What can we learn from charm?

Carolina Bolognani
Dortmund, July 28 2025

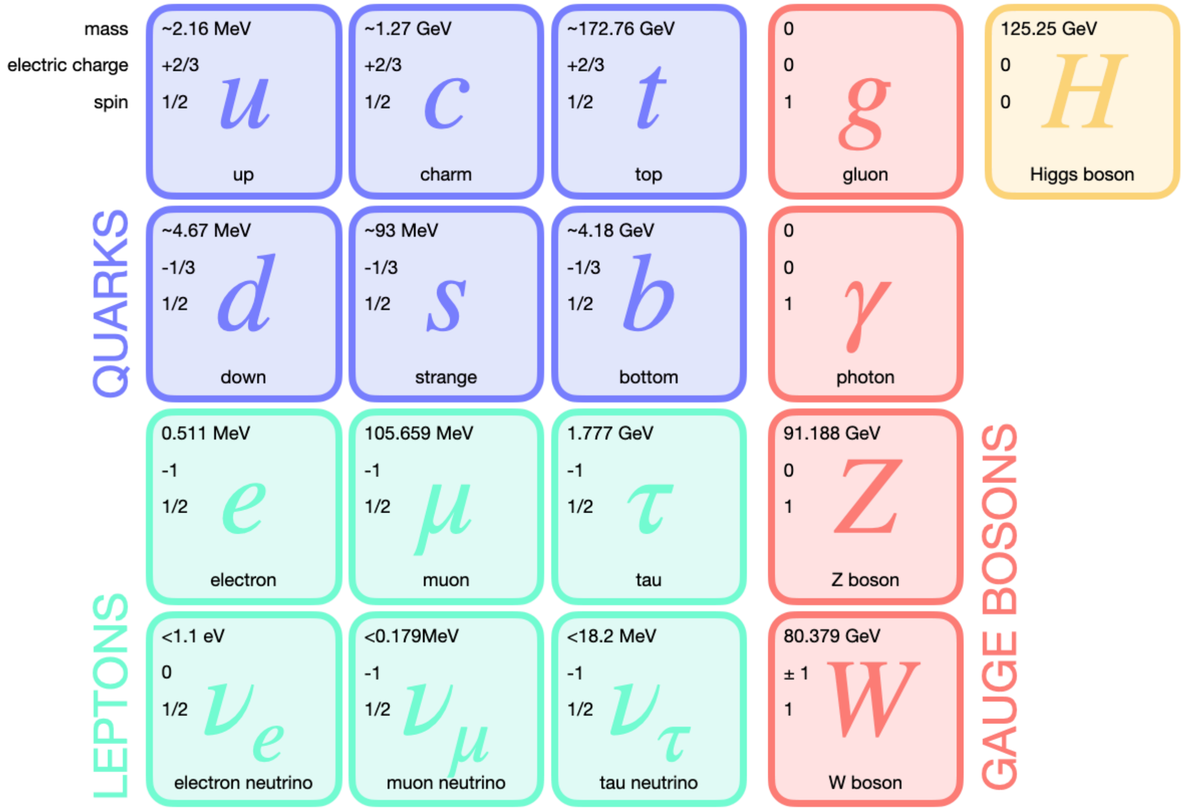
Outline

- Charged weak current charm decays

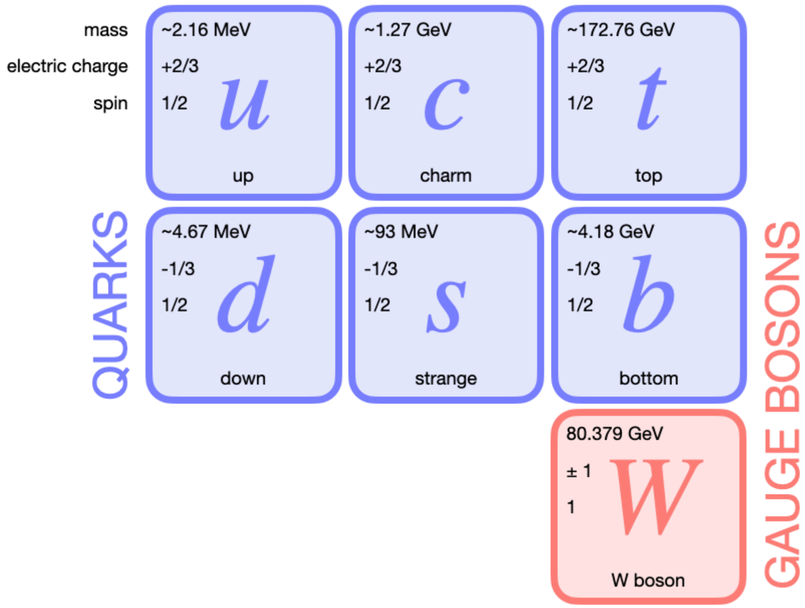
- What can we learn from (semi)leptonic charm decays?
  - ➡ Determination of $|V_{cs}|$ and test of CKM unitarity CB, M. Reboud, D. Van Dyk, K. Vos, *JHEP* 09 (2024) 099

- What can we learn from hadronic charm decays?
  - ➡ Heavy flavour decays under $\mathrm{SU}(3)_\mathrm{F}$ symmetry CB, U. Nierste, S. Schacht, K. Vos, *JHEP* 05 (2025) 148

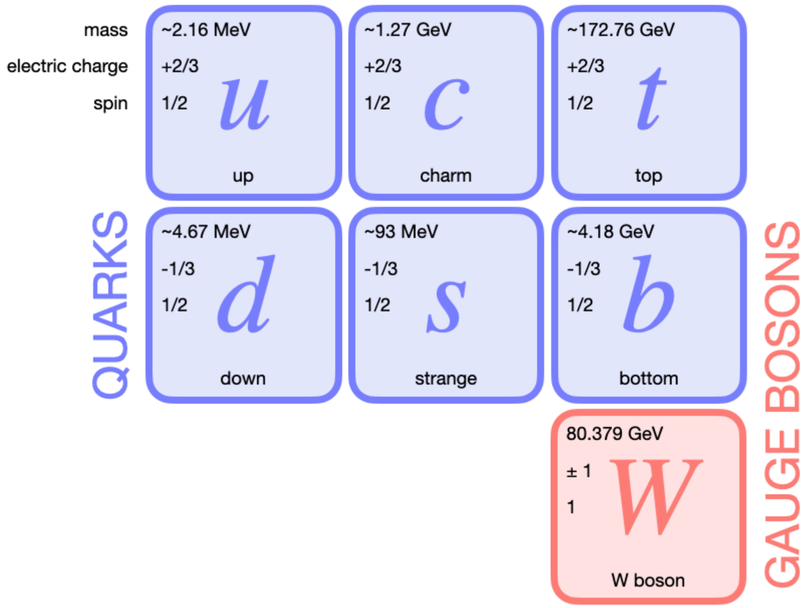
Flavour physics



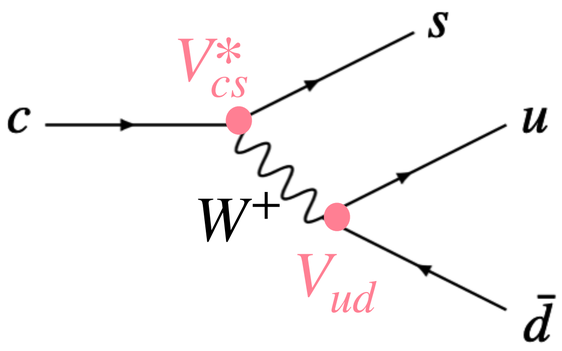
Flavour physics



Flavour physics



- Charged weak interactions induce flavour changes in the Standard Model
- Quark transition probabilities related to Cabibbo-Kobayashi-Maskawa matrix



$$V_{CKM} \equiv \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix}$$

The CKM matrix

- Unitary matrix
- Wolfenstein parametrisation → expansion of $\sin\theta_C \approx V_{us}$

real parameters λ ($= \sin\theta_C \approx 0.22$), A, ρ, η determined from data

$$V_{CKM} = \begin{pmatrix} 1 - \frac{1}{2}\lambda^2 - \frac{1}{8}\lambda^4 & \lambda & A\lambda^3(\rho - i\eta) \\ -\lambda + \frac{1}{2}A^2\lambda^5[1 - 2(\rho + i\eta)] & 1 - \frac{1}{2}\lambda^2 - \frac{1}{8}\lambda^4(1 + 4A^2) & A\lambda^2 \\ A\lambda^3(1 - \bar{\rho} - i\bar{\eta}) & -A\lambda^2 + \frac{1}{2}A\lambda^4[1 - 2(\rho + i\eta)] & 1 - \frac{1}{2}A^2\lambda^4 \end{pmatrix} + \mathcal{O}(\lambda^6)$$

