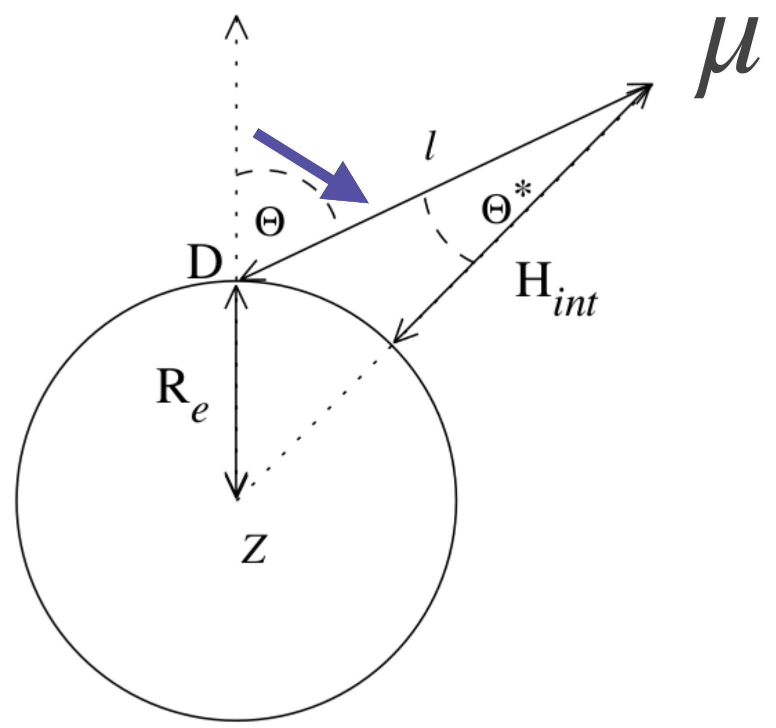
Complementary sensitivity > 50 TeV, $\sigma_{\mathrm{p+air}}$ cannot be measured independently!

- Sensitivity for $\sigma_{\pi+\mathrm{air}}$ **up to 10%** for vertical events (single TeV region) + systematic Uncertainties
- Sensitivity shifts towards higher energies with increasing depth
  —> equivalent to inclination (same length through ice)



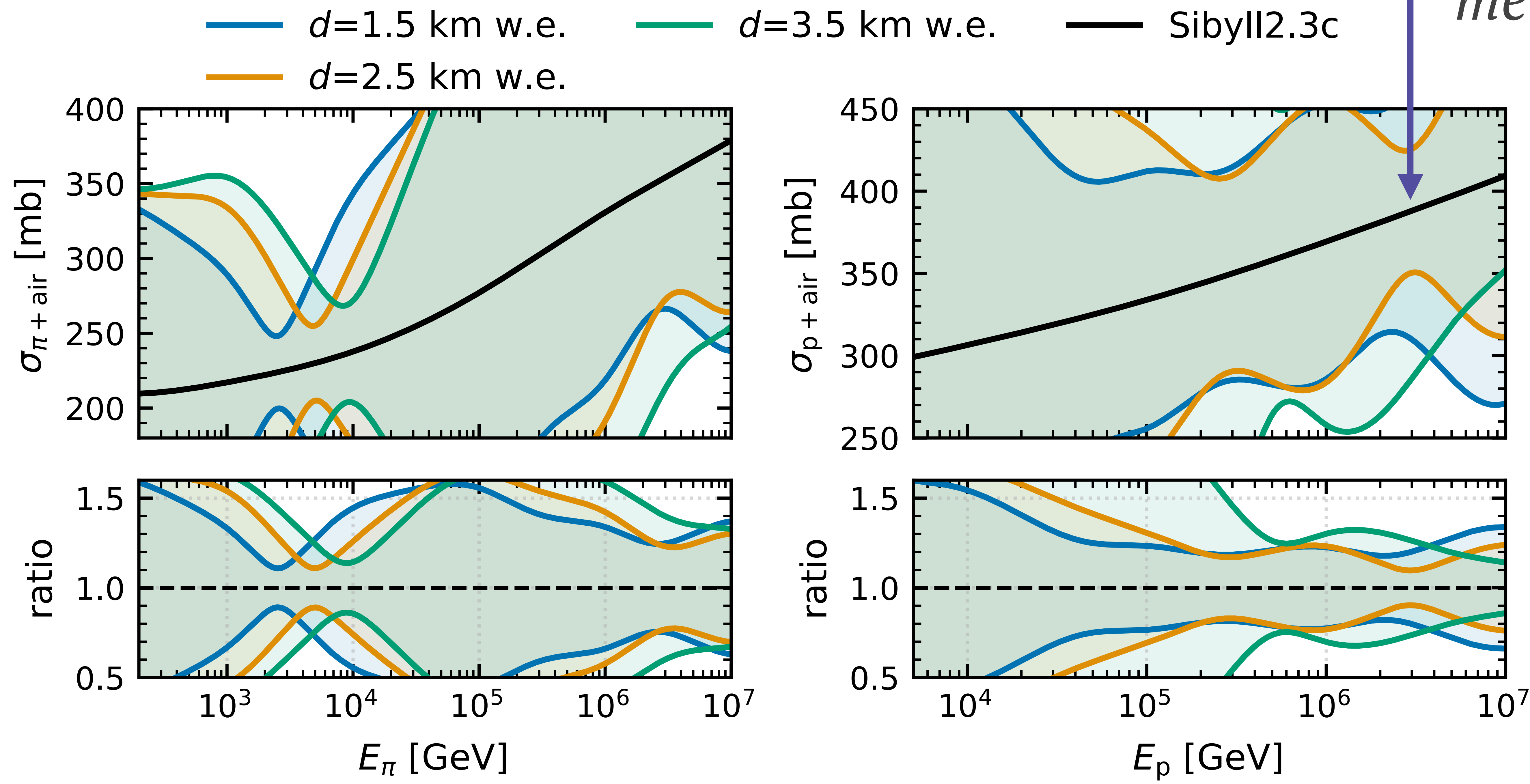
$\theta = 0^\circ$





correlated measurement

$\theta = 60^\circ$



Sensitivity shifts towards higher energy with inclination —> measurement at multiple E

