# Solving Beautiful (and Charming) Puzzles

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Maastricht University & Nikhef

= Dortmund Seminar 2024 =

# The Flavour Puzzle

- Flavour symmetry broken by Yukawa couplings to the Higgs field
- Origin of mixing between families described by unitary CKM matrix
- Visualized by unitary triangles
- Dominant source of CP violation (antiparticle-particle asymmetry)

$$\begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix}$$

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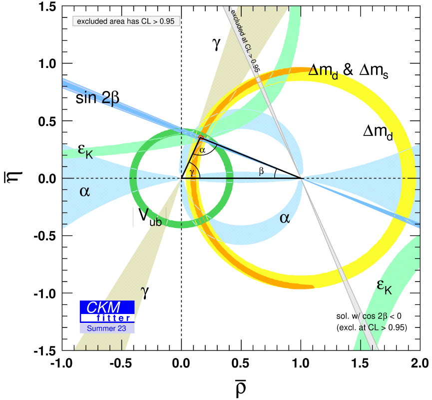
Our understanding of Flavour is unsatisfactory

$$\bar{\rho} + i\bar{\eta} = -\frac{V_{ud}V_{ub}^*}{V_{cd}V_{cb}^*}$$



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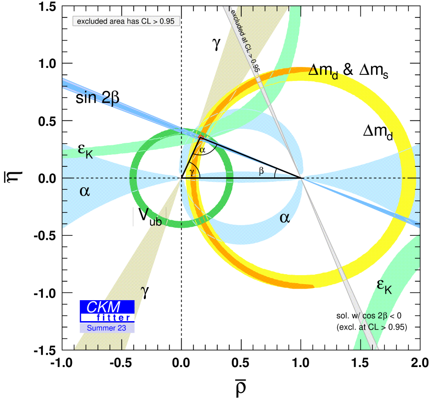
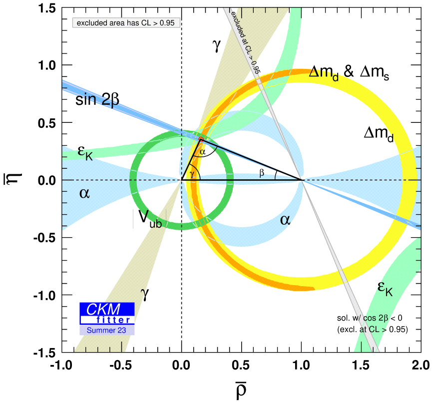



Huge amounts of data + theory advances = Precision frontier

Tiny deviations from SM predictions constrain effects of New Physics

# Puzzles in CKM elements

Key parameters of the Standard Model



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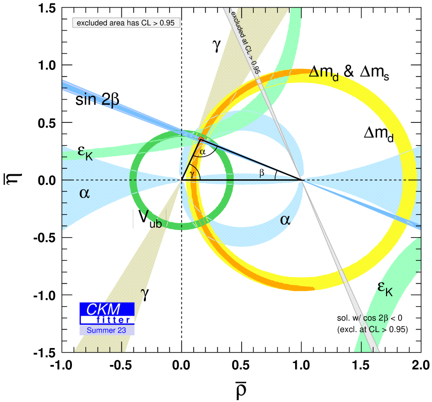


- Several puzzles between determinations
- Discussion on $|V_{us}|$
- CKM unitarity in first row?

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Bernlochner, Prim, KKV Eur.Phys.J.ST 233 (2024) 2, 347-358

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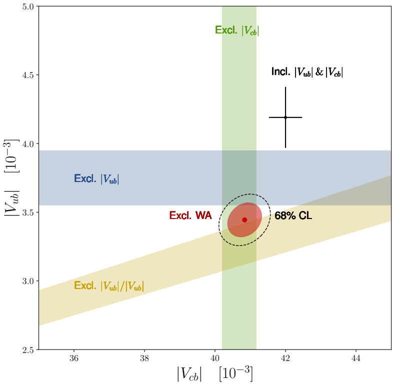


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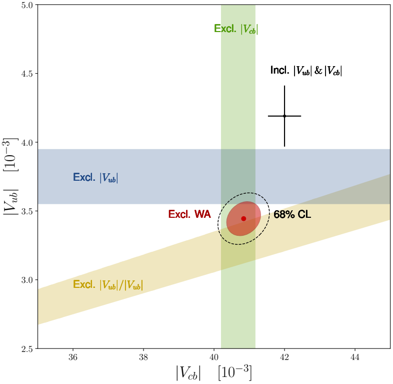


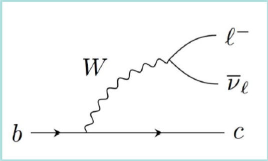

- Several puzzles between determinations
- Discussion on $|V_{us}|$
- CKM unitarity in first row?
- Inclusive versus exclusive decays in $|V_{cb}|$ and $|V_{ub}|$
- $|V_{cs}| \rightarrow$ CKM unitarity in second row/column?

# The Beauty of Semileptonic Decays

## Motivation:

- Theoretically relatively easy to describe: factorization of strong interaction effects

**Quark level process**



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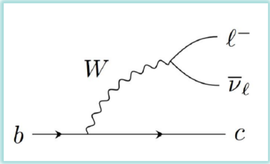
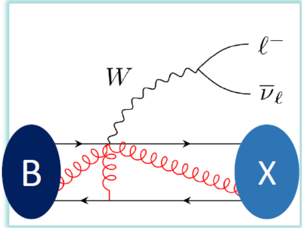



**hadronization** →

Reality: Bound state



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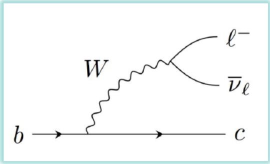


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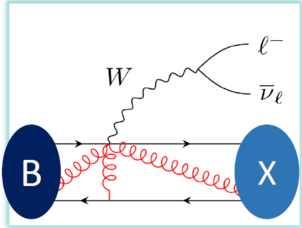


## Two options:

- Exclusive decays: pick one final state with the desired quarks ($V_{cb} \to D^{(*)}$ and $V_{ub} \to \pi$)
- Inclusive decays: everything you can think of! (denoted with $X_c$ or $X_u$)

### Motivation:

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### Challenge:

- Dealing with QCD at large distances/small scales
- Parametrize fundamental mismatch in non-perturbative objects
  - Calculable: Lattice or Light-cone sumrules = Exclusive Decays
  - Measurable: from data = Inclusive Decays

