Model Independent Phase Correction in $D \to K_S\pi^+\pi^-$ Decays for Precise CKM Angle γ Measurement in $B^\pm \to DK^\pm$

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Introduction

CKM Matrix and *CP* Violation

- The Cabibbo-Kobayashi-Maskawa (CKM) matrix describes weak interaction between quarks.
- CKM matrix unitarity reduces the free parameters to 3 rotation angles and 1 complex phase (the 'weak phase').
- This complex phase is the *sole* source of Charge-Parity (*CP*) violation in the Standard Model.
- Triangle geometry constrained by measurements (loop & tree level decays).
- γ is the only CKM angle directly accessible using tree-level decays.

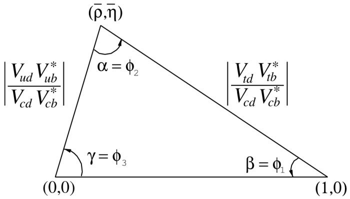


Figure: The CKM Unitarity Triangle.

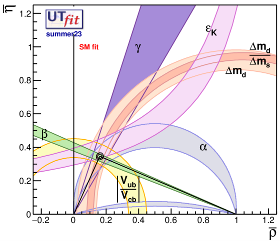


Figure: Global fit constraints on the Unitarity Triangle.

Accessing γ: Tree vs. Loop Level Decays

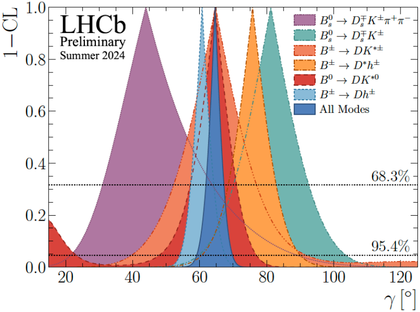


Figure: γ input split by different B decays. (Plot refer to LHCb-CONF-2024-004)

$\gamma = (65.9^{+3.3}_{-3.5})^\circ$ [direct], $\gamma = (66.29^{+0.72}_{-1.86})^\circ$ [indirect] (CKMfitter)

- Good agreement between direct and indirect γ measurements.
- Loop diagrams are generally more sensitive to potential New Physics (NP).
- Theoretical uncertainty in $B^\pm \to DK^\pm$ tree-level decay $\mathcal{O}(10^{-7})$.
- NP could introduce additional CP violation sources.
- Reaching precision comparable to indirect determination with direct methods is crucial for NP searches.

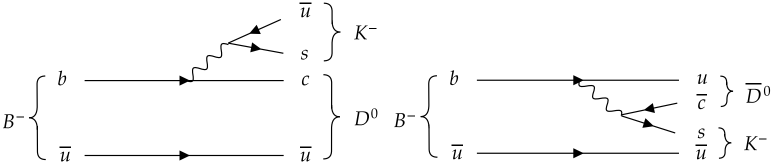


Figure: Feynman diagrams for $B^- \to D^0 K^-$ (favoured, $V_{cb}V_{us}^*$) and $B^- \to \bar{D}^0 K^-$ (suppressed, $V_{ub}V_{cs}^*$) decays.

$$A(B^- \to D^0 K^-) \propto 1(\text{Favoured}) \quad A(B^- \to \bar{D}^0 K^-) \propto r_B e^{i(\delta_B - \gamma)}(\text{Suppressed})$$

$$A(B^+ \to \bar{D}^0 K^+) \propto 1(\text{Favoured}) \quad A(B^+ \to D^0 K^+) \propto r_B e^{i(\delta_B + \gamma)}(\text{Suppressed})$$

- $r_B = |A_{\text{suppressed}}/A_{\text{favoured}}|$: Ratio of magnitudes ($\sim 0.1$).
- δ_B: Relative strong phase between the two B decay amplitudes.
- γ: CKM weak phase to be measured.

Interference between D^0 and $\bar{D}^0$ decaying to a common final state allows access to γ.

BPGGSZ Method (Binned, Model-Independent)

Measurement requires $D^0, \bar{D}^0$ decay to common three-body final state, e.g., $K_S\pi^+\pi^-$. Let $A_D = A(D^0 \to K_S\pi\pi)$, $\bar{A}_D = A(\bar{D}^0 \to K_S\pi\pi)$, $\delta_D = \arg(A_D/\bar{A}_D)$. D is the superposition of $D^0, \bar{D}^0$ and $B^\mp$ decay rates depend on interference:

$$\Gamma(B^- \to D(K_S\pi\pi)K^-) \propto |A_D|^2 + r_B^2|\bar{A}_D|^2 + 2r_B|A_D||\bar{A}_D|\cos(\delta_B - \gamma - \delta_D) \quad (1)$$

$$\Gamma(B^+ \to D(K_S\pi\pi)K^+) \propto r_B^2|A_D|^2 + |\bar{A}_D|^2 + 2r_B|A_D||\bar{A}_D|\cos(\delta_B + \gamma - \delta_D) \quad (2)$$

- $\delta_D(s_-, s_+)$ varies across the Dalitz plot ($s_\pm = m^2_{K_S\pi^\pm}$).
- Divide Dalitz plot into bins i.
- Measure average rates in each bin.
- Extract effective CP parameters per bin. Requires external input for D strong phase information $\langle\cos\delta_D\rangle_i, \langle\sin\delta_D\rangle_i$ (from charm factories).
- Sensitivity loss due to averaging phase variations within bins ($\sim$10%).

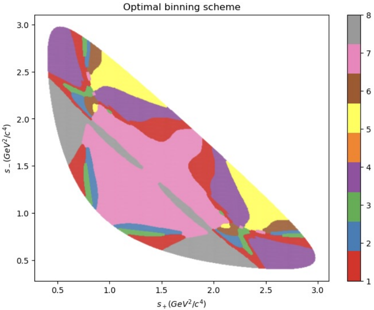


Figure: Example Dalitz plot binning for $D \to K_S\pi^+\pi^-$ (CLEO).

