LCSR application to $D^+ \to \pi^+\ell^+\ell^-$

(Based on 2505.21369 (to appear in JHEP) in collaboration with
Alexander Khodjamirian and Thomas Mannel)

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Outline

- Simplest Charm FCNCs : $D^+ \to \pi^+ \ell^+ \ell^-$
- Our Methodology : LCSR assisted dispersion relation
- LCSR for $D^+ \to \pi^+ \gamma*$
- Hadronic dispersion relation
- Results
- Use of U-spin and Cabibbo favoured modes.
- Discussions and outlook

What are FCNCs?

Flavour Changing Neutral Currents (FCNCs)

- In SM, all neutral currents are flavour diagonal $\Longrightarrow$ FCNCs are absent at tree level
- FCNC are possible at loop levels by the exchange of $W^{\pm}$ boson: **Penguin and Box diagrams.**
- Absence of FCNC at tree level in SM $\Longrightarrow$ sensitive probe for New Physics.

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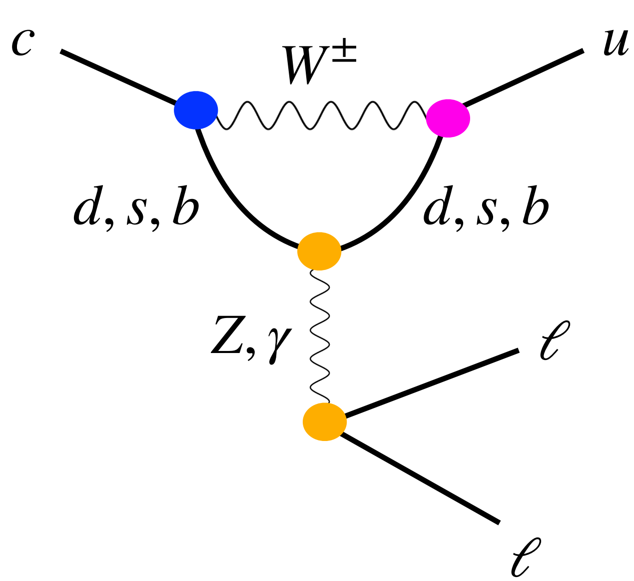
- Unitary of CKM implies: $\lambda_d + \lambda_s + \lambda_b = 0$ (where $\lambda_q = V^*_{cq} V_{uq}$)



Penguin diagram (Leading short distance contribution)

$$\Longrightarrow \mathscr{A}(c \to u) \propto \frac{1}{16\pi^2} \underbrace{V^*_{cs} V_{us}}_{\mathcal{O}(\lambda)} \left(f\left(\frac{m_s^2}{m_W^2}\right) - f\left(\frac{m_d^2}{m_W^2}\right) \right) + \frac{1}{16\pi^2} \underbrace{V^*_{cb} V_{ub}}_{\mathcal{O}(\lambda^5)} \left(f\left(\frac{m_b^2}{m_W^2}\right) - f\left(\frac{m_d^2}{m_W^2}\right) \right)$$

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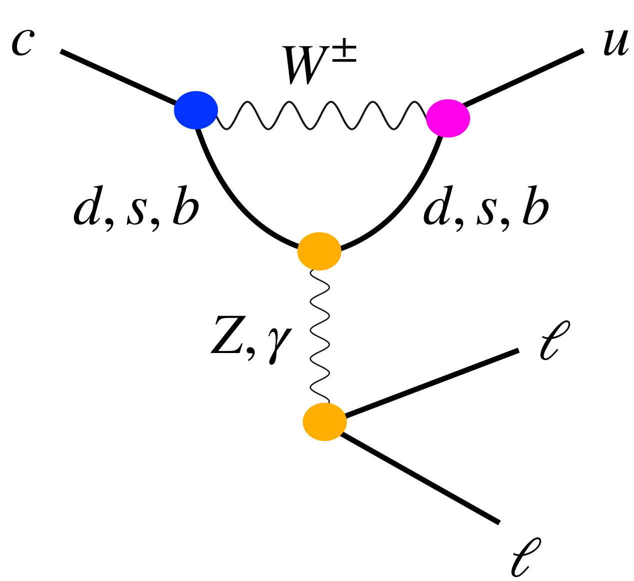

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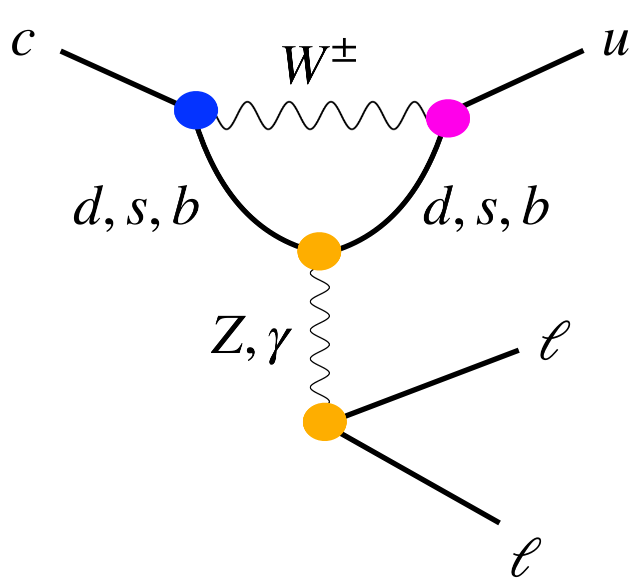

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Very strong GIM and CKM suppression.

- FCNC in b-decays: u, c, t in loop
- CKM factor multiplying t and c loops are $V_{bt}^{*} V_{st} \sim \mathcal{O}(\lambda^2)$ and $V_{bc}^{*} V_{sc} \sim \mathcal{O}(\lambda^2)$

$\Longrightarrow$ GIM cancellation is weaker due to heavy mass of top quark

Flavour Changing Neutral Currents (FCNCs)

Major differences in the FCNCs in bottom and charm decays

FCNC in B-decays ($b \to s$)	FCNC in D-decays ($c \to u$)
FCNC in the down sector	FCNC in the up sector
Short distance dominated	Long distance dominated
Loop contribution is the major source of long-distance uncertainties	Loop contribution is suppressed due to GIM cancellation
Weak annihilation contribution is small.	Weak annihilation is the main contribution
Highly suppressed in SM and provides an excellent opportunity for BSM searches	BSM search is not straightforward because of pollution due to long distance effects.
Cleaner signal at experiments	Experimentally challenging due to resonances
Examples: $B \to K\ell^+\ell^-$,$B \to K^*\gamma$,etc	Examples: $D^0 \to \ell^+\ell^-$, $D^+ \to \pi^+\ell^+\ell^-$, etc

[LHCb, PRL 125 (2020) 011802, 2405.17347, LHCb-paper-2024-022]
[N. Gubernari, M. Rebound, D. V. Dyk, J. Virto, 2206.03797]
[M. Fedele, 2402.03863]
[G. Isidori, Z. Polonsky, A. Tinari, 2405.17551]
⋮

[H. Gisbert, M. Golz, D. Mitzel, 2011.09478],
[G. Hiller et. al., 2202.02331, 2410.00115],
[S. Fajfer, et. al., 2312.07501]
⋮

Charm FCNC: $D \to \pi \ell^+ \ell^-$

- Highly GIM and CKM suppressed.
- Long distance dominated.
- Very challenging : theoretically as well as experimentally.

$D \to \pi\ell^+\ell^-$: Simplest $c \to u\ell^+\ell^-$ mode

- Dominated by weak singly Cabibbo suppressed (SCS) $D \to \pi$ transition combined with an electromagnetic emission of the lepton pair.
- A simple mechanism: $D^+ \to \pi^+\ell^+\ell^- \approx D^+ \to \pi^+ V(\to \ell^+\ell^-)$ (with $V = \rho, \omega, \phi, \ldots$).

V	$BR(D^+ \to \pi^+ V)$	$BR(V \to \mu^+\mu^-)$	$BR(D^+ \to \pi^+ V)_{V \to \mu^+\mu^-}$
$\rho^0(770)$	$(8.4 \pm 0.8) \times 10^{-4}$	$(4.55 \pm 0.28) \times 10^{-5}$	$(3.82 \pm 0.43) \times 10^{-8}$
$\omega(782)$	$(2.8 \pm 0.6) \times 10^{-4}$	$(7.4 \pm 1.8) \times 10^{-5}$	$(2.1 \pm 0.7) \times 10^{-8}$
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[PDG]

- The effective Hamiltonian for $D \to \pi\ell^+\ell^-$ (SCS):

$$\mathcal{H}_{eff}^{\Delta S=0} = \frac{4G_F}{\sqrt{2}} \sum_{\mathcal{D}=d,s} \lambda_{\mathcal{D}} \left[C_1(\mu)O_1^{\mathcal{D}} + C_2(\mu)O_2^{\mathcal{D}}\right] - \lambda_b \sum_{i=3}^{10} C_i(\mu)O_i$$

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suppressing factor

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$\ll C_{1,2} @ \mathcal{O}(m_c)$

suppressing factor

WCs @ $\mu = 1.3$ GeV

	C_1	C_2	C_3	C_4	C_5	C_6
LL	-1.035	1.094	-0.004	-0.061	0.000	0.001
NLL	-0.712	1.038	-0.006	-0.093	0.000	0.001
NNLL	-0.633	1.034	-0.008	-0.093	0.000	0.001
	C_7^{eff}	C_8^{eff}	C_9	C_{10}	C_9^{NNLL}	C_{10}^{NNLL}
LL	0.078	-0.055	-0.098	0	-0.488	0
NLL	0.051	-0.062	-0.309	0		

[S. de Boer, B. Müller, D. Siegel, (1606.05521)]

Amplitude and Hadronic Matrix Element

- Hamiltonian in the GIM limit ($\lambda_b = 0,\ \lambda_d = -\lambda_s$):

$$\mathscr{H}_{eff}^{(\Delta S=0,\lambda_b=0)} = \frac{4G_F}{\sqrt{2}}\lambda_d \left[C_1(O_1^d - O_1^s) + C_2(O_2^d - O_2^s)\right]$$

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The hadronic part (hadronic matrix element)

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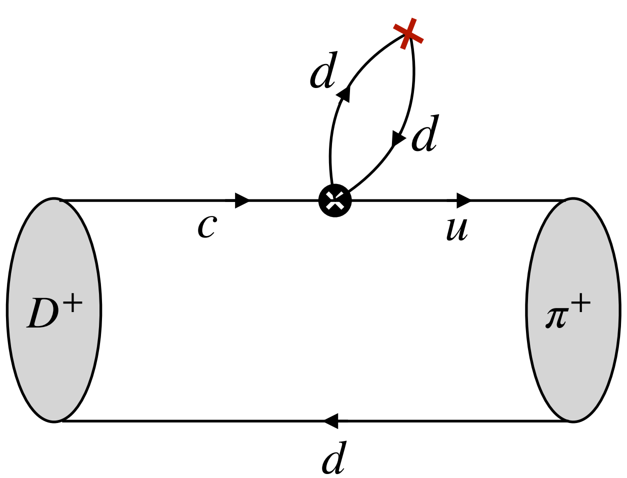
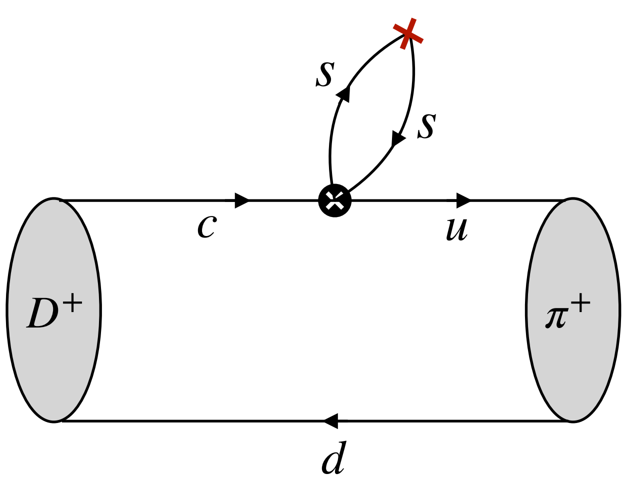
- Vector meson can be created from non-leptonic weak decay before γ^* : **Resonance contributions**
- Largest effect beyond GIM limit $\sim \lambda_b C_9$ ($C_9 = -0.488$) : **short distance contribution.**

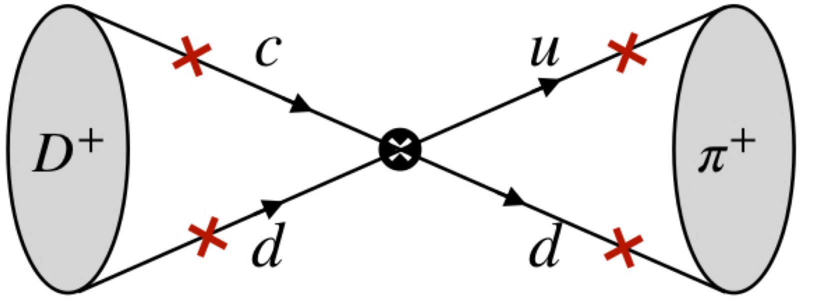
Quark Topologies for $\mathscr{A}_\mu^{D^+\to\pi^+\gamma^*}(p,q)$

Loop Topology
(Only possible in SCS decays)

Annihilation Topology







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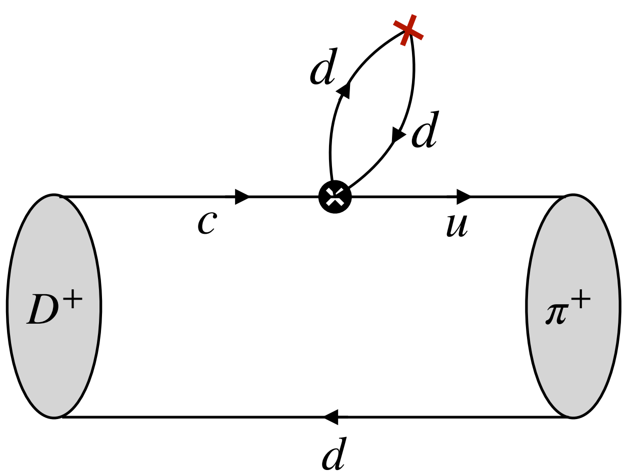

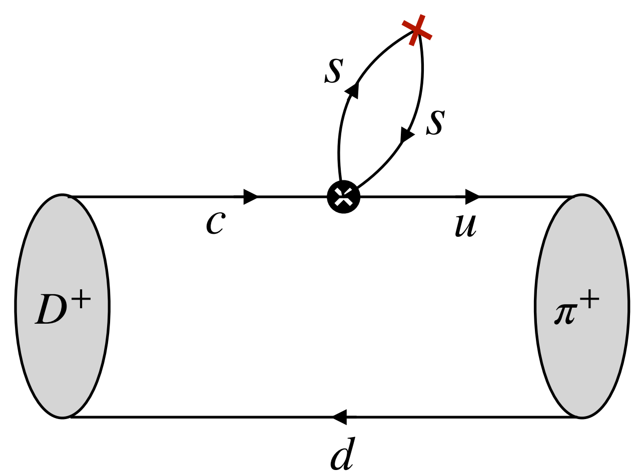

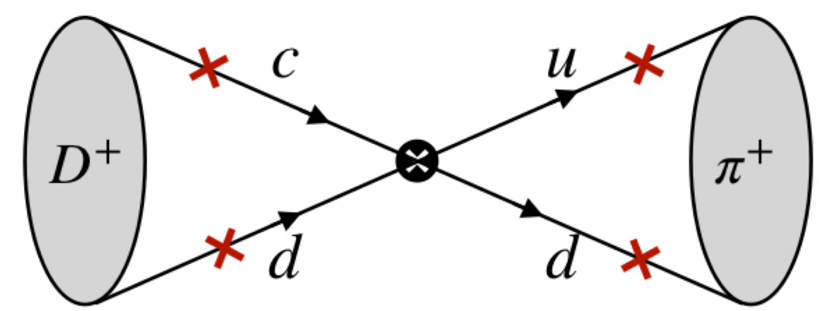

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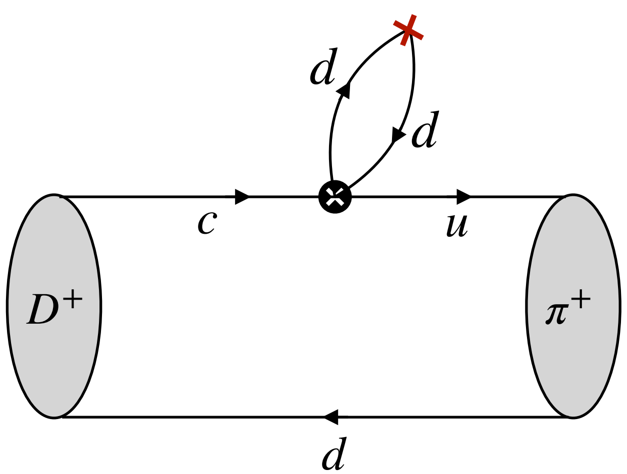

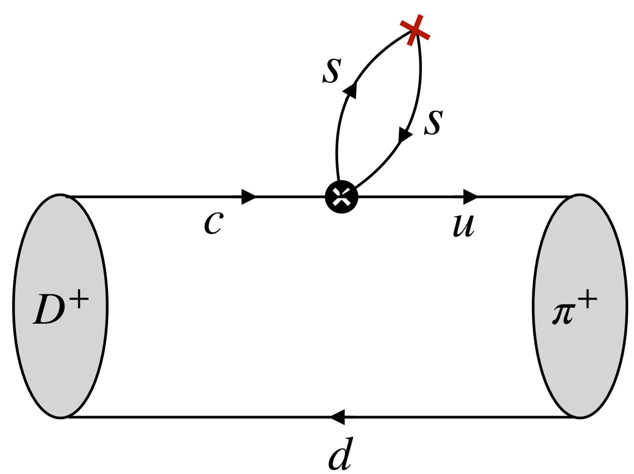

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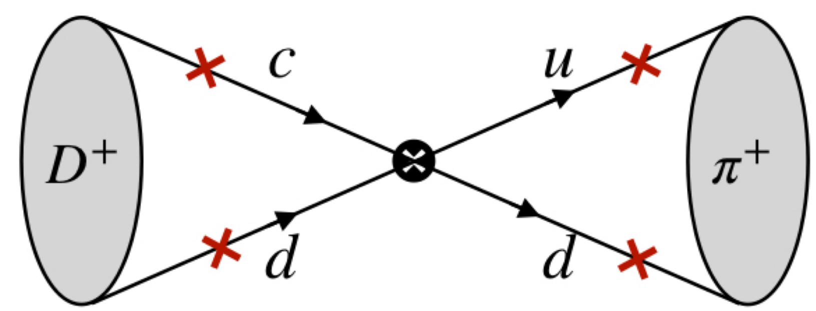

- Only d-quark contributes : No GIM cancellation.

A-topology is the main contribution.

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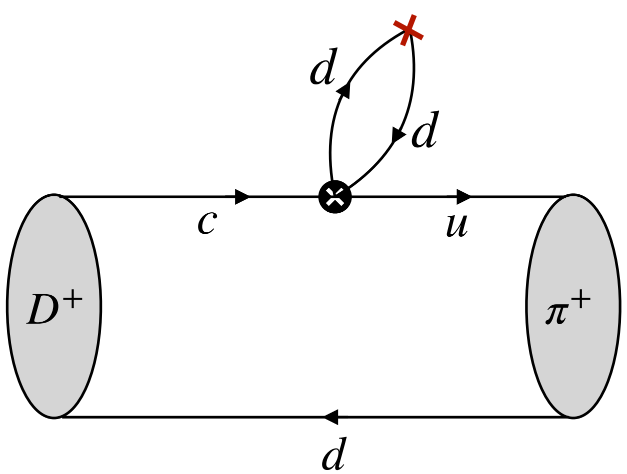
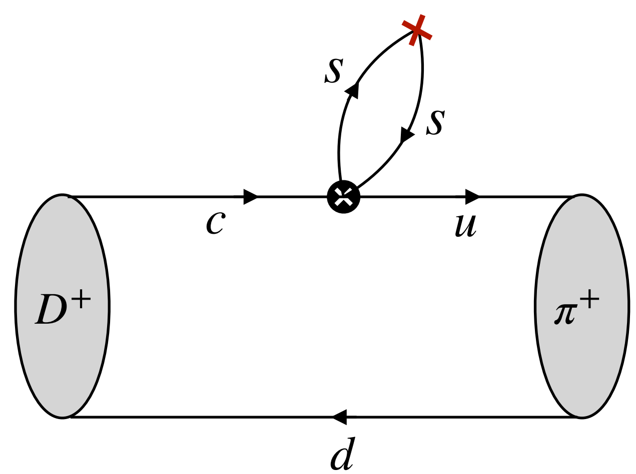
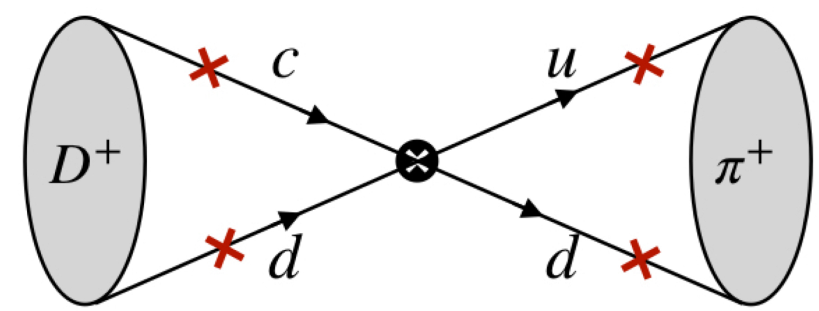




- In $SU(3)_f$ (along with $\lambda_b \approx 0$) limit, complete GIM cancellation

L-topology has non-zero contribution only due to $m_s \neq m_d$.

Annihilation Topology



- Only d-quark contributes : No GIM cancellation.

A-topology is the main contribution.

- At NLO, there will be multiple diagrams with the exchange of virtual gluons : Out of scope of present study.

Previous approach to $D \to P\ell\ell$ in SM

- Non-resonant SM contribution $\sim \mathcal{O}(10^{-12})$
- **Vector resonances:** treated as correction to C_9 ;Wilson Coeff. of $O_9 = (\bar{u}_L \gamma_\mu c_L)(\ell \gamma^\mu \ell)$.

$$C_9^R(q^2) = a_\rho e^{i\delta_\rho} \left(\frac{1}{q^2 - m_\rho^2 + i m_\rho \Gamma_\rho} - \frac{1}{3} \frac{1}{q^2 - m_\omega^2 + i m_\omega \Gamma_\omega} \right) + a_\phi e^{i\delta_\phi} \left(\frac{1}{q^2 - m_\phi^2 + i m_\phi \Gamma_\phi} \right)$$

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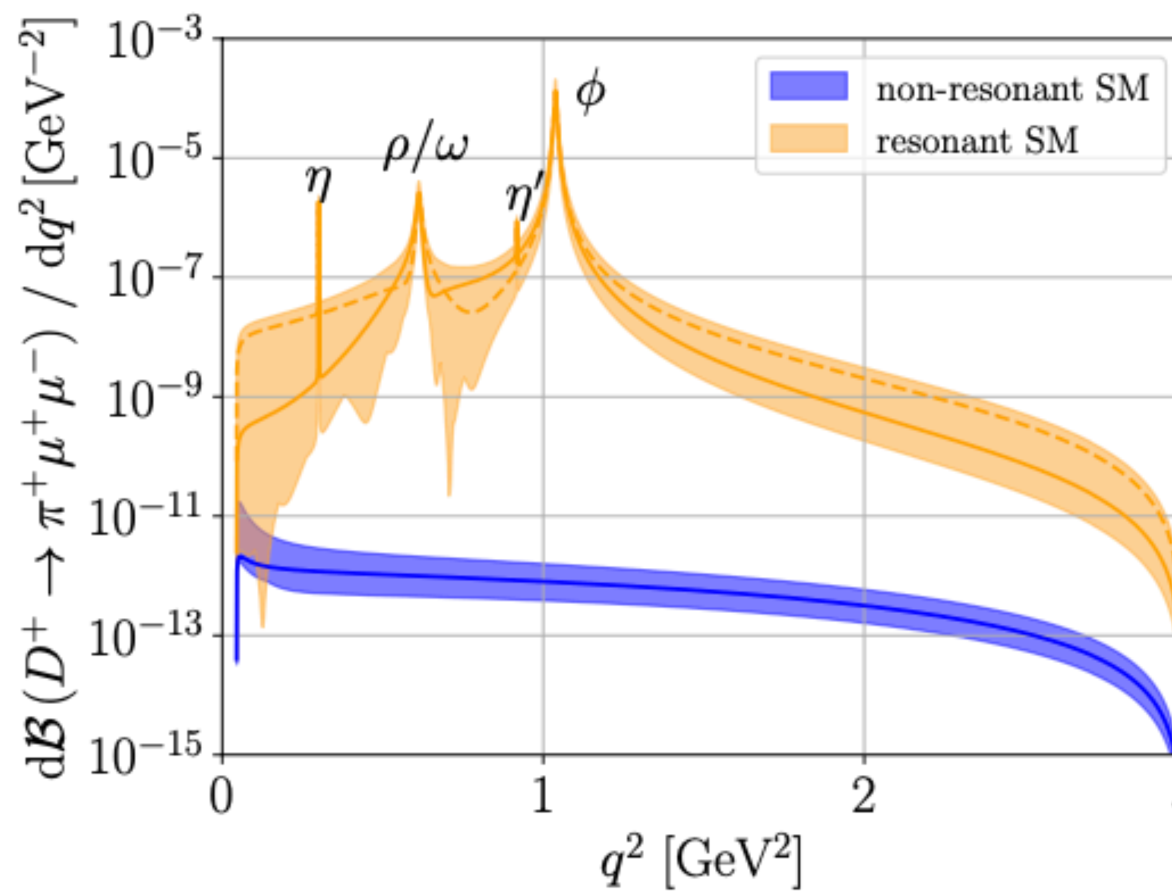

