

# Hadron Physics

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# Plan for the lecture

## 1. Quantum Chromodynamics

- Gluons
- Quarks

## 2. Hadronic excitation ladder:

- Level counting
- Quark model
- Lattice QCD

## 3. Hadron scattering



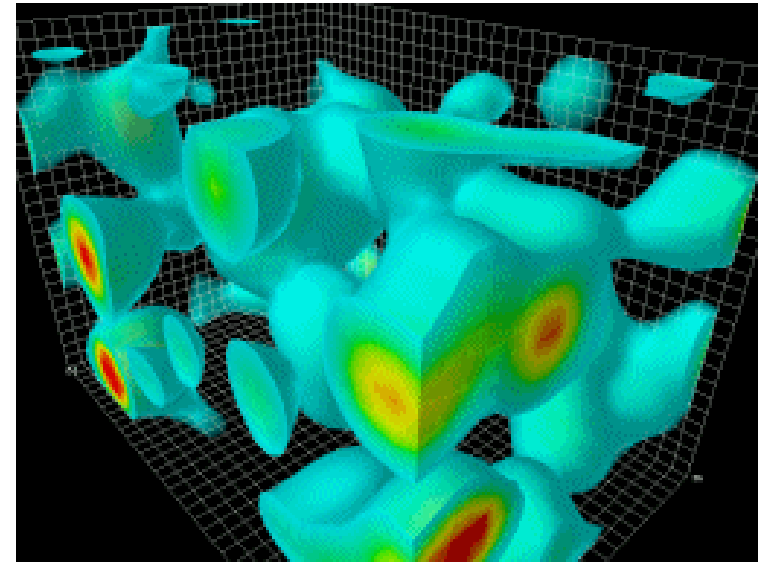
# Understanding Quantum Chromodynamics

[Midjourney 2023, MM]  
Oil pointing, depths of the quantum realm, entangled particles dance gracefully in a cosmic ballet, their movements dictated by the complex interplay of forces and probabilities. Through delicate strokes and swirling colors, capture the particles' intricate choreography, where their trajectories converge and diverge, manifesting the enigmatic beauty of the quantum world.



# QCD vacuum

- Vacuum is not empty, filled with fluctuating gluonic fields.
- Physical quarks appear to move in a gluon mean field they gain mass
- **This is transition from SM massless quarks -> constituent quarks**