# Inclusive Semileptonic Beauty Decays

# Why inclusive decays?

- Set up OPE and heavy quark expansion
- Well established framework for beauty decays!
- Extract important CKM parameters $|V_{cb}|, |V_{ub}|$ (and $|V_{cs}|$?)
- Extract power corrections from data
- Cross check of exclusive decays

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- Cross check of exclusive decays
  - Dominated by lattice determinations

# Inclusive Decays: Heavy Quark Expansion

- $b$ quark mass is large compared to $\Lambda_{\rm QCD}$
- Set up the HQE: momentum of $b$ quark: $p_b = m_b v + k$, expand in $k \sim iD$
- Optical Theorem $\rightarrow$ (local) Operator Product Expansion (OPE)

$$d\Gamma = d\Gamma_0 + \frac{d\Gamma_1}{m_b} + \frac{d\Gamma_2}{m_b^2} + \ldots \qquad d\Gamma_i = \sum_k C_i^{(k)} \left\langle B|O_i^{(k)}|B\right\rangle$$

  - $C_i^{(k)}$ perturbative Wilson coefficients
  - $\langle B|\ldots|B\rangle$ non-perturbative matrix elements $\rightarrow$ string of $iD$
  - operators contain chains of covariant derivatives

HQE elements: $\langle B|\mathcal{O}_i^{(n)}|B\rangle = \langle B|\bar{b}_v(iD_\mu)\ldots(iD_{\mu_n})b_v|B\rangle$

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- Progress on the lattice Juetner et al. [2305.14092]

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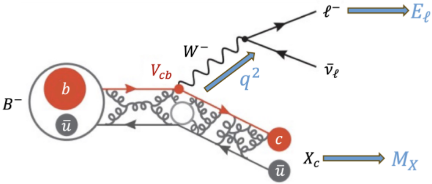
- $\Gamma_2$ :$\mu_\pi^2$ and $\mu_G^2$ at $1/m_b^2$
- $\Gamma_3$ :$\rho_D^3$ and $\rho_{LS}^3$ at $1/m_b^3$
- Many more at $1/m_b^{4,5}$ Mannel, Turczyk, Uraltsev, JHEP 1010 (2011) 109

# Moments of the spectrum

BABAR, PRD 68 (2004) 111104; BABAR, PRD 81 (2010) 032003; Belle, PRD 75 (2007) 032005. Pic from M. Fael

Non-perturbative matrix elements obtained from moments of differential rate




$$\langle O^n \rangle_{\rm cut} = \frac{\int_{\rm cut} dO\, O^n \frac{d\Gamma}{dO}}{\int_{\rm cut} dO \frac{d\Gamma}{dO}}$$

$M_X^2 = (p_B - q)^2, E_\ell = v_B \cdot p_\ell$ and $q^2 = (p_\nu + p_\ell)^2$

hadronic mass, lepton energy and $q^2$ moments

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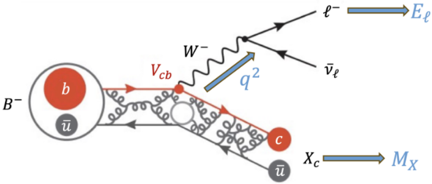


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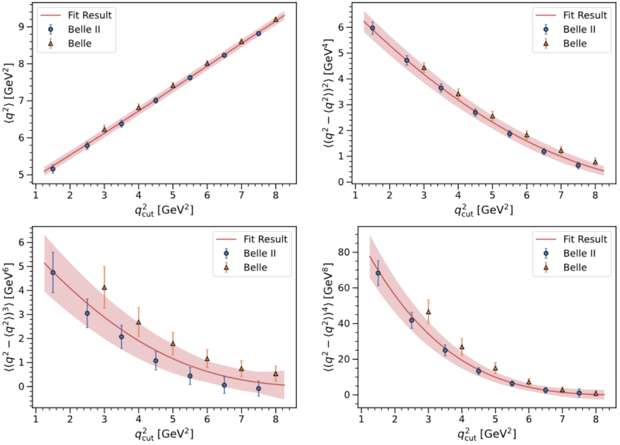
hadronic mass, lepton energy and $q^2$ moments

- Different phase space cuts give additional (correlated) observables
- $\mu_\pi^2, \mu_G^2, \rho_D^3 + \cdots$ extracted from data $\to$ total rate $\to$ $|V_{cb}|$

# Experimental measurements of $q^2$ moments

Bernlochner, Welsch, Fael, Olschewsky, Persson, van Tonder, KKV [2205.10274]




Centralized moments as function of $q^2_{\rm cut}$

# Moments of the spectrum

Moments and total rate are double expansion in $\alpha_s$ and HQE parameters

$$
\begin{aligned}
L_i =& \frac{1}{\Gamma_0} \int_{E_l \geq E_{\rm cut}} \mathrm{d}E_l \, \mathrm{d}q_0 \, \mathrm{d}q^2 \, (E_l)^i \, \frac{\mathrm{d}^3\Gamma}{\mathrm{d}q^2 \, \mathrm{d}q_0 \, \mathrm{d}E_l} \\
=& (m_b)^i \left[ L_i^{(0)} + L_i^{(1)} \frac{\alpha_s(\mu_s)}{\pi} + L_i^{(2)} \left( \frac{\alpha_s(\mu_s)}{\pi} \right)^2 + \frac{\mu_\pi^2}{m_b^2} \left( L_{i,\pi}^{(0)} + L_{i,\pi}^{(1)} \frac{\alpha_s(\mu_s)}{\pi} \right) \right. \\
& + \frac{\mu_G^2(\mu_b)}{m_b^2} \left( L_{i,G}^{(0)} + L_{i,G}^{(1)} \frac{\alpha_s(\mu_s)}{\pi} \right) + \frac{\rho_D^3(\mu_b)}{m_b^3} \left( L_{i,D}^{(0)} + L_{i,D}^{(1)} \frac{\alpha_s(\mu_s)}{\pi} \right) \\
& \left. + \frac{\rho_{LS}^3(\mu_b)}{m_b^3} \left( L_{i,LS}^{(0)} + L_{i,LS}^{(1)} \frac{\alpha_s(\mu_s)}{\pi} \right) + O\left( \frac{1}{m_b^4} \right) \right] ,
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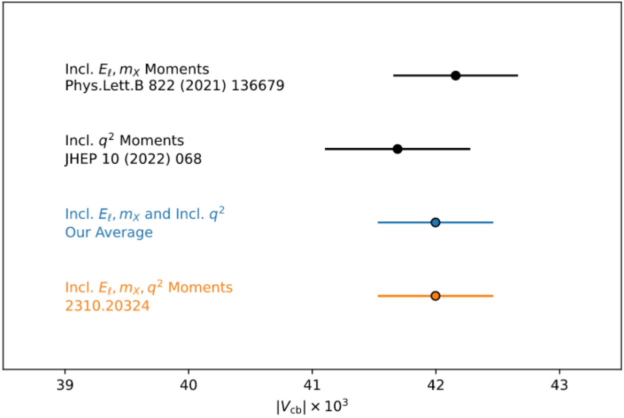
\end{aligned}
$$