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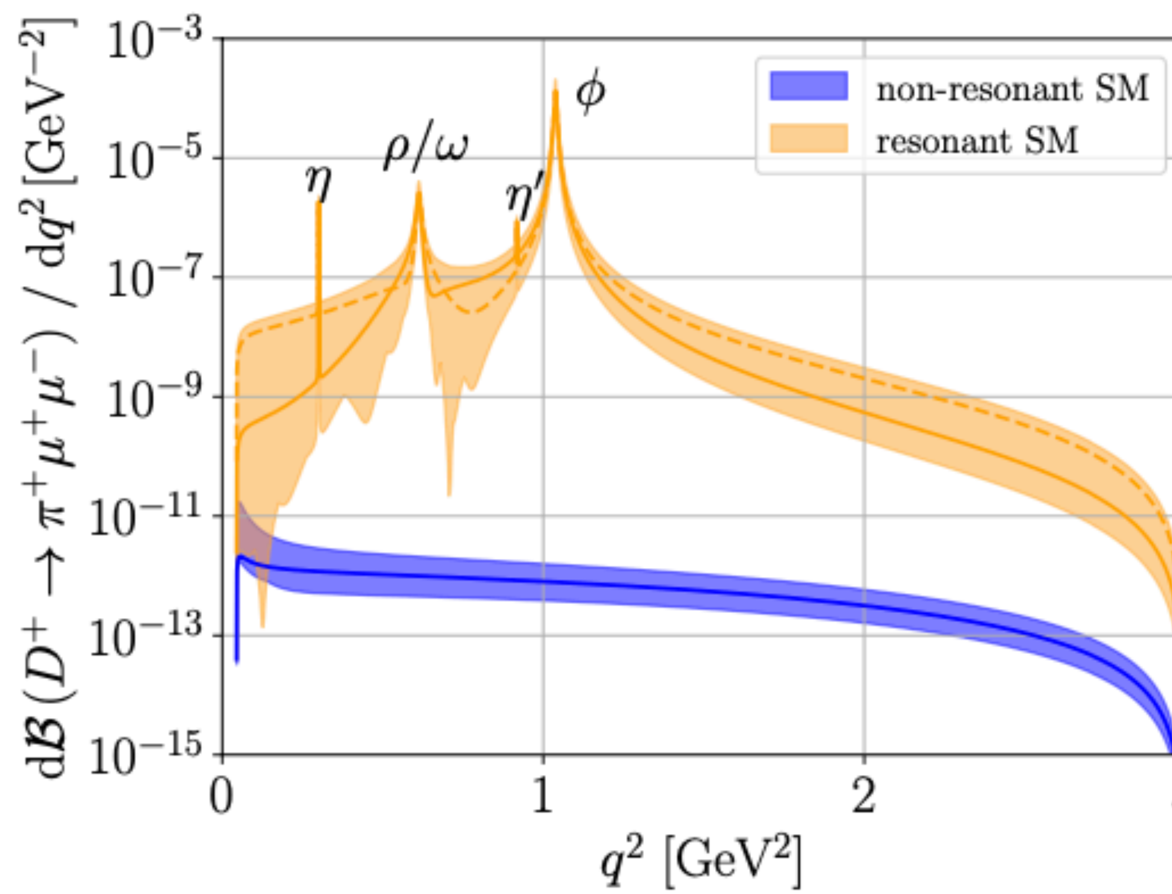

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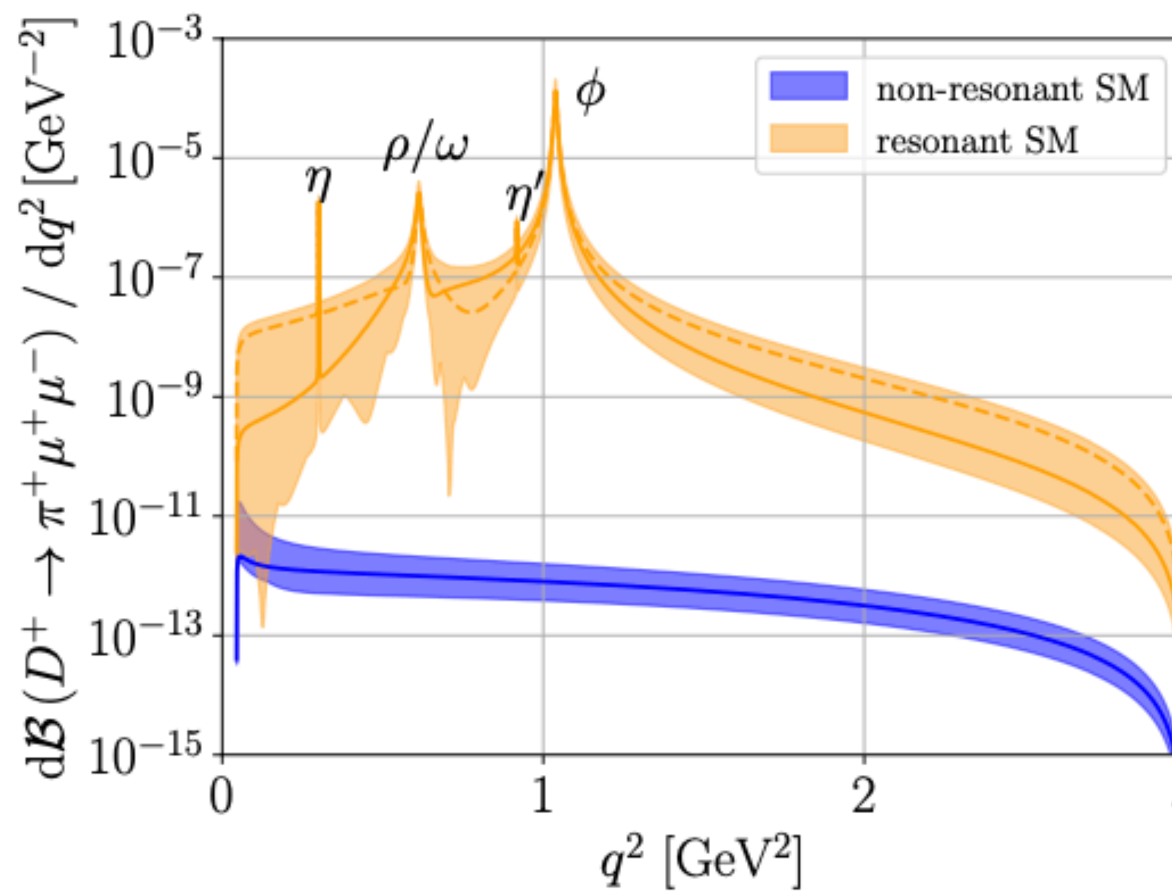

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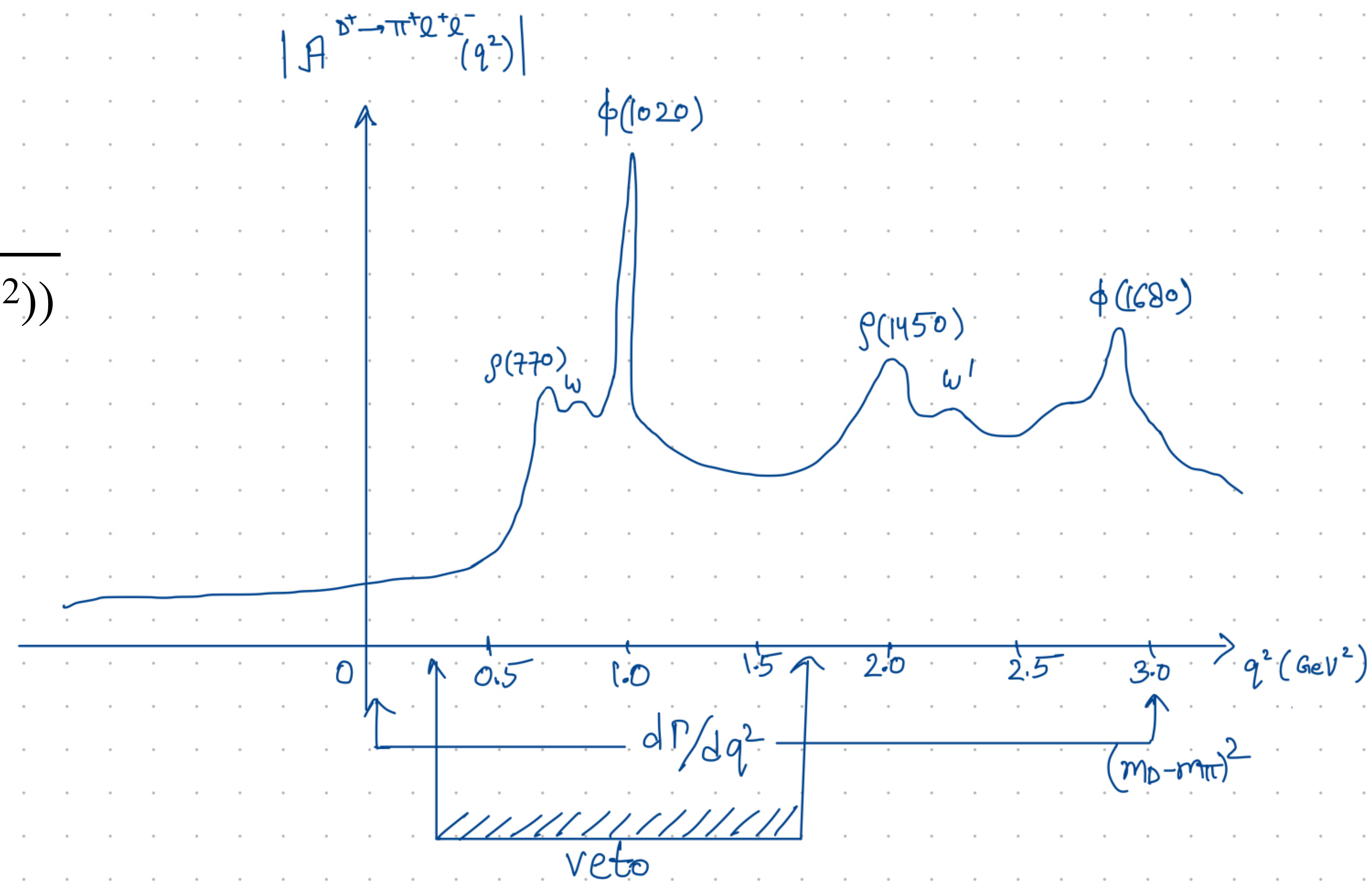
Resonances as indispensable part of DR

- Full amplitude via hadronic dispersion relation (DR) :

$$\mathcal{A}^{(D^+\to\pi^+\gamma^*)}(q^2) = \sum_{V=\rho,\omega,\phi} \frac{\kappa_V f_V |A_{D^+V\pi^+}| e^{i\varphi_V}}{(m_V^2 - q^2 - i\sqrt{q^2}\Gamma_V(q^2))} + \int_{s_{th}}^{\infty} ds \frac{\rho_h(s)}{(s - q^2 - i\epsilon)}$$



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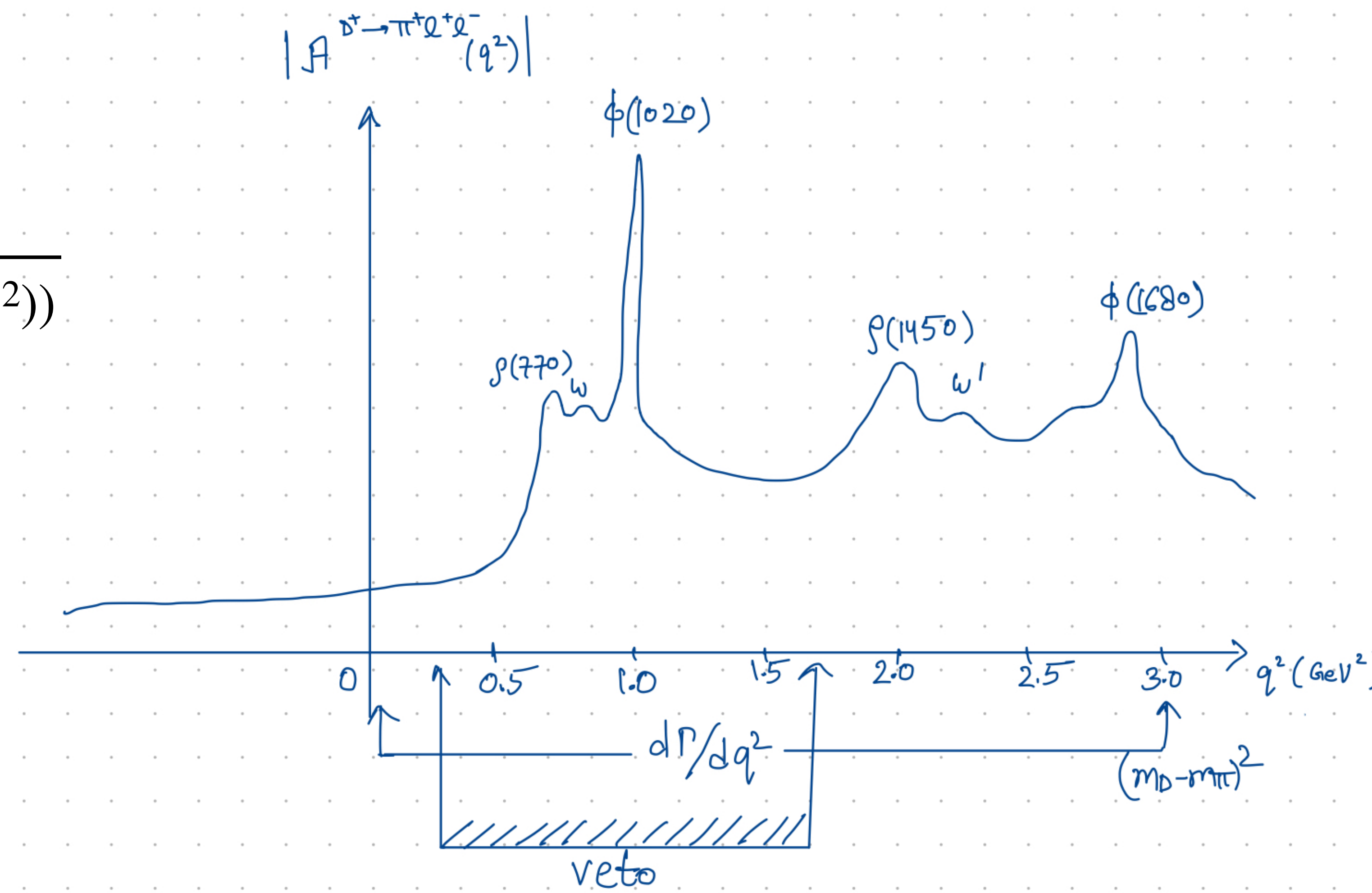

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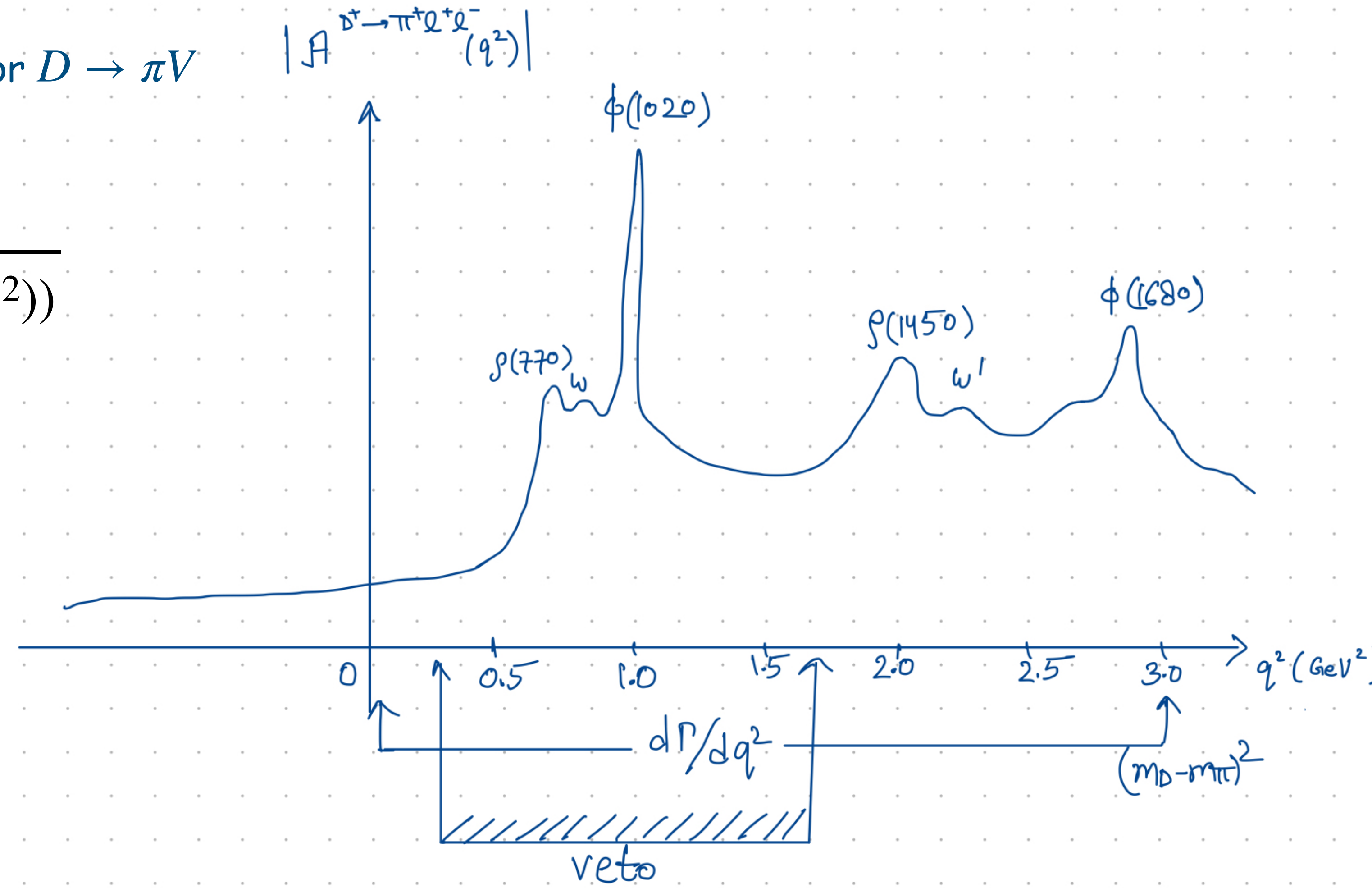

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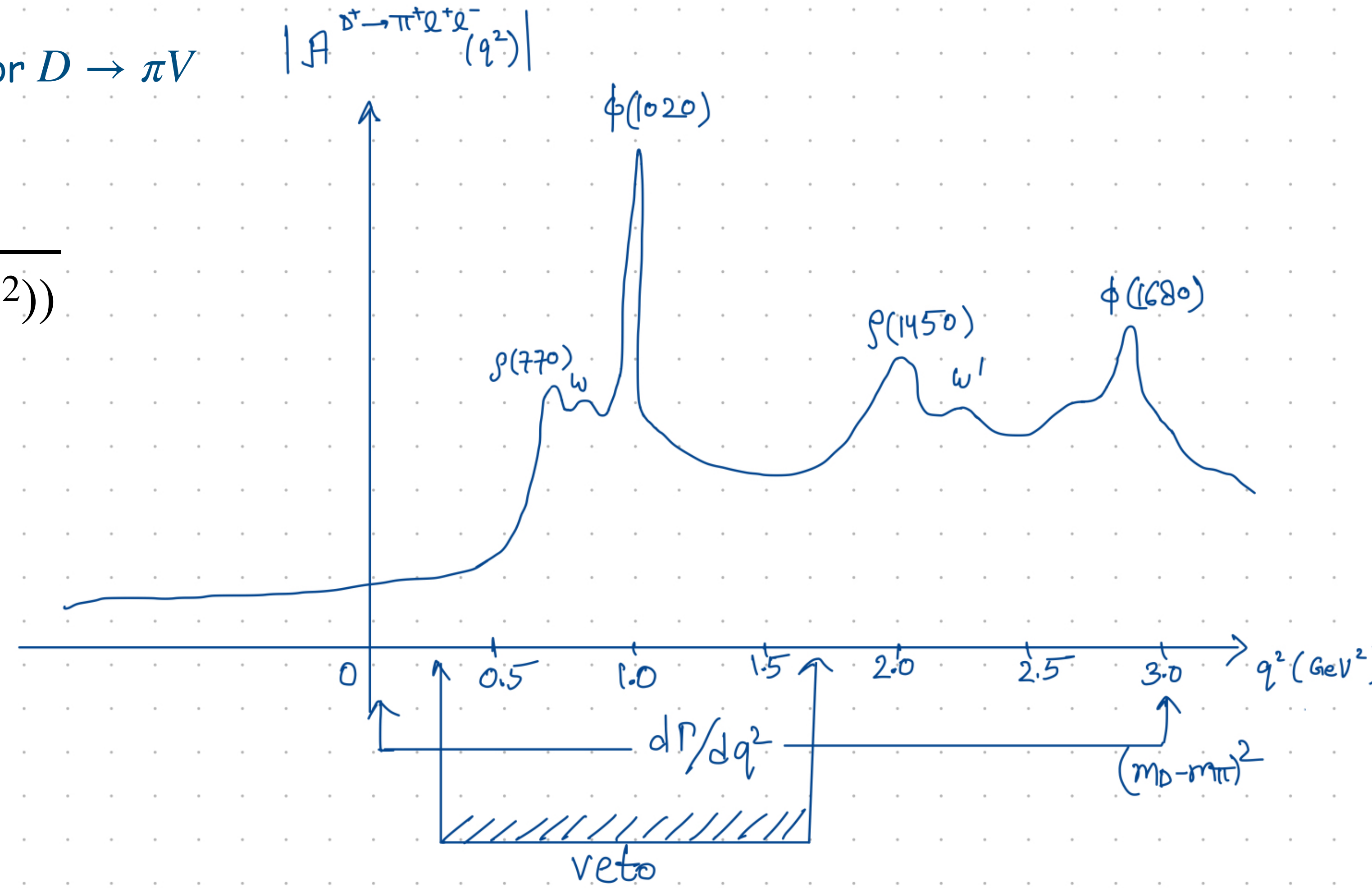

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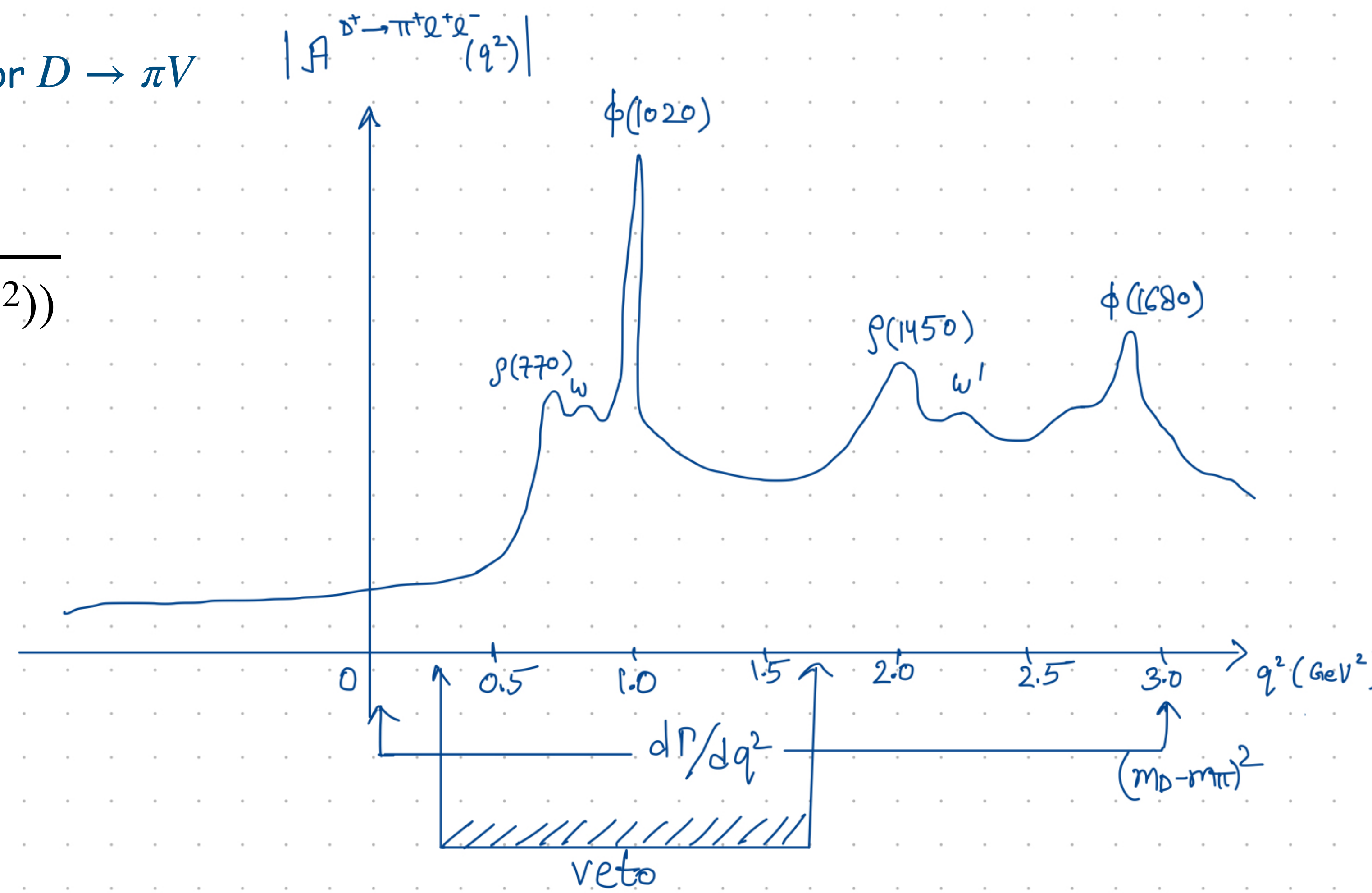