Next step:

For realistic estimation we need multiple **hadronic models**, measurement in **zenith bands**, **p-air parameters**

Towards realistic sensitivity for IceCube

- Estimation of effective area of muon bundles (constant across N_μ & θ) from T and N
- **Fit expected counts** $N(\theta, N_\mu)$ simultaneously for 5 zenith bands to account for correlation
- From χ^2-fit to **Poisson LLH $\mathscr{L}$ fit —>** statistical uncertainty intrinsic to Poisson counts

Livetime T	Trigger rate	Total number of events N	Effective area A	Cos zenith range
15 yrs	2.5 kHz	1E+10	4.475E10 cm2	0.5 to 1.0

"observed" counts

expected counts

$$\log \mathscr{L} = \sum_i - N_i \log \lambda_i + \lambda_i$$

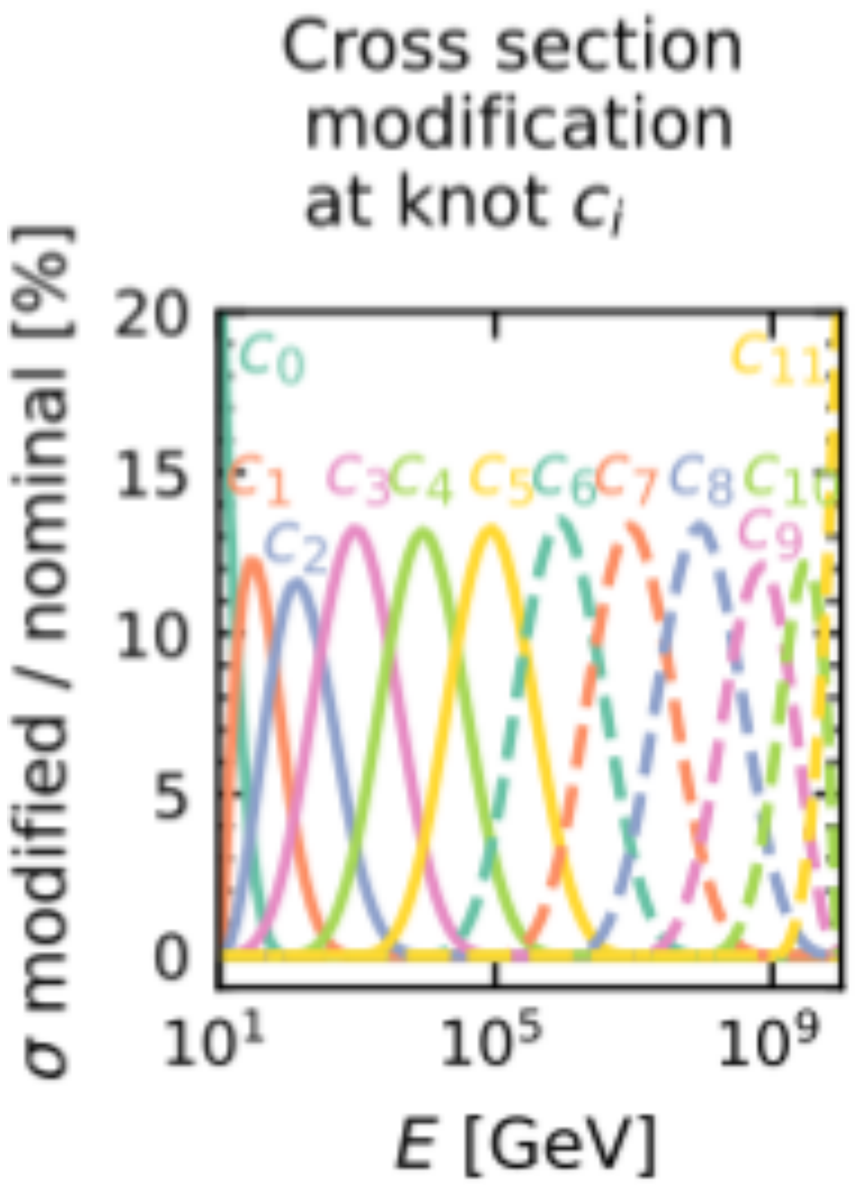
zenith band index