Unbinned Model-Dependent Method

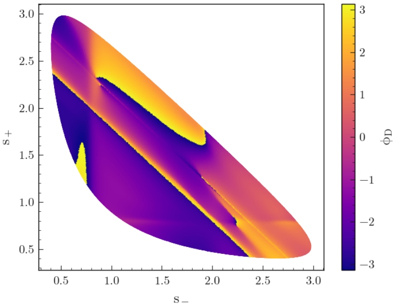


Figure: Strong Phase variation $\delta_D(s_-, s_+)$ from Belle (2018) amplitude model of $D \to K_S\pi^+\pi^-$ (Phys. Rev. D 98, 112012).

- Alternative: Use an amplitude model (fit to flavour-tagged $D^0 \to K_S\pi\pi$ data) to get $\delta_D(s_+, s_-)$ for every event.
- Provides full strong phase information $\implies$ potentially maximal sensitivity to γ.
- Introduces dependence on the amplitude model's correctness, especially the phase.
- Flavour-tagged samples lack D^0-$\bar{D}^0$ interference, so δ_D is inferred indirectly, not directly measured across the phase space.

Quantum-Correlated $D^0\bar{D}^0$ Pairs at BESIII

- At BESIII:
  $e^+e^- \to \psi(3770) \to D^0\bar{D}^0$.
- Initial $\psi(3770)$ ($J^{PC} = 1^{--}$) means $D^0\bar{D}^0$ pair is in a $C = -1$ quantum state.
- This quantum coherence allows access to interference between D^0 and $\bar{D}^0$ decays.
- Decay of one D tags the state of the other D (flavour or CP).
- Provides *direct sensitivity* to the strong phase difference δ_D.

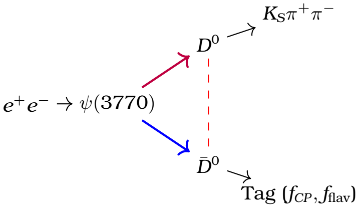


Figure: Production of quantum-correlated (QC) $D^0\bar{D}^0$ pairs at BESIII.

Interference in QC $D^0\bar{D}^0$ System & Phase Correction

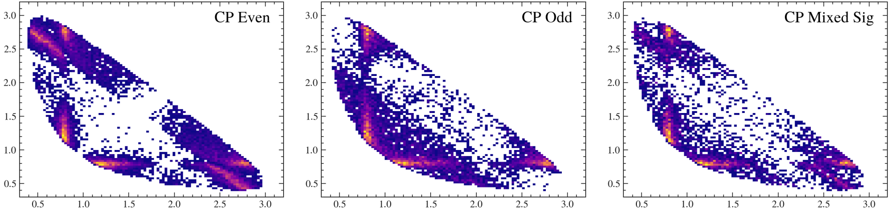


Figure: Simulated Dalitz distributions (s_- vs s_+) for $D \to K_S\pi\pi$ when the other D decays to: (Left) CP-even tag, (Middle) CP-odd tag, (Right) $K_S\pi\pi$ tag.

Decay rate for CP eigenstate tag $f_{CP\pm}$

$$\Gamma(D \to K_S\pi\pi|f_{CP\pm}) \propto |A_D|^2 + |\bar{A}_D|^2 \pm 2|A_D||\bar{A}_D|\cos(\delta_D(s_+, s_-))$$

Decay rate for $K_S\pi\pi$ vs $K_S\pi\pi$

$$\Gamma \propto |A_D|^2|\bar{A}'_D|^2 + |\bar{A}_D|^2|A'_D|^2 - 2|A_D||\bar{A}'_D||\bar{A}_D||A'_D|\cos(\delta_D - \delta'_D)$$

Idea from JHEP 09 (2023) 007:

$$\delta_D^{\text{measured}}(s_-, s_+) = \delta_D^{\text{model}}(s_-, s_+) + \delta_D^{\text{corr}}(s_-, s_+) \quad (3)$$

Phase Correction Parametrisation

Parametrise the phase correction δ_D^{corr} using polynomial functions. We use Legendre polynomials $p_k(z)$.

- To handle boundaries and symmetries, transform coordinates (s_+, s_-).
- $D \to K_S\pi^+\pi^-$ requires Bose symmetry: amplitude must be symmetric under $\pi^+ \leftrightarrow \pi^-$ exchange if K_S is CP-even (mostly true). This implies $A_D(s_+, s_-) = A_D(s_-, s_+)$.
- The phase difference $\delta_D = \arg(A_D/\bar{A}_D)$ has approximate anti-symmetry across $s_+ = s_-$.
- We imply this anti-symmetry in the Parametrisation.

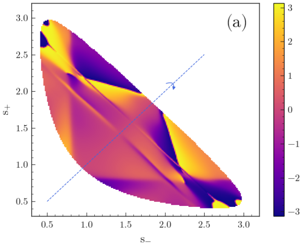


Figure: Dalitz plot for $D \to K_S\pi^+\pi^-$. Phase δ_D has approx. anti-symmetry across $s_+ = s_-$.