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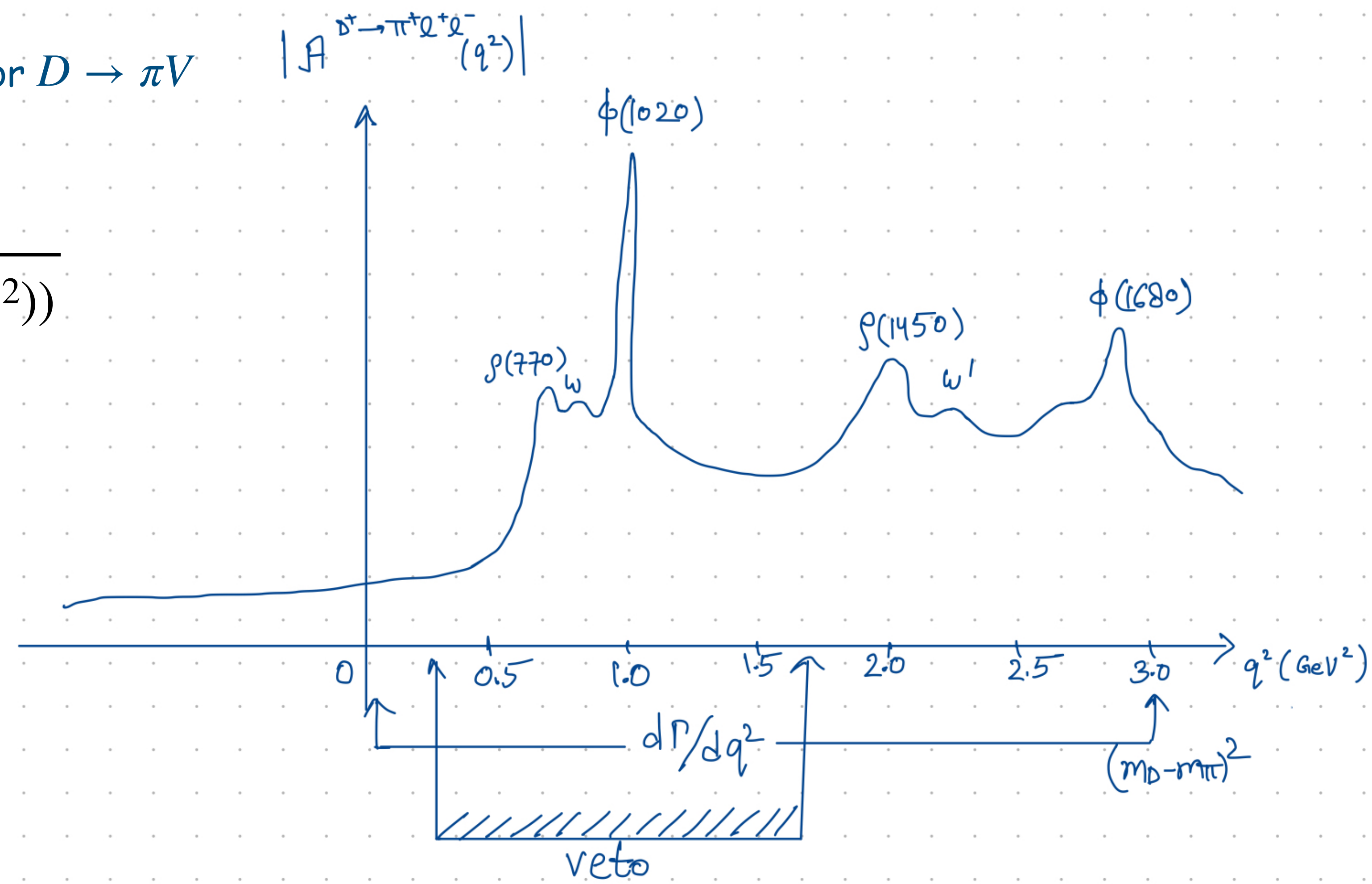

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Is a proper QCD based analysis possible?

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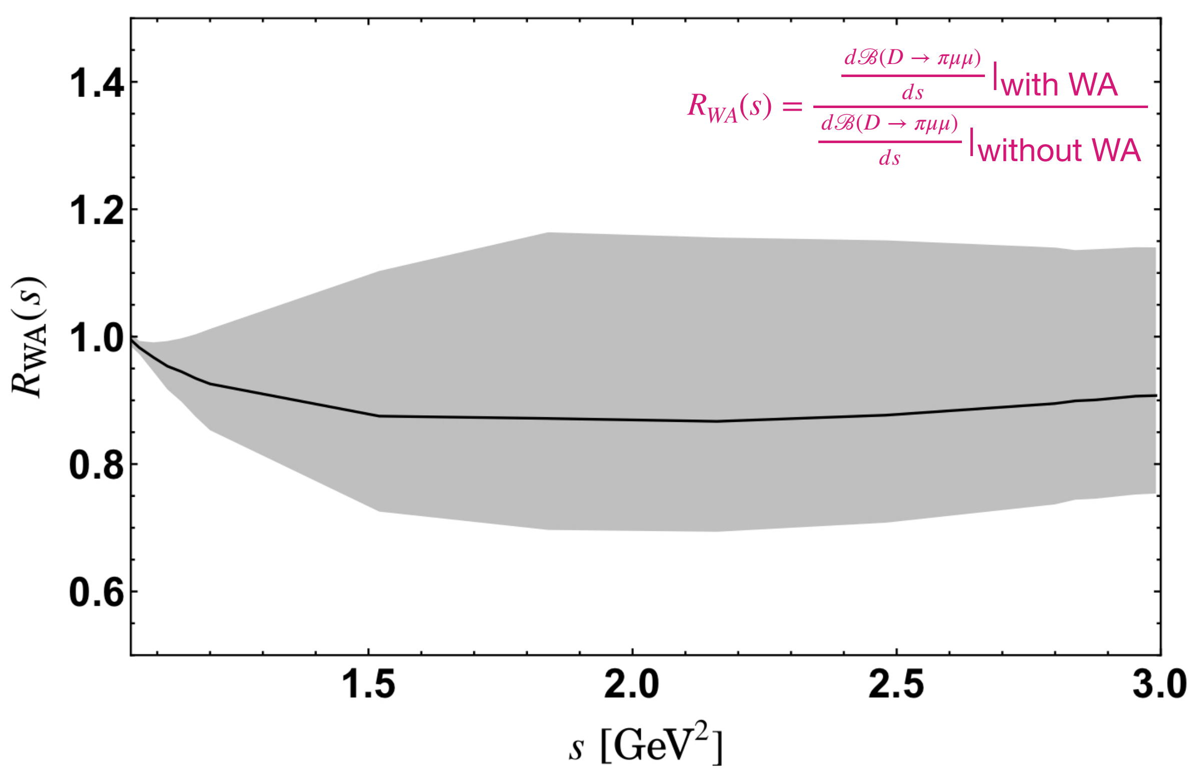

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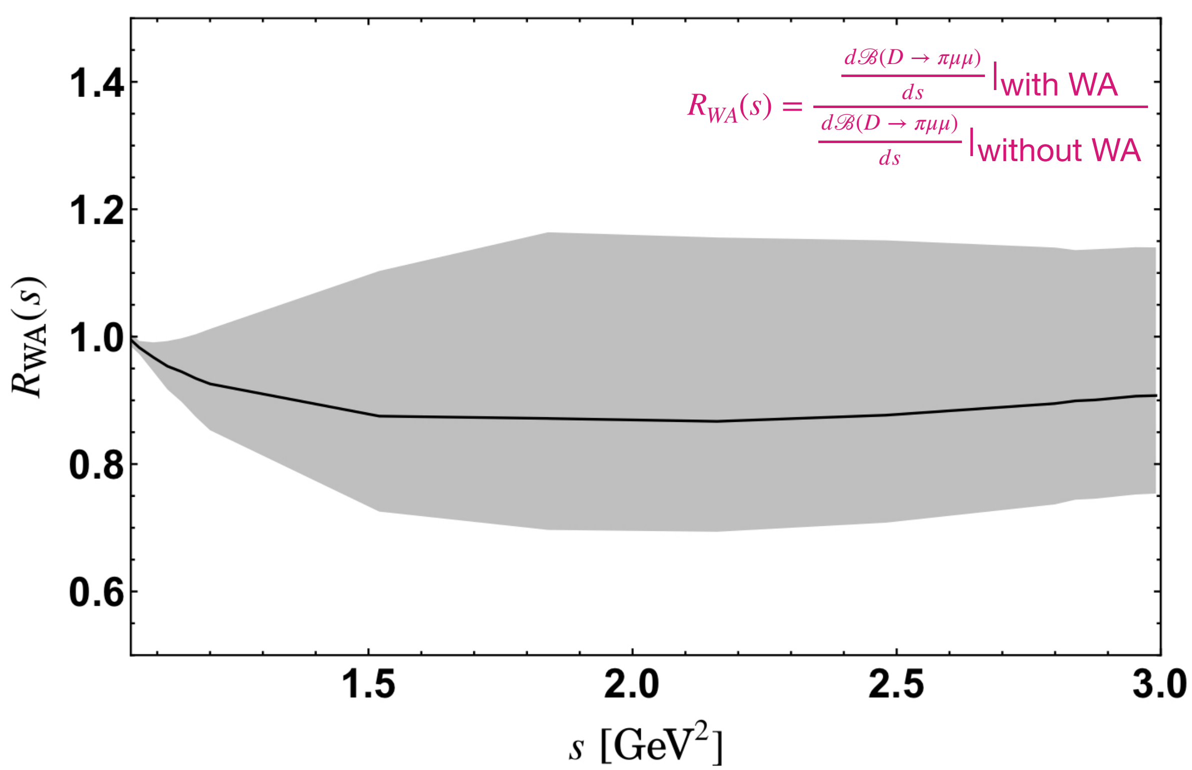

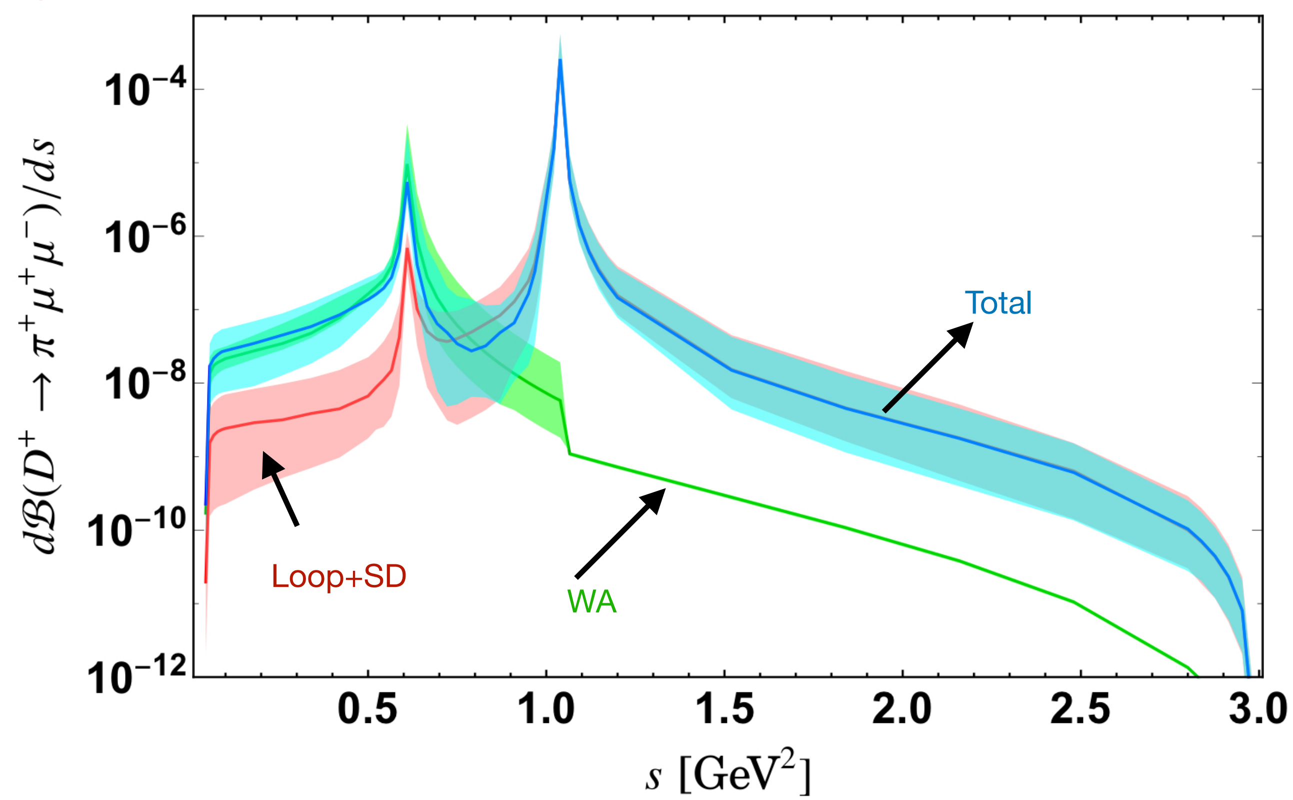

Still open questions!!

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(vetoing the resonance region)

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[LHCb, (JHEP06 (2021) 044)]

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As the Experimental bounds are now approaching theory predictions, it is important to look for alternative QCD based methods.

$D^+ \to \pi^+ \ell^+ \ell^-$ using LCSR assisted Dispersion Relation

[AB, Alexander Khodjamirian and Thomas Mannel, 2505.21369]

- **Benefits:**
  - An independent alternative to QCDf.
  - Finite m_c .
  - Provides estimates for strong phases.

Our methodology: LCSR-assisted dispersion relation

Ways to compute $\mathscr{A}_{\mathscr{D}}^{D^+\to\pi^+\gamma^*}(q^2)$

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Light Cone Sum Rules (LCSR)

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- Amplitude computation using QCD rules.
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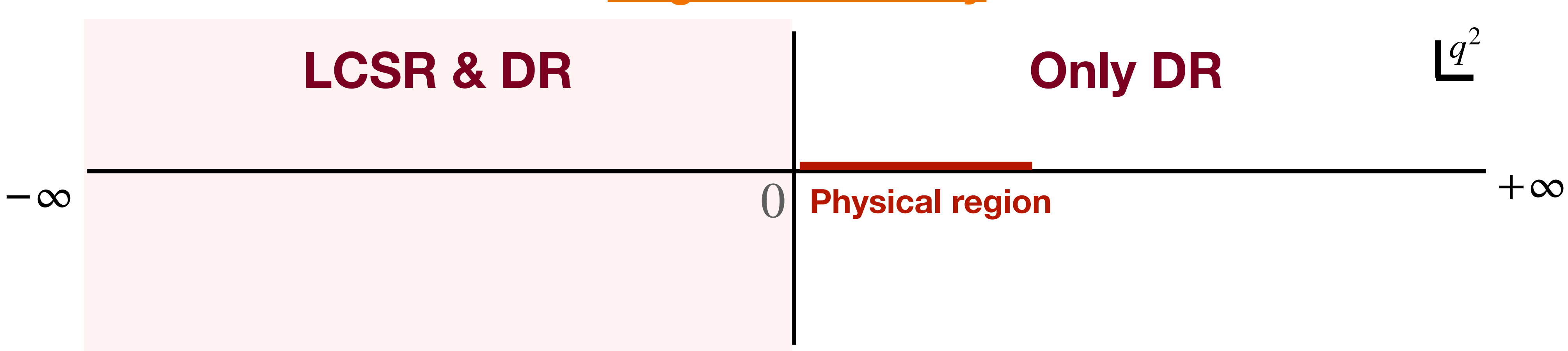
Region of Validity



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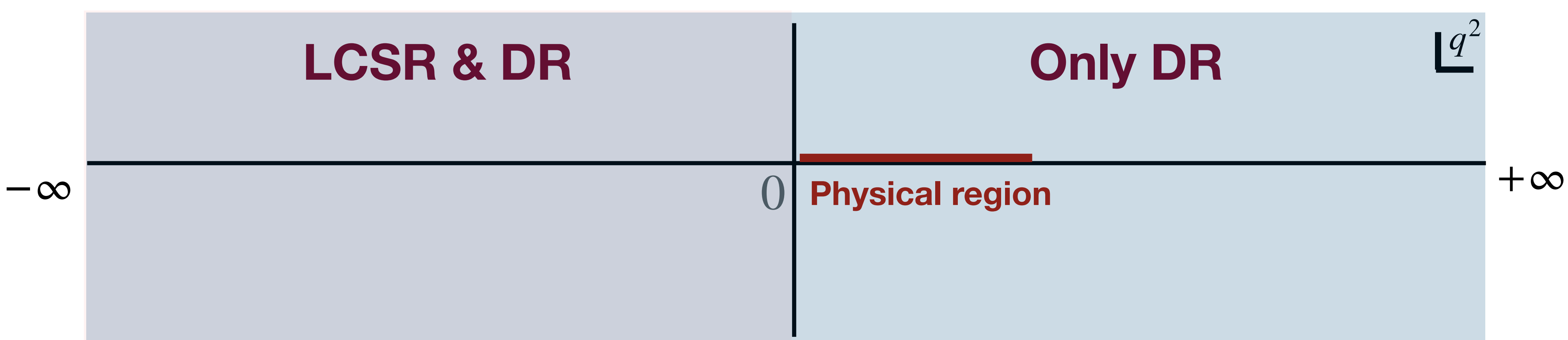
Dispersion relation (DR)

- Valid for all values of q^2.
- Uses analyticity & unitarity of amplitude.
- Written directly in hadronic states.
- Needs parametrisation : Unknown model parameters.

Region of Validity



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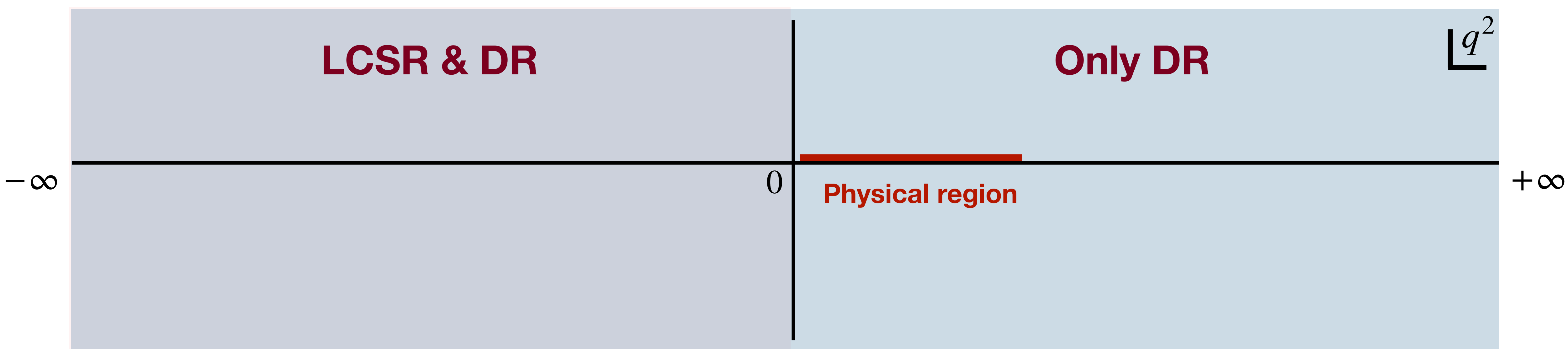
Light Cone Sum Rules (LCSR) | Dispersion relation (DR)



- Fit dispersion relation model to LCSR result in $q^2 < 0 \Longrightarrow$ Estimation of model parameters (strong phases and parametrisation parameters).
- Use these fitted parameters to make predictions in the physical region using the dispersion relation

Summary of the main idea

Step-1: Compute $\mathscr{A}^{(D^+\to\pi^+\gamma^*)}(q^2)$ using LCSR at $q^2 < 0$.

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Step-4: Use these fitted parameters to estimate $\mathscr{A}^{(D^+\to\pi^+\gamma^*)}(q^2)$ using dispersion relation in the physical region.

(Resembling partly the analysis of nonlocal effects in $B \to K^{(*)}\ell^+\ell^-$)

[A. Khodjamirian, T. Mannel, A. Pivovarov, Y. Wang, 1211.0234]

[A. Khodjamirian, A. V. Rusov, 1703.04765] , N. Gubernari, M. Rebound, D. van Dyk, J. Virto, 2011.09813

LCSR for $D^+ \to \pi^+\gamma^*$

LCSR in a Nutshell

- **LCSR:** Analytic method to compute strong interactions dynamics at large distances using QCD.

- **Basic idea:** to compute hadronic parameters using analytic properties of correlation function involved.

Review articles: [A. Khodjamirian, P. Colangelo, hep-ph/0010175], [A. Khodjamirian, B. Melić, YM Wang, 2311.08700]

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- The correlation functions are of **dual nature**:
  - At large Euclidean momentums, short distance quark-anti-quark fluctuations $\Longrightarrow$ pQCD.
  - For all values of momentum, decomposition in terms of hadronic states $\Longrightarrow$ Dispersion relation

Ways to calculate a correlation function

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Dispersion relation

- Uses unitarity & analyticity of correlation function.
- Can be written directly in terms of hadronic states.

Ways to calculate a correlation function

Perturbative QCD

- Uses theory of quarks & gluons.
- Treated in the framework of light-cone operator product expansion (LCOPE).

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Matching the two gives estimates for the hadronic objects

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LCSR for $\mathscr{A}^{D^+\to\pi^+\gamma^*}(q^2)$

TOOLS TO DERIVE LCSR

Light cone OPE

(Computing correlation function as a convolution of perturbative hard scattering kernel and pion DAs)

- **Correlation function (CF):**

$$F_\mu(p,q,k) = -\int d^4x\, e^{iq\cdot x}\int d^4y\, e^{-i(p+q)\cdot y}\langle \pi^+(p-k)\,|\,T\{J_\mu^{em}(x)\mathscr{H}_{eff}^{(\Delta_s=0,\lambda_b=0)}(0)J_5^D(y)\}\,|\,0\rangle$$

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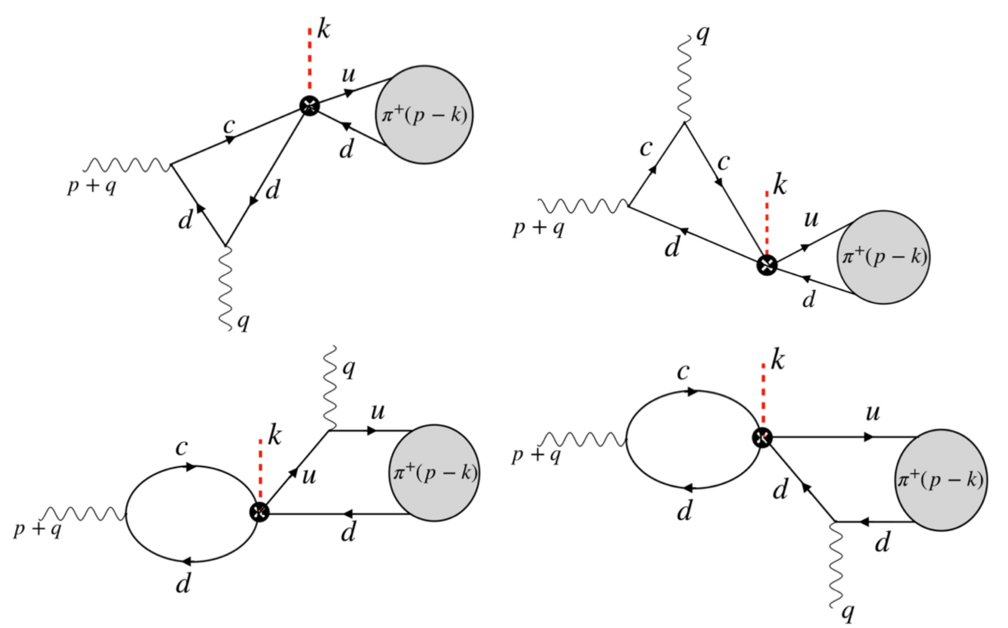
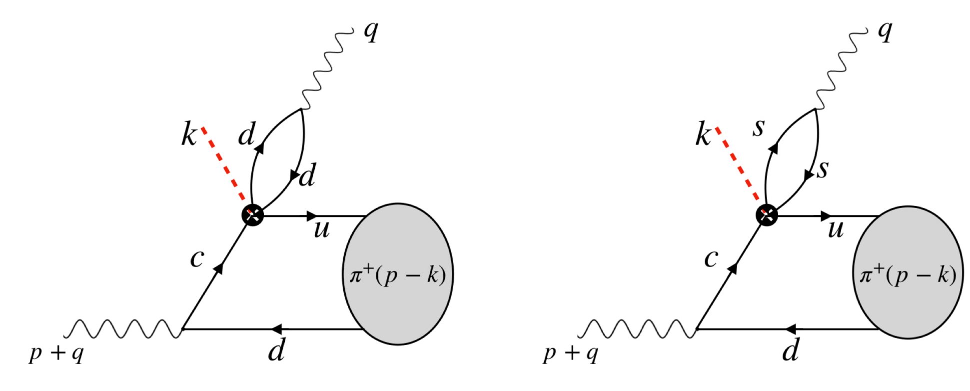
- **Leading contributions:**