CP violating phases at first order

$V_{td} = |V_{td}|e^{-i\beta}$

$V_{ub} = |V_{ub}|e^{-i\gamma}$

Almost diagonal

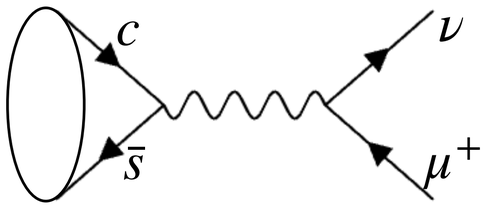
Charm transitions

Cabibbo sub-matrix

Suppressed CP violating phase

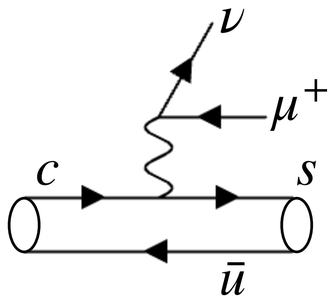
Types of decays
Charged currents

Leptonic



$D_s^+ \to \mu^+\nu$

Semileptonic

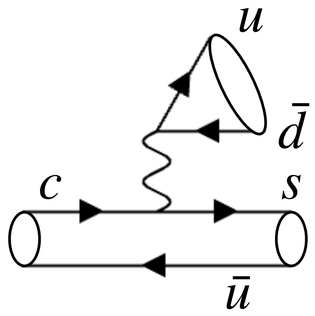


$D^0 \to K^-\mu^+\nu$

Hadronic part factorises from leptonic part

Only one CKM element involved

Hadronic

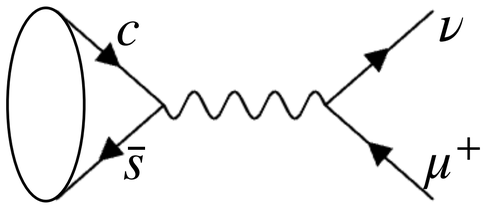


$D^0 \to K^-\pi^+$

Harder to calculate hadronisation effects

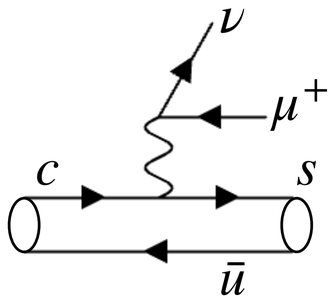
Types of decays
Charged currents

Leptonic



$D_s^+ \to \mu^+\nu$

Semileptonic

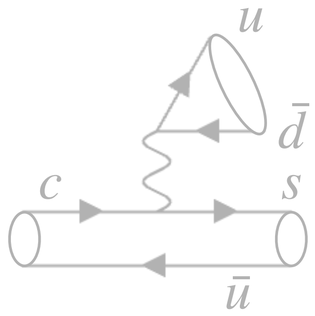


$D^0 \to K^-\mu^+\nu$

Hadronic part factorises from leptonic part

Only one CKM element involved

Hadronic

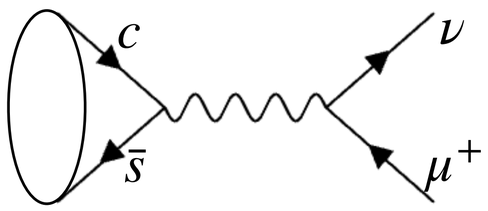


$D^0 \to K^-\pi^+$

Harder to calculate hadronisation effects

(Semi)leptonic decays

Leptonic



$D_s^+ \to \mu^+\nu$

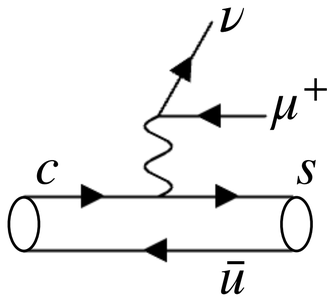
$\langle 0|\,\bar{s}\gamma^\mu\gamma_5 c\,|D_s^+(p)\rangle = i f_{D_s} p^\mu$

Decay constant

All hadronic information factorises from leptonic: hadronic matrix elements

$\mathscr{B} \propto |V_{cs}|^2 |\,\mathrm{Had.\,mat.\,el.}\,|^2$

Semileptonic



$D^0 \to K^-\mu^+\nu$

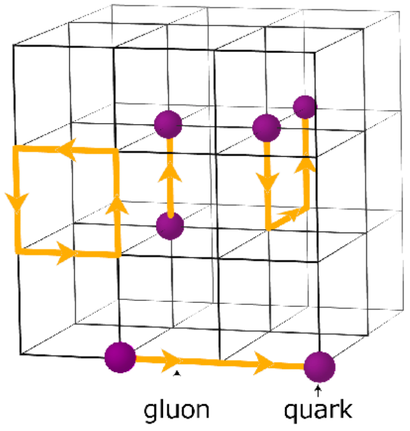
$$\langle K(k)|\,\bar{s}\gamma^\mu c\,|D(p)\rangle = f_+^{D\to K}(q^2)\left[(p+k)^\mu - q^\mu\frac{M_D^2 - M_K^2}{q^2}\right] + f_0^{D\to K}(q^2)q^\mu\frac{M_D^2 - M_K^2}{q^2}$$

Form factors

(Semi)leptonic decays

$c \to s\ell\nu$

- Mostly these hadronic matrix elements are calculated using lattice QCD



Theory calculations

f_{D_s}

$f_{D_s^*}$

FF($\Lambda_c^+ \to \Lambda^0$)

FF($D \to K$)

+

Experimental $\mathscr{B}$

$D_s \to e^+\nu_e$

$D^0 \to K^-\mu^+\nu_\mu$

$D_s \to \mu^+\nu_\mu$

$D_s^* \to e^+\nu_e$

$D^+ \to K_S^0 e^+\nu_e$

$\Lambda_c^+ \to \Lambda^0 e^+\nu_e$

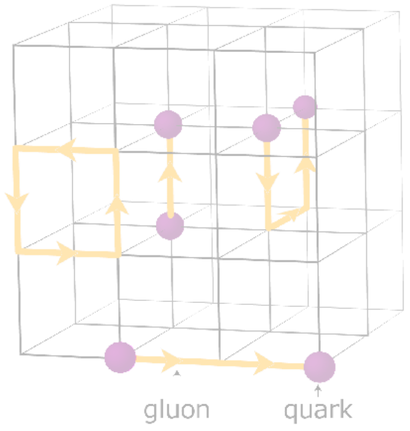
$D^0 \to K^- e^+\nu_e$

$\Lambda_c^+ \to \Lambda^0\mu^+\nu_\mu$

$= |V_{cs}|$

(Semi)leptonic decays

$c \to s\ell\nu$



- Mostly these hadronic matrix elements are calculated using lattice QCD

Correlated! Follow same $c \to s$ currents

Theory calculations

f_{D_s}

$f_{D_s^*}$

$\mathrm{FF}(\Lambda_c^+ \to \Lambda^0)$

$\mathrm{FF}(D \to K)$

$+$

Experimental $\mathscr{B}$

$D_s \to e^+\nu_e$

$D^0 \to K^-\mu^+\nu_\mu$

$D_s \to \mu^+\nu_\mu$

$D_s^* \to e^+\nu_e$

$D^+ \to K_S^0 e^+\nu_e$

$\Lambda_c^+ \to \Lambda^0 e^+\nu_e$

$D^0 \to K^- e^+\nu_e$

$\Lambda_c^+ \to \Lambda^0\mu^+\nu_\mu$

$= |V_{cs}|$

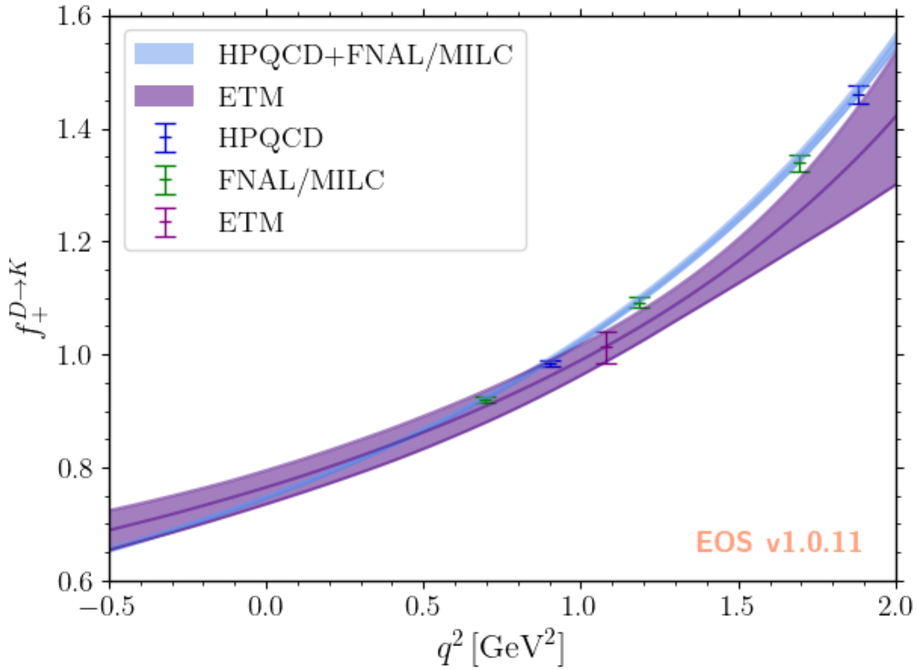
HPQCD, Phys. Rev. D 107 (2023) 014510
FNAL/MILC, Phys. Rev. D 107 (2023) 094516
ETM, Phys. Rev. D 96 (2017) 054514
ETM, Phys. Rev. D 98 (2018) 014516
BESIII, Phys. Rev. D 96 (2017) 012002

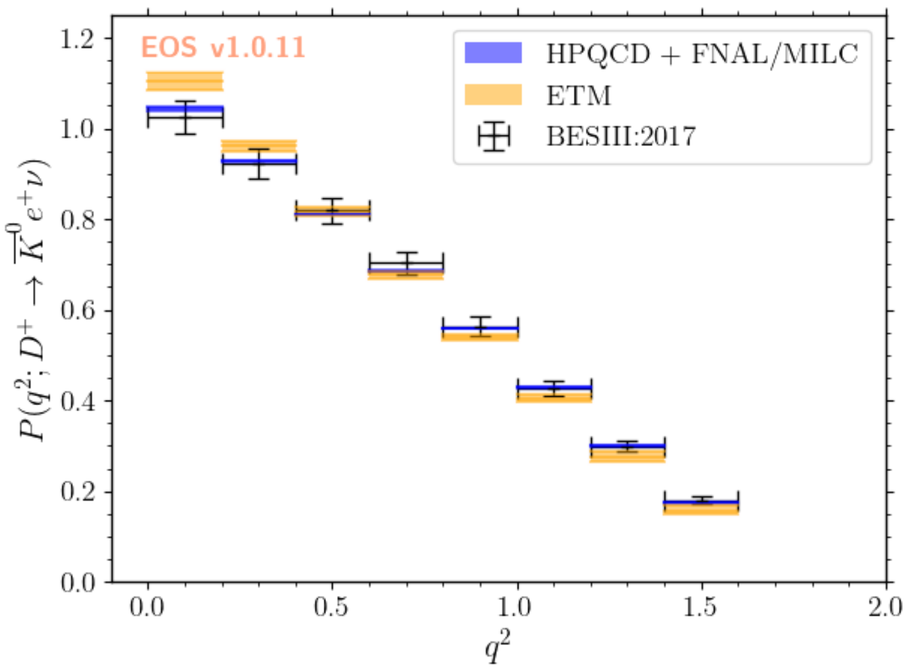
CB, M. Reboud, D. Van Dyk, K. Vos, *JHEP* 09 (2024) 099

Theory calculations

Incompatibilities

- Not all determinations from different Lattice collaborations are compatible
- Some determinations are more compatible with data than others