- Systematic framework for power-corrections
- Higher precision: Include higher-order $1/m_b$ and $\alpha_s$ corrections in rate and moments!
- Proliferation of non-perturbative matrix elements
  - 4 up to $1/m_b^3$
  - 13 up to $1/m_b^4$ Dassinger, Mannel, Turczyk, JHEP 0703 (2007) 087
  - 31 up to $1/m_b^5$ Mannel, Turczyk, Uraltsev, JHEP 1011 (2010) 109

# Summary of $|V_{cb}|$ inclusive

Fael, Prim, KKV, Eur. Phys. J. Spec. Top. (2024). https://doi.org/10.1140/epjs/s11734-024-01090-w




- Up to $1/m_b^3$ HQE terms
- Need new (branching ratio) measurements!

# Experimental Inclusive Prospects

Belle II Physics Week https://indico.belle2.org/event/9402/overview

- New hadronic mass, lepton energy and $q^2$ moments
- Updated branching ratio measurements (with $q^2$ cut)*
- Unconventional cuts (Lepton energy moments with $q^2$ cut?)?
- Forward-Backward asymmetry?

*RPI observable = reduced set of parameters

# NEW: Inclusive decays: The Kolya package

Kolya package, Fael, Milutin, KKV [2409.15007]

Open source Python package:
https://gitlab.com/vcb-inclusive/kolya

- HQE predictions for several observables:
  - Centralized $\langle E_\ell \rangle$ moments
  - Centralized $\langle q^2 \rangle$ moments
  - Centralized $\langle M_X^2 \rangle$ moments
  - Total rate + branching ratio with kinematic cut

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Features:

- Includes power corrections up to $1/m_b^5$ Mannel, Milutin, KKV [2311.1200]
- Employs kinetic scheme for $m_b$ and $\overline{\mathrm{MS}}$ for $m_c$
- Interface with CRunDec for automatic RGE evolution Chetyrkin, Kuhn, Steinhauser, Smidth, Herren
- Includes New Physics effects Fael, Rahimi, KKV [ JHEP 02 (2023) 086]

# The advantage of $q^2$ moments

Mannel, KKV, JHEP 1806 (2018) 115; Fael, Mannel, KKV, JHEP 02 (2019) 177, Mannel, Milutin, KKV [2311.1200]

- Standard lepton energy and hadronic mass moments are not RPI quantities
- Only RPI moments are $q^2$ moments
- Determinations from Belle and Belle II availabe Phys. Rev. D 104, 112011 (2022), Phys. Rev. D 107, 072002 (2023)

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Quirks:

- Setting up the HQE: momentum of $b$ quark: $p_b = m_b v + k$, expand in $k \sim iD$
- Choice of $v$ not unique: Reparametrization invariance (RPI)
- links different orders in $1/m_b \to$ reduction of parameters
- up to $1/m_b^4$: 8 parameters (previous 13)

# $q^2$ moments only analysis

Bernlochner, Welsch, Fael, Olschewsky, Persson, van Tonder, KKV [2205.10274]

$$|V_{cb}|^{q^2}_{\rm incl} = (41.69 \pm 0.27|_{\mathcal{B}} \pm 0.31|_{\Gamma} \pm 0.18|_{\rm exp.} \pm 0.17|_{\rm theo} \pm 0.34|_{\rm const.}) \times 10^{-3}$$

- First extraction using $q^2$ moments with $1/m_b^4$ terms
- NNLO corrections to moments not included
- Higher order coefficients important to check convergence of the HQE

$$r_E^4 = (0.02 \pm 0.34) \cdot 10^{-1}\mathrm{GeV}^4 \quad r_G^4 = (-0.21 \pm 0.69)\mathrm{GeV}^4$$

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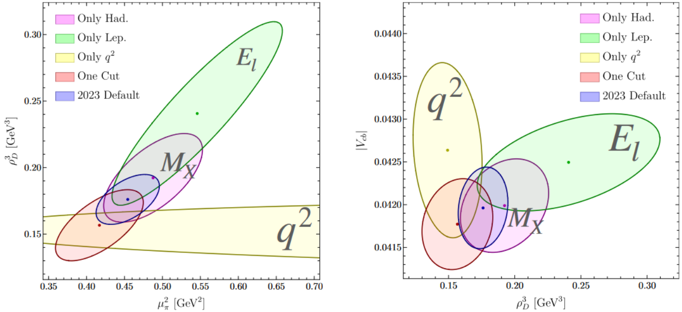
- Inputs for $B \to X_u \ell\nu$, $B$ lifetimes and $B \to X_s \ell\ell$ KKV, Huber, Lenz, Rusov, et al.