Basis function expansion (N=max order):

$$\delta_D^{\text{corr}} = \sum_{i=0}^{N} \sum_{j=0}^{\lfloor (N-i)/2 \rfloor} C_i C_{2j+1} p_i(z'_+) p_{2j+1}(z''_-)$$

New Dalitz Coordinates:

$$z_+ = \frac{1}{2}(s_+ + s_-) \quad z_- = \frac{1}{2}(s_+ - s_-)$$

(z_- is anti-symmetric under $\pi^+ \leftrightarrow \pi^-$ exchange).
Linearly map physical region to approx. $[-1, 1] \times [-1, 1]$ for numerical stability:

$$z'_+ = \frac{2z_+ - (z_+^{\max} + z_+^{\min})}{z_+^{\max} - z_+^{\min}}, \quad z'_- = \frac{2z_- - (z_-^{\max} + z_-^{\min})}{z_-^{\max} - z_-^{\min}}$$

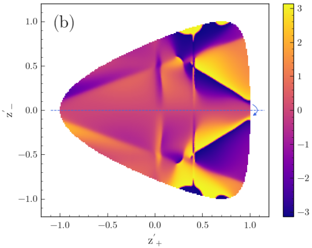


Figure: Dalitz plot transformed to rotated coordinates (z'_+, z'_-). Physical region is now roughly square. Anti-symmetry line is approx. $z'_- = 0$.

Phase Correction: Coordinate Transformation (2)

Further 'stretch' the z'_- coordinate to fill the rectangle more uniformly:

$$z''_- = \frac{2z'_-}{z'_+} + 2$$

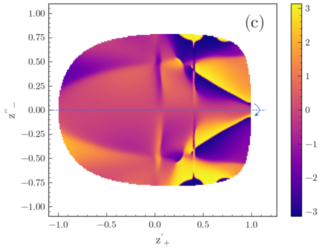
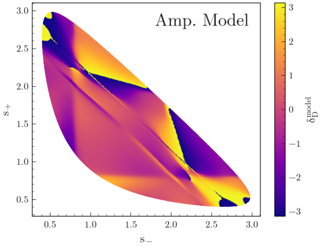


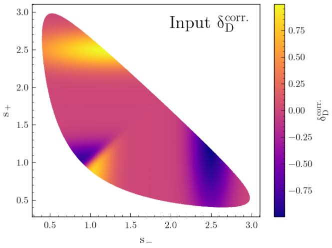
Figure: Dalitz plot transformed to stretched-rotated coordinates (z'_+, z''_-). Physical region fills rectangle better. Anti-symmetry is along z''_-.

Phase correction expressed in these final coordinates (N=max order):

$$\delta_D^{\text{corr}}(z'_+, z''_-) = \sum_{i=0}^{N} \sum_{j=0}^{\lfloor (N-i)/2 \rfloor} C_i C_{2j+1} p_i(z'_+) p_{2j+1}(z''_-)$$

Method Validation with Pseudo-Experiments





- **Validation:** Generate toys using Belle 2018 model + a *known* simulated δ_D^{corr}.
- Fit phase correction polynomial (orders N=0 to 9) to these toys.
- Compare fitted result to known input.
- Check impact on *CP* parameters: $x_\pm = r_B \cos(\delta_B \pm \gamma)$, $y_\pm = r_B \sin(\delta_B \pm \gamma)$.

Residual on *CP* parameters vs. Polynomial Order N

Order	$\Delta x_+ \times 10^2$	$\Delta y_+ \times 10^2$	$\Delta x_- \times 10^2$	$\Delta y_- \times 10^2$
0	1.3(8)	1.2(11)	−1.0(13)	−3.3(13)
1	1.1(8)	0.5(10)	−1.3(8)	−0.6(10)
2	0.5(9)	0.1(10)	−1.0(8)	0.4(10)
3	0.6(8)	0.0(10)	−1.2(8)	0.4(10)
4	0.3(8)	0.4(10)	−0.8(8)	0.3(10)
5	0.4(8)	0.5(10)	−0.7(8)	0.3(10)
6	0.3(8)	0.7(10)	−0.8(8)	0.4(10)
7	0.3(8)	0.5(10)	−0.9(8)	0.7(10)
8	0.3(8)	0.5(10)	−0.9(8)	0.7(10)
9	0.3(8)	0.5(10)	−0.7(8)	0.7(10)

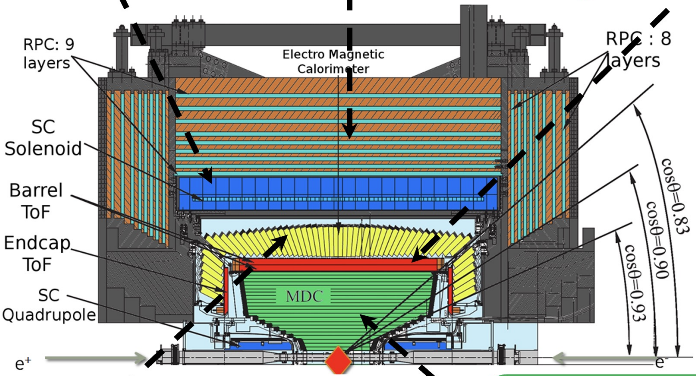
Residual biases decrease significantly with N, validating the method.

Extraction of Strong Phase Correction using BESIII Data

Superconducting solenoid
- 1.0 T

Muon Counter (MUC)
- 9 layers (barrel) + 8 layers (end-cap)

Time Of Flight (TOF)
- $\sigma_t = 90$ ps (barrel)
- $\sigma_t = 65$ ps (end cap)



Electromagnetic Calorimeter(EMC)
- $\Delta E/E = 2.5\%$ @ 1.0 GeV
- $\sigma_{\phi z} = 0.6$ cm @ 1.0 GeV

Main Drift Chamber (MDC)
- $\sigma_{xy} = 130$ μm
- $\Delta P/P = 0.5\%$ @ 1.0 GeV
- $\sigma_{dE/dx} = 6-7\%$

D Tag Modes Used at BESIII

Table: List of D decay modes used to tag the signal $D \to K_S\pi^+\pi^-$ event. Categorised by CP eigenvalue or mixed.