⇨ Add p-air parameters

Simultaneous fit of pion-air & p-air cross section

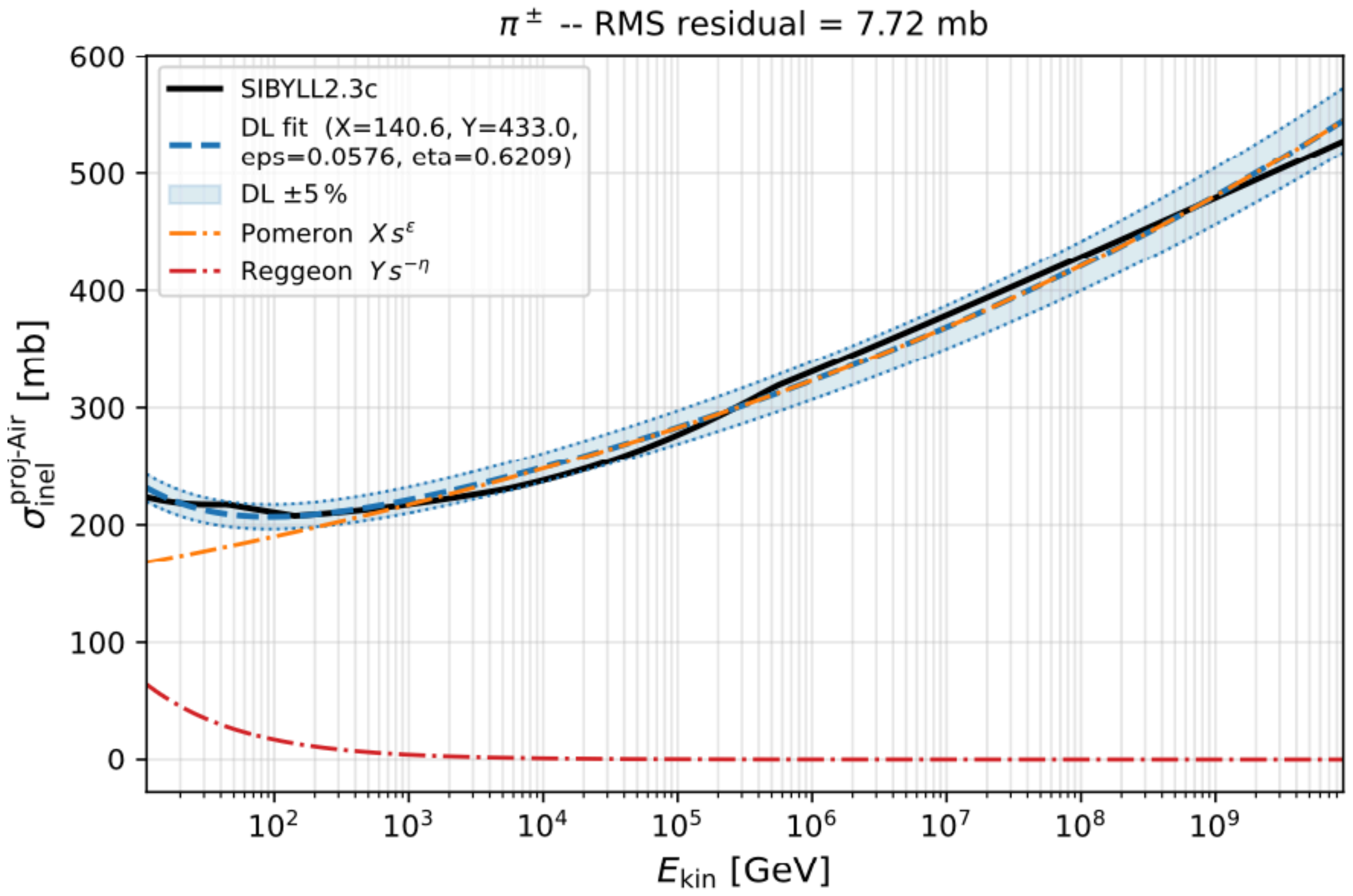
Cross section parameterization options

Add spline for $\sigma_{\mathrm{p+air}}$ modification

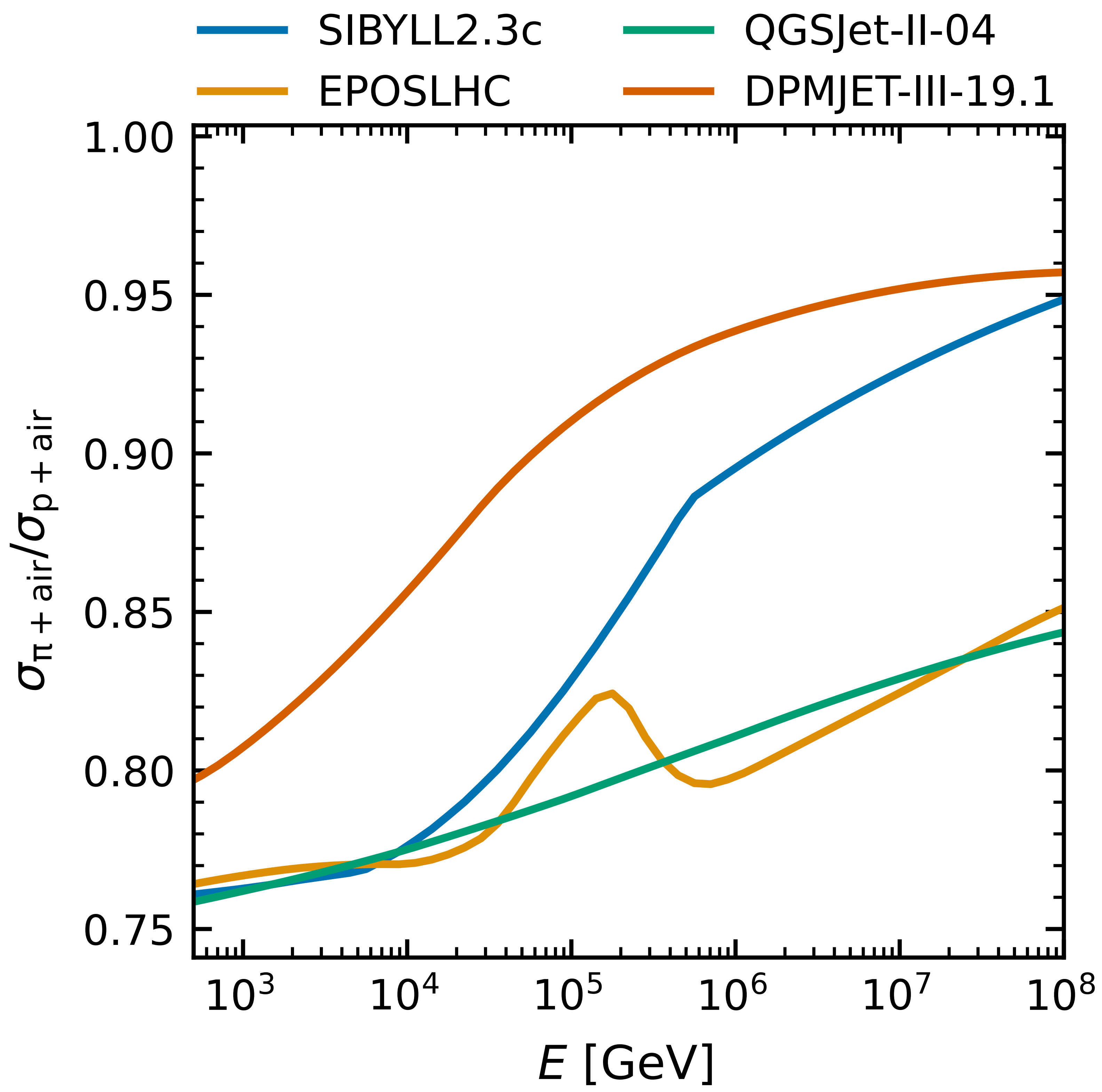


Physics-informed: Donnachie-Landshoff

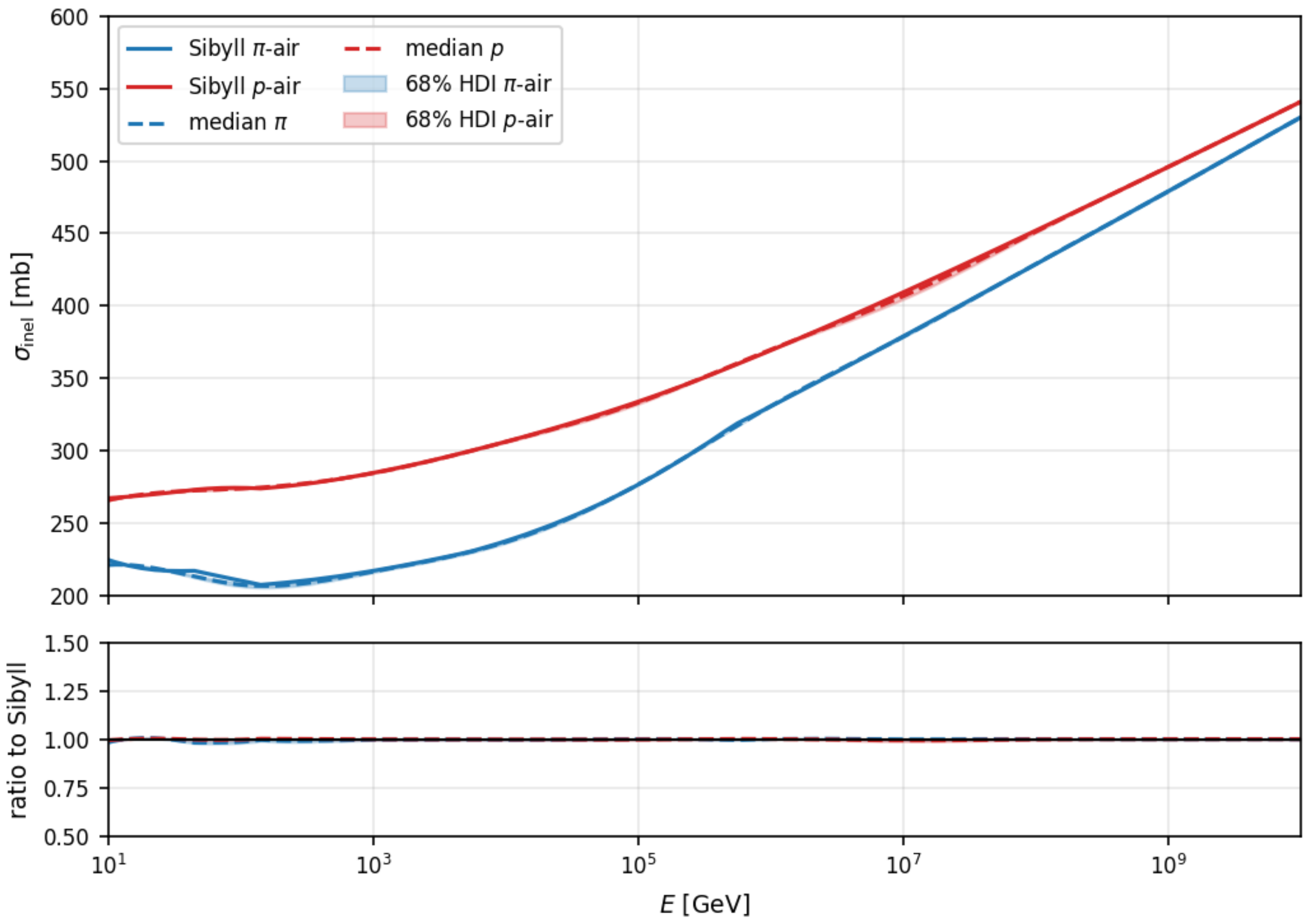
$$\sigma = Xs^{\epsilon} + Ys^{-\eta}$$