Nominal results ⇒ only HPQCD+FNAL/MILC

HPQCD + FNAL/MILC: p-value = 4%

HPQCD + FNAL/MILC + ETM: p-value < 0.1%

The current $|V_{cs}|$ by PDG

- Combines averages of some branching ratios with specific values of the hadronic matrix elements

$$\mathscr{B}(D_s^+ \to \mu^+\nu) + \mathscr{B}(D_s^+ \to \tau^+\nu) + f_{D_s} \quad + \quad \mathscr{B}(D \to K\ell^+\nu) + f_+^{D\to K}(0) \text{ based on ETM and an old HPQCD result} \quad = 0.975 \pm 0.006$$

$$1 + \frac{\alpha_e}{\pi} \ln\left(\frac{M_Z}{\mu}\right) \simeq 1.01$$

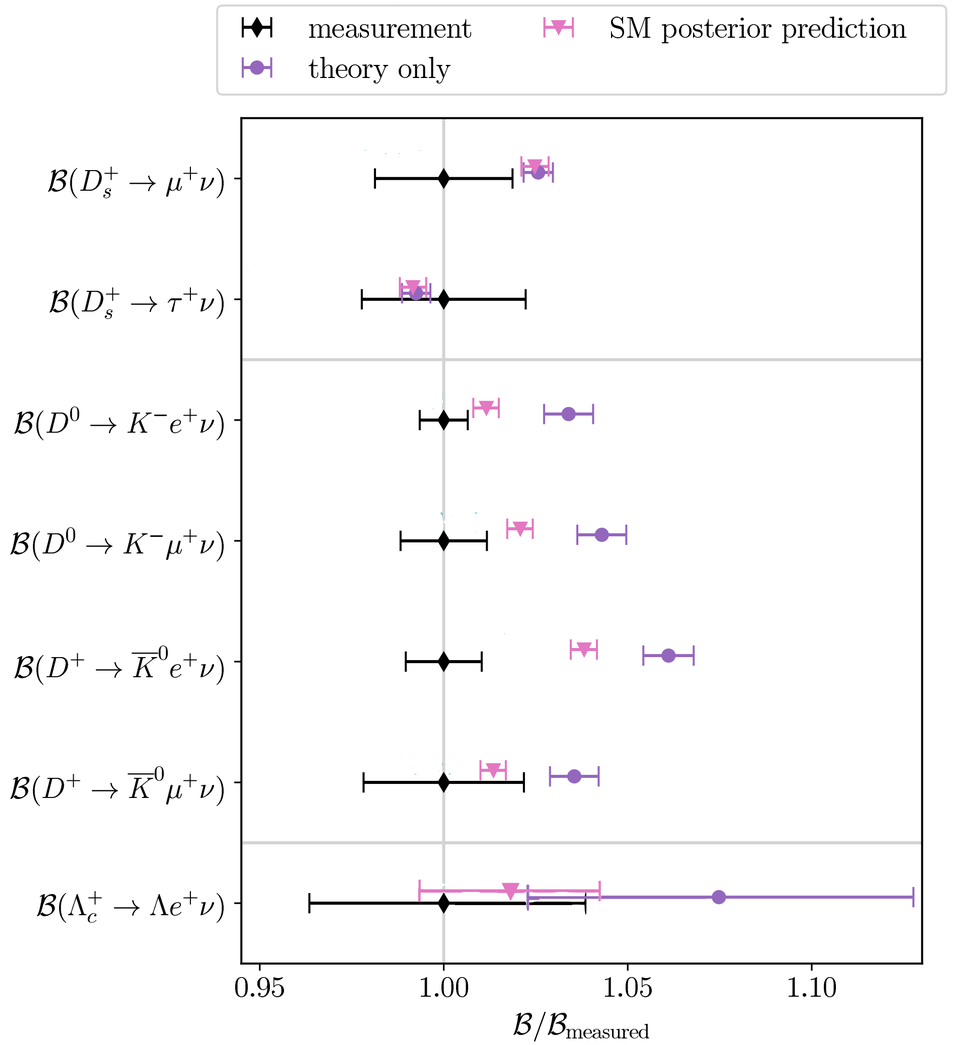
- Does not account for universal electroweak correction (Sirlin factor): 1% effect

Is the current $|V_{cs}|$ compatible with data?

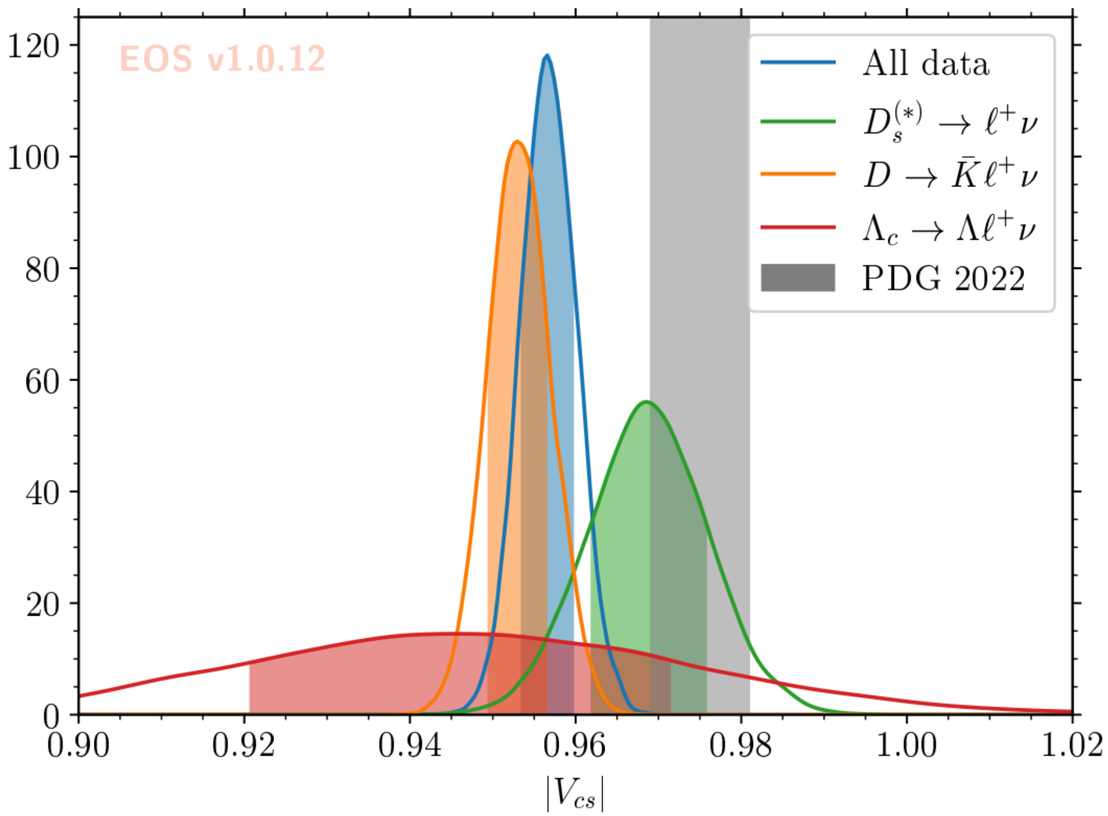
Compatibility with data for "nominal" analysis

- Theory only: calculating the predictions for the branching ratios using the central values of the theoretical determinations

- SM fit: fit to the experimental data allowing the theory determinations to vary within their uncertainties



The value of $|V_{cs}|$



$$|V_{cs}| = 0.957 \pm 0.003$$

2.7σ away from PDG

- All determinations consistently below PDG average: Sirlin + updated form factors
- CKM unitarity

	PDG	nominal
$\lvert V_{cs}\rvert$	0.975 ± 0.006	0.957 ± 0.003
2nd row	1.00 ± 0.014 $(0.08\,\sigma)$	0.966 ± 0.008 $(4.3\,\sigma)$
2nd column	1.00 ± 0.012 $(0.22\,\sigma)$	0.968 ± 0.006 $(5.2\,\sigma)$

Issues in the normalisation of LQCD?
Issue in experimental determinations?
New physics at play?

New Physics

How can it enter?

- In the SM the operator that describes the interaction is $\mathcal{O}^{\ell}_{V,L} = [\bar{s}\gamma^{\mu}P_L c]\,[\bar{\nu}\gamma_{\mu}P_L \ell]$
- Through New Physics, other kinds of interactions can contribute

$$\mathcal{O}^{\ell}_{V,R} = [\bar{s}\gamma^{\mu}P_R c]\,[\bar{\nu}\gamma_{\mu}P_L \ell] \qquad \mathcal{O}^{\ell}_{S,L} = [\bar{s}P_L c]\,[\bar{\nu}P_L \ell]$$

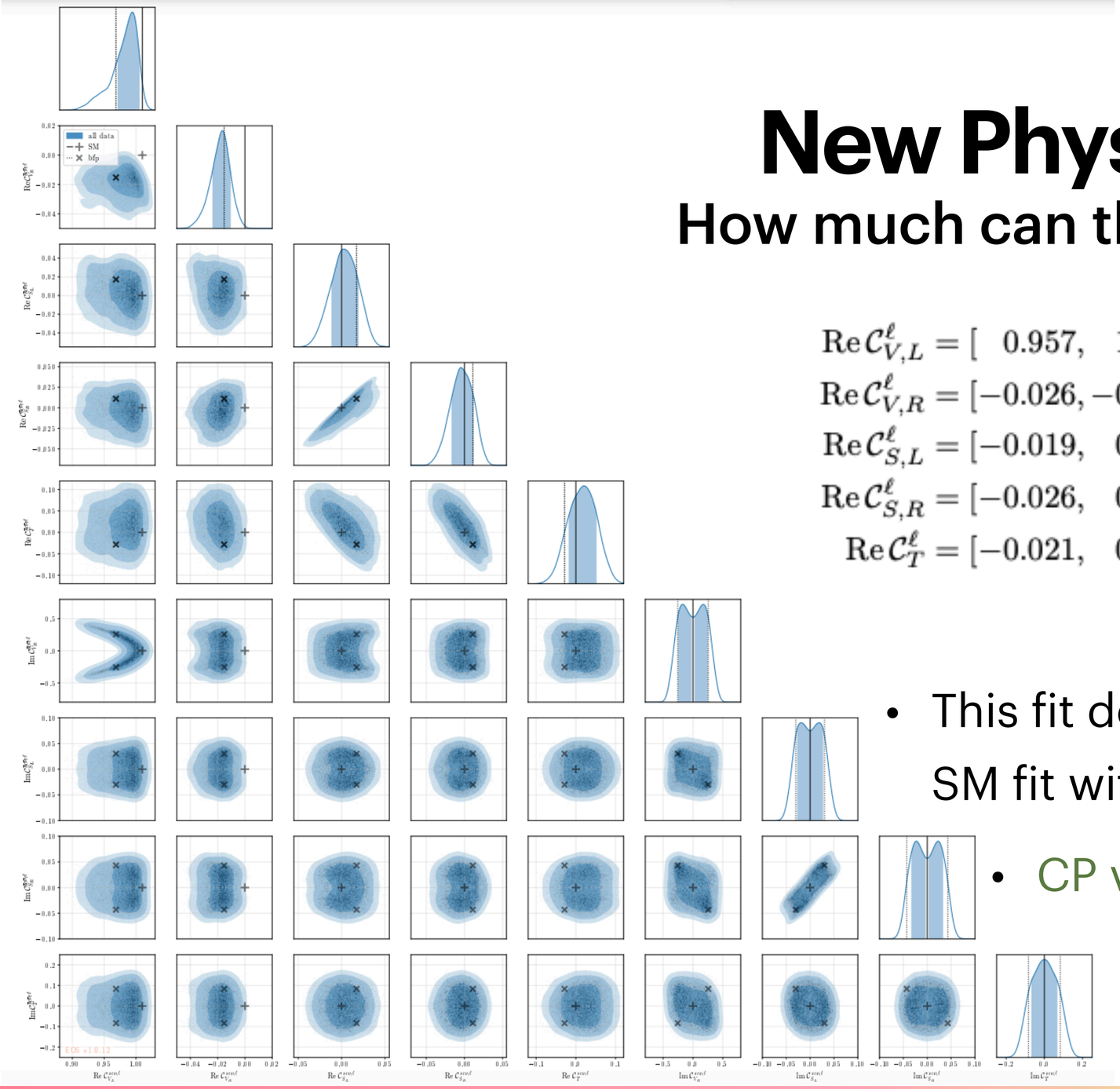
$$\mathcal{O}^{\ell}_{T} = [\bar{s}\sigma^{\mu\nu}b]\,[\bar{\nu}\sigma_{\mu\nu}P_L \ell] \qquad \mathcal{O}^{\ell}_{S,R} = [\bar{s}P_R c]\,[\bar{\nu}P_L \ell]$$