Tag Type	Decay Modes
CP odd ($CP = -1$)	$D \to K_S\pi^0$, $D \to K_L\pi^0\pi^0$ $D \to K_S\omega(\omega \to \pi^+\pi^-\pi^0)$ $D \to K_S\eta(\eta \to \gamma\gamma)$ $D \to K_S\eta(\eta \to \pi^+\pi^-\pi^0)$ $D \to K_S\eta'(\eta' \to \pi^+\pi^-\eta)$ $D \to K_S\eta'(\eta' \to \rho\gamma)$
CP even ($CP = +1$)	$D \to K^+K^-$ $D \to \pi^+\pi^-$ $D \to \pi^+\pi^-\pi^0$ ($F_{\text{even}} \approx 0.94$) $D \to K_L\pi^0$ $D \to K_S\pi^0\pi^0$
Mixed	$D \to K_S\pi^+\pi^-$ vs $D \to K_S\pi^+\pi^-$ $D \to K_S(\pi^0\pi^0)\pi\pi$ miss π^0 $D \to K_S\pi^+\pi^-$ miss $\pi^{+(-)}$

Signal Yield Extraction using discriminant variable

Define Beam Constrained Mass: $M_{\mathrm{BC}} = \sqrt{E^2_{\mathrm{beam}} - |\vec{p}_D|^2}$.

Missing Mass: $M^2_{\mathrm{miss}} = E^2_{\mathrm{miss}} - p^2_{\mathrm{miss}}$

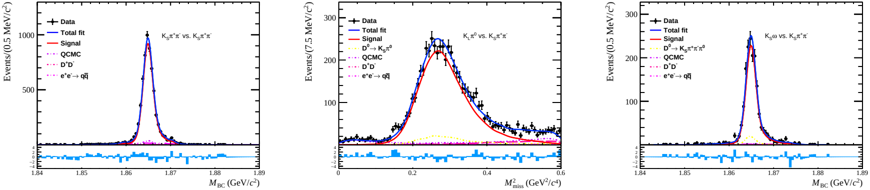


Figure: Fit to M_{BC} distribution for events tagged by (Left) $D \to K_S\pi\pi$, (Middle) $D \to K_L\pi^0$, (Right) $D \to K_S\omega$. Data (points), total fit (blue), signal (red), background components shown.

- Signal and background shapes from Monte Carlo(MC) simulation, smoothed using Kernel Density Estimation (KDE).
- Data/MC resolution difference compensate via Gaussian smearing (parameters shared).
- Yields of signal/background components determined by unbinned maximum likelihood fits.

Background Validation in Dalitz Projections (CP Tags)

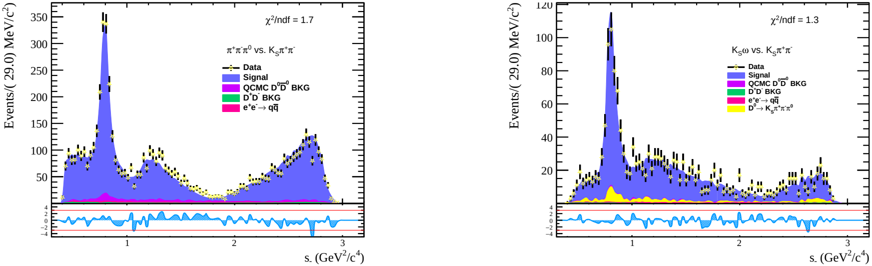
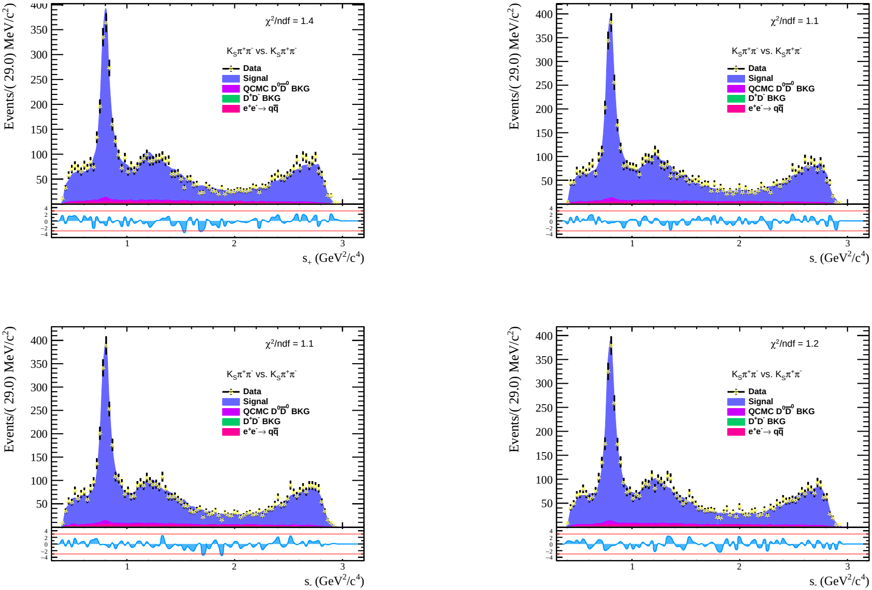


Figure: Dalitz projection $m^2(K_S\pi^-)$ for signal candidates tagged by (Left) $D \to \pi^+\pi^-\pi^0$ and (Right) $D \to K_S\omega$. Data compared to simulation sum (Stack: signal+background), normalized by M_{BC} fit yields.

- Check background modeling in Dalitz variables.
- Signal MC: Belle 2018 model + QC effects.
- Components summed according to yields from M_{BC} fit. Good agreement observed.
- Background shape for Dalitz fit: KDE PDF based on inclusive background MC simulation.

Background Validation in Dalitz Projections (Mixed Tag)



Dalitz Plot Likelihood for Phase Correction Fit

Simultaneous fit to Dalitz distributions $\Omega = (s_+, s_-)$ of all tag categories j.