Relative fit to $\sigma_{\mathrm{p+air}}$



Injection test



- Fit only sensitive spline knots obtained through PCA

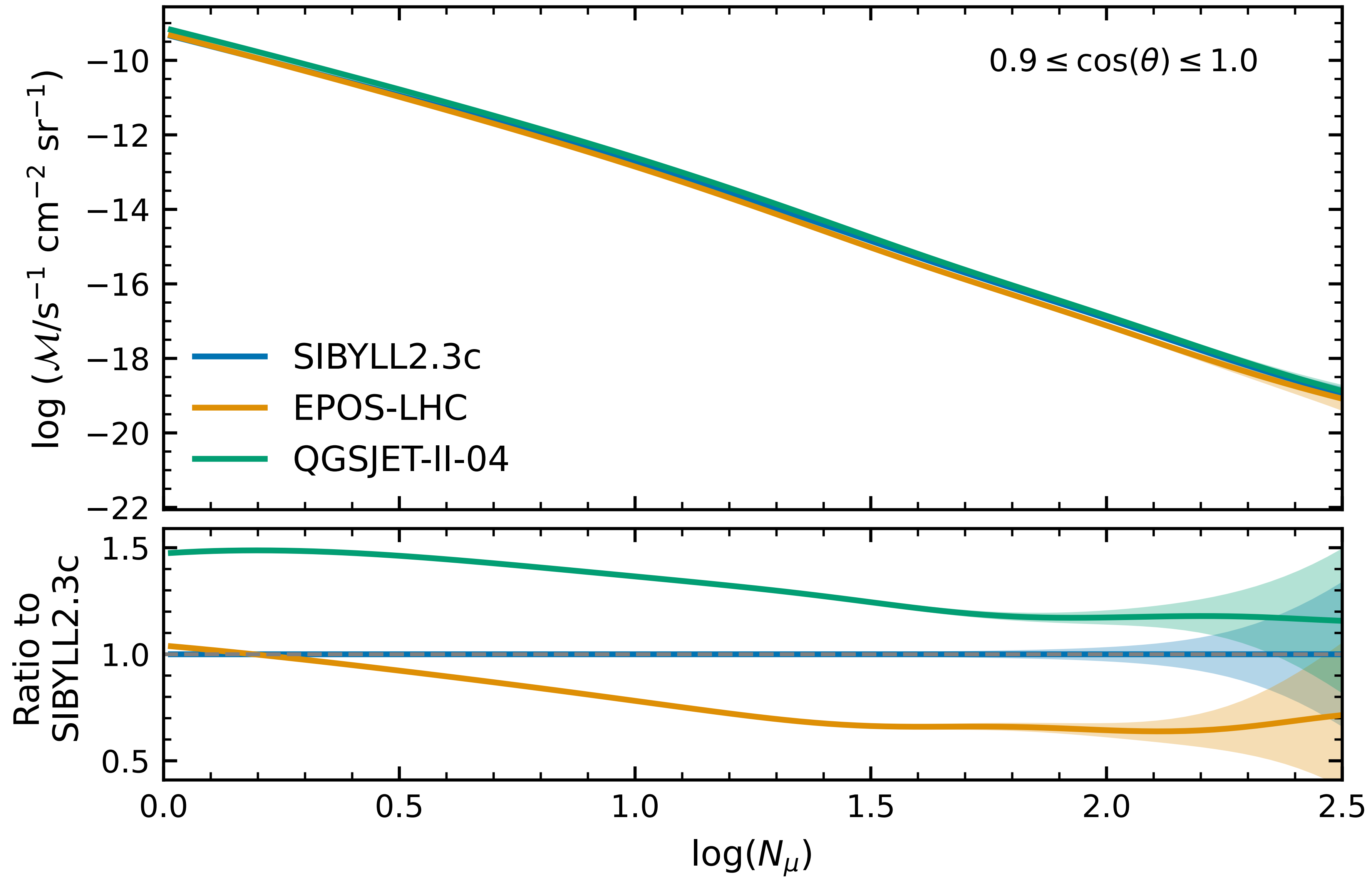
- Injection tests: pseudo data fitted with same hadronic interaction model

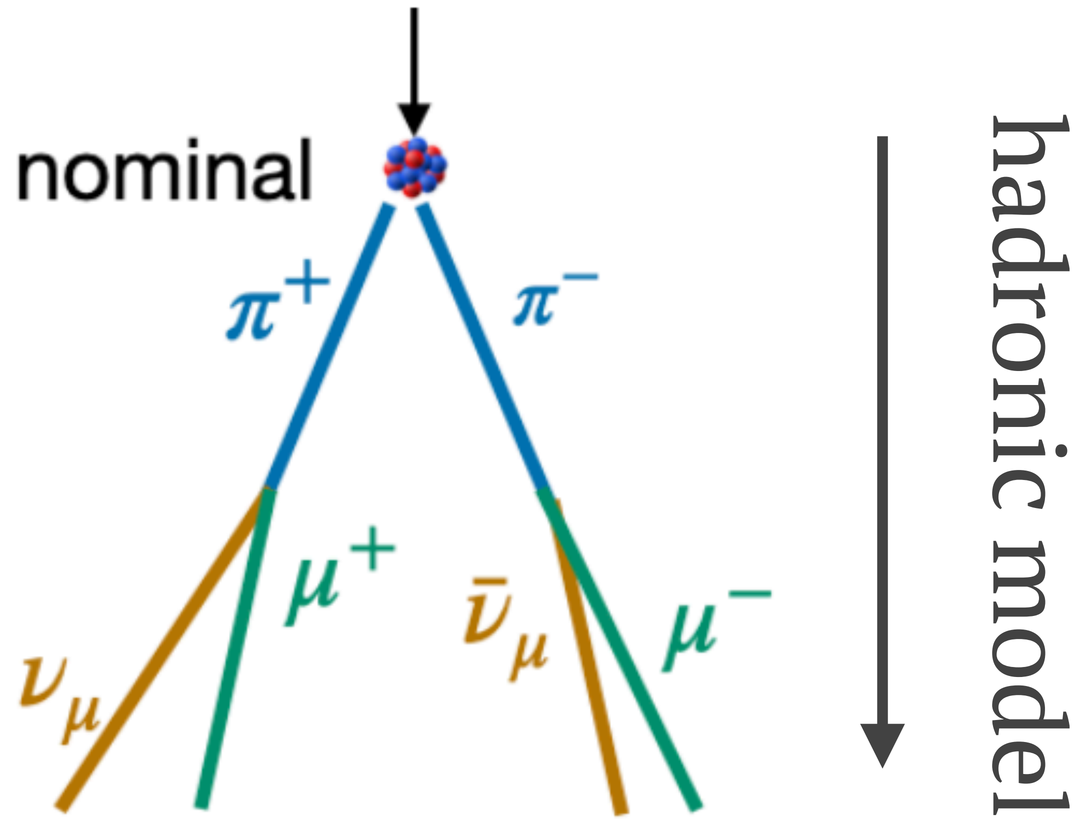
- Fit fails if pseudo data generated from different hadronic interaction model —> problematic for real measurement