Weak Annihilation contributions



Loop contributions



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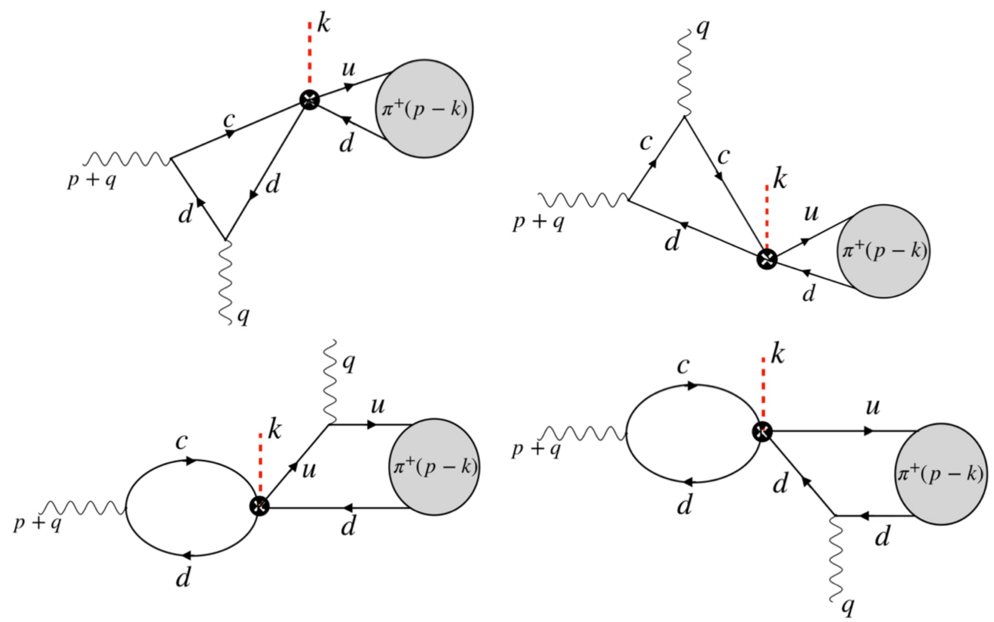

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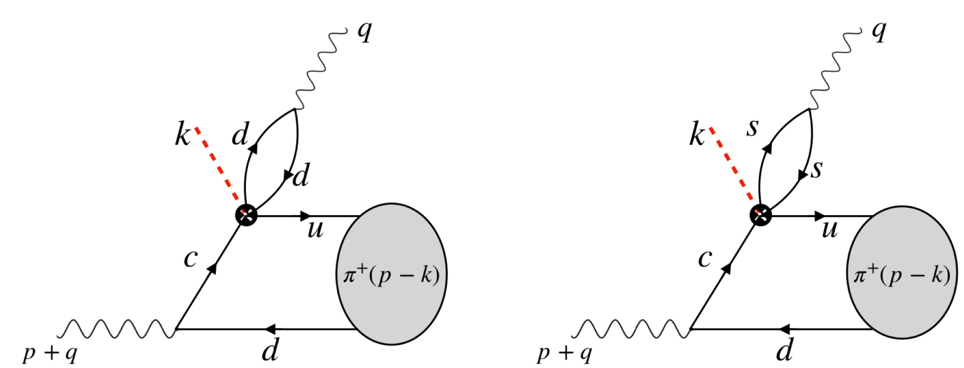

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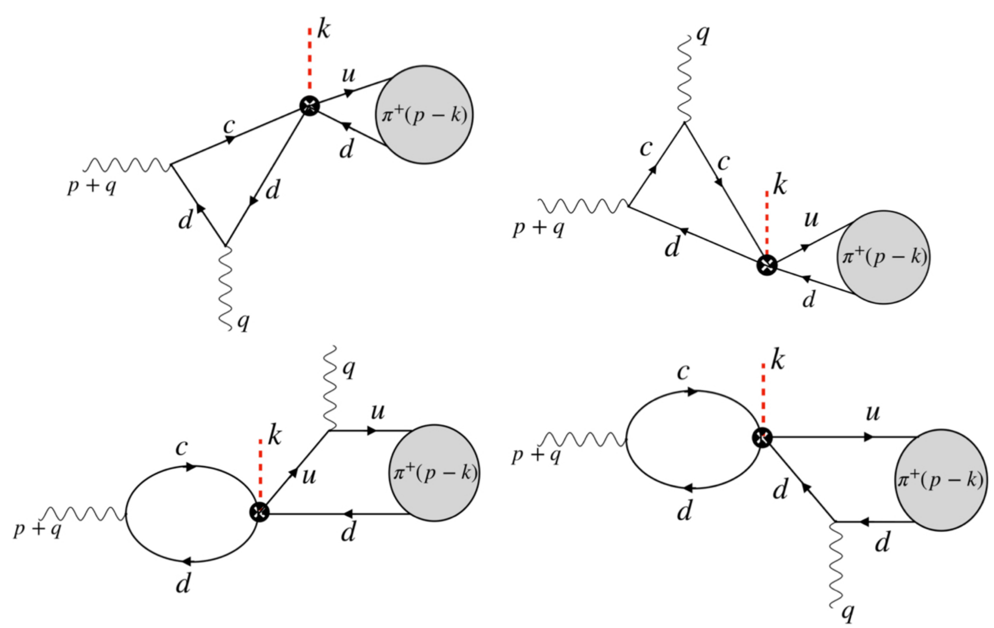

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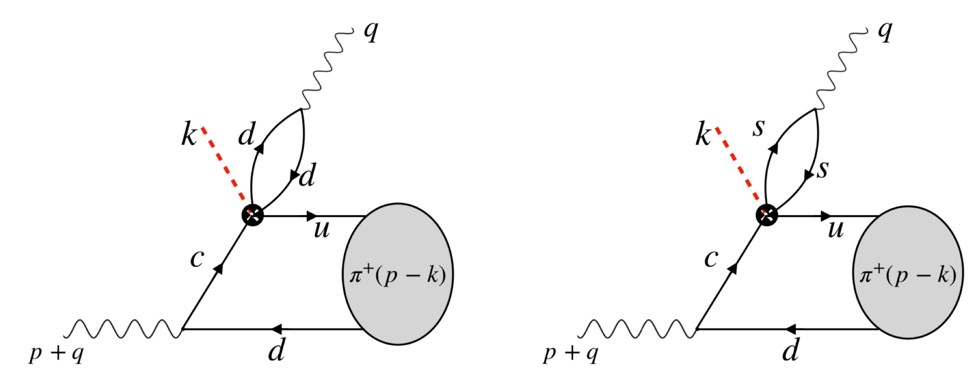

- Both O_1^d and O_2^d contribute.
- Small due to GIM suppression.

❖ The artificial momentum k is introduced at the four vertex to avoid parasitic contributions in the dispersion relation.

(Method used before in LCSR analysis of $B\to 2\pi$ and $D\to 2\pi, K\bar{K}$)

A. Khodjamirian et. al., hep-ph/0012271, hep-ph/0304179, hep-ph/0509049, 1706.07780

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Dispersion Relation in D-meson channel

(Enables to relate the calculated correlation function to the sum over $D \to \pi\gamma^*$ hadronic matrix elements.)

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Light Cone OPE | **Dispersion relation in $(p+q)^2$**

$$\frac{1}{\pi}\int_{m_c^2}^{\infty} ds \frac{\text{Im}F^{(OPE)}(s,q^2,P^2=m_D^2)}{s-(p+q)^2} = \frac{m_D^2 f_D A^{(D^+\to\pi^+\gamma^*)}(q^2)}{m_D^2-(p+q)^2} + \int_{s_{h_d}}^{\infty} ds \frac{\rho_{h_D}(s,q^2,P^2=m_D^2)}{s-(p+q)^2}$$

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contribution from D-meson state

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contribution from D-meson state

Unknown contribution from higher and continuum state ($s > s_{h_d}$)

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Quark Hadron Duality
(Approximates the integral over the spectral density to the integral over imaginary of the perturbatively calculated correlation function)

$$\int_{s_{h_d}}^{\infty} ds \frac{\rho_{h_D}(s, q^2, P^2 = m_D^2)}{s-(p+q)^2} \approx \frac{1}{\pi}\int_{s_0^D}^{\infty} ds \frac{\mathrm{Im}F^{(OPE)}(s, q^2, P^2 = m_D^2)}{s-(p+q)^2}$$

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Borel Transformation
(To suppress the effect of continuum and higher resonances to reduce the uncertainty due to duality approximation)

Results for LCSR

- **Final Sum Rule (valid for $q^2 < 0$):**

$$\mathscr{A}^{(LCSR)}_{(D^+\pi^+\gamma^*)}(q^2) = \frac{1}{\pi m_D^2 f_D} \int\limits_{m_c^2}^{s_0^D} ds\, e^{(m_D^2 - s)/M^2}\, \mathrm{Im} F^{(OPE)}(s, q^2, m_D^2)$$

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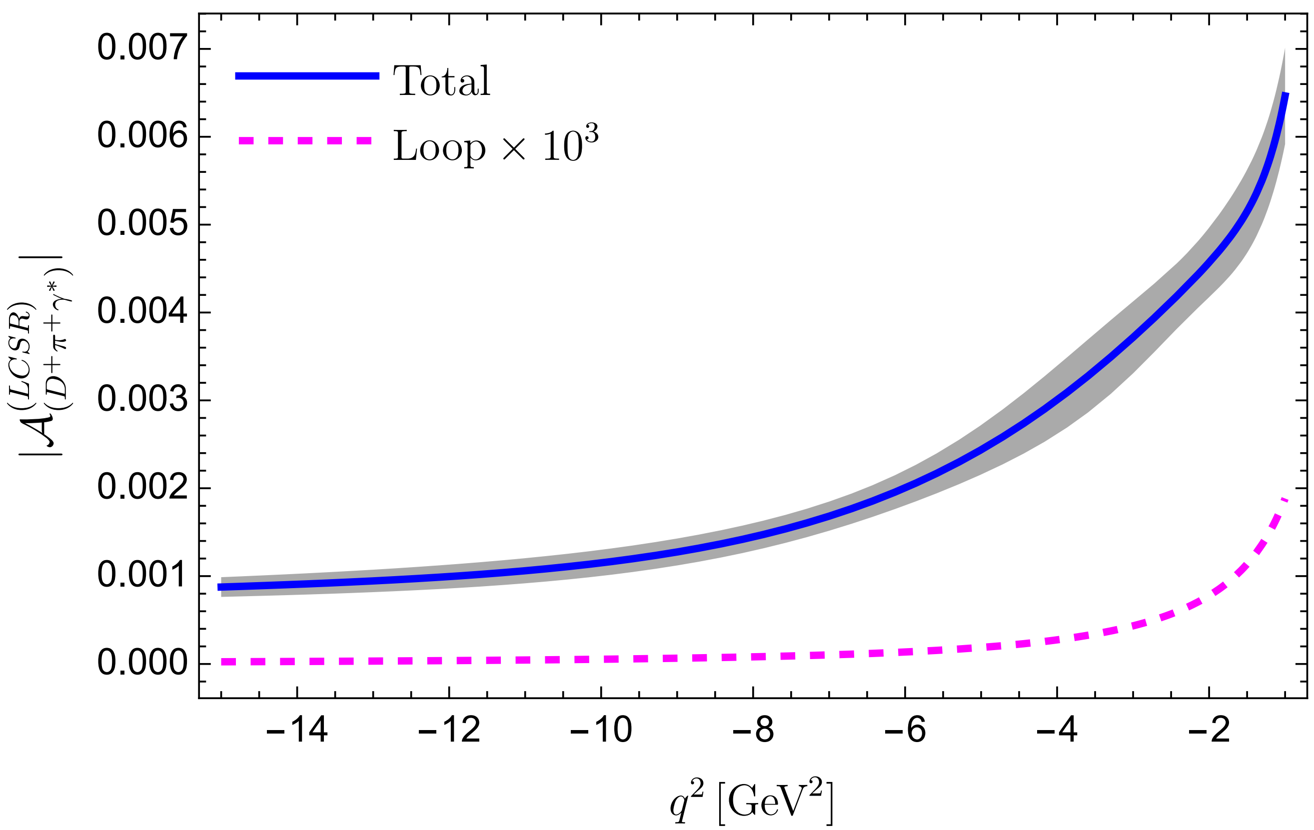
- Sum rule parameters:

  Borel Mass: $M^2 = (2.0 \pm 0.5)$ GeV^2

  Effective Threshold: $s_0^D = (5.5 \pm 0.5)$ GeV^2



Remarks:

- Error band shows only parametric uncertainties.
- F^{OPE} include contribution from twist-2 distribution amplitude (DA) of pion (using 2 Gegenbauer moments).
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- Contribution from O_9 is roughly three order of magnitudes smaller than total amplitude.

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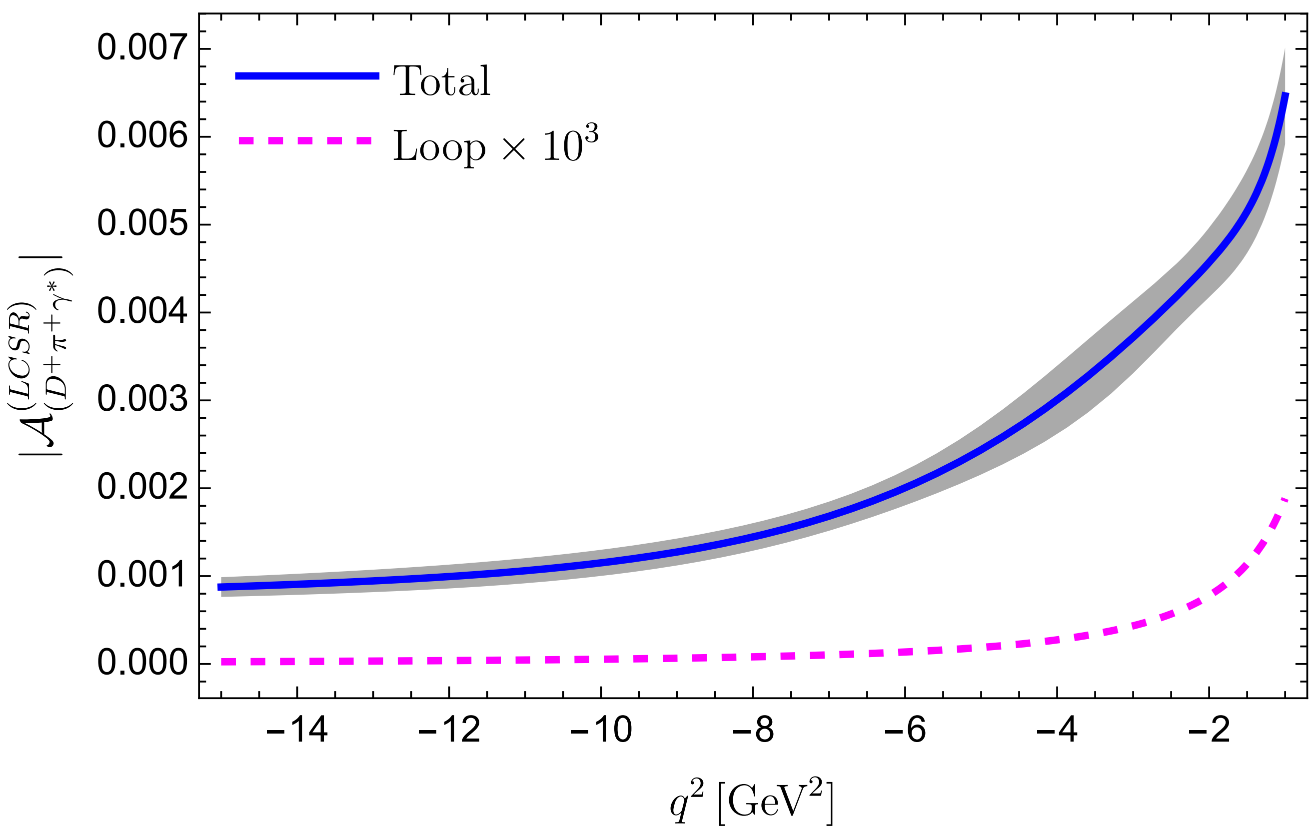
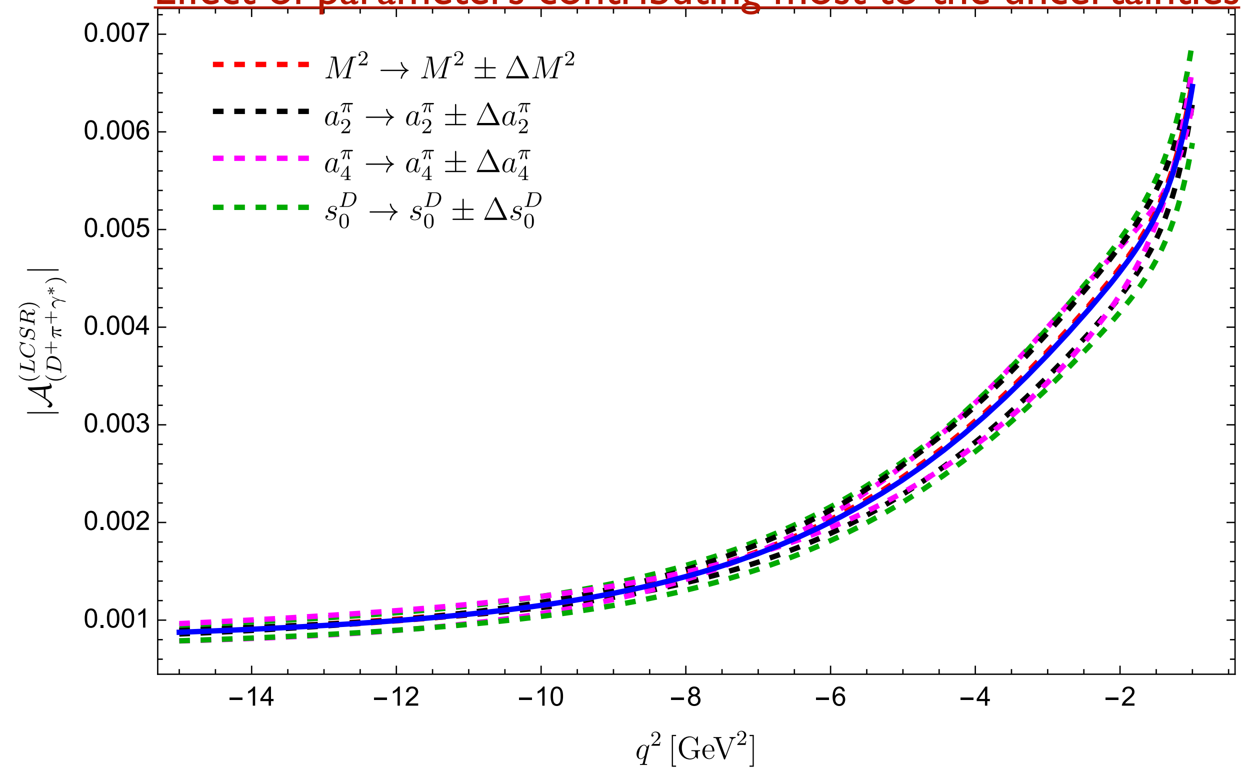


Effect of parameters contributing most to the uncertainties



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Hadronic Dispersion Relation

Hadronic dispersion relation

$$\mathscr{A}^{(D^+\to\pi^+\gamma^*)}(q^2) = \sum_{V=\rho,\omega,\phi} \frac{r_V\, e^{i\varphi_V}}{(m_V^2 - q^2 - i\sqrt{q^2}\Gamma_V(q^2))} + \int_{s_0^h}^{\infty} ds \frac{\rho_h(s)}{(s - q^2 - i\epsilon)}$$

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$$r_V = \kappa_V f_V |A_{D^+V\pi^+}|$$

$k_\rho = 1/\sqrt{2}, k_\omega = 1/(3\sqrt{2}), k_\phi = -1/3$: Normalised to valence quark content of V

$$|f_V| = \frac{1}{|k_V|}\sqrt{\frac{3m_V BR(V\to e^+e^-)\Gamma_{tot}^V}{4\pi\alpha_{em}^2}}$$

$$|A_{D^+V\pi^+}| = \left(\frac{8\pi BR(D^+\to V\pi^+)}{\tau(D)G_F^2|V_{cs}|^2|V_{ud}|^2 m_{D^+}^3 \lambda_{D^+V\pi^+}^{3/2}}\right)^{1/2}$$

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- Unknown spectral densities (too complicated) : Need to be modelled/parametrised.

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We consider two models: **z-parametrisation** and **extended resonance model**.

Z-parametrization

- Using once subtracted dispersion relation with q_0^2 as subtraction point.

$$\mathcal{A}^{(D^+\to\pi^+\gamma^*)}(q^2) = \mathcal{A}^{(D^+\to\pi^+\gamma^*)}(q_0^2) + \left[(q^2-q_0^2)\sum_{V=\rho,\omega,\phi}\frac{r_V\, e^{i\varphi_V}}{(m_V^2-q_0^2)(m_V^2-q^2-im_V\Gamma_V^{tot})} + \int_{s_{th}}^{\infty} ds\,\frac{(q^2-q_0^2)\,\rho_h(s)}{(s-q_0^2)(s-q^2-i\epsilon)}\right]$$

$$\int_{s_{th}}^{\infty} ds\,\frac{(q^2-q_0^2)\,\rho_h(s)}{(s-q_0^2)(s-q^2-i\epsilon)} = \sum_{k=1}^{K} a_k\left([z(q^2)]^k - [z(q_0^2)]^k\right)$$

with,

$$z(q^2) = \frac{\sqrt{s_{th}-q^2}-\sqrt{s_{th}}}{\sqrt{s_{th}-q^2}+\sqrt{s_{th}}}$$

With $s_{th} = \left(m_{\rho'} - \frac{\Gamma_{\rho'}}{2}\right)^2 \approx 1.60 \pm 0.15\ \text{GeV}^2$

and $a_k =$ Complex coefficients

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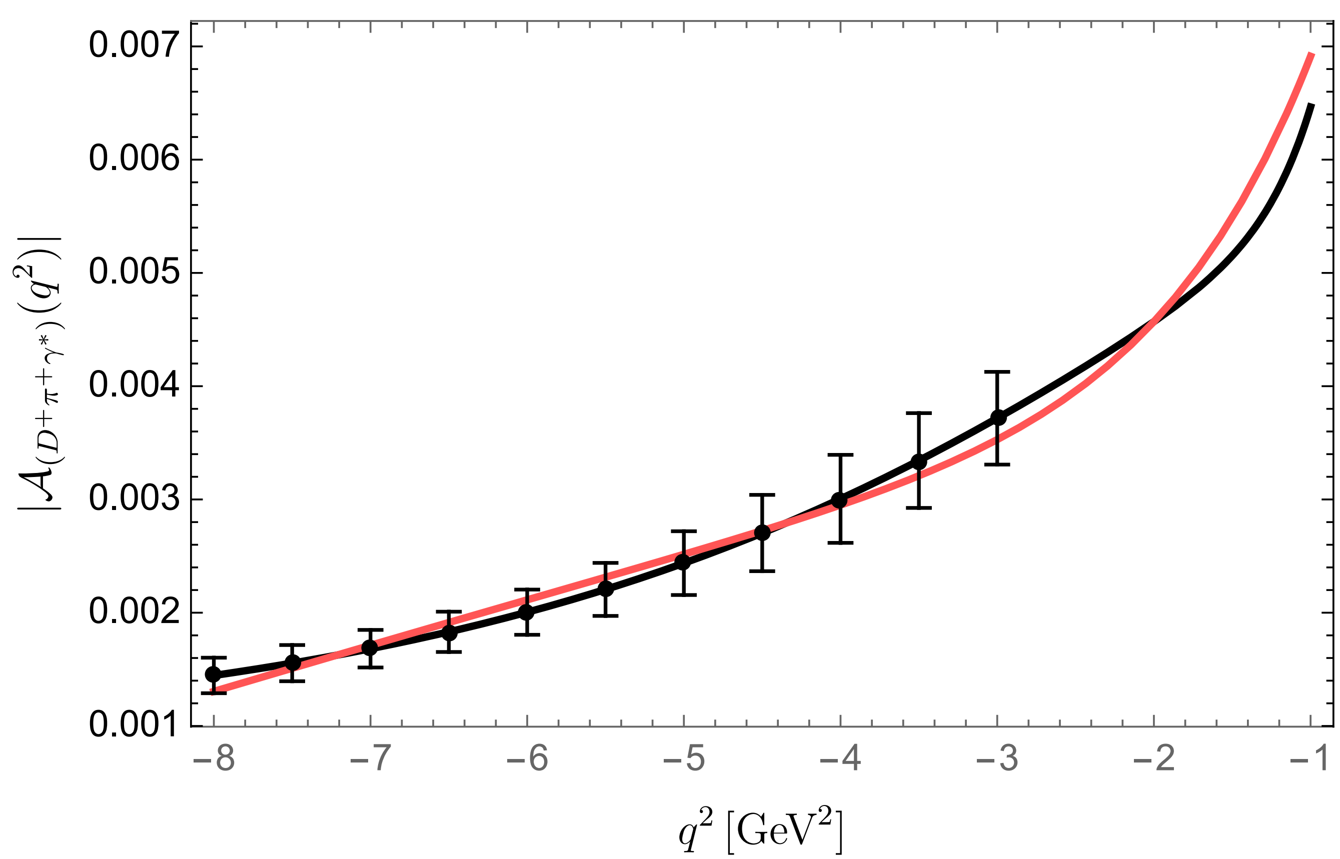
- We truncate the z-parametrisation at $K = 2$

$\Longrightarrow$ 7 **fit parameters**: 3 phases + 4 z-parameters

- Fit results suggests:

$$\phi_\rho - \phi_\omega \sim 0 \text{ and } \phi_\rho - \phi_\phi \sim \pi$$



Z-parametrization

- Using once subtracted dispersion relation with q_0^2 as subtraction point.