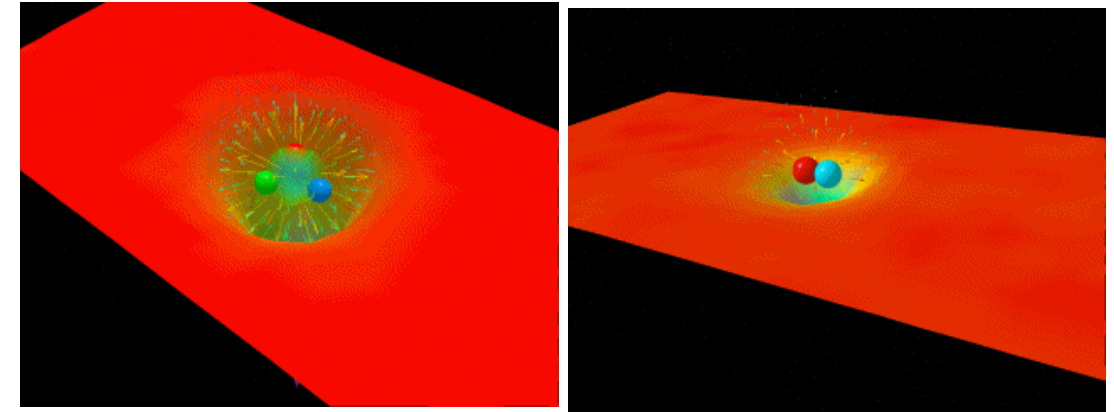
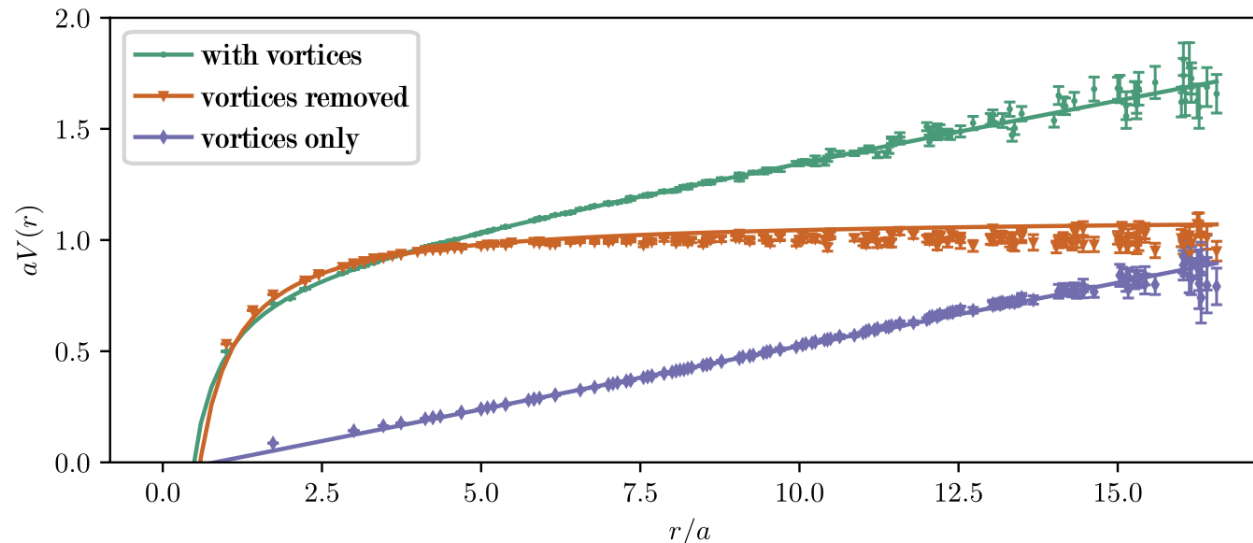


The animations illustrate the typical four-dimensional structure of gluon-field configurations averaged over in describing the vacuum properties of QCD. The volume of the box is 2.4 by 2.4 by 3.6 fm, big enough to hold a couple of protons. Contrary to the concept of an empty vacuum, QCD induces chromo-electric and chromo-magnetic fields throughout space-time in its lowest energy state. After a few sweeps of smoothing the gluon field (50 sweeps of APE smearing), a lumpy structure reminiscent of a lava lamp is revealed. This is the QCD Lava Lamp. The action density, which is similar to an energy density, is displayed.