- Test NP contribution ⇒ determine the phase space for Wilson Coefficients

$$\mathcal{H}^{sc\nu\ell} = -\frac{4G_{\mathrm{F}}}{\sqrt{2}}\tilde{V}^{*}_{cs}\sum_i \mathcal{C}^{\ell}_i(\mu_c)\mathcal{O}^{\ell}_i + \mathrm{h.c.}$$

New Physics

How much can there be?



$$\mathrm{Re}\,\mathcal{C}^{\ell}_{V,L} = [\ \ 0.957,\ \ 1.002]\,,$$
$$\mathrm{Re}\,\mathcal{C}^{\ell}_{V,R} = [-0.026, -0.012]\,, \quad \mathrm{Im}\,\mathcal{C}^{\ell}_{V,R} = [-0.225, 0.225]\,,$$
$$\mathrm{Re}\,\mathcal{C}^{\ell}_{S,L} = [-0.019,\ \ 0.014]\,, \quad \mathrm{Im}\,\mathcal{C}^{\ell}_{S,L} = [-0.030, 0.030]\,,$$
$$\mathrm{Re}\,\mathcal{C}^{\ell}_{S,R} = [-0.026,\ \ 0.006]\,, \quad \mathrm{Im}\,\mathcal{C}^{\ell}_{S,R} = [-0.028, 0.028]\,,$$
$$\mathrm{Re}\,\mathcal{C}^{\ell}_{T} = [-0.021,\ \ 0.046]\,, \quad \mathrm{Im}\,\mathcal{C}^{\ell}_{T} = [-0.068, 0.068]\,.$$

- This fit does not provide an improvement over the SM fit with our obtained value of $|V_{cs}|$
- CP violating effects in right-handed currents?

What did we learn?

(Semi)leptonic charm decays

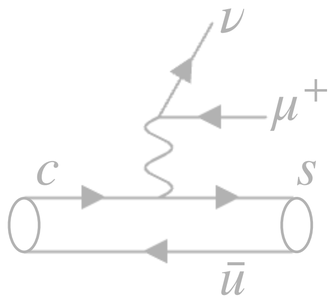
- We found a smaller $|V_{cs}|$: global look at the available data, correlated hadronic matrix elements, electroweak corrections
- CKM unitarity needs to be further investigated
  - ➡ Global analysis with charm, kaon, mixing effects
- Discrepancies in lattice calculations require further investigation
- New data may help resolve preference for Standard Model x New Physics
  - ➡ Differential decay rates!

Types of decays
Charged currents

Leptonic

$D_s^+ \to \mu^+\nu$

Semileptonic

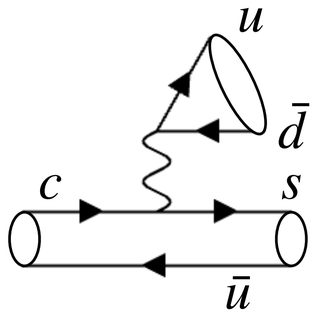


$D^0 \to K^-\mu^+\nu$

Hadronic part factorises from leptonic part

Only one CKM element involved

Hadronic

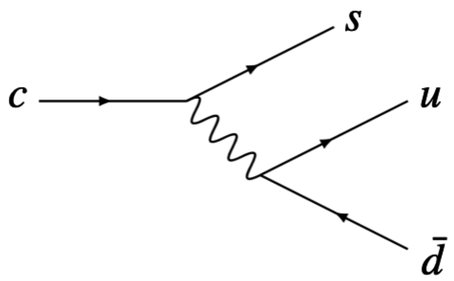


$D^0 \to K^-\pi^+$

Harder to calculate hadronisation effects

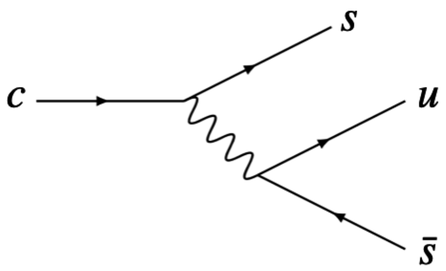
Hadronic charm decays

Tree-level



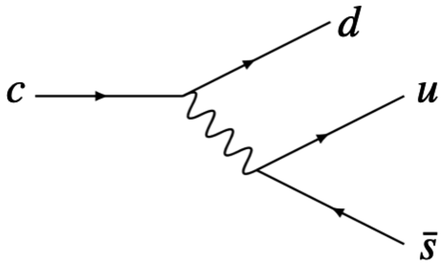
$V_{cs}^* V_{ud} \sim \mathcal{O}(1)$

Cabibbo-favoured



$V_{cs}^* V_{us} \sim V_{cd}^* V_{ud} \sim \mathcal{O}(\lambda)$

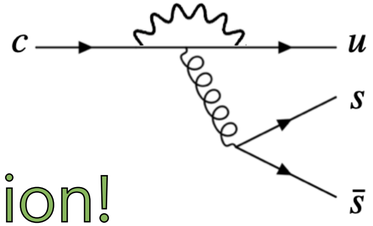
singly Cabibbo-suppressed



$V_{cd}^* V_{us} \sim \mathcal{O}(\lambda^2)$

doubly-Cabibbo-suppressed

Additional penguin topology due to $q\bar{q}$ pair

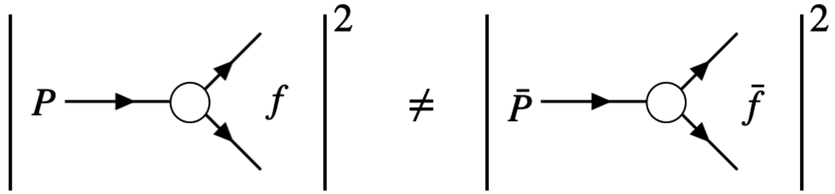


CP violation!

Hadronic charm decays

Direct CP violation

- CP violation happens due to interference between multiple processes



$$\mathscr{A}(P \to f) = |M_1| e^{i\delta_1} e^{i\phi_1} + |M_2| e^{i\delta_2} e^{i\phi_2}$$

Strong phases → CP-even
Weak phases → CP-odd

$$\bar{\mathscr{A}}(\bar{P} \to \bar{f}) = |M_1| e^{i\delta_1} e^{-i\phi_1} + |M_2| e^{i\delta_2} e^{-i\phi_2}$$

$$A_{CP} = \frac{|\mathscr{A}(f)|^2 - |\bar{\mathscr{A}}(\bar{f})|^2}{|\mathscr{A}(f)|^2 + |\bar{\mathscr{A}}(\bar{f})|^2} = \frac{2|M_1||M_2|\sin(\delta_1 - \delta_2)\sin(\phi_1 - \phi_2)}{|M_1|^2 + |M_2|^2 + 2|M_1||M_2|\cos(\delta_1 - \delta_2)\cos(\phi_1 - \phi_2)}$$

Hadronic charm decays

SCS decays and direct CP violation

- The CKM elements that enter SCS decays are of the form $\lambda_q = V_{cq}^* V_{uq}$

$$\lambda_d = -0.21874 + 2.51 \times 10^{-5} i\,, \qquad \lambda_s = 0.21890 + 0.13 \times 10^{-5} i\,,$$

$$\lambda_b = 6.3 \times 10^{-5} - 1.4 \times 10^{-4} i\,.$$

- Leading amplitude $\boxed{\lambda_s = -\lambda_d \in \mathbb{R}}$

- Largest complex phase ⇒ suppressed in magnitude ⇒ small CPV

HFLAV, Phys. Rev. D 107 (2023) 052008

LHCb Collaboration, Phys. Rev. Lett. 122 (2019) 211803

One observation so far of direct CPV: $A_{CP}(D^0 \to K^- K^+) - A_{CP}(D^0 \to \pi^- \pi^+) = (-16.1 \pm 2.8) \times 10^{-4}$

Idea: understand the leading amplitude and then look at CPV!

Hadronic charm decays

Why is charm so hard?

- b-hadron decays $\Rightarrow$ HQET, QCDF often used

- These techniques rely on expansions but parameters are bigger in charm

$$\frac{\Lambda_{QCD}}{m_c} \approx 0.3 \qquad \frac{\Lambda_{QCD}}{m_b} \approx 0.1$$

$$\alpha_s(m_c) \approx 0.35 \qquad \alpha_s(m_b) \approx 0.22$$

First principle calculations are very challenging but even without them we can still study these decays and make predictions!