# First combined Fit

Gambino, Finauri [2310.20324]




- Complementarity between different measurements
- Full analysis including $1/m_b^{4,5}$ in progress Bernlochner, Prim, Fael, Milutin, KKV

# Inclusive $B_s$ decays?

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De Cian, Feliks, Rotondo, KKV [2312.05147]. Pic from M. Fael

Full $m(X_c)$ spectrum can be reconstructed as sum-over-exclusives

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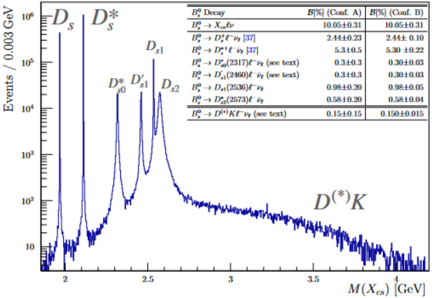
**Why measure $B_s$ decays?**

- HQE parameters depend on the initial state meson
- Study $SU(3)$ breaking of HQE
- Necessary to study $f_s/f_d$ and lifetimes of $B_s$ decays

# Inclusive $B_s$ decays?

De Cian, Feliks, Rotondo, KKV [2312.05147]. Pic from M. Fael. PRD 101 (2020),072004

First study of the possibilities using sum-over-exclusive technique




| $B_s^0$ Decay | $\mathcal{B}$[%] (Conf. A) | $\mathcal{B}$[%] (Conf. B) |
|---|---|---|
| $B_s^0 \to X_{cs}\ell\nu$ | 10.05±0.31 | 10.05±0.31 |
| $B_s^0 \to D_s^+\ell^-\nu_\ell$ [37] | 2.44±0.23 | 2.44± 0.10 |
| $B_s^0 \to D_s^{*+}\ell^-\bar{\nu}_\ell$ [37] | 5.3±0.5 | 5.30 ±0.22 |
| $B_s^0 \to D_{s0}^*(2317)\ell^-\nu_\ell$ (see text) | 0.3±0.3 | 0.30±0.03 |
| $B_s^0 \to D'_{s1}(2460)\ell^-\bar{\nu}_\ell$ (see text) | 0.3±0.3 | 0.30±0.03 |
| $B_s^0 \to D_{s1}(2536)\ell^-\nu_\ell$ | 0.98±0.20 | 0.98±0.05 |
| $B_s^0 \to D_{s2}^*(2573)\ell^-\bar{\nu}_\ell$ | 0.58±0.20 | 0.58±0.04 |
| $B_s^0 \to D^{(*)}K\ell^-\nu_\ell$ (see text) | 0.15±0.15 | 0.150±0.015 |

Constructed $M_X$ spectrum:

- $B_s \to D_s^{(*)}\ell\nu$ (LHCb)
- First higher excited states $B_s \to D_{s0}(D'_{s1})\ell\nu$ (not known)
- Second higher excited states $B_s \to D_{s1}(D_{s2}^*)\ell\nu$ (measured with $20 - 35\%$ unc)
- Non-resonant decays modelled modified Goity-Roberts

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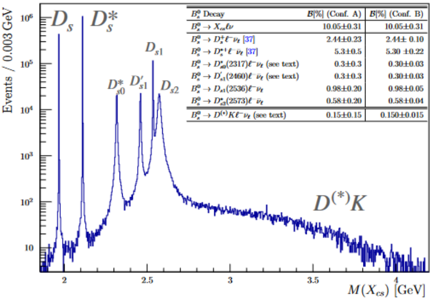


| $B_s^0$ Decay | $\mathcal{B}$[%] (Conf. A) | $\mathcal{B}$[%] (Conf. B) |
|---|---|---|
| $B_s^0 \to X_{cs}\ell\nu$ | 10.05±0.31 | 10.05±0.31 |
| $B_s^0 \to D_s^+\ell^-\nu_\ell$ [37] | 2.44±0.23 | 2.44± 0.10 |
| $B_s^0 \to D_s^{*+}\ell^-\bar{\nu}_\ell$ [37] | 5.3±0.5 | 5.30 ±0.22 |
| $B_s^0 \to D_{s0}^*(2317)\ell^-\nu_\ell$ (see text) | 0.3±0.3 | 0.30±0.03 |
| $B_s^0 \to D'_{s1}(2460)\ell^-\bar{\nu}_\ell$ (see text) | 0.3±0.3 | 0.30±0.03 |
| $B_s^0 \to D_{s1}(2536)\ell^-\nu_\ell$ | 0.98±0.20 | 0.98±0.05 |
| $B_s^0 \to D_{s2}^*(2573)\ell^-\bar{\nu}_\ell$ | 0.58±0.20 | 0.58±0.04 |
| $B_s^0 \to D^{(*)}K\ell^-\nu_\ell$ (see text) | 0.15±0.15 | 0.150±0.015 |

Constructed $M_X$ spectrum:

- $B_s \to D_s^{(*)}\ell\nu$ (LHCb)
- First higher excited states $B_s \to D_{s0}(D'_{s1})\ell\nu$ (not known)
- Second higher excited states $B_s \to D_{s1}(D^*_{s2})\ell\nu$ (measured with $20 - 35\%$ unc)
- Non-resonant decays modelled modified Goity-Roberts

- Extracted moments depend highly on non-resonant moment
- Estimate for HQE parameters provided
- $V_{cb}$ extraction requires Branching ratio from Belle II! $\to$ 5% extraction from current data!

# Inclusive $B_s$ decays as a precision measurement?

De Cian, Feliks, Rotondo, KKV [2312.05147]. Pic from M. Fael. PRD 101 (2020),072004

First study of the possibilities using sum-over-exclusive technique

**Improvements:**

- First measurements of $B_s \to D^*_{s0}(D'_{s1})\ell\nu$
- Updated measurements of higher excited states $B_s \to D_{s1}(D^*_{s2})\ell\nu$
- Improved knowledge $D_s^{**}$ decays
- Understand non-resonant contribution

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With these improvements a precise measurement of the HQE parameters in $B_s$ decays can be obtained!

# Exclusive Charm decays and the $|V_{cs}|$ puzzle

# How to determine exclusive $|V_{cs}|$?

Modes included in PDG

## Pure leptonic modes

- $D_s^+ \to \{\mu^+, \tau^+\}\nu$

## Semileptonic modes

- $D^0 \to K^-\{e^+, \mu^+\}\nu$
- $D^+ \to \bar{K}^0\{e^+, \mu^+\}\nu$

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Bolognani, Reboud, van Dyk, KKV JHEP 09 (2024) 099 [2407.06145]

Data from Belle, BES, BESIII, CLEO-c

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## Differential $q^2$ Semileptonic rate

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$\to D \to K$ form factor ETM, FNAL/MILC

$\to \Lambda_c \to \Lambda$ form factor Meinel, Our work

## Differential $q^2$ Semileptonic rate

Total of 51 observations

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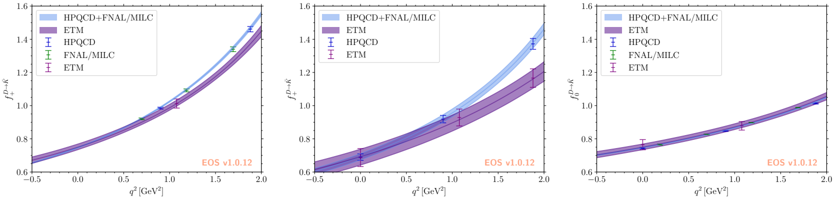
Total of 51 observations

# Form factors for semileptonic charm decays

Bolognani, Reboud, van Dyk, KKV JHEP 09 (2024) 099 [2407.06145] EOS/DATA-2024-01: Supplementary material for EOS/ANALYSIS-2023-08, 10.5281/zenodo.12688257.