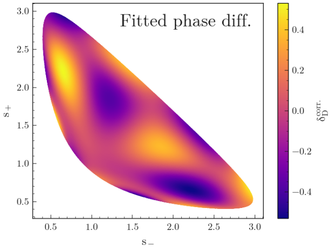
- **Signal PDF:** $P_{\text{sig}}(\Omega|\text{tag } j; \{C_{i,j}\})$
  - Proportional to $|\mathcal{A}(\Omega|\text{tag } j; \{C_{i,j}\})|^2$.
  - Amplitude $\mathcal{A}$ includes Belle 2018 model + $\delta_D^{\text{corr}}(\Omega; \{C_{i,j}\})$.
  - $\{C_{i,j}\}$ are the Legendre coefficients to be fitted.
  - Normalization $N_j = \int |\mathcal{A}|^2 d\Omega$ from Phase Space MC.
  - $PDF_{\text{sig}}(\Omega|j; \{C_{i,j}\}) = |\mathcal{A}|^2 / N_j$.
- **Background PDF:** $PDF_{\text{bkg}}(\Omega)$
  - Obtained from simulation using KDE, validated previously.
- **Total Likelihood:** For each category j:

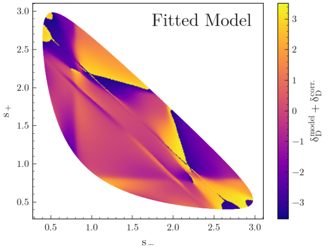
$$\mathcal{L}_j = \prod_{k=1}^{N_{\text{obs},j}} [N_j^{\text{sig}} PDF_{\text{sig}}(\Omega_k|j; \{C_{i,j}\}) + N_j^{\text{bkg}} PDF_{\text{bkg}}(\Omega_k)]$$

  ($N_j^{\text{sig}}, N_j^{\text{bkg}}$ yields fixed from M_{BC} fit or floated with constraints).
- Minimize total negative log-likelihood using MINUIT package(iminuit).

Phase Correction Fit Results from BESIII Data

- Simultaneous fit performed on BESIII $20.3 fb^{-1}$ data using CP-even, CP-odd, and mixed tags.
- Phase correction δ_D^{corr} determined using Legendre polynomial basis (N=7).
- Fit converges well up to N=7. N=8 failed. $\Longrightarrow$ Baseline: N=7.
- Correction δ_D^{corr} typically $\mathcal{O}$(few degrees), varies across Dalitz plot.





Fit Projections: CP-Even Tags

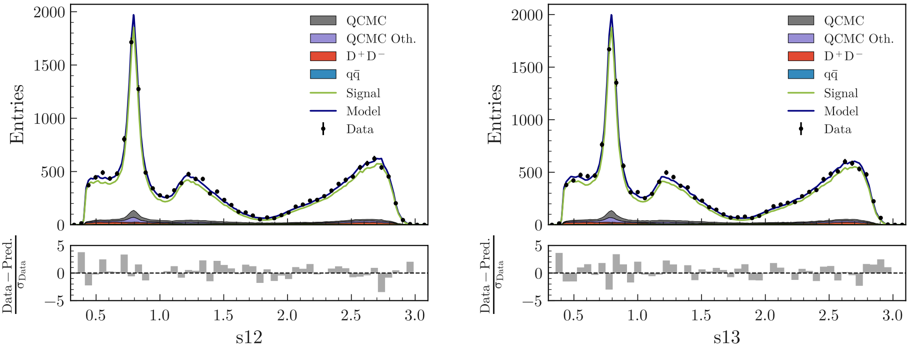


Figure: Fit projection onto $m^2(K_S\pi^+)$ (left) and $m^2(K_S\pi^-)$ (right) for CP-even tagged events. Data (points), total fit PDF (blue), signal (red), background (green). (Add pulls/residuals if available).

- Good description of data by the fit model.

Fit Projections: CP-Odd Tags

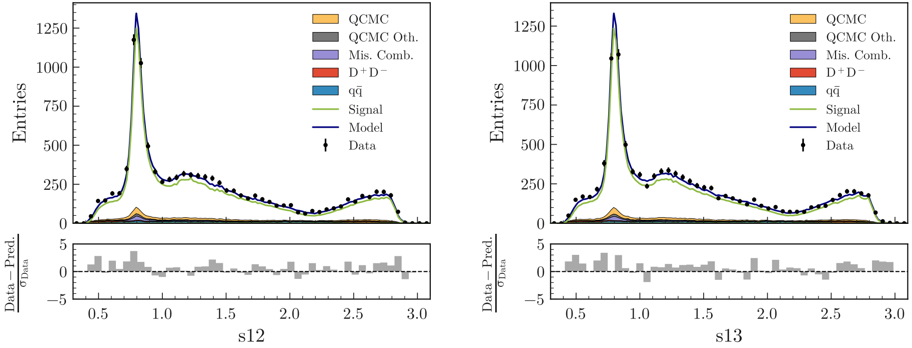
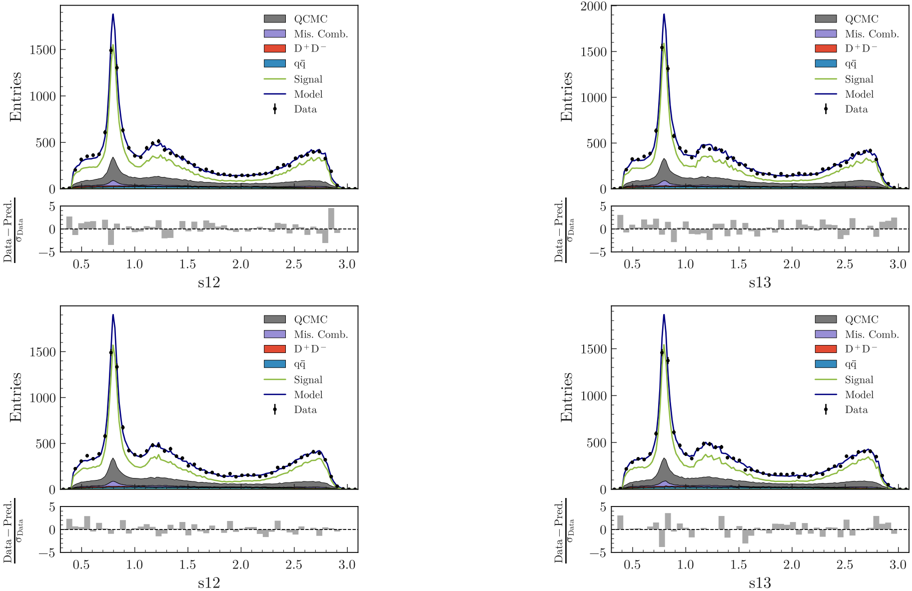