$$\mathcal{A}^{(D^+\to\pi^+\gamma^*)}(q^2) = \mathcal{A}^{(D^+\to\pi^+\gamma^*)}(q_0^2) + \left[(q^2-q_0^2)\sum_{V=\rho,\omega,\phi}\frac{r_V\, e^{i\varphi_V}}{(m_V^2-q_0^2)(m_V^2-q^2-im_V\Gamma_V^{tot})} + \int_{s_{th}}^{\infty} ds\frac{(q^2-q_0^2)\,\rho_h(s)}{(s-q_0^2)(s-q^2-i\epsilon)}\right]$$

$$\int_{s_{th}}^{\infty} ds\frac{(q^2-q_0^2)\,\rho_h(s)}{(s-q_0^2)(s-q^2-i\epsilon)} = \sum_{k=1}^{K} a_k\left([z(q^2)]^k - [z(q_0^2)]^k\right)$$

with, $z(q^2) = \dfrac{\sqrt{s_{th}-q^2}-\sqrt{s_{th}}}{\sqrt{s_{th}-q^2}+\sqrt{s_{th}}}$ With $s_{th} = \left(m_{\rho'} - \dfrac{\Gamma_{\rho'}}{2}\right)^2 \approx 1.60 \pm 0.15\ \text{GeV}^2$

and $a_k =$ Complex coefficients

- We truncate the z-parametrisation at $K = 2$

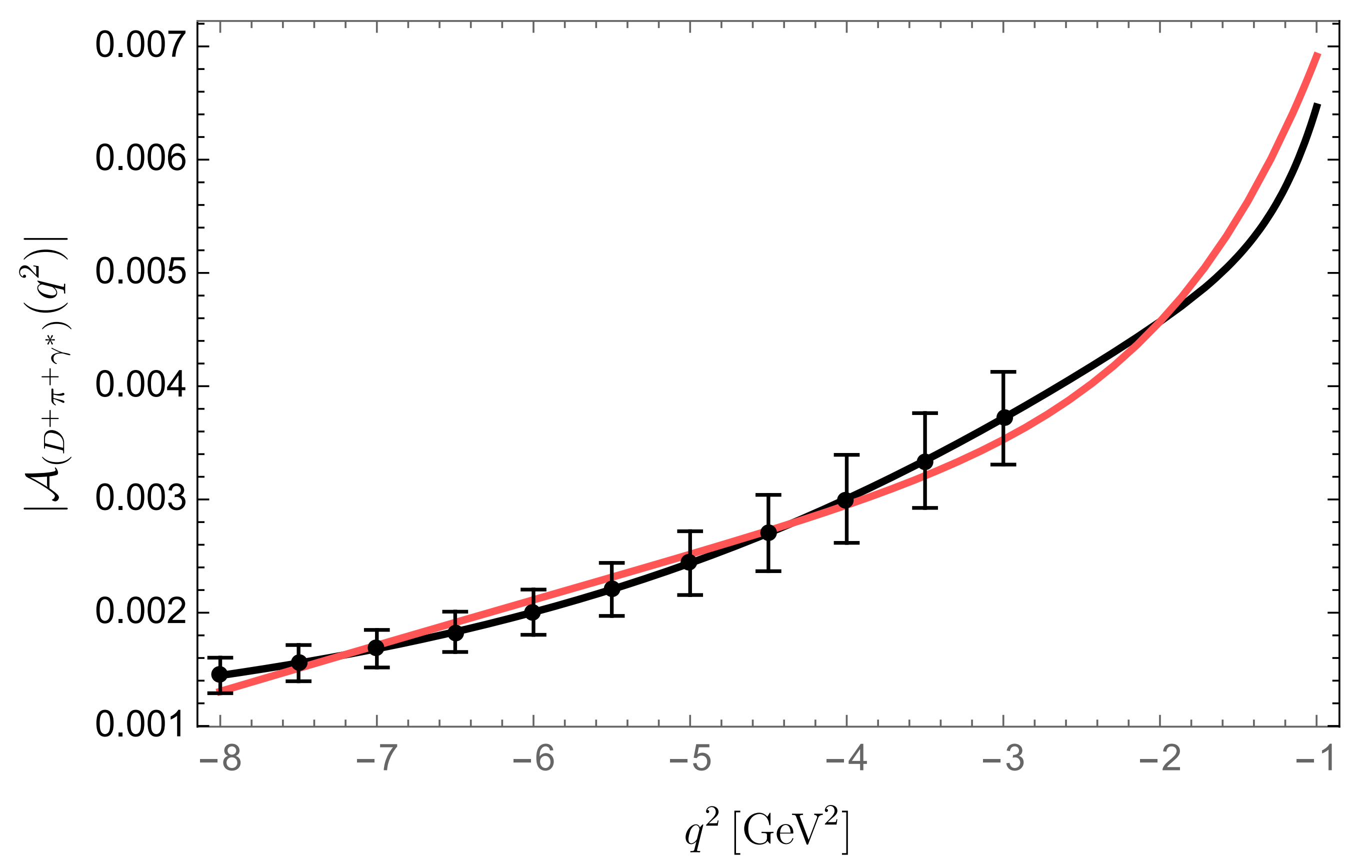
$\Longrightarrow$ 7 **fit parameters**: 3 phases + 4 z-parameters

- Fit results suggests:

$\phi_\rho - \phi_\omega \sim 0$ and $\phi_\rho - \phi_\phi \sim \pi$

Limitations

- can provide predictions only at $q^2 < m_{\rho'}^2$.
- Unknown systematics due to series truncation : Expected to be small.



Extended resonance model

- Using unsubtracted dispersion relation:

$$\mathscr{A}^{(D^+\to\pi^+\gamma^*)}(q^2) = \sum_{V=\rho,\omega,\phi} \frac{r_V\, e^{i\varphi_V}}{(m_V^2 - q^2 - i\sqrt{q^2}\Gamma_V(q^2))} + \int_{s_0^h}^{\infty} ds \frac{\rho_h(s)}{(s - q^2 - i\epsilon)}$$

$$\int_{s_0^h}^{\infty} ds \frac{\rho_h(s)}{(s - q^2 - i\epsilon)} = \sum_{V'=\rho',\phi'} \frac{r'_V e^{i\varphi'_V}}{(m_V^2 - q^2 - i\sqrt{q^2}\Gamma_V(q^2))} + \frac{r_{eff}\, e^{i\varphi_{eff}}}{m_{eff}^2 - q^2 - i\sqrt{q^2}\Gamma_{eff}(q^2)}$$

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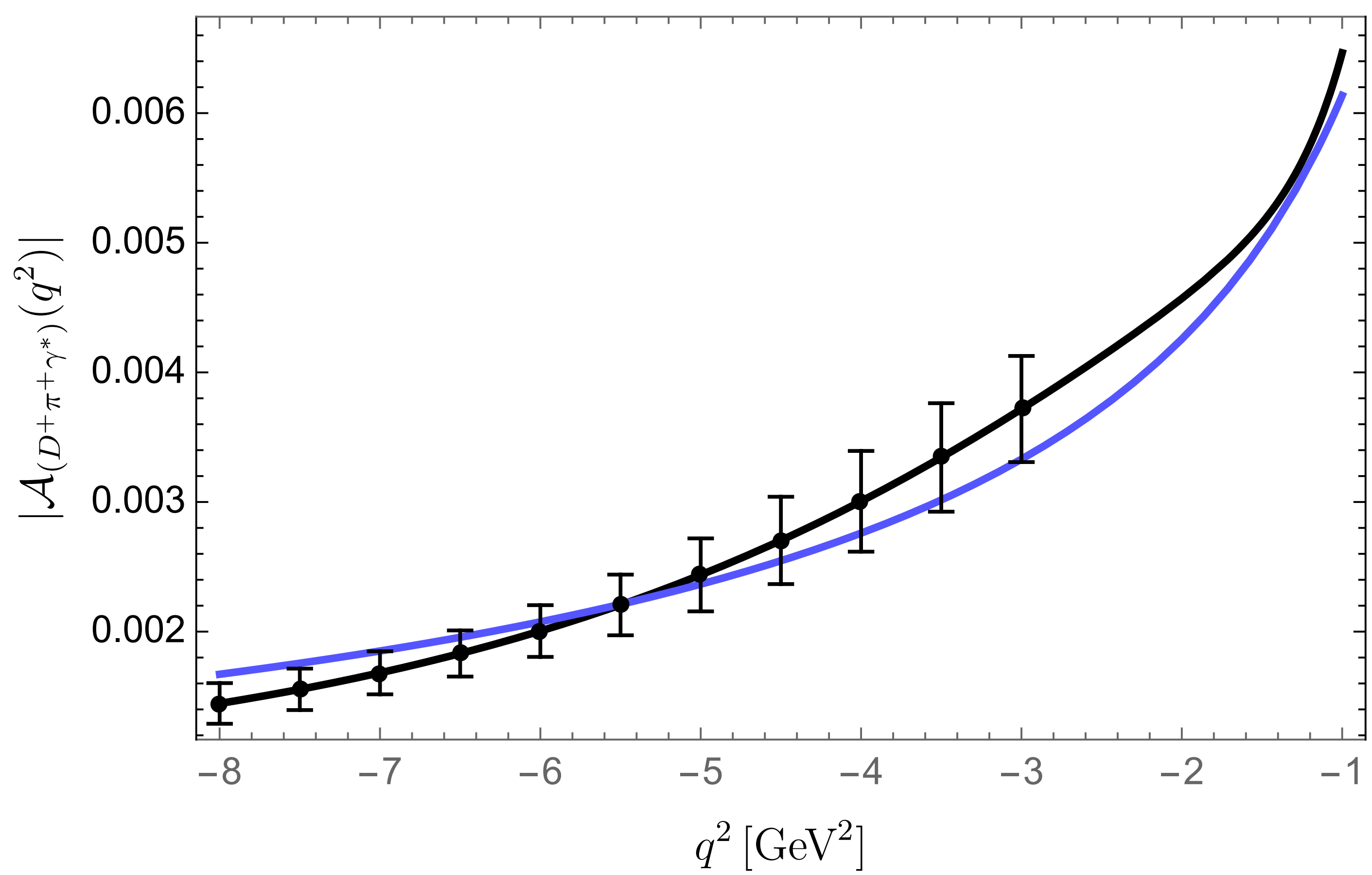
- Fit results:

$$\varphi_\rho = 5.944\,,\quad \varphi_\omega = 5.944\,,\quad \varphi_\phi = 2.797\,,$$
$$\varphi_{\rho'} = 5.929\,,\quad \varphi_{\phi'} = 5.925\,,$$
$$|r_{\rm eff}| = 3.538\times 10^{-2}\,,\quad Arg[r_{\rm eff}] = 3.312\,.$$



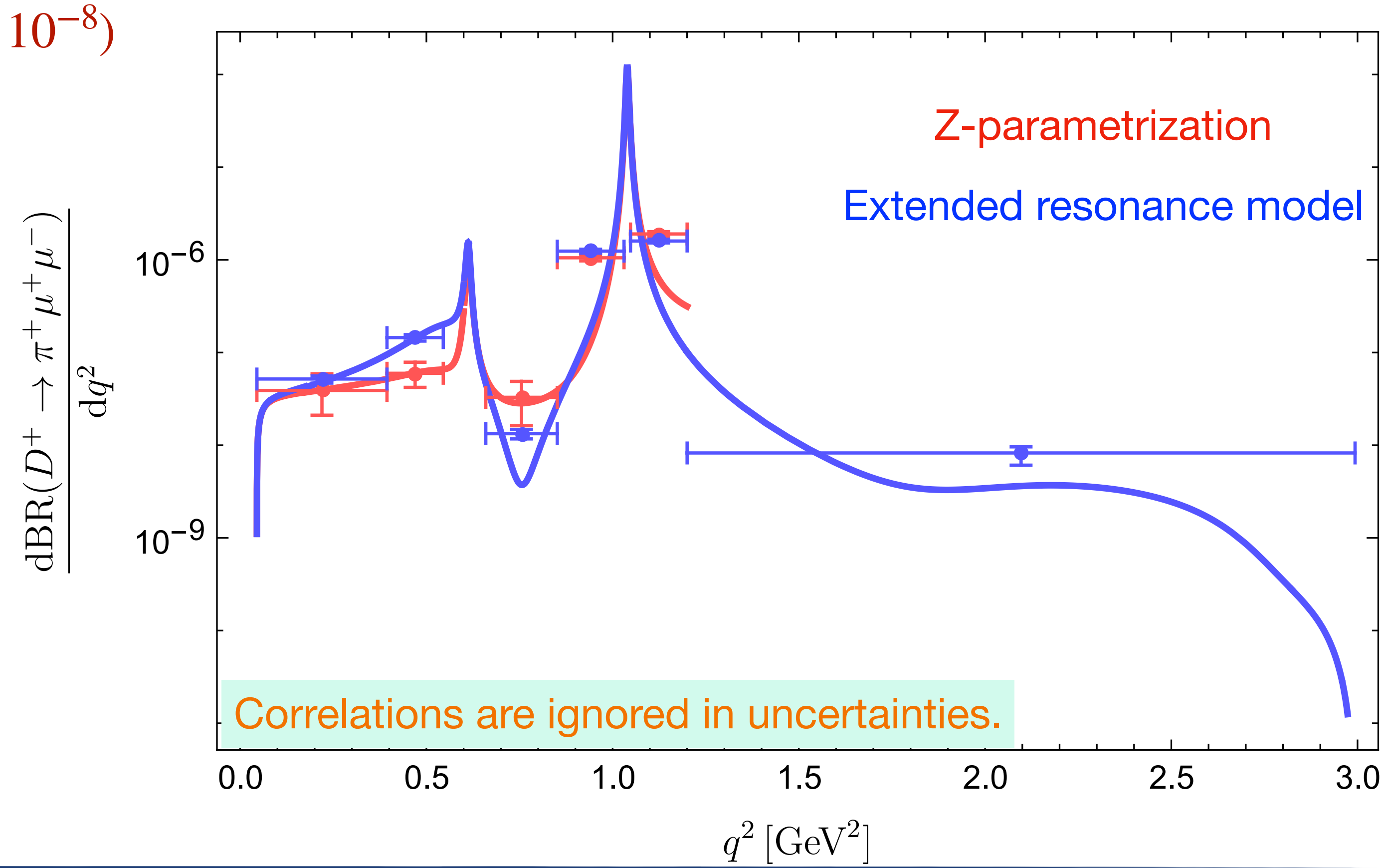
Results

- Binned branching fraction:

$$\Delta\mathcal{B}_{(D^+\to\pi^+\mu^+\mu^-)}(q^2_{min}, q^2_{max}) \equiv \frac{1}{(q^2_{max}-q^2_{min})}\int_{q^2_{min}}^{q^2_{max}} ds\, \frac{d\mathrm{BR}(D^+\to\pi^+\mu^+\mu^-)}{ds},$$

Bin	$q^2_{\rm min}$	$q^2_{\rm max}$	$(q^2_{\rm max}-q^2_{\rm min})\,\Delta\mathcal{B}^{(disp-z)}$	$(q^2_{\rm max}-q^2_{\rm min})\,\Delta\mathcal{B}^{(disp-res)}$
I	$4m_\mu^2$	$(m_\rho-\Gamma_\rho)^2$	$1.36^{+0.70}_{-0.63}$	$1.81^{+0.14}_{-0.17}$
II	$(m_\rho-\Gamma_\rho)^2$	$(m_\rho-\Gamma_\rho/4)^2$	$0.90^{+0.29}_{-0.27}$	$2.19^{+0.17}_{-0.18}$
III	$(m_\rho+\Gamma_\rho/4)^2$	$(m_\rho+\Gamma_\rho)^2$	$0.63^{+0.31}_{-0.32}$	$0.26^{+0.03}_{-0.03}$
IV	$(m_\rho+\Gamma_\rho)^2$	$(m_\phi-\Gamma_\phi)^2$	$18.88^{+1.39}_{-1.40}$	$22.27^{+1.08}_{-1.09}$
V	$(m_\phi+\Gamma_\phi)^2$	$1.2\ \mathrm{GeV}^2$	$28.93^{+1.70}_{-2.16}$	$24.27^{+1.12}_{-1.13}$
VI	$1.2\ \mathrm{GeV}^2$	$(m_{D^+}-m_{\pi^+})^2$	–	$1.48^{+0.88}_{-0.39}$
LHCb regions	$4m_\mu^2$ $(1.250\ \mathrm{GeV})^2$	$(0.525\ \mathrm{GeV})^2$ $(m_{D^+}-m_{\pi^+})^2$	$0.81^{+0.43}_{-0.38}$ –	$0.93^{+0.08}_{-0.10}$ $0.40^{+0.29}_{-0.25}$

(In units 10^{-8})



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- Results for two models agrees within uncertainties (except bin-II).
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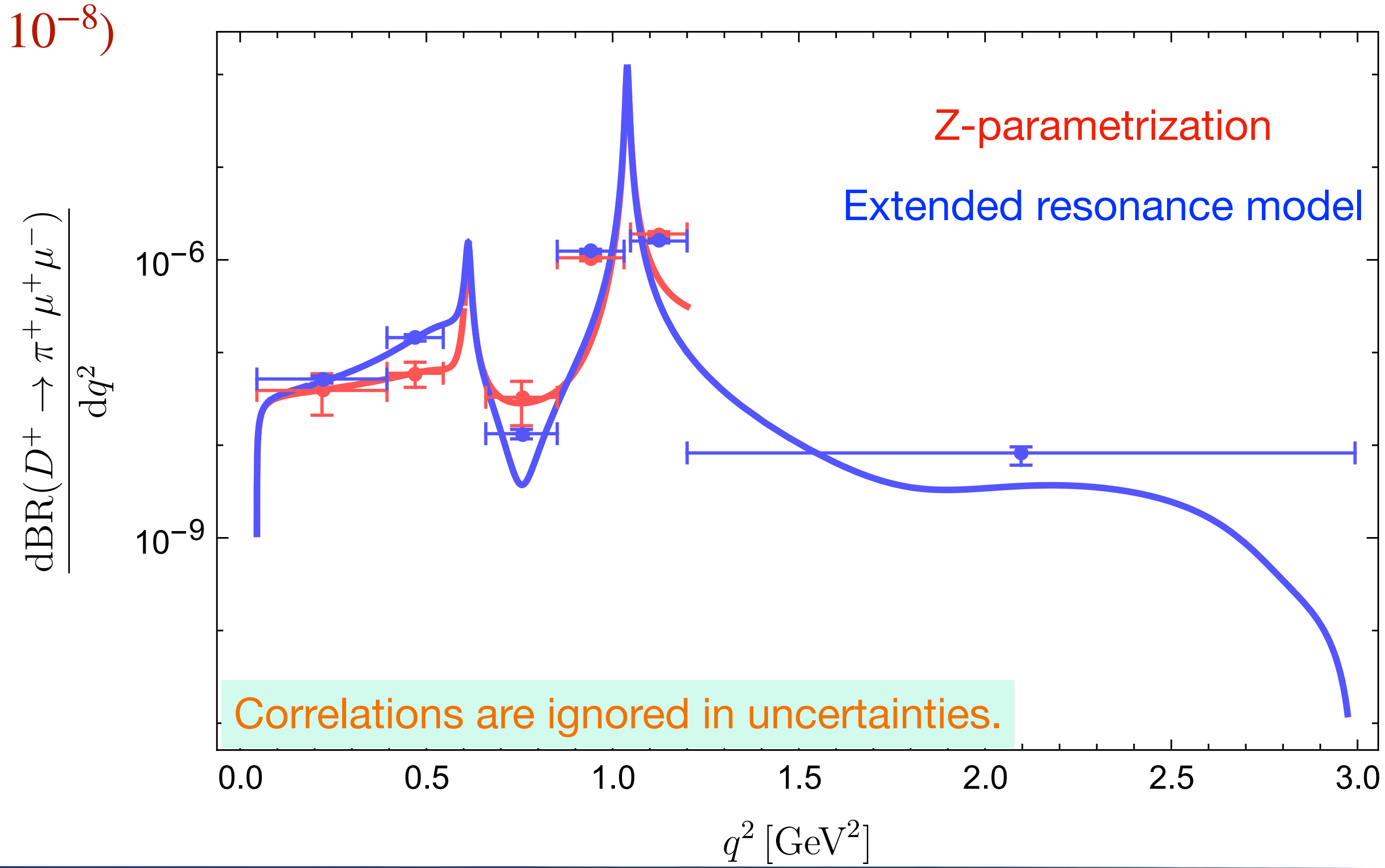

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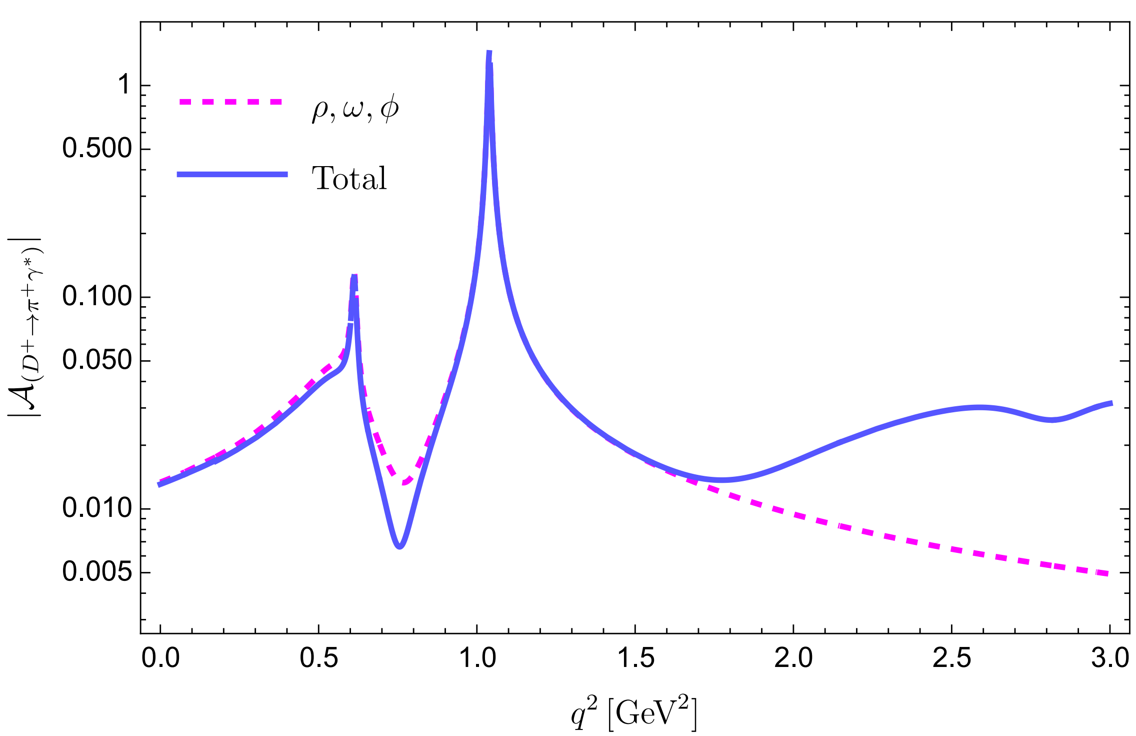

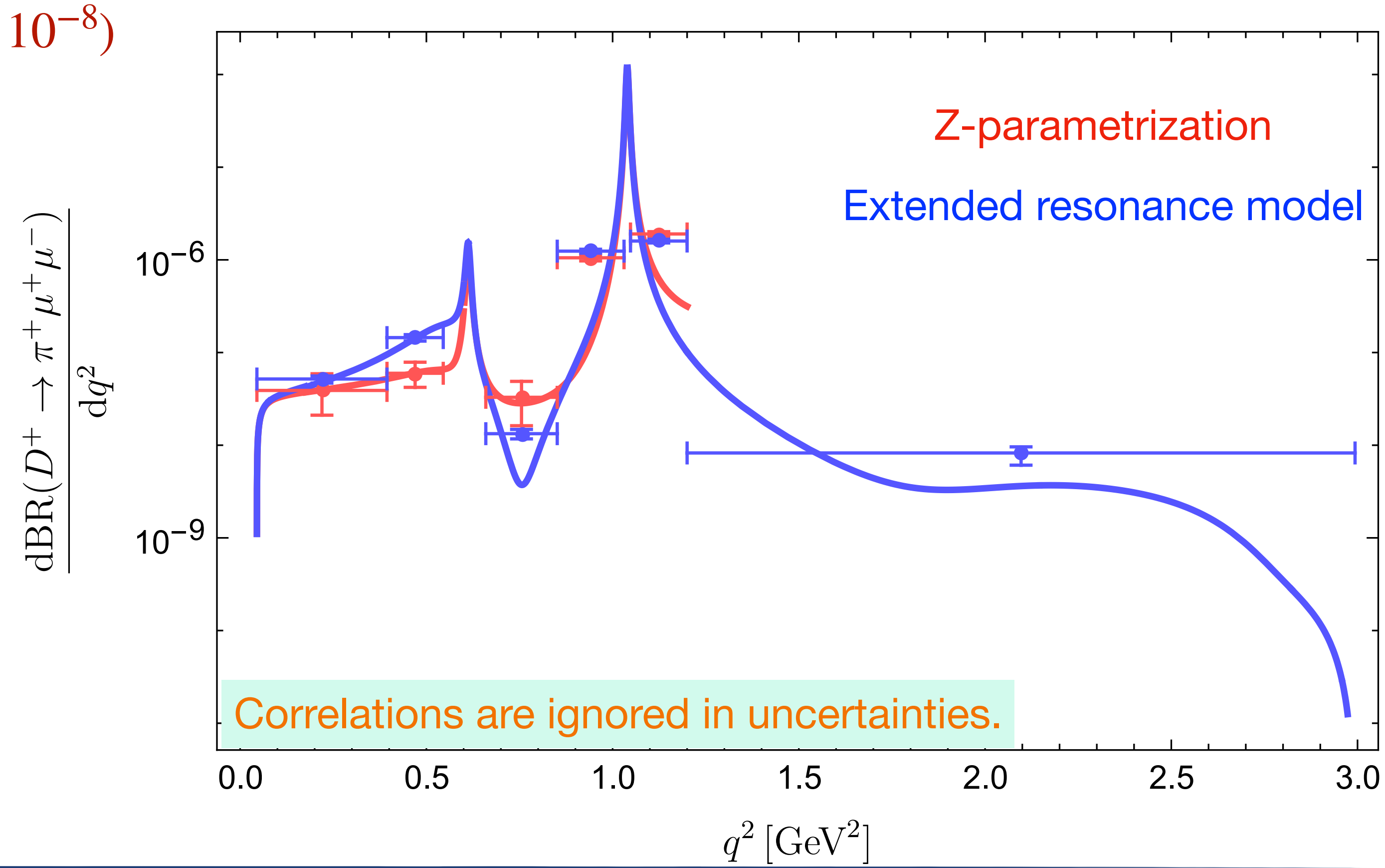

Can we probe this dynamics somewhere else?