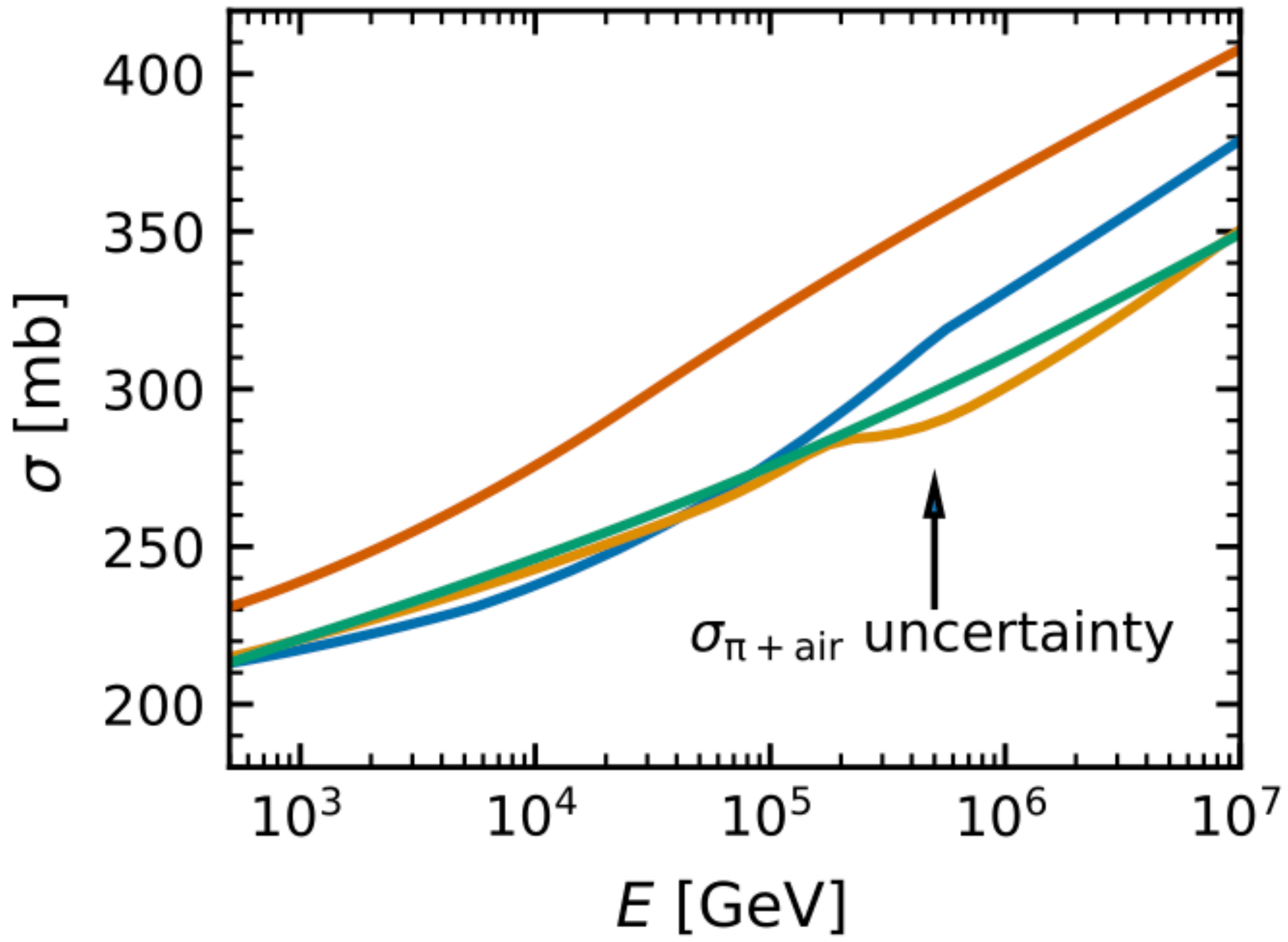
Hadronic Model Discrepancy

Multiplicity flux can can be calculated with different hadronic models
—> sensitivity to cross section can change by model

Reminder







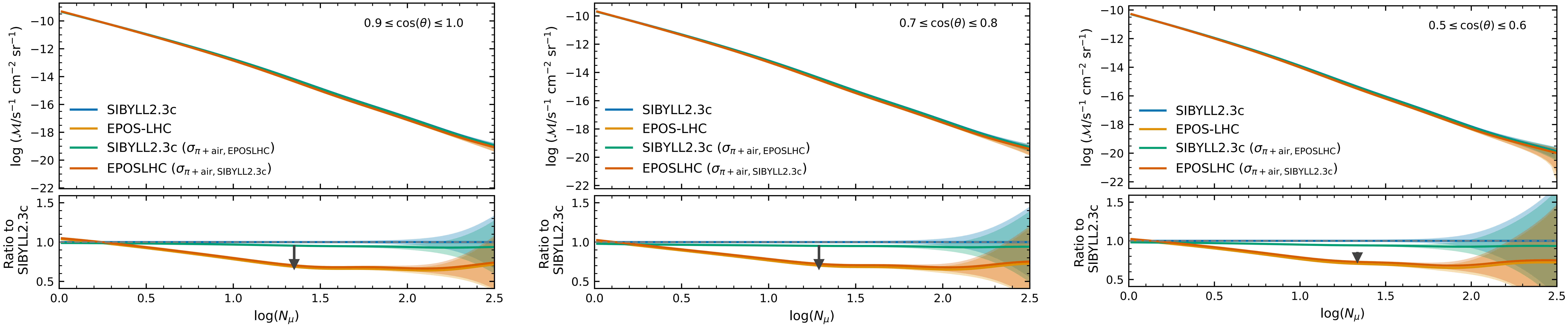
Differences larger than $\Delta\sigma_{\pi+\mathrm{air}}$ between models

Hadronic Model Discrepancy

Condition for model independent fit of cross section spline coefficients:

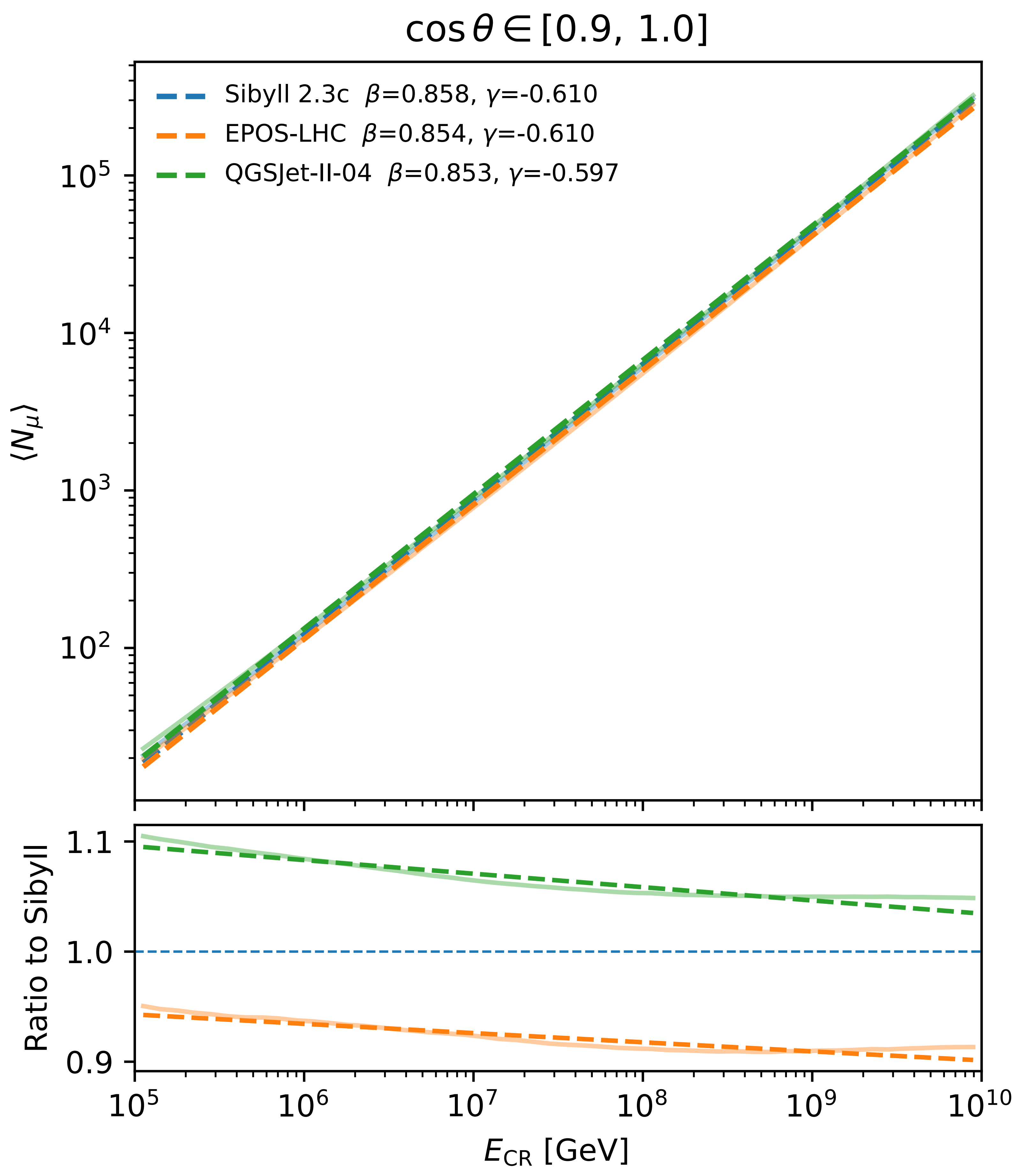
$$\mathscr{M}_{\text{sibyll}}(N_\mu, d, \theta, \sigma_{\pi+\text{air, SIBYLL}}) = \mathscr{M}_{\text{EPOSLHC}}(N_\mu, d, \theta, \sigma_{\pi+\text{air, SIBYLL}})$$

$$\mathscr{M}_{\text{model}}(N_\mu, d, \theta, c_1, \dots, c_n) = \mathscr{M}_{\text{nominal}} + \sum_i \frac{\partial \mathscr{M}}{\partial \hat{c}_i} c_i$$



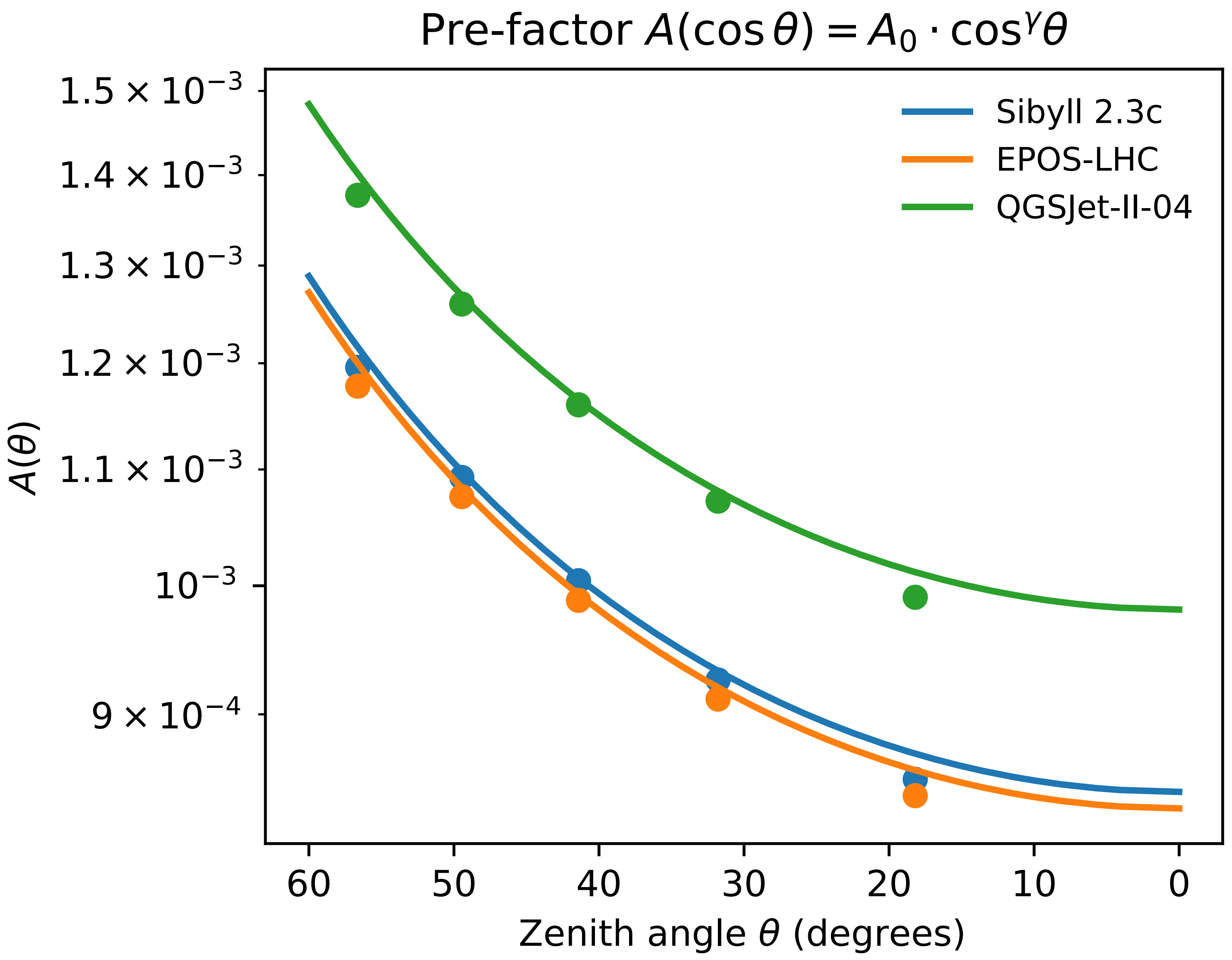
- Model dependence shifts with zenith band & slope decreases —> no suitable parameterization found
- Different strategies: relative fit, parameterization of zenith band difference, **model-dependent fit**, additional fit variables (bundle energy), …