Figure: Fit projection onto $m^2(K_S\pi^+)$ (left) and $m^2(K_S\pi^-)$ (right) for CP-odd tagged events (e.g., $K_S\pi^0, K_S\omega$). Data, fit, components shown. (Add pulls/residuals if available).

- Good description of data.

Fit Projections: CP-Mixed Tags ($K_S\pi\pi$ vs $K_S\pi\pi$)



Validation: Pull Study for Correction Coefficients

- Check for bias in fitted coefficients $C_{i,j}$ and their uncertainties.
- Generate 1000 pseudo-experiments:
  - Statistics fluctuated (Poisson) around BESIII data yields.
  - Events generated using the *result* of the fit to data (N=7 polynomial).
- Fit each toy using the same N=7 polynomial basis.
- Calculate pulls: $pull(C_{i,j}) = (C_{i,j}^{\text{fit}} - C_{i,j}^{\text{gen}})/\sigma(C_{i,j}^{\text{fit}})$.

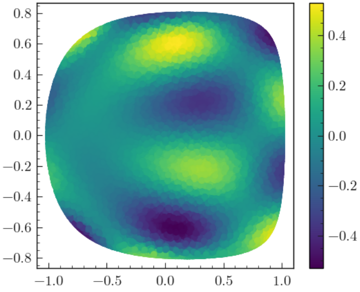
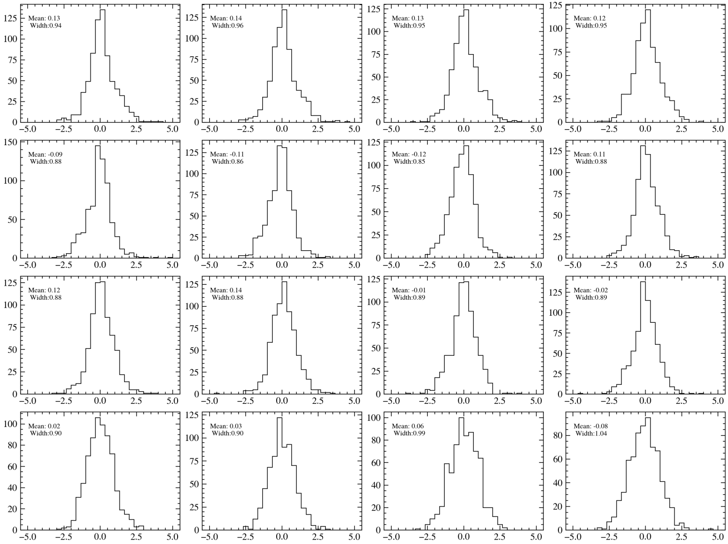


Figure: Distribution of fitted δ_D^{corr} from data fit result.

Validation: Pull Distributions for Coefficients



- Observing potential bias on fit, still understanding.

Impact of BESIII Phase Correction Uncertainty on γ

- Estimate systematic uncertainty on γ due to BESIII statistical uncertainty on δ_D^{corr}.
- Method:
  1. Sample 1000 sets of coefficients $\{C_{i,j}\}$ from BESIII fit covariance matrix.
  2. For each set, generate large LHCb-like $B \to DK$ pseudo-experiment using $\delta_D^{\text{meas}} = \delta_D^{\text{model}} + \delta_D^{\text{corr}}$.
  3. Fit each toy for CP parameters $(x_\pm, y_\pm)$.
  4. Study the spread/shift induced in derived γ due to the variation of $\{C_{i,j}\}$.

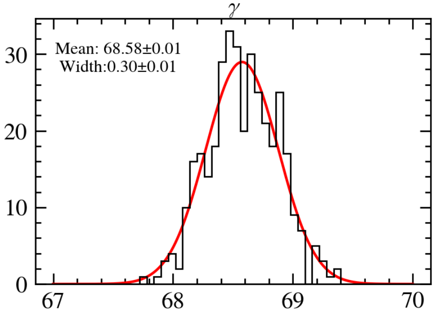
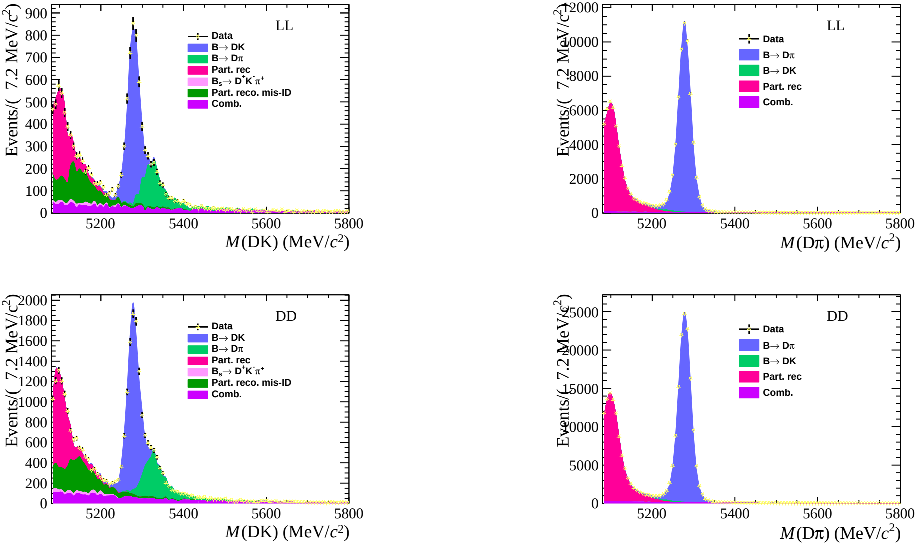