The use of U-spin

- Combining : GIM limit, $\lambda_b = 0, \lambda_d = -\lambda_s$ with $SU(3)_{fl}$ limit, $m_s = m_{u,d}$ **(Only annihilation topology)**
- The Hamiltonians of Cabibbo favoured (CF), Singly Cabibbo Suppressed (CS), and Doubly CS modes form a U-triplet:

$$O_1^{(U=1)} \equiv \begin{pmatrix} (\bar{u}_L\gamma_\mu s_L)(\bar{d}_L\gamma^\mu c_L) \\ \frac{1}{\sqrt{2}}\left[(\bar{u}_L\gamma_\mu d_L)(\bar{d}_L\gamma^\mu c_L) - (\bar{u}_L\gamma_\mu s_L)(\bar{s}_L\gamma^\mu c_L)\right] \\ (\bar{u}_L\gamma_\mu d_L)(\bar{s}_L\gamma^\mu c_L) \end{pmatrix} = \begin{pmatrix} |1,+1\rangle \\ -|1,0\rangle \\ |1,-1\rangle \end{pmatrix}$$

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$\langle P^+_{(U=1/2)} | j_\mu^{em}(x) O_1^{(U=1)} | D^+_{(U=1/2)} \rangle$

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U-spin relations

$$\mathscr{A}^{(D^+\to\pi^+\gamma^*)}(q^2) = -\mathscr{A}^{(D_s^+\to K^+\gamma^*)}(q^2) = \mathscr{A}^{(D_s^+\to\pi^+\gamma^*)}(q^2) = \mathscr{A}^{(D^+\to K^+\gamma^*)}(q^2)$$

$$\mathscr{A}^{(D^0\to\bar{K}^0\gamma^*)}(q^2) = \mathscr{A}^{(D^0\to K^0\gamma^*)}(q^2) = -\frac{1}{2}\mathscr{A}^{(D^0\to\pi^0\gamma^*)}(q^2) + \frac{\sqrt{3}}{2}\mathscr{A}^{(D^0\to\eta^0\gamma^*)}(q^2)$$

$$\mathscr{A}^{(D^0\to\eta_8\gamma^*)}(q^2) = -\sqrt{3}\mathscr{A}^{(D^0\to\pi^0\gamma^*)}(q^2)$$

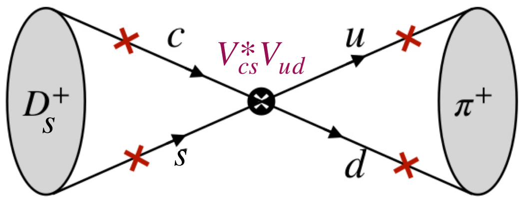
$$\mathscr{A}^{(D^0\to\eta'\gamma^*)}(q^2) = 0$$

D^0, η': U-spin singlets.

- Other $D_{(s)} \to P\ell^+\ell^-$ $(P = \pi, K, \eta)$; CF and doubly CS modes also interesting : can help to disentangle annihilation topology.

Byproduct: $D_s^+ \to \pi^+\ell^+\ell^-$

- Cabibbo favoured (CF) mode $\Longrightarrow$ larger branching fraction.
- Can be simply computed as an extension of the method: Setting $m_s \neq 0$
- Better chance at experiments $\Longrightarrow$ can be helpful in better understanding of annihilation topologies and overall strong dynamics involved.



only annihilation topologies

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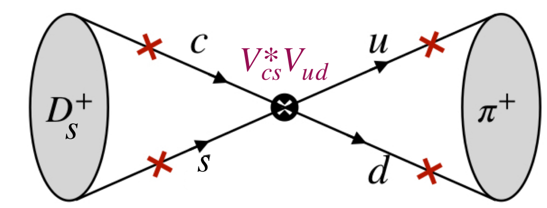

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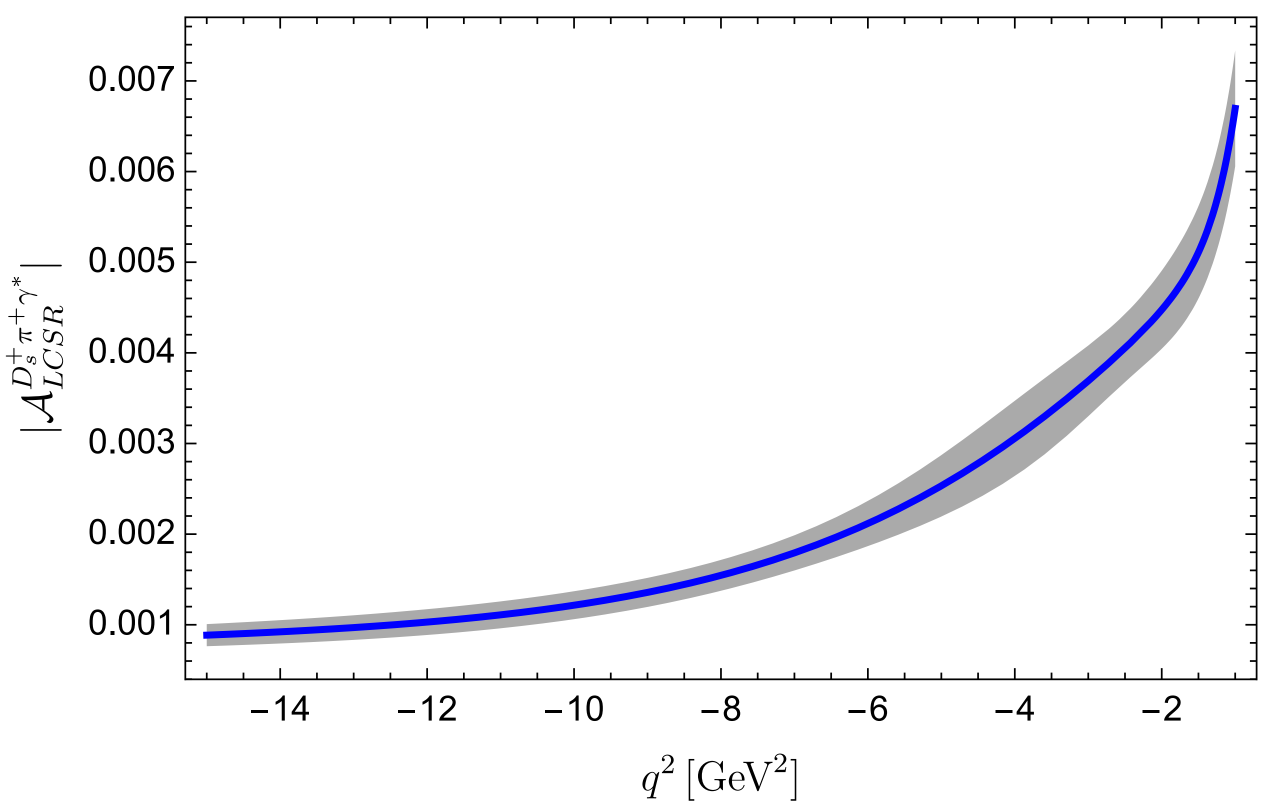

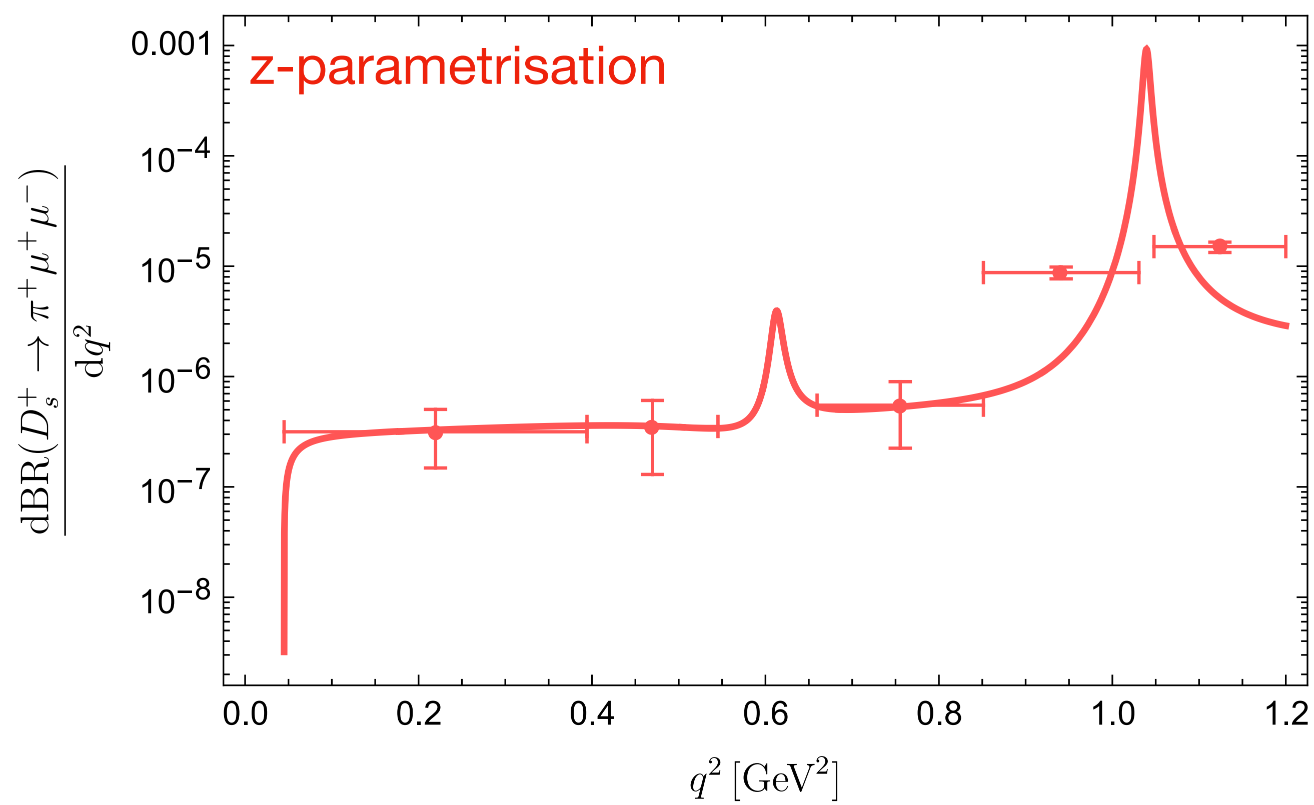

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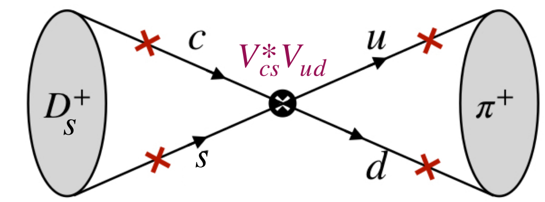

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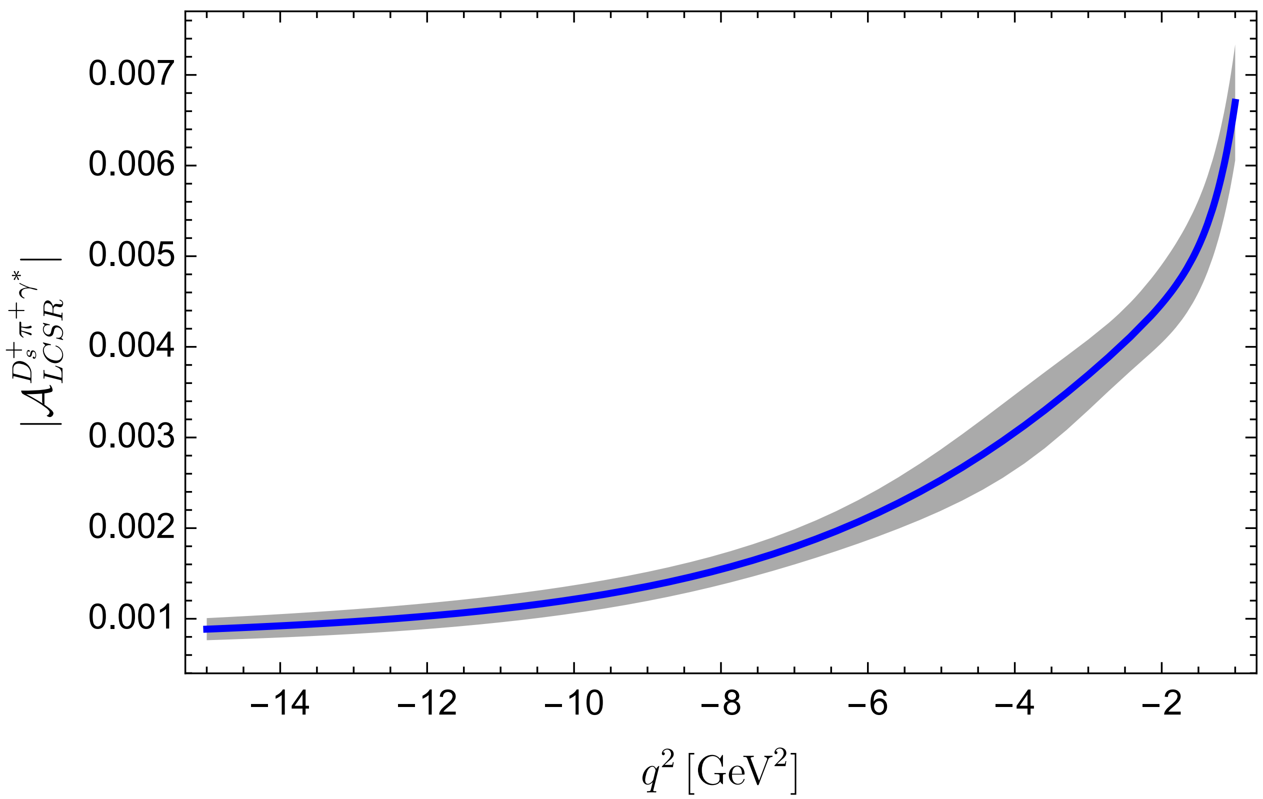

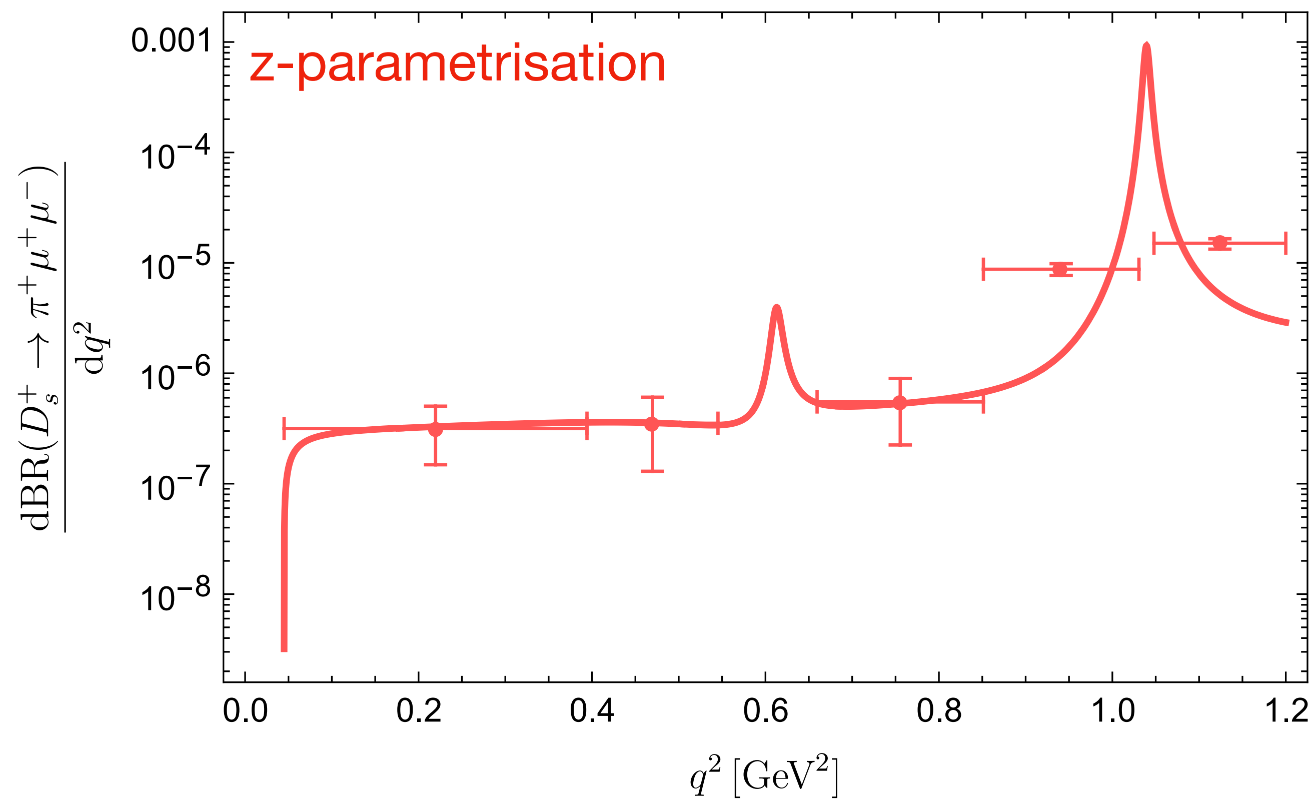

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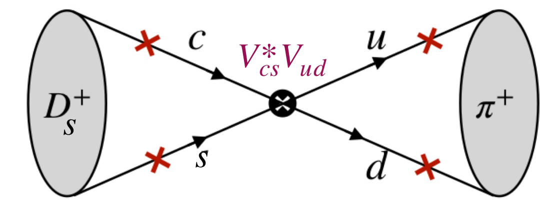

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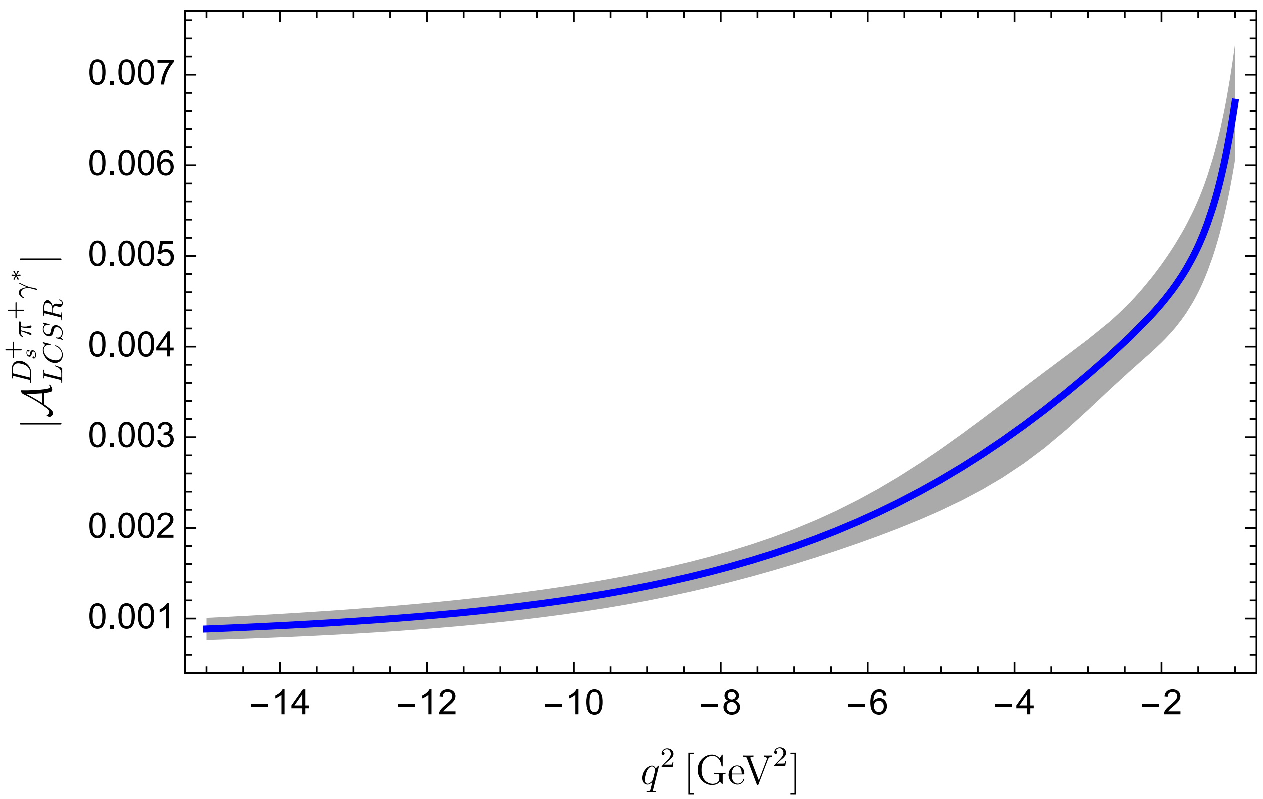

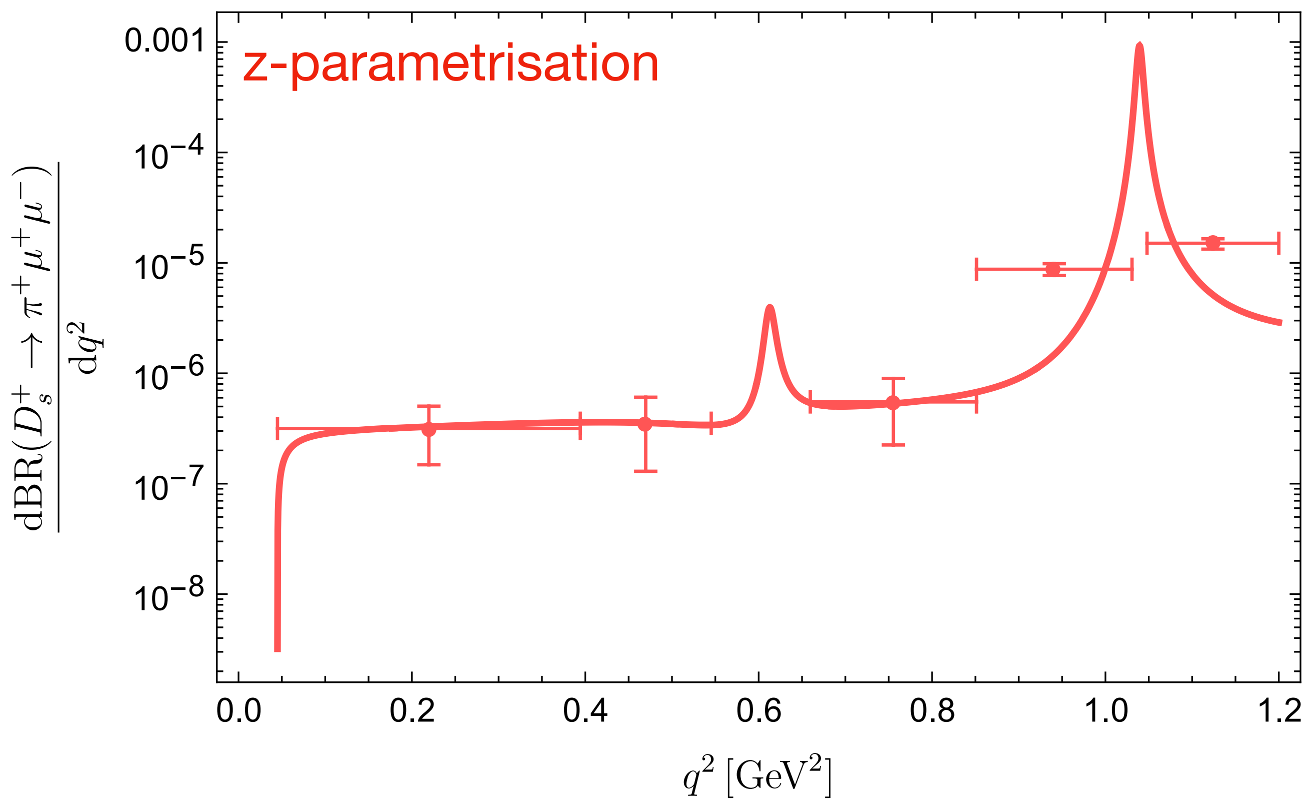

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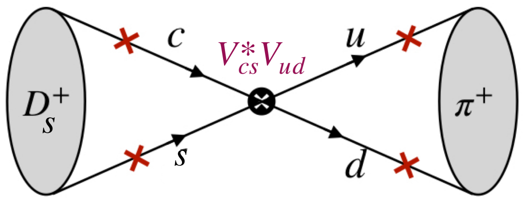