Using the $\mathrm{SU}(3)_\mathrm{F}$ symmetry

Defining the pseudoscalars of $D \to PP$

- Approximately $m_u = m_d = m_s$
- Combining three quarks as $q\bar{q}$ pairs : $3 \otimes \bar{3} = 8 \oplus 1$

$$K^0 = d\bar{s} \qquad K^+ = u\bar{s}$$

$$\pi^0 = \frac{u\bar{u} - d\bar{d}}{\sqrt{2}}$$

$$\pi^- = \bar{u}d \qquad \eta_8 = \frac{u\bar{u} + d\bar{d} - 2s\bar{s}}{\sqrt{6}} \qquad \pi^+ = u\bar{d}$$

$$\eta_0 = \frac{u\bar{u} + d\bar{d} + s\bar{s}}{\sqrt{3}}$$

$$K^- = \bar{u}s \qquad \bar{K}^0 = \bar{d}s$$

Separating decays under $\mathrm{SU}(3)_\mathrm{F}$

Treatment of the singlet

- Similar quantum numbers and $\mathrm{SU}(3)_\mathrm{F}$-breaking $\Rightarrow \eta_8 - \eta_0$ mixing into $\eta - \eta'$

$$
\begin{aligned}
|\eta_8> &= \quad |\eta> \cos\theta + |\eta'> \sin\theta \\
|\eta_0> &= -\,|\eta> \sin\theta + |\eta'> \cos\theta
\end{aligned}
$$

Different!

$$
\begin{aligned}
<P\eta|H|D> &= \cos\theta \boxed{<P\eta_8|H|D>} - \sin\theta \boxed{<P\eta_0|H|D>} \\
<P\eta'|H|D> &= \sin\theta \boxed{<P\eta_8|H|D>'} + \cos\theta \boxed{<P\eta_0|H|D>'}
\end{aligned}
$$

Separating decays under $\mathrm{SU}(3)_\mathrm{F}$

Treatment of the singlet

- Similar quantum numbers and $\mathrm{SU}(3)_\mathrm{F}$-breaking $\Rightarrow \eta_8 - \eta_0$ mixing into $\eta - \eta'$

$$|\eta_8> \;=\; |\eta> \cos\theta + |\eta'> \sin\theta$$
$$|\eta_0> \;= -|\eta> \sin\theta + |\eta'> \cos\theta$$

$$\theta = \mathcal{O}(\varepsilon) \Rightarrow \varepsilon \sim 1 - \frac{f_\pi}{f_K} \sim 20 - 30\,\%$$

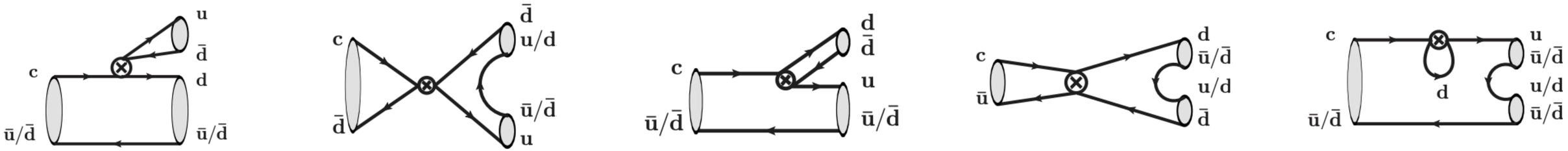
$$|\eta> \;= |\eta_8> + \mathcal{O}(\varepsilon)$$
$$|\eta'> \;= |\eta_0> + \mathcal{O}(\varepsilon)$$

Separate $D \to P_8 P_8$ from $D \to P_8 \eta'$

Topological parametrisation

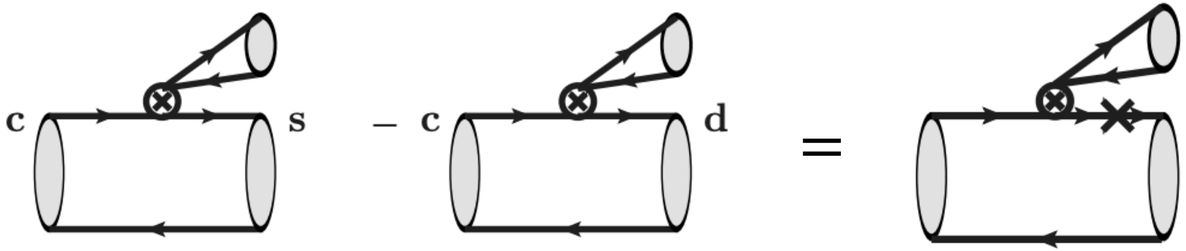
- Write the amplitudes in terms of the contributing diagrams

T A C E P



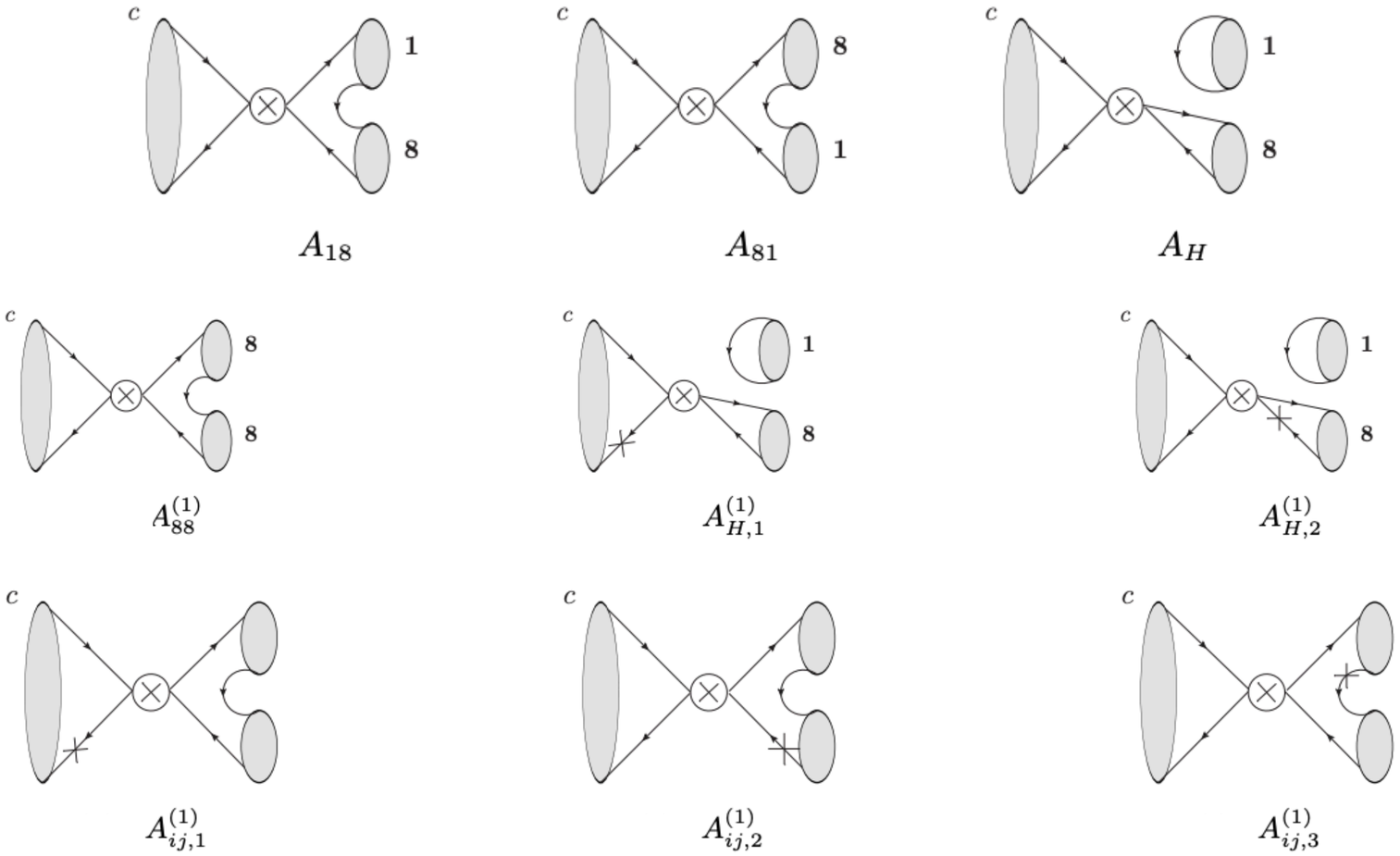
- Account for $\mathrm{SU}(3)_\mathrm{F}$ breaking corrections $(m_s - m_d)s\bar{s}$ to the diagrams

$T_1^{(1)}$



Topological parametrisation

$$D \to P_8 \eta'$$



Topological parametrisation

- Linearly dependent parameters
- Sum rules in the $\mathrm{SU}(3)_\mathrm{F}$ limit and with $\mathrm{SU}(3)_\mathrm{F}$ breaking
- 17 parameters, 8 measurements

Decay ampl. $\mathcal{A}(d)$	$\hat{T}_{18}$	$\hat{T}^{(1)}_{18,1}$	$\hat{T}^{(1)}_{18,2}$	$\hat{T}^{(1)}_{88}$	$\hat{C}^{(1)}_{18,1}$	$\hat{C}_{81}$	$\hat{C}^{(1)}_{81,1}$	$\hat{C}^{(1)}_{81,2}$	$\hat{C}^{(1)}_{88}$
SCS									
$\mathcal{A}(D^0 \to \pi^0\eta')$	0	0	0	0	$\frac{1}{\sqrt{6}}$	$\frac{1}{\sqrt{6}}$	0	0	$-\frac{1}{\sqrt{3}}$
$\mathcal{A}(D^0 \to \eta\eta')$	0	0	0	0	$\frac{1}{3\sqrt{2}}$	$-\frac{1}{\sqrt{2}}$	$-\frac{\sqrt{2}}{3}$	$-\frac{\sqrt{2}}{3}$	-1
$\mathcal{A}(D^+ \to \pi^+\eta')$	$-\frac{1}{\sqrt{3}}$	0	0	$-\frac{1}{\sqrt{6}}$	$\frac{1}{\sqrt{3}}$	0	0	0	$-\sqrt{\frac{3}{2}}$
$\mathcal{A}(D_s^+ \to K^+\eta')$	$\frac{1}{\sqrt{3}}$	$\frac{1}{\sqrt{3}}$	$\frac{1}{\sqrt{3}}$	$-\sqrt{\frac{2}{3}}$	$\frac{1}{\sqrt{3}}$	0	0	0	$-\sqrt{\frac{3}{2}}$
CF									
$\mathcal{A}(D^0 \to \overline{K}^0\eta')$	0	0	0	0	0	$\frac{1}{\sqrt{3}}$	$\frac{1}{\sqrt{3}}$	0	$\frac{1}{\sqrt{6}}$
$\mathcal{A}(D_s^+ \to \pi^+\eta')$	$\frac{1}{\sqrt{3}}$	$\frac{1}{\sqrt{3}}$	0	$-\sqrt{\frac{2}{3}}$	0	0	0	0	0
DCS									
$\mathcal{A}(D^0 \to K^0\eta')$	0	0	0	0	0	$\frac{1}{\sqrt{3}}$	0	$\frac{1}{\sqrt{3}}$	$\frac{1}{\sqrt{6}}$
$\mathcal{A}(D^+ \to K^+\eta')$	$\frac{1}{\sqrt{3}}$	0	$\frac{1}{\sqrt{3}}$	$\frac{1}{\sqrt{6}}$	0	0	0	0	0

$$\mathcal{A}(D^+ \to K^+\eta') = \mathcal{A}(D_s^+ \to \pi^+\eta') = \mathcal{A}(D_s^+ \to K^+\eta') = -\mathcal{A}(D^+ \to \pi^+\eta')$$

$$\mathcal{A}(D^0 \to K^0\eta') = \mathcal{A}(D^0 \to \overline{K}^0\eta') = \sqrt{2}\mathcal{A}(D^0 \to \pi^0\eta') = -\sqrt{\frac{2}{3}}\mathcal{A}(D^0 \to \eta\eta')$$