BGL like fit with dispersive bounds in EOS flavour software




- HPQCD + FNAL/MILC: p-value $= 4\%$
- HPQCD + FNAL/MILC + ETM: p-value $< 0.1\%$

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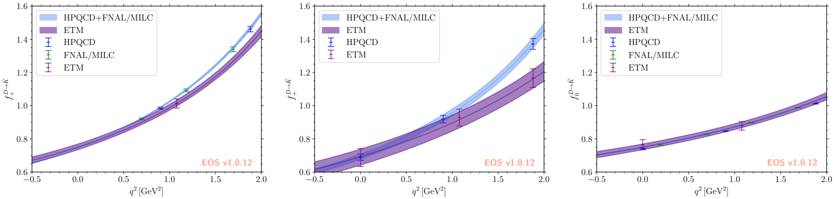


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Incompatible data: use also PDG-like scale factor* $S^2 \equiv \frac{\chi^2}{N_{\text{d.o.f.}}} = 6.34$

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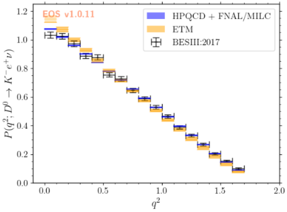


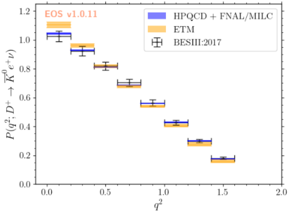


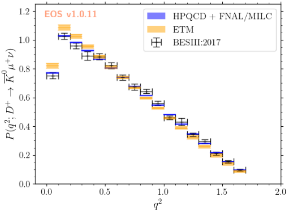


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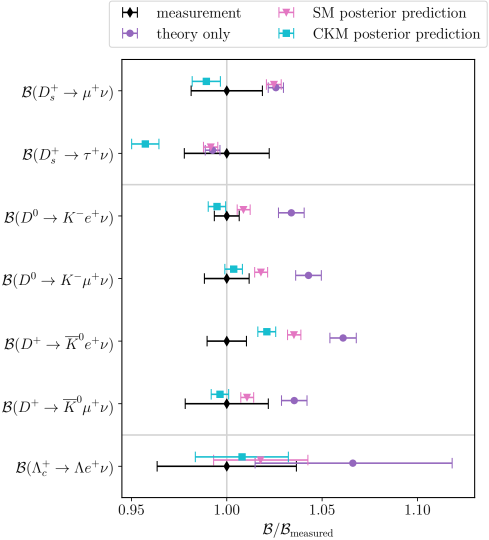
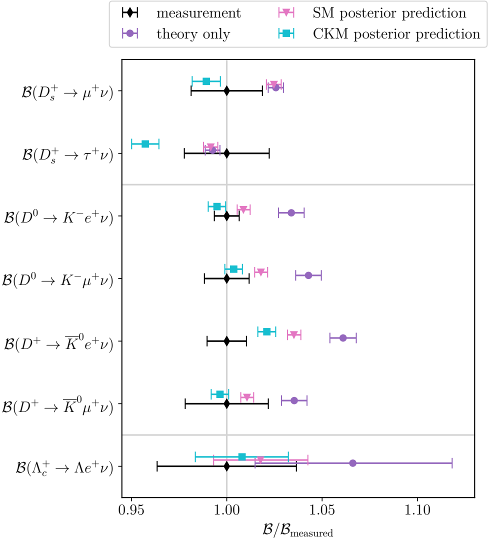



- Theory only: use PDG $|V_{cs}| = 0.975$
  - large $> 5\sigma$ tension in $B \to K$ modes

# Branching ratios

Bolognani, Reboud, van Dyk, KKV JHEP 09 (2024) 099 [2407.06145]

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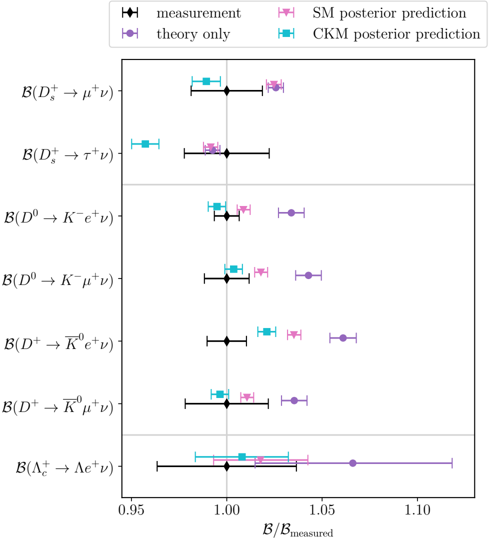
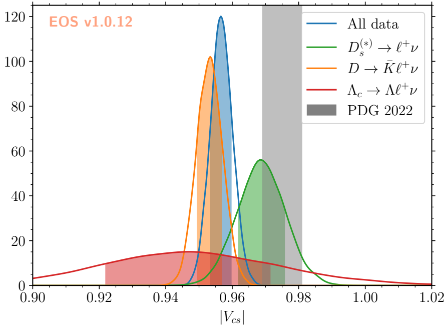



- Theory only: use PDG $|V_{cs}| = 0.975$
- SM: No parameters of interest, $|V_{cs}|$ fixed: *null hypothesis*
- CKM: Fit $|V_{cs}|$

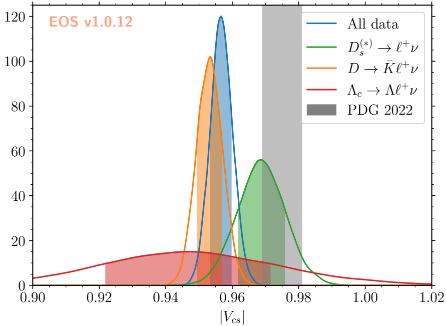
- Nominal result: $|V_{cs}| = 0.957 \pm 0.003$ with p-value 41%