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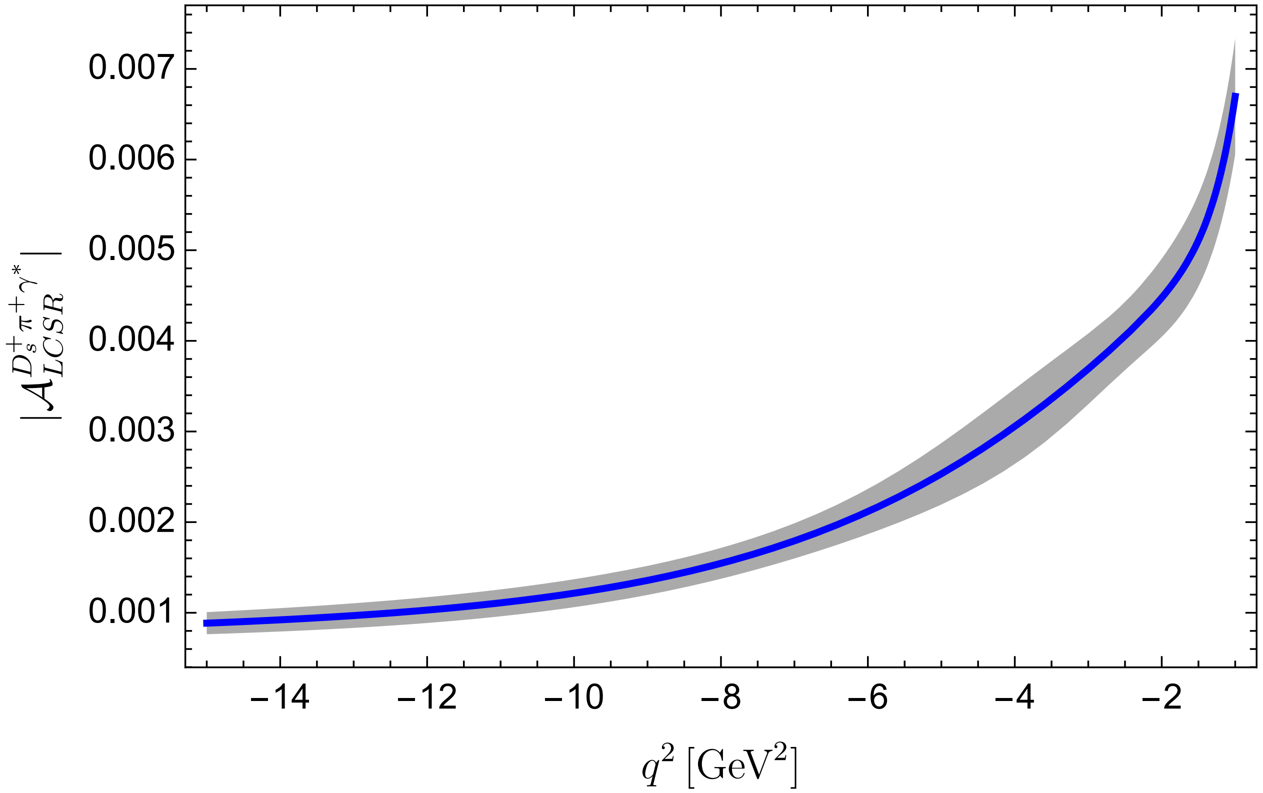

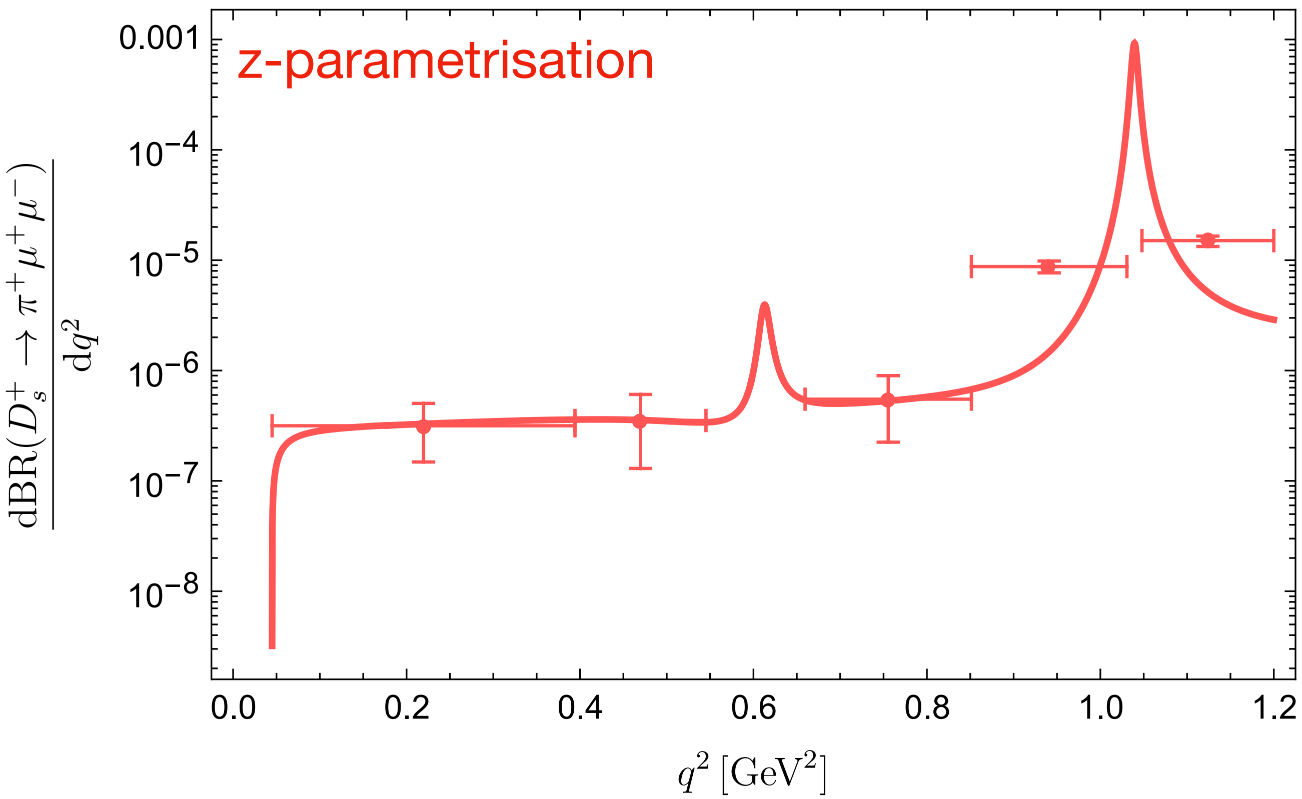

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Numerical results suggests ~20% U-spin violation at amplitude level.

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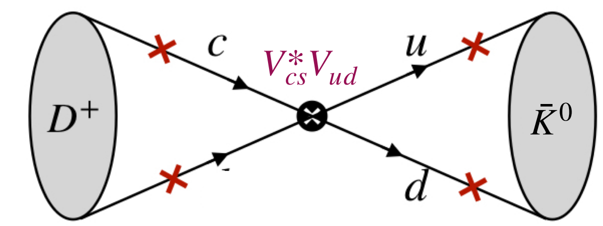
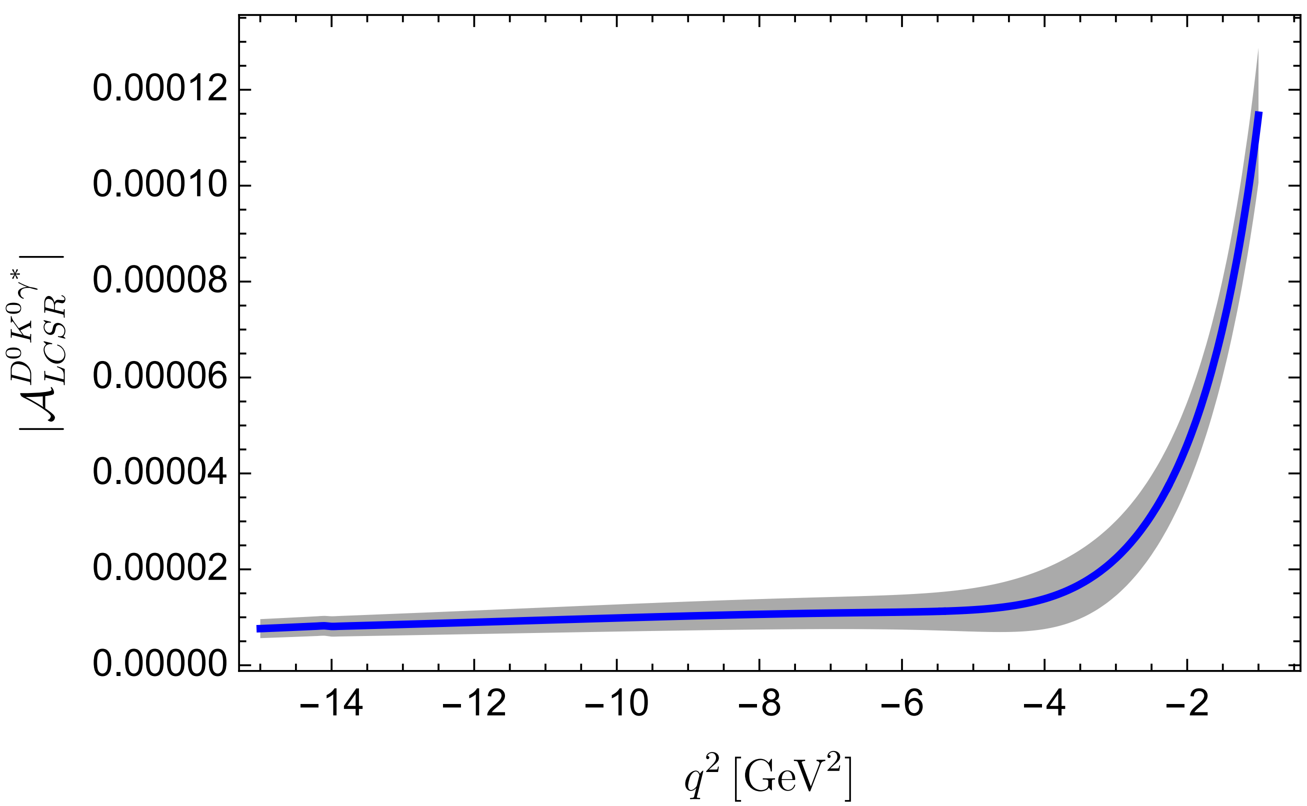
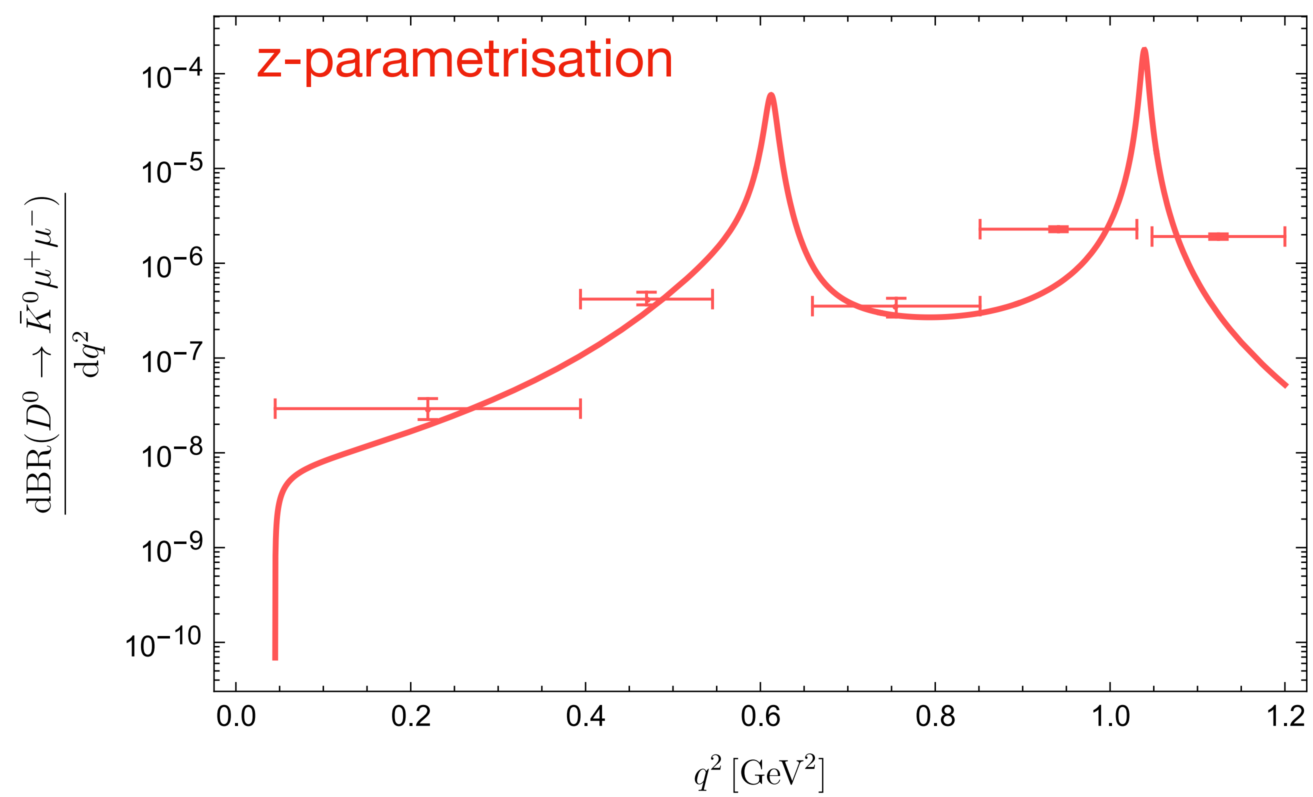


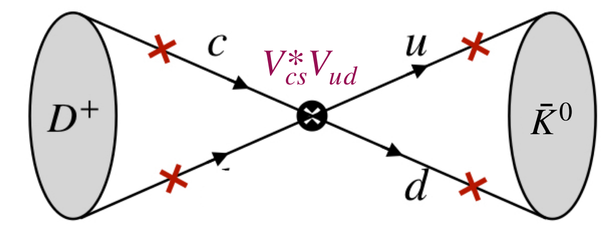
only annihilation topologies





Byproduct: $D^0 \to \bar{K}^0 \ell^+ \ell^-$

- Cabibbo favoured (CF) mode $\Longrightarrow$ larger branching fraction.
- Can be computed as an extension of method after some changes.
- Better chance at experiments $\Longrightarrow$ can be helpful in better understanding of annihilation topologies and overall strong dynamics involved.



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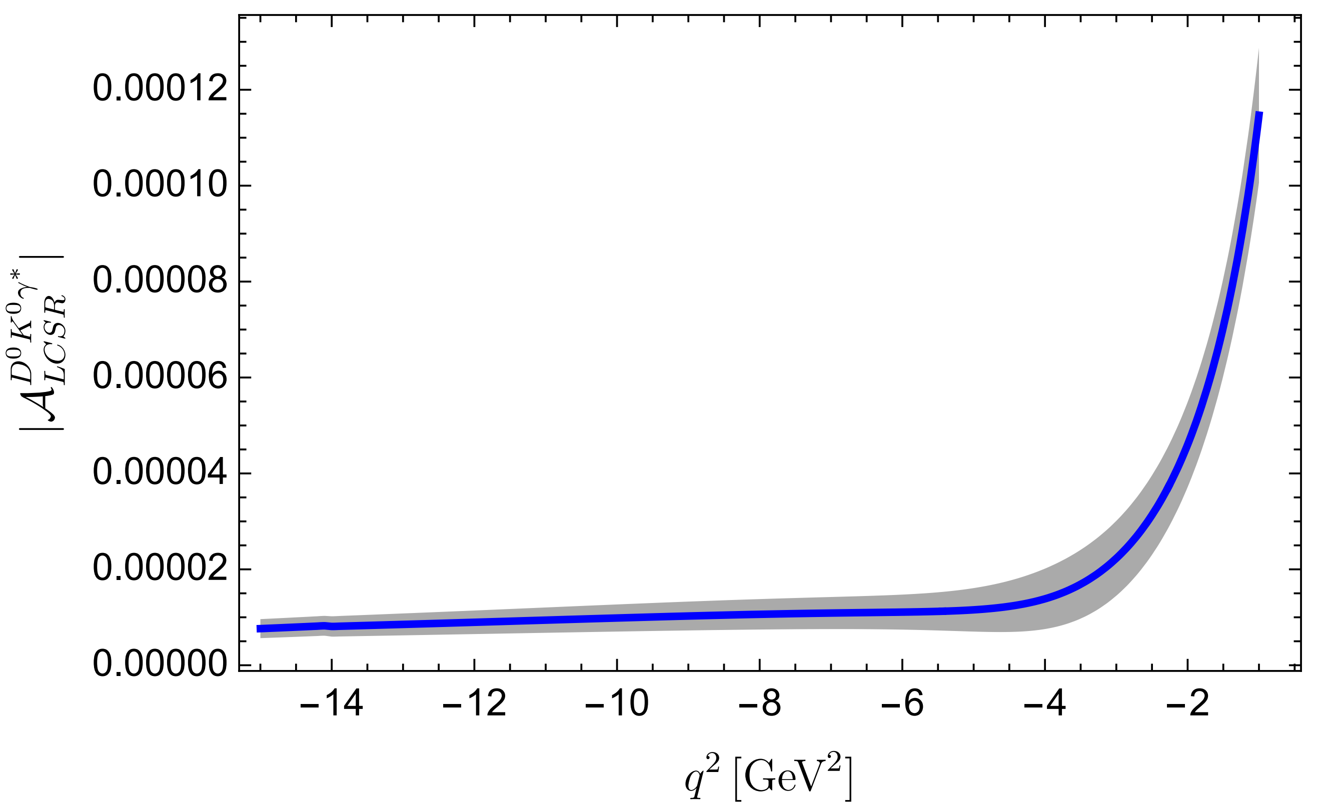

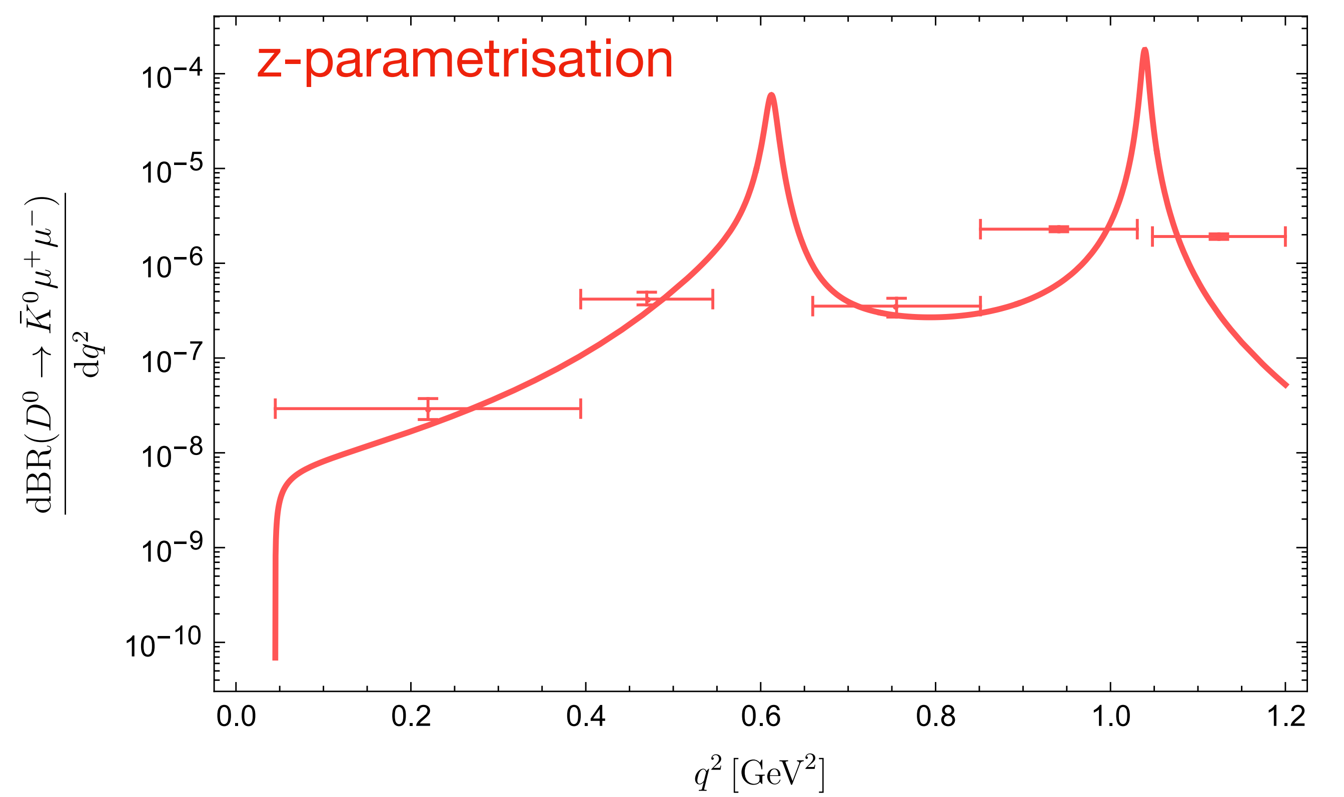

- Not enough experimental data for extended resonance model.
- Present experimental bounds are very far from the predicted values.

Summary and Outlook

- We study $D^+ \to \pi^+\ell^+\ell^-$ decays using LCSR assisted dispersion relation.
- Amplitude for $D^+ \to \pi^+\ell^+\ell^-$ is mainly dominated by weak annihilation topology generated by $O_{1,2}$."Loop" and "Short distance" contributions e.g. due to O_9 are tiny.
- Contribution from higher resonances is important especially for intermediate and high q^2 region.
- Fits suggest a constructive interference between ρ/ω and ϕ resonances.
- Perform U-spin analysis to relate $D^+ \to \pi^+\ell^+\ell^-$ to Cabibbo Favoured (CF) modes.
- CF modes includes only A-topologies $\Longrightarrow$ Can be helpful to understand QCD dynamics involved.
- CF modes ($D_s^+ \to \pi^+\ell^+\ell^-$ and $D^0 \to K^0\ell^+\ell^-$) are also presented.
- Comparison of $D^+ \to \pi^+\ell^+\ell^-$ & $D_s^+ \to \pi^+\ell^+\ell^-$ suggests ~20% U-spin violation at amplitude level.

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- Future prospects:
  - Perturbative and soft-gluon corrections to annihilation: corrections to the LCSR results.
  - Estimates for other CF and SCS modes.
  - Varying resonance ansatz in the dispersion relation (including ρ', ω', ϕ').
  - Use of other statistical methods : Bayesian fits

Suggestions/Questions for Experiments

- Can we obtain the full q^2 spectrum?
  - Can be useful as an independent check for the method.
  - Hadronic dispersion model can be fitted directly to the data to estimate the model parameters.
- The bin between ρ and ϕ resonances is very interesting.
- Look for the CF modes.
  - Larger branching fractions.
  - Presents bounds for $D_s^+ \to \pi^+ \ell\ell$ is very close to predictions while for $D^0 \to \bar{K}^0 \ell\ell$ is very far.
  - Not enough data on the excited vector resonances.

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Thank you for your attention !!

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SUBATOMIC HEROES

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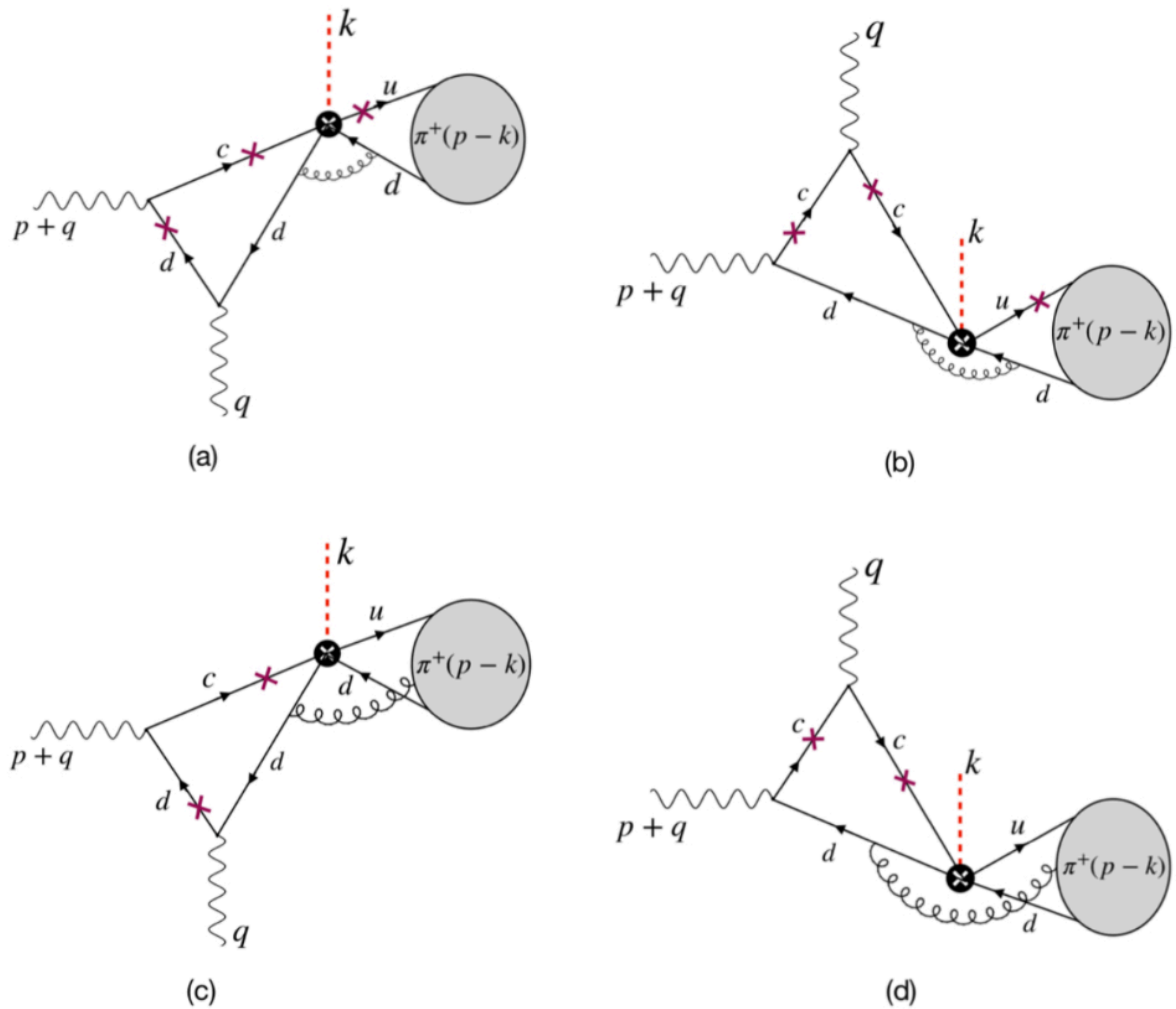
Experimental upper limits