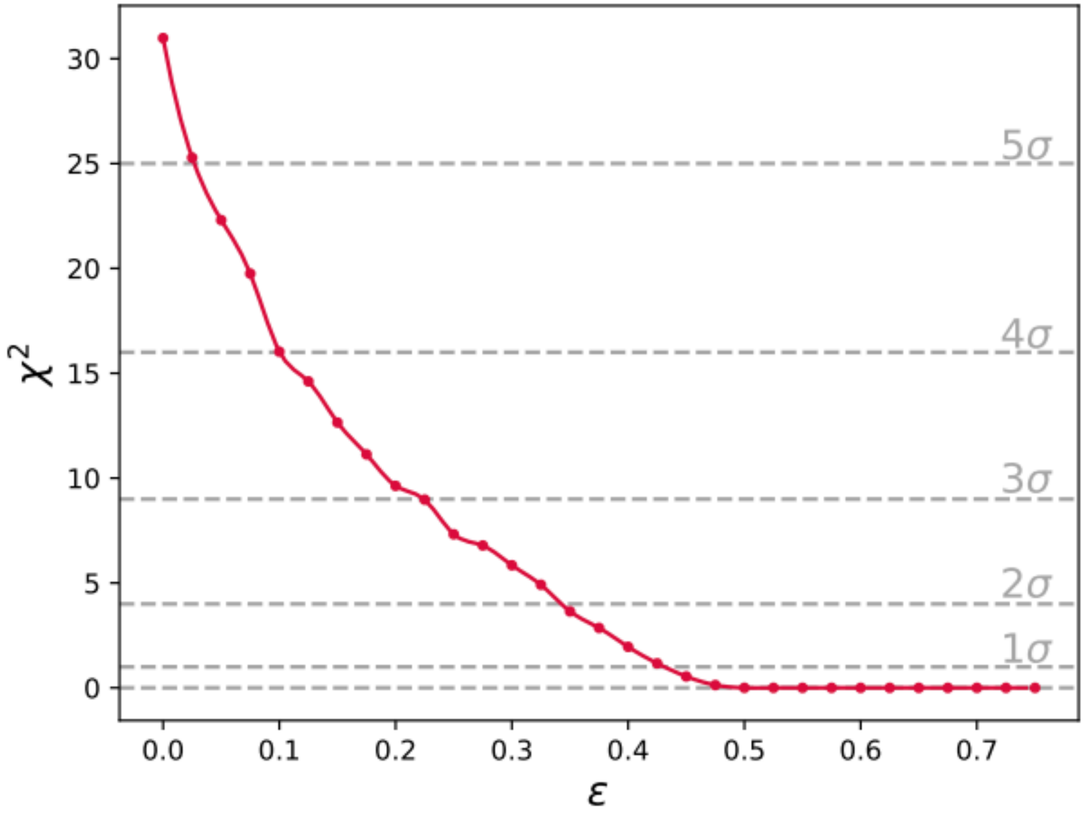
$$\mathcal{A}(D^+ \to K^+\eta') + \mathcal{A}(D_s^+ \to \pi^+\eta') - \mathcal{A}(D_s^+ \to K^+\eta') + \mathcal{A}(D^+ \to \pi^+\eta') = 0$$

$$\mathcal{A}(D^0 \to K^0\eta') + \mathcal{A}(D^0 \to \overline{K}^0\eta') - \frac{\mathcal{A}(D^0 \to \pi^0\eta')}{\sqrt{2}} + \sqrt{\frac{3}{2}}\mathcal{A}(D^0 \to \eta\eta') = 0$$

Does it fit experimental data?

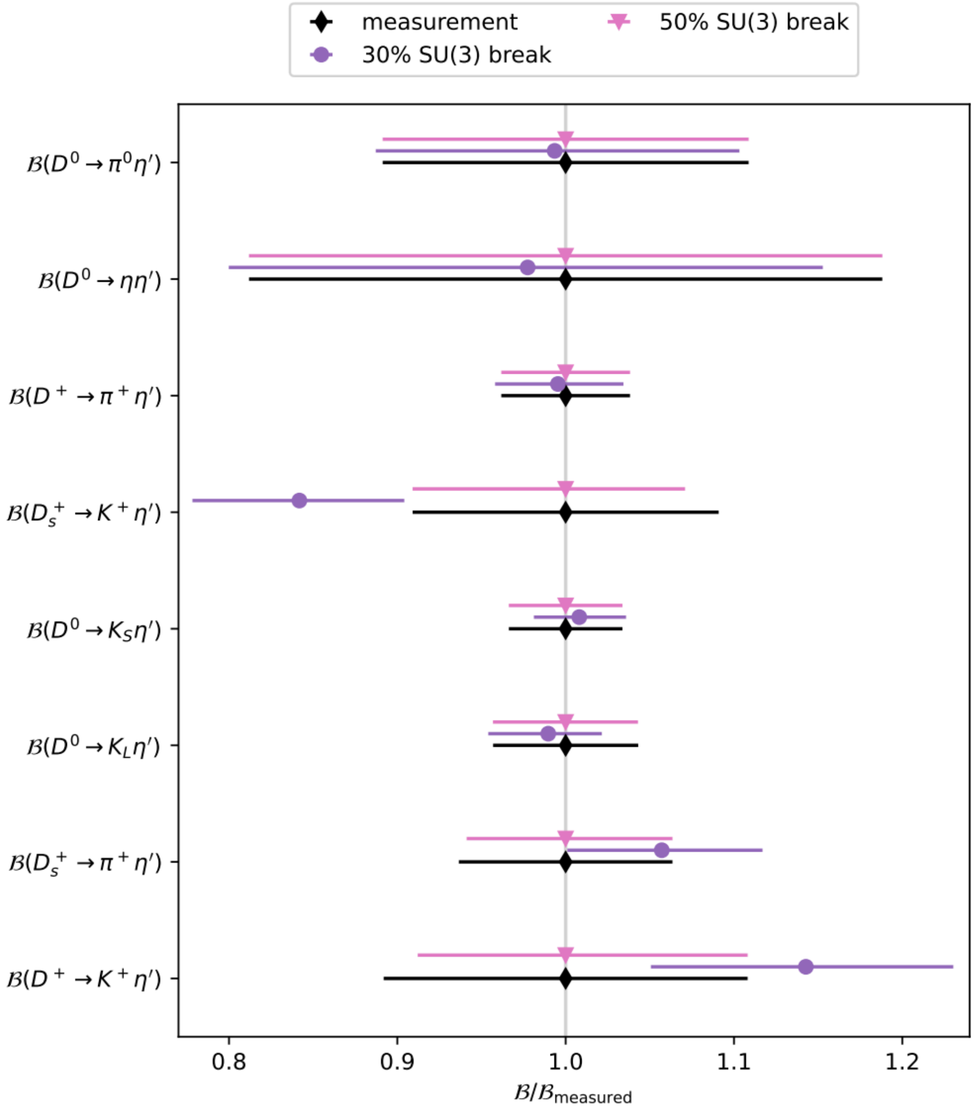
Constrain amount of SU(3) breaking

$$\left| \frac{\mathcal{A}_{\cancel{\mathrm{SU(3)}}}(D \to P\eta')}{\mathcal{A}_{\mathrm{SU(3)-lim}}(D \to P\eta')} \right| \leq \varepsilon$$

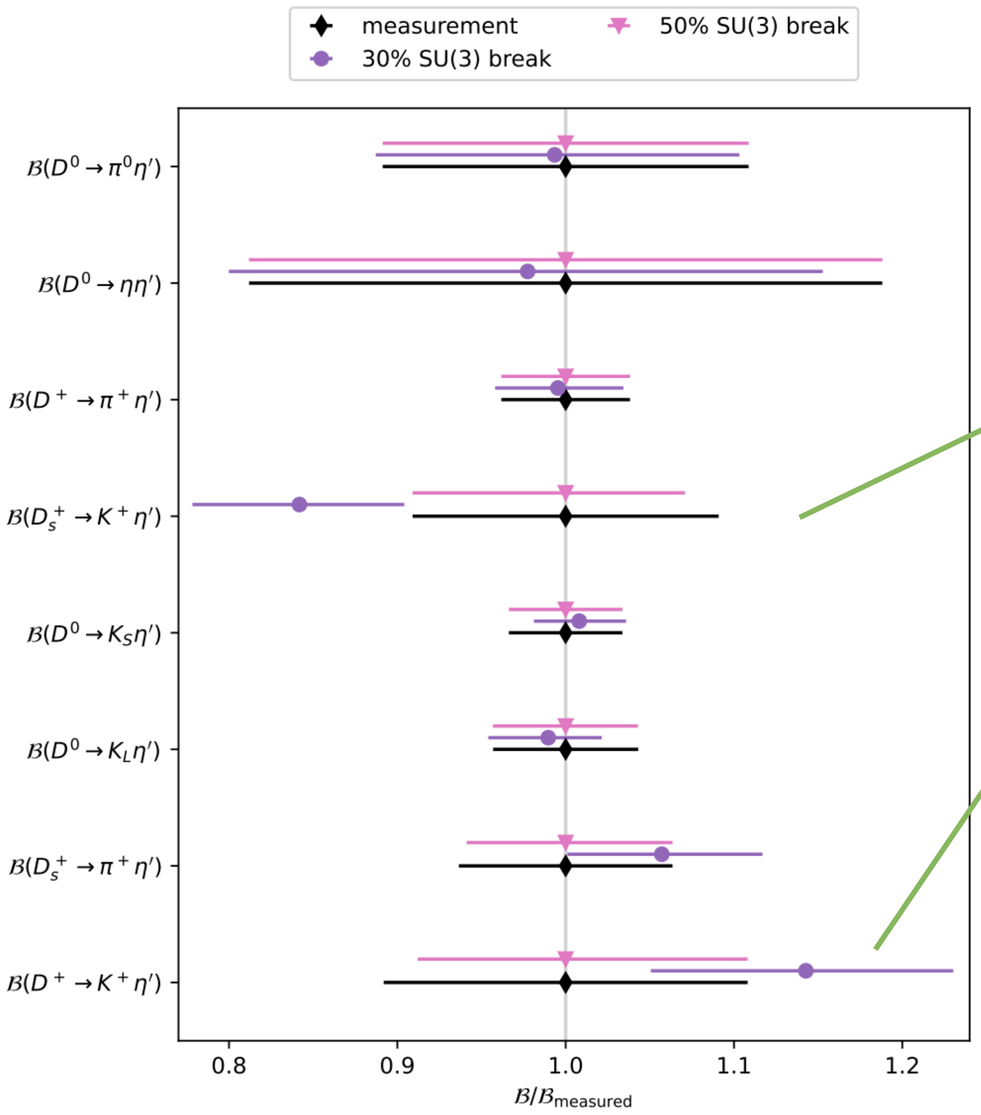


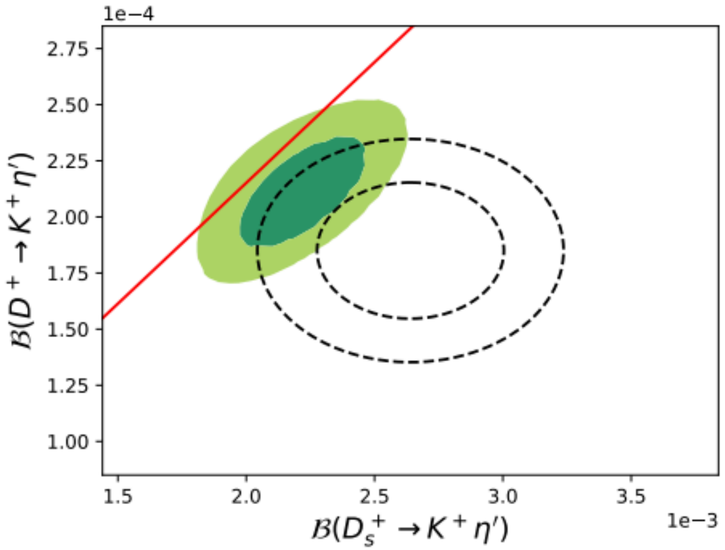
$\mathrm{SU(3)_F}$ limit ruled out!