Figure: Distribution of γ in LHCb-like toys, variation due solely to BESIII δ_D^{corr} statistical uncertainty. The width corresponds to the uncertainty contribution.

Estimated impact on γ measurement: $\sim 0.3^\circ$
(from BESIII δ_D^{corr} statistical uncertainty)

Feasibility Study at LHCb

Simulate expected signals and backgrounds according to JHEP02(2021)169.



LHCb Feasibility Study: Simulation Model Details

Input CP parameters from JHEP02(2021)169:

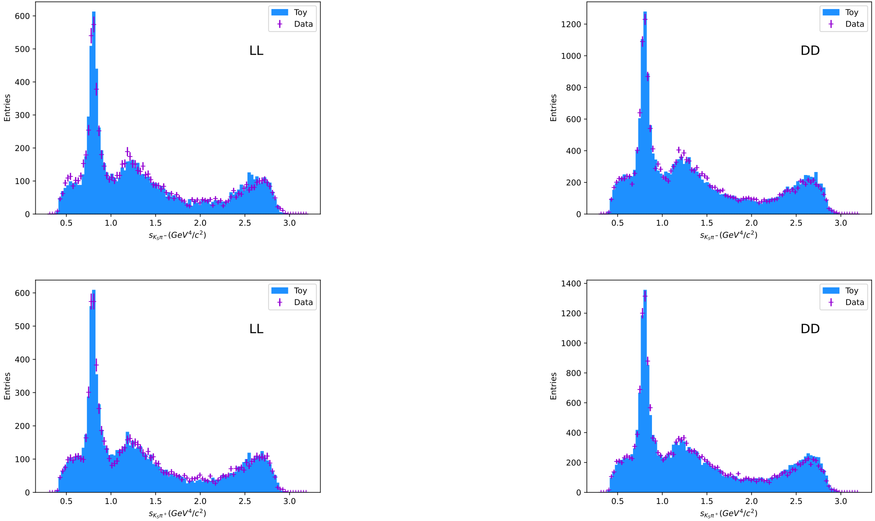
$$x_+^{DK} = -0.0897,\ x_-^{DK} = 0.0586,\ y_+^{DK} = -0.0110,\ y_-^{DK} = 0.0688,$$

$$x_\xi^{D\pi} = -0.0549,\ y_\xi^{D\pi} = 0.0070(4)$$

Table: Dalitz distribution models for components in LHCb simulation.

Component	Dalitz Model
$B^\mp \to DK^\mp(\pi^\mp)$ Signal	Belle2018 Amp. + CP params. Includes LHCb eff. map.
Misid $B^\mp \to D\pi^\mp(K^\mp)$	Belle2018 Amp. + $D\pi$ CP params (derived via ξ). Includes LHCb eff. map.
Combinatorial bkg	Assumed symmetric $D^0, \bar{D}^0$ production: $0.5 \cdot f_{\text{comb}}^D \cdot (D^0 + \bar{D}^0) + (1 - f_{comb}^D)$.
Partially Reco. bkg	Assumed D^0-like for B^- sample, $\bar{D}^0$-like for B^+.
Part. Reco. Misid. bkg	Assumed D^0-like for B^- sample, $\bar{D}^0$-like for B^+.
$B_s^0 \to DK^+\pi^-$ bkg	Assumed $\bar{D}^0$-like for B^- sample, D^0-like for B^+.

LHCb Feasibility Study: Simulated Dalitz Projections



- Dalitz distributions generated with Belle 2018 model (no δ_D^{corr}) and CP parameters.

LHCb Analysis Strategy: Simultaneous DK and $D\pi$ Fit

- **Motivation:** $B \to D\pi$ and $B \to DK$ are backgrounds to each other (mis-ID). Simultaneous fit constrains cross-feeds, improves precision.
- Uses shared $D \to K_S\pi\pi$ model.
- Different hadronic parameters for DK and $D\pi$. Relate them via relative parameters ξ:
  - $\xi = (r_B^{D\pi}/r_B^{DK})e^{i(\delta_B^{D\pi} - \delta_B^{DK})}$
  - Cartesian coords: $x_\xi = \mathrm{Re}(\xi)$, $y_\xi = \mathrm{Im}(\xi)$.
  - $D\pi$ parameters related to DK params:

$$x_\pm^{D\pi} = x_\xi x_\pm^{DK} - y_\xi y_\pm^{DK}$$
$$y_\pm^{D\pi} = x_\xi y_\pm^{DK} + y_\xi x_\pm^{DK}$$

  - Total physics parameters: 4 ($x_\pm^{DK}, y_\pm^{DK}$) + 2 (x_ξ, y_ξ) = **6 parameters**.

LHCb Analysis Strategy: Likelihood Construction

Simultaneous unbinned maximum likelihood fit to $M_{B^\pm}$ and Dalitz variables $\Omega = (s_+, s_-)$ across all categories j (DK/Dπ, LL/DD, B^+/B^-).