# Extracting $|V_{cs}|$




| Scenario | Data set | $\chi^2$ | d.o.f. | $p$ value [%] | $\lvert V_{cs}\rvert$ |
|---|---|---|---|---|---|
| | $D_s^{(*)+} \to \ell^+\nu$ | 2.5 | 2 | 28.0 | $0.969 \pm 0.007$ |
| | $\Lambda_c \to \Lambda\ell\nu$ | 0.1 | 1 | 81.2 | $0.947^{+0.027}_{-0.026}$ |
| nominal | $D \to \bar{K}\ell\nu$ | 44.1 | 45 | 50.9 | $0.953 \pm 0.004$ |
| | joint fit | 51.7 | 50 | 40.9 | $0.957 \pm 0.003$ |
| scale factor | $D \to \bar{K}\ell\nu$ | 42.7 | 45 | 57.0 | $0.957 \pm 0.007$ |
| | joint fit | 48.2 | 50 | 54.5 | $0.963 \pm 0.005$ |

# Extracting $|V_{cs}|$




- Our results are compatible with PDG at $2.5\sigma$ level
- Only reproduce PDG results if we do not include universal EW corrections (Sirlin factor)*
- Specifically: update form factors + Sirlin factor shift $|V_{cs}| = 0.952$

# CKM Unitarity?

- Interesting to check second row and column unitarity
- Use PDG average for other elements*

$$|V_{cd}|^{\rm PDG} = 0.221 \pm 0.004\,, \qquad |V_{cb}|^{\rm PDG} = (40.8 \pm 1.4) \times 10^{-3}\,,$$
$$|V_{us}|^{\rm PDG} = 0.2243 \pm 0.0008\,, \quad |V_{ts}|^{\rm PDG} = (41.5 \pm 0.9) \times 10^{-3}\,.$$

| | PDG | nominal | scale factor |
|---|---|---|---|
| $\lvert V_{cs}\rvert$ | $0.975 \pm 0.006$ | $0.957 \pm 0.003$ | $0.963 \pm 0.005$ |
| $2^{\rm nd}$ row | $1.00 \pm 0.014$ $(0.08\,\sigma)$ | $0.966 \pm 0.008$ $(4.3\,\sigma)$ | $0.978 \pm 0.012$ $(1.9\,\sigma)$ |
| $2^{\rm nd}$ column | $1.00 \pm 0.012$ $(0.22\,\sigma)$ | $0.968 \pm 0.006$ $(5.2\,\sigma)$ | $0.979 \pm 0.010$ $(2.0\,\sigma)$ |

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- Requires looking into normalization of $B \to K$ modes
- Or New Physics?
  - Strong constraints on potential (pseudo)scalar and tensor effects
  - Large CP-violating effects in right-handed currents allowed

# Inclusive charm decays?

# Why HQE for charm?

- Expansion parameters $\alpha_s(m_c)$ and $\Lambda_{\rm QCD}/m_c$ less than unity, but not so small ...
- Turn vice into virtue: more sensitive to higher $1/m_Q$ corrections
- Exploit the full physics potential of BES III, LHCb, Belle II.
- Constrain Weak Annihilation (WA) contributions

  $\rightarrow$ $B_d \rightarrow s\ell\ell$ [Huber, Hurth, Lunghi, Jenkins, KKV, Qin ]

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  $\rightarrow$ $V_{ub}$
- Extraction of $|V_{cs}|$ and $|V_{cd}|$?
- Test HQE parameters across species and test SU(3) symmetry

**Open Questions:**

- Valence and non-valence WA operators at higher orders
- Scale for radiative corrections
- Charm mass definition

# The HQE for charm

$D \to X_q \ell \nu$ is not a copy of $B \to X_c \ell \nu$!

OPE for $b \to c \ell \bar{\nu}$: $m_Q \sim m_q \gg \Lambda_{\rm QCD}$

- $q$ is treated as a heavy degree of freedom
- two-quarks operators: $\bar{Q}_v (iD^\alpha \cdots iD^\beta) Q_v$
- IR sensitivity to mass $m_q$

$$\Gamma\Big|_{1/m_Q^3} = \left[\frac{34}{3} + 8 \log \rho + \ldots \right] \frac{\rho_D^3}{m_Q^3}, \quad \text{with } \rho = (m_q/m_Q)^2$$

# The HQE for charm

Fael, Mannel, KKV, JHEP 12 (2019) 067 [1910.05234]

OPE for $c \to s\ell\bar{\nu}$: $m_Q \gg m_q \sim \Lambda_{\mathrm{QCD}}$

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- Additional HQE parameters for $c \to q$: $T_i \equiv \frac{1}{2m_D}\langle D|O_i^{\rm 4q}|D\rangle$
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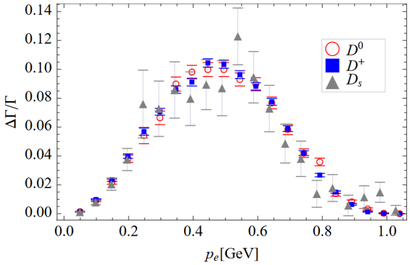
Let's test the HQE for charm on data!

# Extracting weak annihilation from data

CLEO data, Gambino, Kamenik [1004.0114]




- Lepton energy moments extracted from spectrum
- Kinetic mass for charm at $\mu = 0.5$ GeV threshold, HQE parameters as input
- Max 2% weak annihilation (WA) contribution to $B \to X_u \ell \nu$

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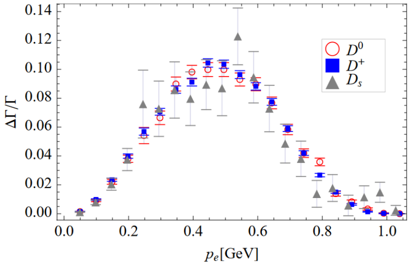


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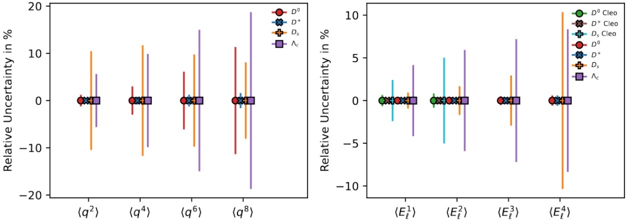
- Future prospects: Feasibility study to measure $q^2$ moments at BESIII Bernlochner, Gilman, Malde, Prim, KKV, Wilkinson [2408.10063]