Decay mode	Cabibbo hierarchy	BR, upper limit [12]
$D^+ \to \pi^+\ell^+\ell^-$	SCS	$1.1\times10^{-6}\,(\ell=e)$ $6.7\times10^{-8}\,(\ell=\mu)$ (*)
$D^+ \to K^+\ell^+\ell^-$	DCS	$8.5\times10^{-7}\,(\ell=e)$ (*) $5.4\times10^{-8}\,(\ell=\mu)$ (*)
$D^0 \to \bar{K}^0\ell^+\ell^-$	CF	$2.4\times10^{-5}\,(\ell=e)$ $2.6\times10^{-4}\,(\ell=\mu)$
$D^0 \to \pi^0\ell^+\ell^-$	SCS	$4\times10^{-6}\,(\ell=e)$ $1.8\times10^{-4}\,(\ell=\mu)$
$D^0 \to \eta\ell^+\ell^-$	SCS	$3\times10^{-6}\,(\ell=e)$ $5.3\times10^{-4}\,(\ell=\mu)$
$D^0 \to \eta'\ell^+\ell^-$	SCS	-
$D^0 \to K^0\ell^+\ell^-$	DCS	-
$D_s^+ \to \pi^+\ell^+\ell^-$	CF	$5.5\times10^{-6}\,(\ell=e)$ (*) $1.8\times10^{-7}\,(\ell=\mu)$ (*)
$D_s^+ \to K^+\ell^+\ell^-$	SCS	$3.7\times10^{-6}\,(\ell=e)$ $1.4\times10^{-7}\,(\ell=\mu)$ (*)

[PDG]

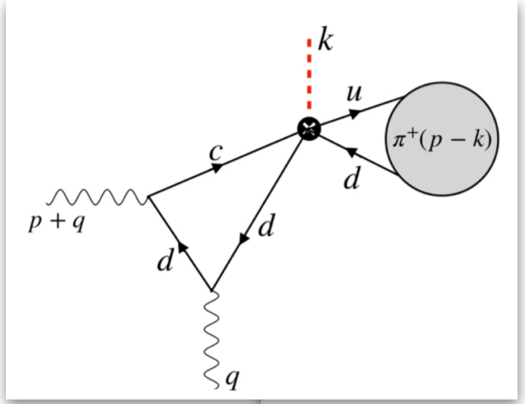
Diagrams for higher order correction to LCSR



Light Cone OPE for $D^+ \to \pi^+\gamma^*$

$$\frac{1}{\pi}\mathrm{Im}F^{(OPE)}(s,q^2,P^2) = \frac{1}{\pi}\mathrm{Im}F^{(a)}(s,q^2,P^2) + \frac{1}{\pi}\mathrm{Im}F^{(b)}(s,q^2,P^2) + \frac{1}{\pi}\mathrm{Im}F^{(c\oplus d)}(s,q^2,P^2) + \frac{1}{\pi}\mathrm{Im}F^{(L)}(s,q^2,P^2)\,.$$

$$\begin{aligned}\frac{1}{\pi}\mathrm{Im}F^{(a)}(s,q^2,P^2) = \; & (3C_1Q_dm_c^2f_\pi)\Bigg\{\frac{-\rho_{cd}(s)}{2q^2(s-q^2)^4}\Big[4m_c^2m_\pi^2\left(s^2+4sq^2+(q^2)^2\right) \\ & +2m_c^2(s-q^2)\left(-2P^2(s+2q^2)+s^2+2sq^2+3(q^2)^2\right)-\left(s^3\left(3m_\pi^2-3P^2+s\right)\right) \\ & +s^2q^2\left(-11m_\pi^2+P^2-3s\right)+(q^2)^3\left(m_\pi^2+P^2+s\right)+s(q^2)^2\left(m_\pi^2-5P^2+3s\right)\Big] \\ & -\frac{\rho_{cdd}(s,q^2)}{2(s-q^2)^4}\Big[-2m_c^4\left(3m_\pi^2(s+q^2)-(s-q^2)(3P^2-s-2q^2)\right) \\ & +2m_c^2(s-q^2)\left(3m_\pi^2(s+q^2)-(s-q^2)(3P^2-s-2q^2)\right) \\ & +(s-q^2)^2\left((P^2-s)(s-q^2)-m_\pi^2(s+q^2)\right)\Big] \\ & -\frac{2\rho_{cdfq}(s,q^2)}{q^2(s-q^2)^4}\Big[-3m_\pi^2s(s+q^2)+P^2(s-q^2)(2s+q^2)-s^3+(q^2)^3\Big]\Bigg\}, \quad \text{(A.42)}\end{aligned}$$



$$\rho_{cd}(s) = -\frac{1}{16\pi^2}\frac{(s-m_c^2)}{s}\,;$$

$$\rho_{cdd}(s,q^2) = -\frac{1}{16\pi^2}\frac{1}{(s-q^2)}\log\left[\frac{-q^2m_c^2}{s(-q^2+s-m_c^2)}\right];$$

$$\rho_{cdfq}(s,q^2) = \frac{1}{64\pi^2}\frac{(s-m_c^2)^2(s+q^2)}{s^2}\,.$$

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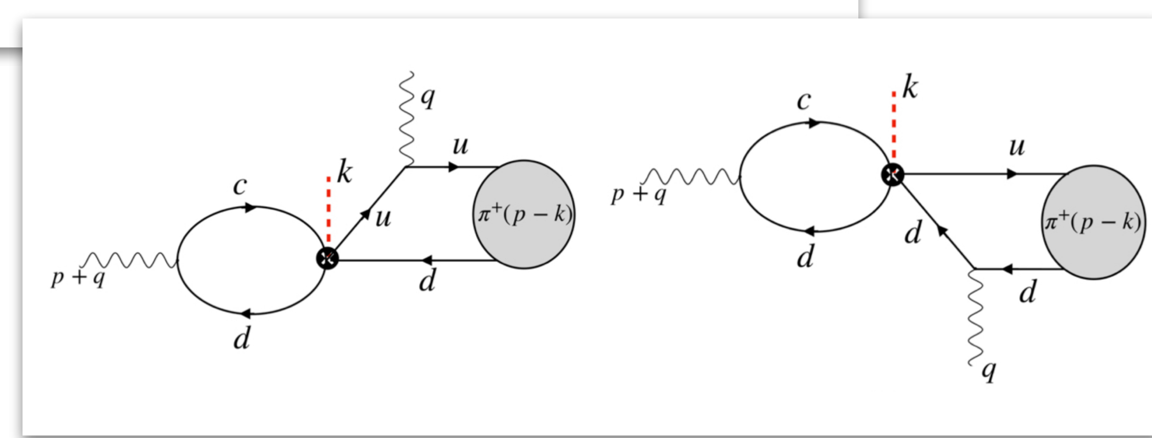
$$\frac{1}{\pi}\int_{m_c^2}^{s_0^D} ds e^{-s/M^2}\mathrm{Im}F^{(c\oplus d)}(s,q^2,m_D^2) = \frac{3}{4}C_1(Q_u - Q_d)m_c^2 f_\pi\left[I_0(M^2)J_0(q^2) + I_1(M^2)J_1(q^2)\right]$$

where the integrals over u are

$$J_n(q^2) = \frac{1}{(m_D^2 - q^2)(-q^2)}\int_0^1 du\,\varphi_\pi(u)\frac{[-m_D^2(2u+1) + 2q^2(u+1)]^{1-n}}{(u-u_*)}\,,$$

with

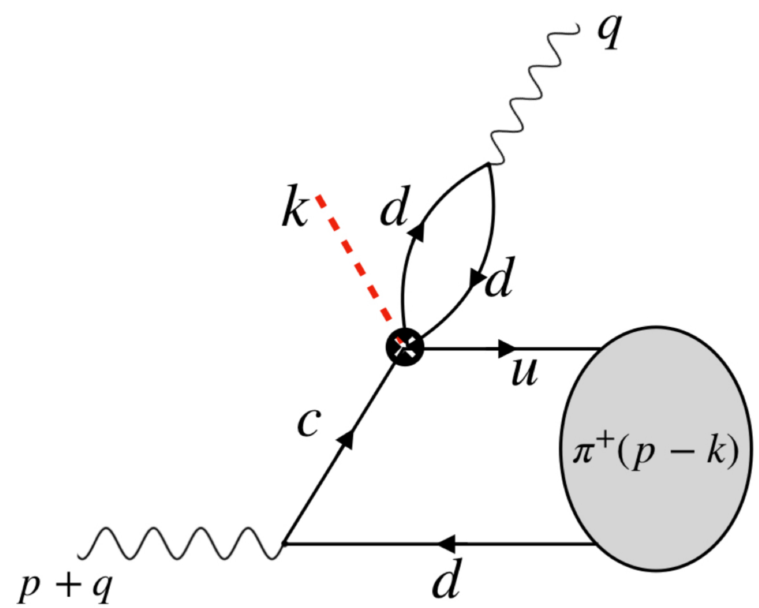
$$u_* \equiv \frac{-q^2}{m_D^2 - q^2}\,.$$

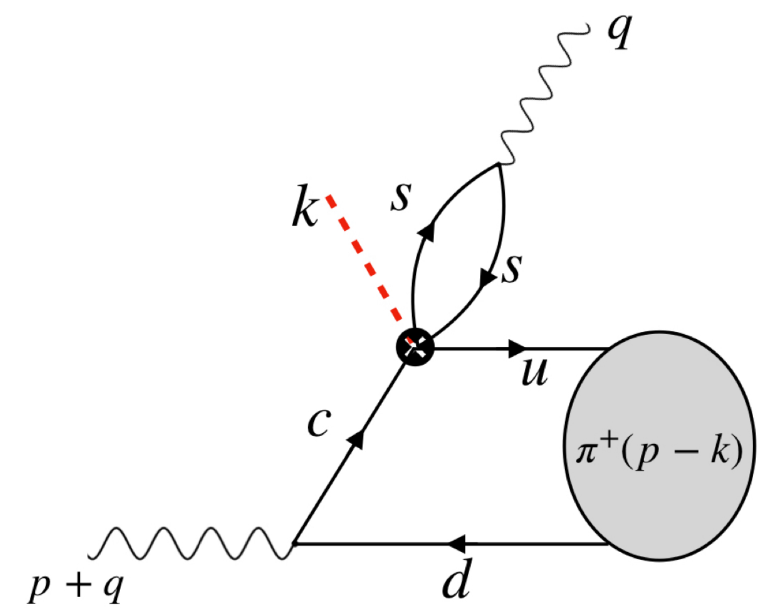


Loop diagram from LCSR

- The correlation function reads as:

$$\mathcal{F}^{(L)}_{\mu}(p,q,k) = -\left[(p\cdot q)q_\mu - q^2 p_\mu\right]\frac{1}{9}\left(C_1+\frac{4}{3}C_2\right)\Pi^{(d-s)}(q^2)G((p+q)^2,q^2,P^2)$$





Loop diagram from LCSR

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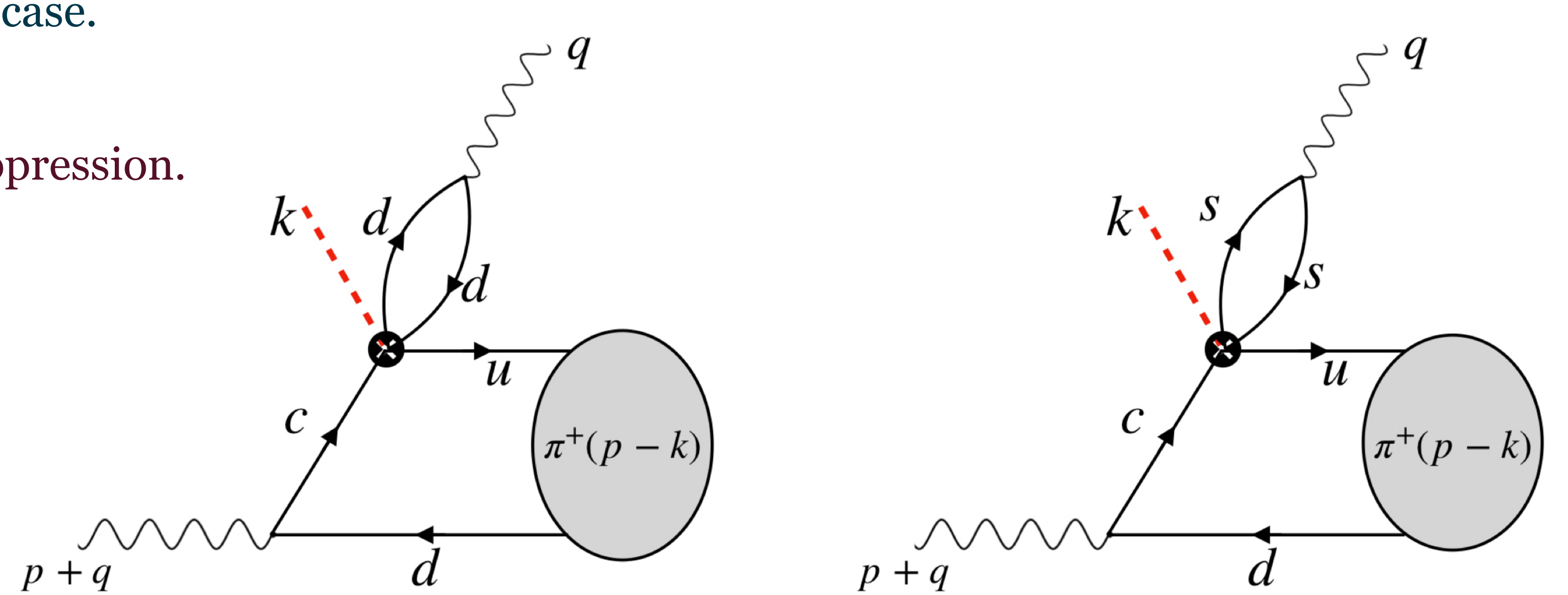
$$\Pi^{d}(q^2) - \Pi^{s}(q^2) \equiv \Pi^{(d-s)}(q^2) = \frac{3}{4\pi^2}\int_0^1 dx\, x\,(1-x)\log\left(\frac{m_s^2 - q^2x(1-x)}{m_d^2 - q^2x(1-x)}\right)$$

$$G_{\rho}(p,q,k) = i\int d^4y\, e^{-i(p+q)\cdot y}\langle \pi^+(p-k)\,|\,T\left\{\left(\bar{u}_L(0)\gamma_{\rho}c_L(0)\right)j_5^D(y)\right\}|0\rangle$$

- Both WCs (C_1 and C_2) contribute in this case.

- The contribution is small due to GIM suppression.



Experimental data for $D^+ \to \pi^+ V$

- **Ground resonances:**

Vector meson V	$\rho^0(770)$	$\omega(782)$	$\phi(1020)$
m_V (MeV) Γ_V^{tot}(MeV)	775.26 ± 0.23 147.4 ± 0.8	782.66 ± 0.13 8.68 ± 0.13	1019.46 ± 0.02 4.249 ± 0.013
$BR(V \to \mu^+\mu^-)$ $\lvert f_V\rvert$ (MeV)	$(4.55 \pm 0.28) \times 10^{-5}$ 219.85 ± 1.31	$(7.4 \pm 1.8) \times 10^{-5}$ 201.09 ± 1.12	$(2.85 \pm 0.19) \times 10^{-4}$ 228.12 ± 0.57
$BR(D^+ \to \pi^+ V)$ $\lvert A_{D^+\pi^+V}\rvert$ (MeV)	$(8.4 \pm 0.8) \times 10^{-4}$ 24.33 ± 1.25	$(2.8 \pm 0.6) \times 10^{-4}$ 14.14 ± 1.54	$(5.7 \pm 0.14) \times 10^{-3}$ 81.72 ± 1.83
$r_V = \kappa_V f_V \lvert A_{D^+\pi^+V}\rvert$ (in 10^{-3} GeV^2)	3.783 ± 0.195	0.670 ± 0.073	$-(6.214 \pm 0.140)$
$BR(D^+ \to \pi^+ V)_{V \to \mu^+\mu^-}$	$(3.82 \pm 0.43) \times 10^{-8}$	$(2.1 \pm 0.7) \times 10^{-8}$	$(1.62 \pm 0.12) \times 10^{-6}$

- **Excited resonances:**

[PDG]

Resonance V'	$\rho' \equiv \rho(1450)$	$\phi' \equiv \phi(1680)$
$m_{V'}$ (MeV) $\Gamma_{V'}^{tot}$ (MeV)	1465 ± 25 400 ± 60	1680 ± 20 150 ± 50
$BR(D^+ \to \pi^+ V')_{V' \to \pi^+\pi^-}$ $BR(D^+ \to \pi^+ V')_{V' \to K^+K^-}$	$(1.8 \pm 0.5) \times 10^{-4}$ –	– $(4.9^{+4.0}_{-1.9}) \times 10^{-5}$
$\lvert r_{V'}\rvert = \lvert \kappa_V f_{V'} A_{D^+\pi^+V'}\rvert$ (GeV^2)	$(9.64^{+2.92}_{-4.76}) \times 10^{-3}$	$(11.87^{+4.74}_{-3.79}) \times 10^{-3}$
$\lvert f_{V'}\rvert$ (MeV)	140^{+15}_{-35}	$\lvert f_{\phi'}\rvert = \lvert f_{\rho'}\rvert$

Fit Results

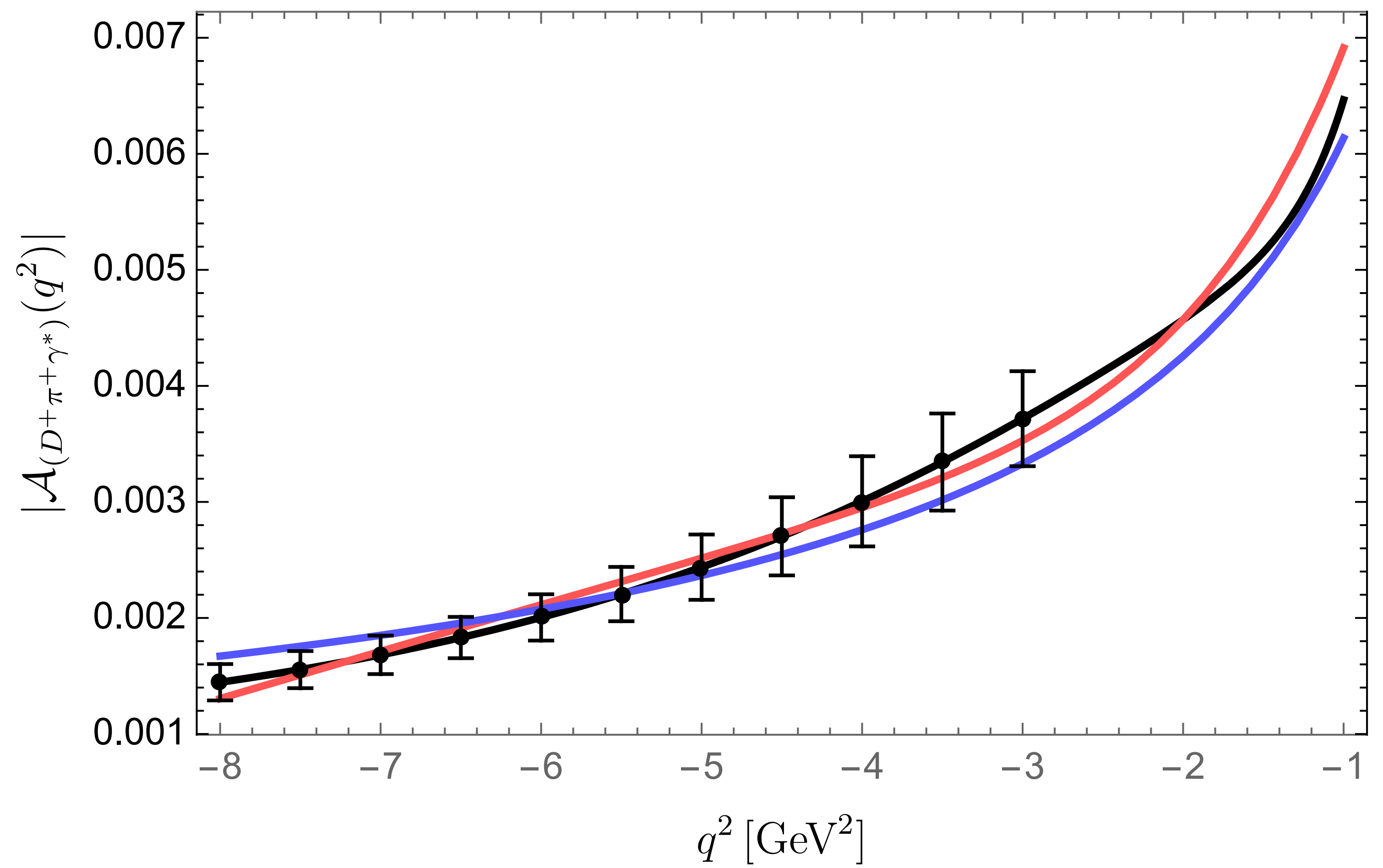
$$\sum_{q^2<0} \left| \mathcal{A}^{(disp-z)}_{(D^+\pi^+\gamma^*)}(q^2) - \mathcal{A}^{(LCSR)}_{(D^+\pi^+\gamma^*)}(q^2) \right|^2 = min\,,$$

- **Z-parametrisation:**

$$\varphi_\rho = 1.809, \quad \varphi_\omega = 1.804, \quad \varphi_\phi = -1.339, \quad \text{(in radians)}\,,$$
$$\alpha_1 = (-7.858 + 0.727\,i) \times 10^{-2}, \quad \alpha_2 = (9.548 + 2.184\,i) \times 10^{-2}\,.$$

- **Extended resonance model:**

$$\varphi_\rho = 5.944\,, \quad \varphi_\omega = 5.944\,, \quad \varphi_\phi = 2.797\,,$$
$$\varphi_{\rho'} = 5.929\,, \quad \varphi_{\phi'} = 5.925\,,$$
$$|r_{\rm eff}| = 3.538 \times 10^{-2}\,, \quad Arg[r_{\rm eff}] = 3.312\,.$$



Experimental data for Cabibbo favoured modes

Vector meson V	$\rho^0(770)$	$\omega(782)$	$\phi(1020)$
$BR(D_s^+ \to \pi^+ V)$ $\lvert A_{D_s\pi^+V}\rvert$ (MeV)	$(1.12 \pm 0.17) \times 10^{-4}$ 2.59 ± 0.20	$(1.92 \pm 0.30) \times 10^{-3}$ 10.79 ± 0.85	$(4.5 \pm 0.4)\%$ 64.86 ± 2.91
$r_V^{(D_s\pi^+)} = \kappa_V f_V \lvert A_{D_s\pi^+V}\rvert$ (in 10^{-3} GeV^2)	0.403 ± 0.031	0.512 ± 0.040	$-(4.932 \pm 0.222)$
$BR(D_s^+ \to \pi^+ V)_{V\to\mu^+\mu^-}$	$(5.096 \pm 0.835) \times 10^{-9}$	$(1.421 \pm 0.411 \times 10^{-7}$	$(1.282 \pm 0.142) \times 10^{-5}$
$BR(D^0 \to \bar{K}^0 V)$ $\lvert A_{D^0\bar{K}^0V}\rvert$ (MeV)	$1.26^{+0.12}_{-0.16}$ % 41.13 ± 2.62	2.32 ± 0.08 % 56.29 ± 1.03	0.83 ± 0.05 % 49.10 ± 1.40
$r_V^{(D^0\bar{K}^0)} = \kappa_V f_V \lvert A_{D^0\bar{K}^0V}\rvert$ (in 10^{-3} GeV^2)	6.39 ± 0.41	2.67 ± 0.05	$-(3.73 \pm 0.11)$
$BR(D^0 \to \bar{K}^0 V)_{V\to\mu^+\mu^-}$	$(5.73 \pm 0.81) \times 10^{-7}$	$1.72 \pm 0.42) \times 10^{-6}$	$(2.36 \pm 0.20) \times 10^{-6}$

[PDG]

Results for Cabibbo favoured modes

- **Fit results:**
  - for the $D_s^+ \to \pi^+ \gamma^*$ transition:

$$\varphi_\rho = 2.154\,, \quad \varphi_\omega = 2.148\,, \quad \varphi_\phi = -0.997,$$
$$\alpha_1 = (-7.121 - 1.697i) \times 10^{-2}, \quad \alpha_2 = (8.092 + 4.799i) \times 10^{-2},$$

  - for the $D^0 \to \bar{K}^0 \gamma^*$ transition:

$$\varphi_\rho = -0.008\,, \quad \varphi_\omega = 3.139\,, \quad \varphi_\phi = -0.015,$$
$$\alpha_1 = (0.261 + 0.038i) \times 10^{-2}, \quad \alpha_2 = (-0.313 - 0.046i) \times 10^{-2},$$

- **Bin results:**

Bin	$q^2_{\rm min}$	$q^2_{\rm max}$	$(q^2_{\rm max} - q^2_{\rm min})\, \Delta\mathcal{B}^{(disp-z)}_{D_s\to\pi^+\mu^+\mu^-}$	$(q^2_{\rm max} - q^2_{\rm min})\, \Delta\mathcal{B}^{(disp-z)}_{D^0\to\bar{K}^0\mu^+\mu^-}$
I	$4m_\mu^2$	$(m_\rho - \Gamma_\rho)^2$	$1.10^{+0.66}_{-0.58}$	$0.10^{+0.03}_{-0.02}$
II	$(m_\rho - \Gamma_\rho)^2$	$(m_\rho - \Gamma_\rho/4)^2$	$0.53^{+0.38}_{-0.34}$	$0.63^{+0.12}_{-0.08}$
III	$(m_\rho + \Gamma_\rho/4)^2$	$(m_\rho + \Gamma_\rho)^2$	$1.06^{+0.67}_{-0.63}$	$0.68^{+0.14}_{-0.16}$
IV	$(m_\rho + \Gamma_\rho)^2$	$(m_\phi - \Gamma_\phi)^2$	$15.69^{+1.93}_{-1.92}$	$4.11^{+0.25}_{-0.25}$
V	$(m_\phi + \Gamma_\phi)^2$	1.2 GeV^2	$22.88^{+2.24}_{-2.64}$	$2.91^{+0.19}_{-0.20}$
LHCb	$4m_\mu^2$	$(0.525$ GeV$)^2$	$0.69^{+0.39}_{-0.35}$	$0.03^{+0.01}_{-0.01}$

(In units 10^{-7})