- **PDF for component i in category j:**

$$\mathcal{P}_{i,j}(M_{B^\pm}, \Omega|\theta) = N_{i,j} \cdot \mathcal{P}_{i,j}(M_{B^\pm}) \cdot \mathcal{P}_{i,j}(\Omega|\theta_{\text{phys}})$$

  ($N_{i,j}$: yield, $\mathcal{P}(M)$: mass PDF, $\mathcal{P}(\Omega)$: Dalitz PDF including efficiency).

- **Dalitz PDF** $\mathcal{P}_{i,j}(\Omega|\theta_{\mathbf{phys}})$: Depends on physics parameters $\theta_{\text{phys}} = \{x_\pm^{DK}, y_\pm^{DK}, x_\xi, y_\xi\}$. Uses Belle2018. Normalized: $\int \mathcal{P}_{i,j}(\Omega) d\Omega = 1$.
- **Total PDF for event k in category j:**

$$\mathcal{P}_j^{\text{total}}(k|\theta) = \sum_i \mathcal{P}_{i,j}(k|\theta)$$

- **Negative Log-Likelihood (Extended):**

$$-\ln \mathcal{L} = \sum_j \left[\left(\sum_i N_{i,j} \right) - \ln \left(\sum_i N_{i,j} \mathcal{P}_{i,j}(M_B^\pm) \mathcal{P}_{i,j}(\Omega|\theta_{\text{phys}}) \right) \right]$$

LHCb Feasibility: Example Fit Projections (Mass)

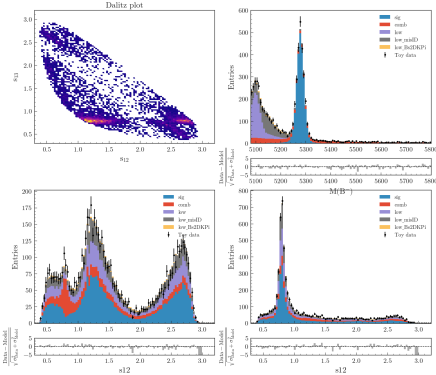


Figure: $B^+ \to DK$, LL Fit

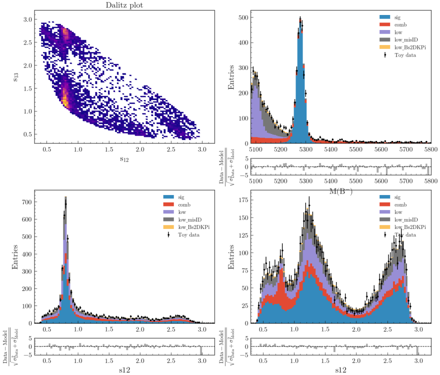


Figure: $B^- \to DK$, LL Fit

Figure: Simultaneous fit projection onto B mass for $B \to DK$, LL category using pseudo-data. Shows data, total fit, signal, cross-feed, backgrounds.

- Fit model provides good description of the pseudo-data.

LHCb Feasibility: Fit Validation with Pulls

Table: Input values and pull results (mean, width) for the 6 physics parameters from fits to 1000 pseudo-experiments simulating LHCb run1,2 data.

Parameter	Input value	Pull mean	Pull width
x_+^{DK}	-0.09	0.02(3)	0.97(3)
x_-^{DK}	0.06	0.01(3)	1.02(3)
y_+^{DK}	-0.01	0.07(3)	1.03(3)
y_-^{DK}	0.07	0.00(3)	1.04(3)
x_ξ	-0.05	0.08(3)	1.00(3)
y_ξ	0.01	0.03(3)	1.00(3)

- Pull means consistent with 0, widths consistent with 1.
- Indicates the simultaneous fit strategy is unbiased and uncertainties are correctly estimated for the simulated scenario.

LHCb Feasibility: Interpretation results

The measured values of $(x_\pm^{DK}, y_\pm^{DK}, x_\xi^{D\pi}, y_\xi^{D\pi})$ are then used to put constraints on the possible values of the CKM angle γ and the hadronic parameters r_B^{DK}, δ_B^{DK}, $r_B^{D\pi}$, and $\delta_B^{D\pi}$:

$$\chi^2(\theta|A_{obs}) = (A_{obs} - A(\theta))^T \Sigma_{A_{obs}}^{-1} (A_{obs} - A(\theta)) + const., \quad (5)$$

Table: The mean and width for the measured γ and hadronic parameters from 1000 pesudo-experiments.

CP observable	Input value	Mean	Width
γ [°]	68.7	68.37	4.65
r_B^{DK} [$\times 10^{-2}$]	9.04	9.122	0.663
δ_B^{DK} [°]	118.3	118.19	4.83
$r_B^{D\pi}$ [$\times 10^{-2}$]	0.5	0.545	0.155
$\delta_B^{D\pi}$ [°]	291	290.6	20.9

Summary

- Precise γ measurement is crucial for SM tests and NP searches.
- Developed model-independent method to measure strong phase phase correction δ_D^{corr} for amplitude model.
- BESIII QC $D\bar{D}$ data allows direct access to $D \to K_S\pi\pi$ strong phase δ_D.
- Applied to $20.3 fb^{-1}$ BESIII data: Determined δ_D^{corr} (N=7 polynomial), validated fit procedure.
- **Expectation:** Reduces uncertainty on γ due to δ_D knowledge to $\sim \mathbf{0.3}^\circ$ (from BESIII stats).
- LHCb feasibility study shows simultaneous $DK/D\pi$ fit and background subtraction is robust and promising for high precision γ.
- Ongoing work: Finalizing BESIII analysis, incorporating into LHCb analyses.

Thank you! Stay tuned!