Future Strategy: constraints through IceTop



$$\langle N_\mu \rangle = A \log_{10} E_{\mathrm{CR}} + b$$

$$A = A_0 \cos^{\gamma}(\theta)$$



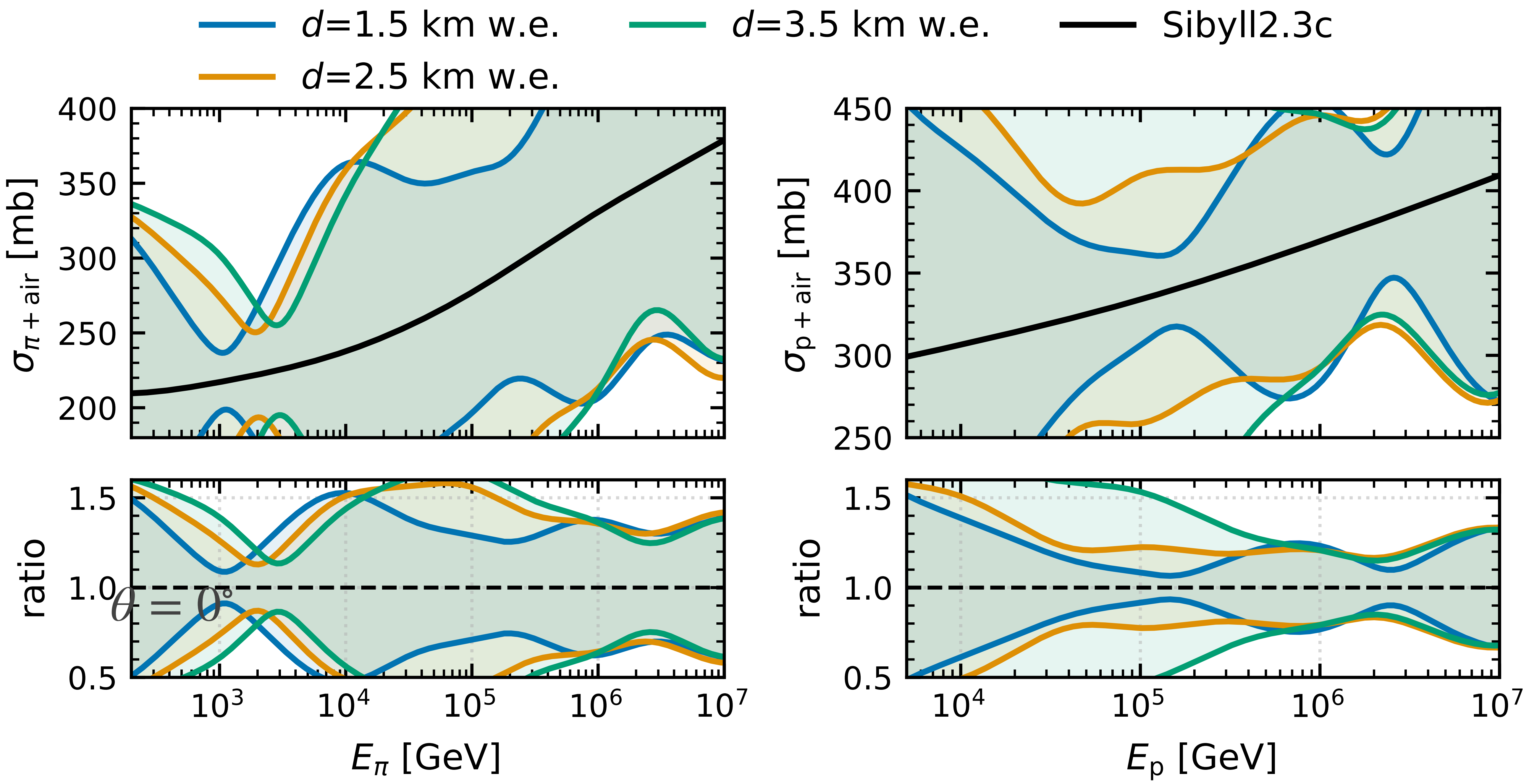
- Take cosmic ray energy spectrum as known observation from IceTop
- Simple parameterization as indicator of hadronic model
- $\langle N_\mu \rangle$ can be replaced in likelihood

Can we measure the pion-air cross section?

… Cannot be fully answered yet

Take-home message:

- Underground experiments have unique capability to study hadronic cross sections in air showers in collider inaccessible regime
- **Muon bundle flux** as new **proxy for pion-air & p-air interactions**
- Hadronic model degeneracy in the muon bundle flux —> model-dependent measurement
- **Future Strategy**: implement cosmic ray energy spectrum constraint from IceTop



ICRC proceeding