# Confinement

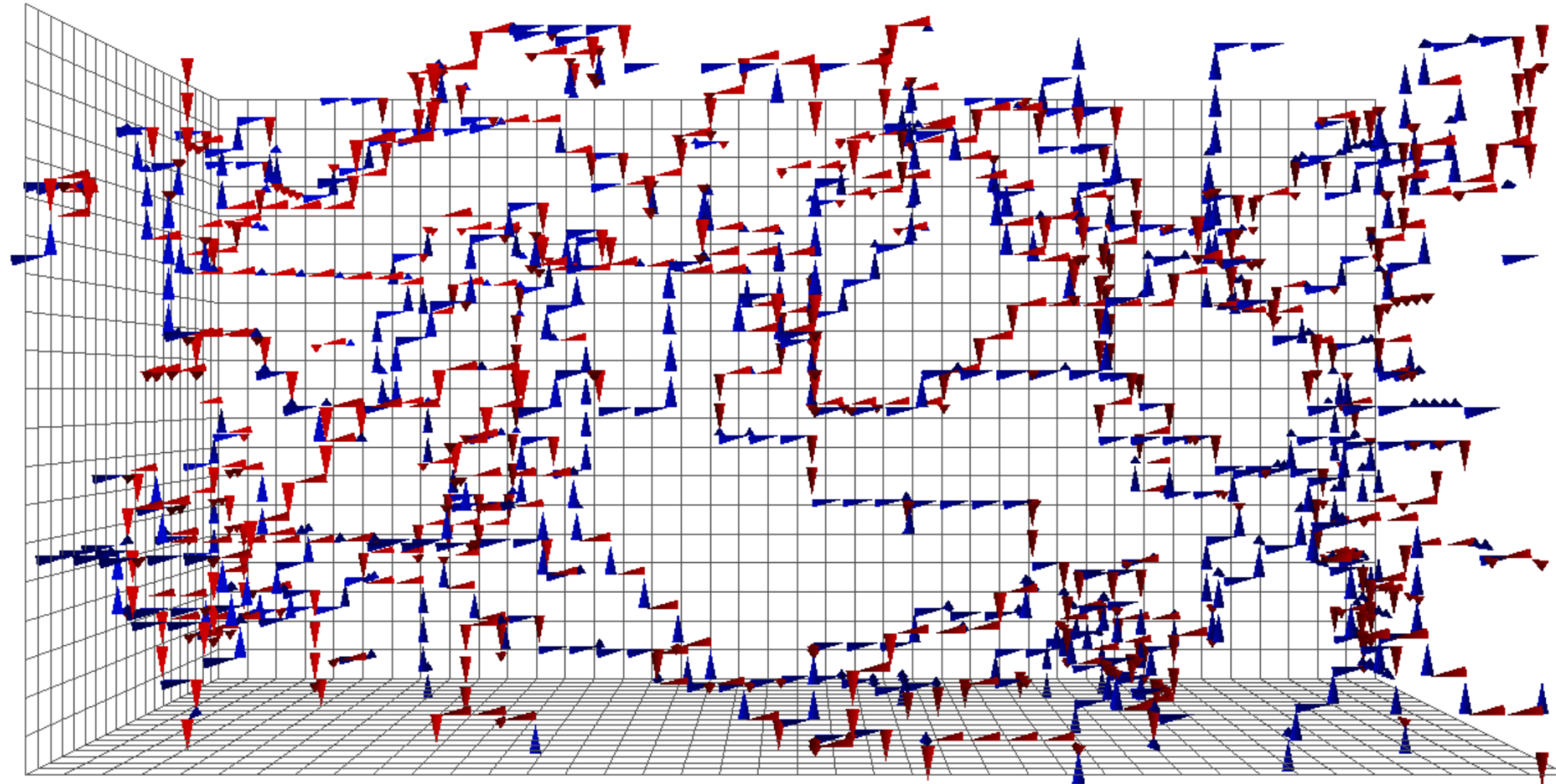
- Gluon self-interaction
- Flux tube
- Energy increases with distance
- Many strictures in QCD vacuum fields: instantons, merons, abelian monopoles, **centre vortices**



This animation shows the suppression of the QCD vacuum from the region between a quark-antiquark pair illustrated by the colored spheres. The separation of the quarks varies from 0.125 fm to 2.25 fm, the latter being about 1.3 times the diameter of a proton. The surface plot illustrates the reduction of the vacuum action density in a plane passing through the centers of the quark-antiquark pair. The vector field illustrates the gradient of this reduction. The tube joining the two quarks reveals the positions in space where the vacuum action is maximally expelled and corresponds to the famous "flux tube" of QCD. As the separation between the quarks changes the tube gets longer but the diameter remains approximately constant. As it costs energy to expel the vacuum field fluctuations, a linear confinement potential is felt between quarks.

# QCD Vortices

Penetrate vacuum and cause the confinement



# SM and QCD Lagrangian







**Gluons! – reason of the confinement**

**Light quarks: u,d,s**

- massless(!)
- cheap
- mixed

**Heavy quarks: c,b,t**

- massive
- expensive
- conserved

$\approx 2.3 \text{ MeV}/c^2$ $2/3$ $1/2$  u up	$\approx 1.275 \text{ GeV}/c^2$ $2/3$ $1/2$  c charm	$\approx 173.07 \text{ GeV}/c^2$ $2/3$ $1/2$  t top
$\approx 4.8 \text{ MeV}/c^2$ $-1/3$ $1/2$  d down	$\approx 95 \text{ MeV}/c^2$ $-1/3$ $1/2$  s strange	$\approx 4.18 \text{ GeV}/c^2$ $-1/3$ $1/2$  b bottom