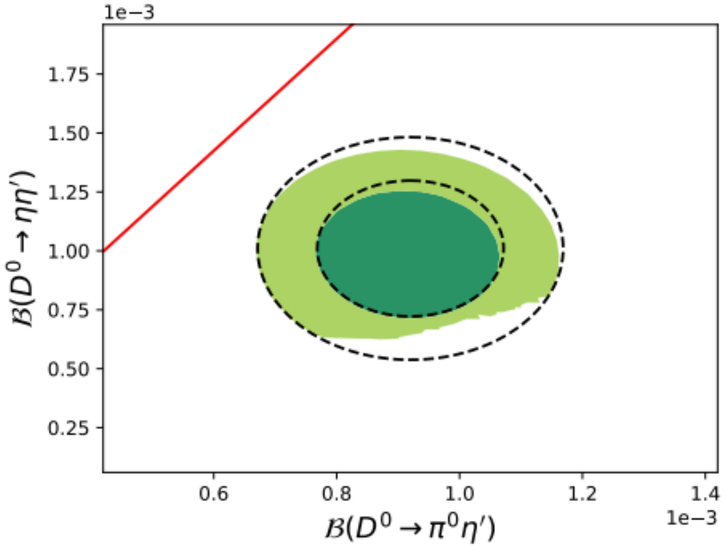
Does it fit experimental data?



Does it fit experimental data?



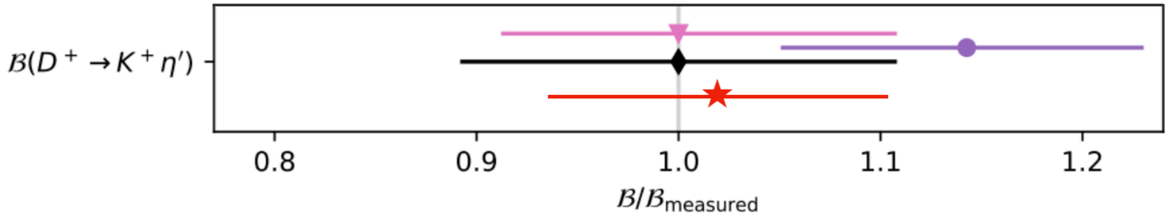




What did we learn?

Hadronic charm decays

- $\mathrm{SU}(3)_\mathrm{F}$ does not accommodate these decays ⇒ due to data on $D^0 \to \pi^0\eta'$ and $D^0 \to \eta\eta'$
- Important to get updated measurements on $D^0 \to \pi^0\eta'$, $D^0 \to \eta\eta'$, $D_s^+ \to K^+\eta'$
- Interesting! Update of $D^+ \to K^+\eta'$ by BESIII in-line with prediction



arXiv:2506.15533

- Next steps:
  - ➡ Investigate CP violation in these decays to predict unmeasured asymmetries
  - ➡ Update analysis of $D \to P_8P_8$ decays in light of new data! Same picture?

Conclusions

What did we learn from charm?

- Charm decays are our way of studying decays of up-type quarks
- Investigating the values of CKM elements may lead to potential New Physics
  - New $|V_{cs}| \Rightarrow$ issue with CKM unitarity? New Physics?
  - Future: investigate $|V_{cd}|$?
- CP violation in charm is still a puzzle! Many models arise but they cannot fully describe the available data
  - $\mathrm{SU}(3)_\mathrm{F}$ applied to topological decomposition $\Rightarrow$ $\mathrm{SU}(3)_\mathrm{F}$ limit excluded!
  - Currently: CPV in $D \to P\eta'$!
  - Future: what about $D \to PP$?

Thank you!

Carolina Bolognani
Dortmund, July 28 2025

Theory calculations

Combining them all

- Unitarity bounds ⇒ correlate most hadronic parameters
- Currents describing decays $\langle 0 | \mathcal{T} J_\mu(x) J_\nu^\dagger(0) | 0 \rangle$
- Perturbative quantities γ ⇒ hadronic representation of correlators

$$\chi_\Gamma^{(i)}(Q^2) = \frac{1}{n!}\left[\frac{\partial}{\partial q^2}\right]^n \Pi_\Gamma^{(i)}(q^2)\Big|_{q^2=Q^2} = \frac{1}{\pi}\int_0^\infty ds\, \frac{\mathrm{Im}\,\Pi_\Gamma^{(i)}(s)}{(s-Q^2)^{n+1}}$$

$$J\mu(x) = \bar{s}(x)\gamma^\mu c(x) \Rightarrow \chi = \frac{M_{D_s^*}^2 f_{D_s^*}^2}{(M_{D_s^*}^2 - Q^2)^3} = \frac{1}{8\pi^2}\int_{(M_D+M_K)^2}^\infty ds \frac{\lambda^{3/2}(M_D^2, M_K^2, s)}{s^2(s-Q^2)^3} |f_+^{D\to K}(s)|^2$$

- Form factor parametrisation normalised and bound

$$f(q^2) = \frac{1}{\phi_f(z) B(z)} \sum_k a_k^f p_k^f(z)\Big|_{z=z(q^2)} \qquad \sum_f \sum_k \left|a_k^f\right|^2 < 1$$

Further discussion on form factor approach: Gubernari, (Reboud), van Dyk, Virto 2021 & 2022; Blake et al. 2022; Flynn et al. 2023

Form factor parametrisation

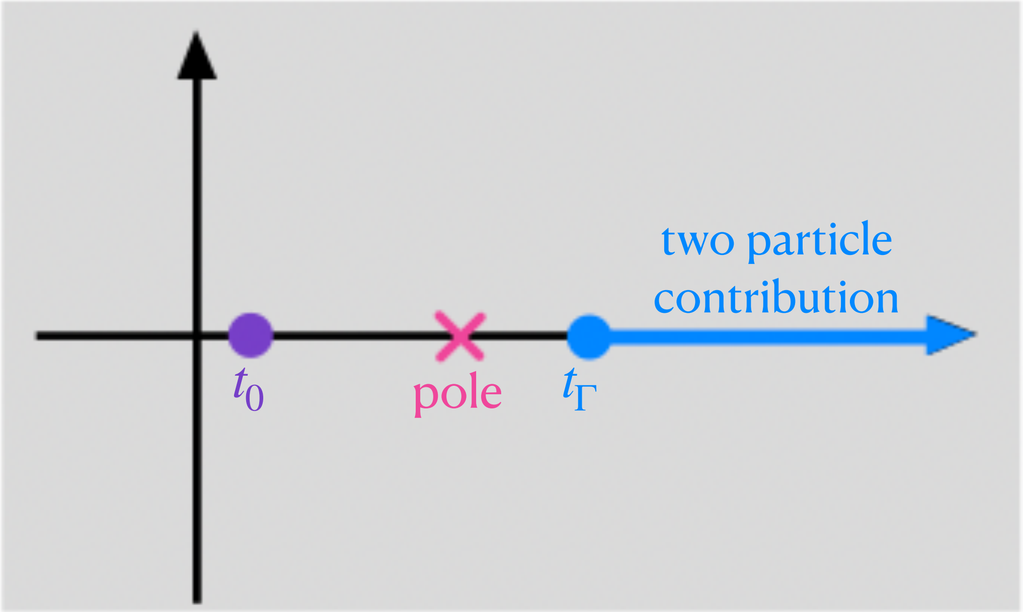
$z\left(q^2 = t_0\right) = 0$

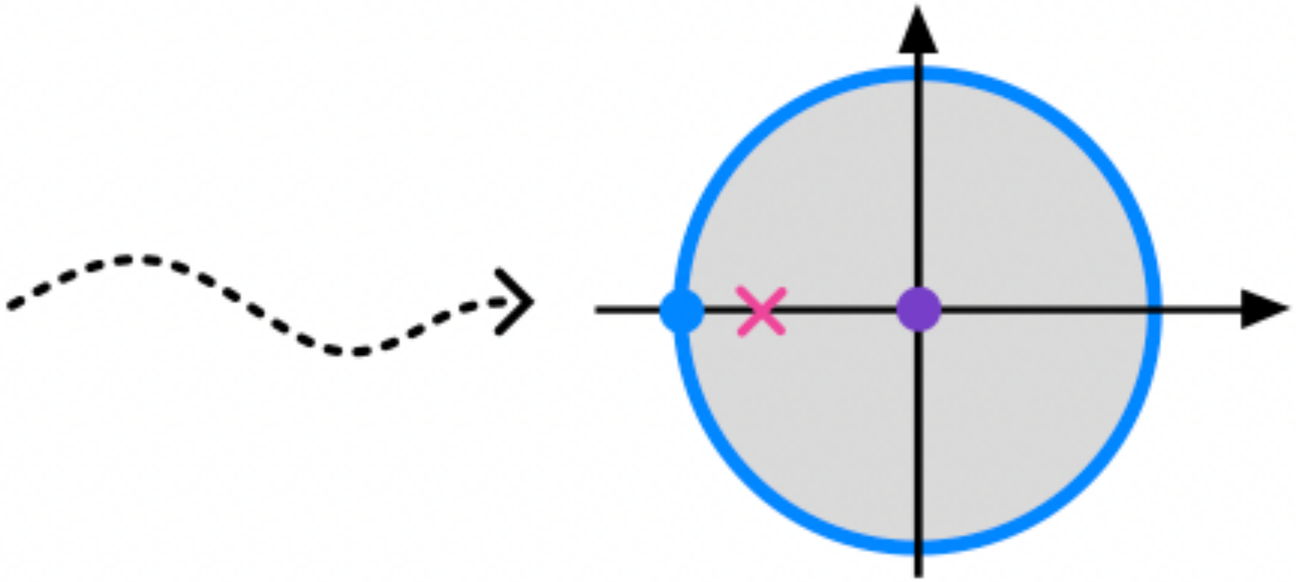
- Analyticity + unitarity

$$q^2 \mapsto z(q^2) = \frac{\sqrt{t_\Gamma - q^2} - \sqrt{t_\Gamma - t_0}}{\sqrt{t_\Gamma - q^2} + \sqrt{t_\Gamma - t_0}}$$