# Prospects for BESIII

Bernlochner, Gilman, Malde, Prim, KKV, Wilkinson [2408.10063]




- Future prospects: Experimental and Theory program for inclusive charm
- Interesting experimental prospects

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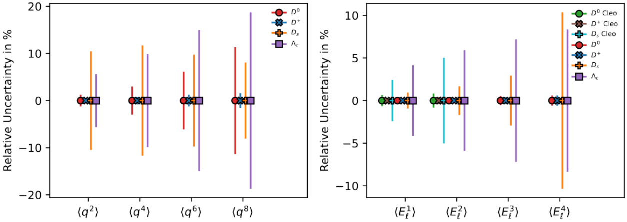


- Future prospects: Experimental and Theory program for inclusive charm
- Interesting experimental prospects
- Quite some theory challenges

# Thank you for your attention!

# Backup

# Comparison with PDG

$$\mathcal{H}^{sc\nu\ell} = -\frac{4G_F}{\sqrt{2}}\tilde{V}_{cs}^* \sum_i \mathcal{C}_i^\ell(\mu_c)\mathcal{O}_i^\ell + \text{h.c.}$$

$$\mathcal{O}_{V,L}^\ell = [\bar{s}\gamma^\mu P_L c]\,[\bar{\nu}\gamma_\mu P_L \ell]\,, \qquad \mathcal{O}_{V,R}^\ell = [\bar{s}\gamma^\mu P_R c]\,[\bar{\nu}\gamma_\mu P_L \ell]\,,$$

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- Account for universal electroweak corrections via Sirlin factor $\mathcal{C}_{V,L}^\ell(\mu) = 1 + \frac{\alpha_e}{\pi}\ln\left(\frac{M_Z}{\mu}\right) \simeq 1.01,$
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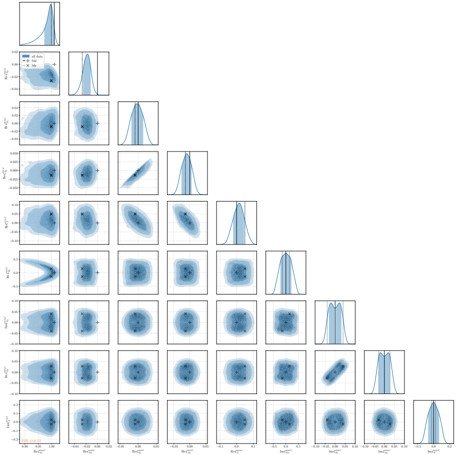
# WET correlations

Bolognani, Reboud, van Dyk, KKV JHEP 09 (2024) 099 [2407.06145]



# WET NP analysis

$$\mathcal{H}^{sc\nu\ell} = -\frac{4G_F}{\sqrt{2}}\tilde{V}_{cs}^{*}\sum_i \mathcal{C}_i^{\ell}(\mu_c)\mathcal{O}_i^{\ell} + \text{h.c.}$$

$$\begin{aligned}
\mathcal{O}_{V,L}^{\ell} &= [\bar{s}\gamma^{\mu}P_L c]\,[\bar{\nu}\gamma_{\mu}P_L\ell]\,, & \mathcal{O}_{V,R}^{\ell} &= [\bar{s}\gamma^{\mu}P_R c]\,[\bar{\nu}\gamma_{\mu}P_L\ell]\,,\\
\mathcal{O}_{S,L}^{\ell} &= [\bar{s}P_L c]\,[\bar{\nu}P_L\ell]\,, & \mathcal{O}_{S,R}^{\ell} &= [\bar{s}P_R c]\,[\bar{\nu}P_L\ell]\,,\\
\mathcal{O}_{T}^{\ell} &= [\bar{s}\sigma^{\mu\nu}b]\,[\bar{\nu}\sigma_{\mu\nu}P_L\ell]\,.
\end{aligned}$$

$$\begin{aligned}
\mathrm{Re}\,\mathcal{C}_{V,L}^{\ell} &= [\;\;0.957,\;\;\;1.002]\,, & & \\
\mathrm{Re}\,\mathcal{C}_{V,R}^{\ell} &= [-0.026,-0.012]\,, & \mathrm{Im}\,\mathcal{C}_{V,R}^{\ell} &= [-0.225,0.225]\,,\\
\mathrm{Re}\,\mathcal{C}_{S,L}^{\ell} &= [-0.019,\;\;\;0.014]\,, & \mathrm{Im}\,\mathcal{C}_{S,L}^{\ell} &= [-0.030,0.030]\,,\\
\mathrm{Re}\,\mathcal{C}_{S,R}^{\ell} &= [-0.026,\;\;\;0.006]\,, & \mathrm{Im}\,\mathcal{C}_{S,R}^{\ell} &= [-0.028,0.028]\,,\\
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\end{aligned}$$

# WET NP analysis

$$\mathcal{H}^{sc\nu\ell} = -\frac{4G_F}{\sqrt{2}}\tilde{V}^*_{cs}\sum_i \mathcal{C}_i^\ell(\mu_c)\mathcal{O}_i^\ell + \text{h.c.}$$

$$\mathcal{O}^\ell_{V,L} = [\bar{s}\gamma^\mu P_L c]\,[\bar{\nu}\gamma_\mu P_L \ell]\,, \qquad \mathcal{O}^\ell_{V,R} = [\bar{s}\gamma^\mu P_R c]\,[\bar{\nu}\gamma_\mu P_L \ell]\,,$$
$$\mathcal{O}^\ell_{S,L} = [\bar{s}P_L c]\,[\bar{\nu}P_L \ell]\,, \qquad \mathcal{O}^\ell_{S,R} = [\bar{s}P_R c]\,[\bar{\nu}P_L \ell]\,,$$
$$\mathcal{O}^\ell_{T} = [\bar{s}\sigma^{\mu\nu} b]\,[\bar{\nu}\sigma_{\mu\nu} P_L \ell]\,.$$

$$\text{Re}\,\mathcal{C}^\ell_{V,L} = [\;0.957,\;1.002]\,,$$
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$$\text{Re}\,\mathcal{C}^\ell_{T} = [-0.021,\;0.046]\,, \quad \text{Im}\,\mathcal{C}^\ell_{T} = [-0.068, 0.068]\,.$$

- Strong constraints on potential (pseudo)scalar and tensor effects, large CP-violating effects in right-handed currents allowed
- Results provide in EOS $\to$ fit your favorite NP model

# What mass to use?

Bigi, Shifman, Uraltsev, Vainshtein, hep-ph/9704245, hep-ph/9405410; Czarnecki, Melnikov, Uraltsev, hep-ph/9708372.