$$f(q^2) = \frac{1}{\phi_f(z)B(z)} \sum_k a_k^f p_k^f(z) \Big|_{z=z(q^2)}$$

$$f_+(q^2 = 0) = f_0(q^2 = 0)$$





$\eta - \eta'$

$$\langle 0|J^8|\eta\rangle = \langle 0|J^8|\eta\rangle_8 \to f_\eta^8, \quad \langle 0|J^8|\eta'\rangle = \langle 0|J^8|\eta'\rangle_8 \to f_{\eta'}^8,$$

$$\langle 0|J^0|\eta\rangle = \langle 0|J^0|\eta\rangle_0 \to f_\eta^0, \quad \langle 0|J^0|\eta'\rangle = \langle 0|J^0|\eta'\rangle_0 \to f_{\eta'}^0.$$

$$< P\eta\,|\,H\,|\,D > \;= \cos\theta < P\eta_8\,|\,H\,|\,D > - \sin\theta < P\eta_0\,|\,H\,|\,D >$$

$$< P\eta\,|\,H\,|\,D >' \;= \sin\theta < P\eta_8\,|\,H\,|\,D >' + \cos\theta < P\eta_0\,|\,H\,|\,D >'$$

Does it fit experimental data?

$$\mathcal{A}^{\rm CF}(d) \equiv V_{cs}^* V_{ud} \mathcal{A}(d) \equiv V_{cs}^* V_{ud} \sum_i c_i^d \mathcal{T}_i,$$

$$\mathcal{A}^{\rm SCS}(d) \equiv \lambda_{sd} \mathcal{A}(d) \equiv \lambda_{sd} \sum_i c_i^d \mathcal{T}_i,$$

$$\mathcal{A}^{\rm DCS}(d) \equiv V_{cd}^* V_{us} \mathcal{A}\ (d) \equiv V_{cd}^* V_{us} \sum_i c_i^d \mathcal{T}_i,$$

$$\lambda_{sd} = (\lambda_s - \lambda_d)/2 = (V_{cs}^* V_{us} - V_{cd}^* V_{ud})/2 \simeq \lambda_s \simeq -\lambda_d.$$

Constraining $\left| \frac{\mathcal{A}_{\cancel{\rm SU(3)}}(D \to P\eta')}{\mathcal{A}_{\rm SU(3)-lim}(D \to P\eta')} \right| \leq \varepsilon$

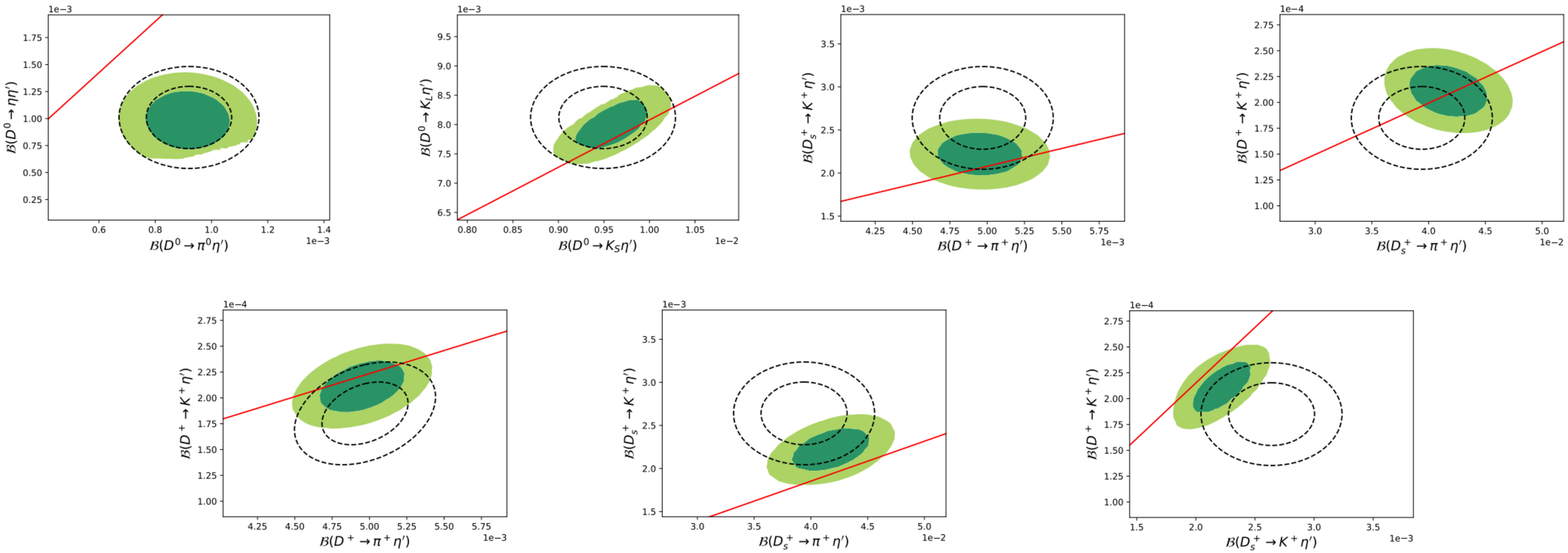
$$\mathcal{B}(D \to P\eta') = |\mathcal{A}^X(D \to P\eta')|^2 \times \mathcal{P}(D \to P\eta')\,,$$

$$\mathcal{P}(D \to P\eta') = \frac{\tau_D}{16\pi m_D^3} \times \sqrt{(m_D^2 - (m_P - m_{\eta'})^2)(m_D^2 - (m_P + m_{\eta'})^2)}$$

Observable	Experiment
$\mathcal{B}(D^0 \to \pi^0\eta')$	$(9.2 \pm 1.0) \cdot 10^{-4}$
$\mathcal{B}(D^0 \to \eta\eta')$	$(1.01 \pm 0.19) \cdot 10^{-3}$
$\mathcal{B}(D^+ \to \pi^+\eta')$	$(4.97 \pm 0.19) \cdot 10^{-3}$
$\mathcal{B}(D_s^+ \to K^+\eta')$	$(2.64 \pm 0.24) \cdot 10^{-3}$
$\mathcal{B}(D^0 \to K_S\eta')$	$(9.49 \pm 0.32) \cdot 10^{-3}$
$\mathcal{B}(D^0 \to K_L\eta')$	$(8.12 \pm 0.35) \cdot 10^{-3}$
$\mathcal{B}(D_s^+ \to \pi^+\eta')$	$(3.94 \pm 0.25) \cdot 10^{-2}$
$\mathcal{B}(D^+ \to K^+\eta')$	$(1.85 \pm 0.20) \cdot 10^{-4}$

Does it fit experimental data?

Interesting correlations



CP violation

Parametrising SCS decays

- For branching ratios, contributions proportional to $\lambda_b = V_{cb}^* V_{ub}$ are suppressed compared to $\lambda_s = V_{cs}^* V_{us} \approx -\lambda_d = -V_{cd}^* V_{ud}$
- For CP violation we need to take them into account by rewriting

$$\mathcal{A}^{SCS} = \lambda_{sd}\mathcal{A}_{sd} - \frac{\lambda_b}{2}\mathcal{A}_b$$

$$A_{CP} = \frac{\mathrm{Im}(\lambda_b)\,\mathrm{Im}(e^{-i\arg\mathcal{A}_{sd}}\mathcal{A}_b)}{\lambda_{sd}|\mathcal{A}_{sd}|}$$

CP violation

Parametrising SCS decays

Decay ampl. $\mathcal{A}_b(d)$	$\mathcal{A}_{sd}$	$\bar{C}_{18}$	$\hat{C}_{81}$	$\hat{C}^{(1)}_{18,1}$	$\bar{C}^{(1)}_{18,3}$	$\hat{C}^{(1)}_{88}$	$\bar{C}^{(1)}_{11}$	$\bar{E}^{(1)}_{88}$	$P^{(1)}_{88}$
$D^0 \to \pi^0\eta'$	1	$\frac{2}{\sqrt{6}}$	$\frac{-2}{\sqrt{6}}$	0	0	0	0	$-\frac{2}{\sqrt{3}}$	$\frac{1}{\sqrt{3}}$
$D^0 \to \eta\eta'$	1	$\frac{2}{\sqrt{18}}$	$\frac{2}{\sqrt{18}}$	0	0	$\frac{2}{3}$	$\frac{4}{3}$	$\frac{4}{3}$	$\frac{1}{3}$
$D^+ \to \pi^+\eta'$	-1	$\frac{2}{\sqrt{3}}$	0	$\frac{2}{\sqrt{3}}$	0	$-\frac{4}{\sqrt{6}}$	0	$-\frac{4}{\sqrt{6}}$	$\frac{1}{\sqrt{6}}$
$D_s^+ \to K^+\eta'$	1	$\frac{2}{\sqrt{3}}$	0	0	$\sqrt{\frac{2}{3}}$	$\sqrt{\frac{2}{3}}$	0	$\sqrt{\frac{2}{3}}$	$-\frac{1}{\sqrt{6}}$

$$a_{CP}^{\rm dir}(D^+ \to \pi^+\eta')\Big|_{\rm HFLAV} = (0.4 \pm 0.2)\%$$

$$a_{CP}^{\rm dir}(D_s^+ \to K^+\eta')\Big|_{\rm HFLAV} = (6.0 \pm 18.9)\%$$

$$a_{CP}^{\rm dir}(D^0 \to \pi^0\eta')\Big|_{SU(3)_F-\,\rm lim.} = -3\ a_{CP}^{\rm dir}(D^0 \to \eta\eta')\Big|_{SU(3)_F-\,\rm lim.}\ ,$$

$$a_{CP}^{\rm dir}(D^+ \to \pi^+\eta')\Big|_{SU(3)_F-\,\rm lim.} = -\ a_{CP}^{\rm dir}(D_s^+ \to K^+\eta')\Big|_{SU(3)_F-\,\rm lim.}\ .$$