- Renormalon issues require short-distance mass
- Kinetic mass: relating hadron versus quark mass
  QCD corrections using hard cut off $\mu$

$$m_Q(\mu)^{\rm kin} = m_Q^{\rm Pole} - \left[\overline{\Lambda}\right]_{\rm pert} + \left[\frac{\mu_\pi^2}{2m_Q}\right]_{\rm pert} + \ldots$$

$$[\overline{\Lambda}]_{\rm pert} = \frac{4}{3} C_F \frac{\alpha_s(m_c)}{\pi}\mu \qquad [\mu_\pi^2]_{\rm pert} = C_F \frac{\alpha_s(m_c)}{\pi}\mu^2$$

- Higher-order terms in the HQE generate corrections $(\alpha_s/\pi)\mu^n/m_Q^n$.

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- Higher-order terms in the HQE generate corrections $(\alpha_s/\pi)\mu^n/m_Q^n$.
- $\boldsymbol{\Lambda_{\rm QCD} < \mu < m_Q}$: expansion parameters $\mu/m_Q$
  - Well established for $m_B$: $\mu/m_B \simeq 0.2$
  - Charm??

    $\rightarrow$ $\mu = 1$ GeV $\rightarrow \mu/m_c \simeq 1$

    $\rightarrow$ $\mu = 0.5$ GeV $\rightarrow \mu/m_c \simeq 0.4$

Challenge: $\mu = 0.5$ GeV touches upon the non-perturbative regime?

# Spectral-Density Mass

Chetyrkin, Kuehn, Steinhauser hep-ph/9705254, Penin, Pivovarov hep-ph/9805344 Boushmelev, Mannel, KKV [2301.05607]

- $m_c$ not observable $\to$ no physical meaning
- Extracted from data: moments of the spectral density in $e^+e^- \to$ hadrons

$$R(s) = \frac{\sigma(e^+e^- \to \text{hadrons})}{\sigma(e^+e^- \to \mu^+\mu^-)}$$

- Replace $m_c$ by moments of the spectral density!

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- Replace $m_c$ by moments of the spectral density!
- First study shows small improvement in pert. series

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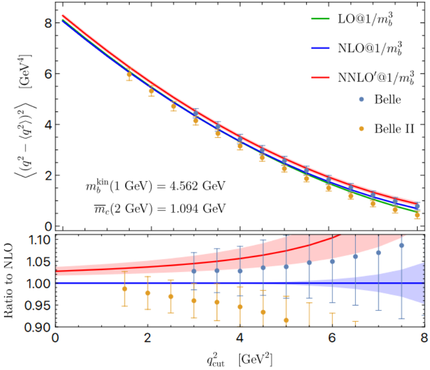
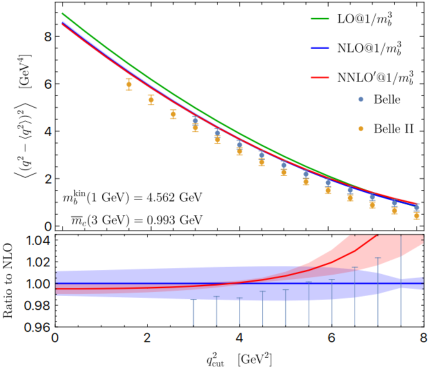
- Replace $m_c$ by moments of the spectral density!

- In progress: Similar approach for the charm + power corrections

# NEW: NNLO corrections to $q^2$ momemts

Herren, Fael [2403.03976]




$\overline{m_c}(2\text{ GeV})$ not ideal choice

$\overline{m_c}(3\text{ GeV})$ better

# NEW: NNLO corrections to $q^2$ momemts

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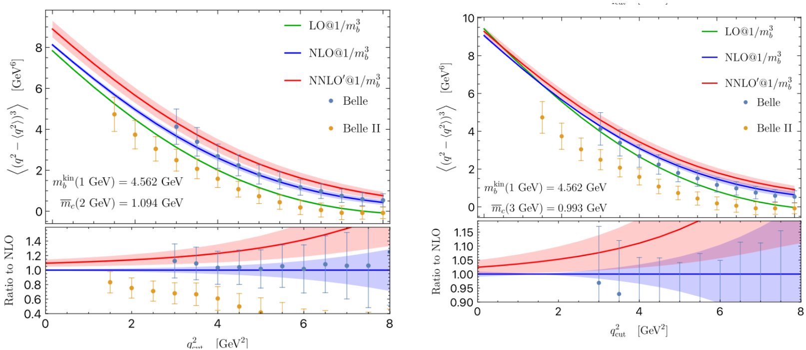


NNLO effects mainly re-absorbed in the fit into a shift of $\rho_D^3$, $r_E^4$ and $r_G^4$.
No major shift in $|V_{cb}|$.

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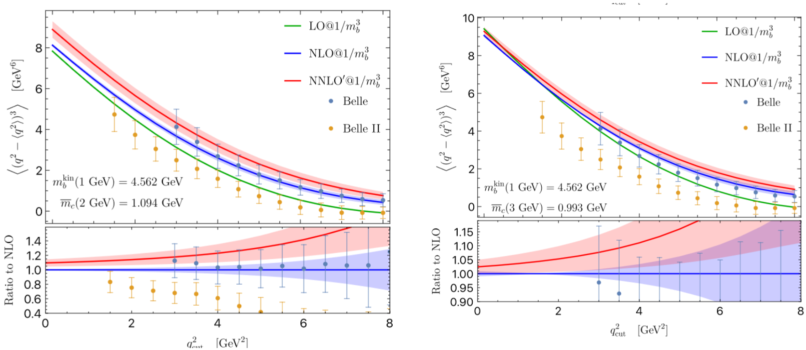


NNLO effects mainly re-absorbed in the fit into a shift of $\rho_D^3$, $r_E^4$ and $r_G^4$.
No major shift in $|V_{cb}|$.

Full combined analysis and updated $q^2$ fits in progress!