$$\begin{aligned}
 \mathcal{L}_{SM} = & -\frac{1}{2}\partial_\nu g_\mu^a \partial_\nu g_\mu^a - g_s f^{abc} \partial_\mu g_\nu^a g_\mu^b g_\nu^c - \frac{1}{4}g_s^2 f^{abc} f^{ade} g_\mu^b g_\nu^c g_\mu^d g_\nu^e - \partial_\nu W_\mu^+ \partial_\nu W_\mu^- \\
 & M^2 W_\mu^+ W_\mu^- - \frac{1}{2}\partial_\nu Z_\mu^0 \partial_\nu Z_\mu^0 - \frac{1}{2c_w^2} M^2 Z_\mu^0 Z_\mu^0 - \frac{1}{2}\partial_\mu A_\nu \partial_\mu A_\nu - ig_{c_w} (\partial_\nu Z_\mu^0 (W_\mu^+ W_\nu^- - \\
 & W_\nu^+ W_\mu^-) - Z_\nu^0 (W_\mu^+ \partial_\nu W_\mu^- - W_\mu^- \partial_\nu W_\mu^+) + Z_\nu^0 (W_\nu^+ \partial_\mu W_\mu^- - W_\mu^- \partial_\nu W_\nu^+)) - \\
 & ig_{s_w} (\partial_\nu A_\mu (W_\mu^+ W_\nu^- - W_\nu^+ W_\mu^-) - A_\nu (W_\mu^+ \partial_\nu W_\mu^- - W_\mu^- \partial_\nu W_\mu^+) + A_\nu (W_\nu^+ \partial_\mu W_\mu^- - \\
 & W_\mu^- \partial_\nu W_\nu^+)) - \frac{1}{2}g^2 W_\mu^+ W_\nu^+ W_\nu^- + \frac{1}{2}g^2 W_\mu^- W_\nu^- W_\nu^+ + g^2 c_w^2 (Z_\mu^0 W_\mu^+ Z_\nu^0 W_\nu^- - \\
 & Z_\nu^0 Z_\mu^0 W_\nu^+ W_\nu^-) + g^2 s_w^2 (A_\mu W_\nu^+ A_\nu W_\nu^- - A_\mu A_\nu W_\nu^+ W_\nu^-) + g^2 s_w c_w (A_\mu Z_\nu^0 (W_\mu^+ W_\nu^- - \\
 & W_\nu^+ W_\mu^-) - 2A_\mu Z_\nu^0 W_\nu^+ W_\nu^-) - \frac{1}{2}\partial_\mu H \partial_\mu H - 2M^2 \alpha_h H^2 - \partial_\mu \phi^+ \partial_\mu \phi^- - \frac{1}{2}\partial_\mu \phi^0 \partial_\mu \phi^0 - \\
 & \beta_h \left( \frac{2M^2}{g^2} + \frac{2M}{g} H + \frac{1}{2}(H^2 + \phi^0 \phi^0 + 2\phi^+ \phi^-) \right) + \frac{2M^4}{g^2} \alpha_h - \\
 & g\alpha_h M (H^3 + H\phi^0 \phi^0 + 2H\phi^+ \phi^-) - \\
 & \frac{1}{8}g^2 \alpha_h (H^4 + (\phi^0)^4 + 4(\phi^+ \phi^-)^2 + 4(\phi^0)^2 \phi^+ \phi^- + 4H^2 \phi^+ \phi^- + 2(\phi^0)^2 H^2) - \\
 & gMW_\mu^+ W_\mu^- H - \frac{1}{2}g \frac{M}{c_w} Z_\mu^0 Z_\mu^0 H - \\
 & \frac{1}{2}ig (W_\mu^+ (\phi^0 \partial_\mu \phi^- - \phi^- \partial_\mu \phi^0) - W_\mu^- (\phi^0 \partial_\mu \phi^+ - \phi^+ \partial_\mu \phi^0)) + \\
 & \frac{1}{2}g (W_\mu^+ (H \partial_\mu \phi^- - \phi^- \partial_\mu H) + W_\mu^- (H \partial_\mu \phi^+ - \phi^+ \partial_\mu H)) + \frac{1}{2}g \frac{1}{c_w} (Z_\mu^0 (H \partial_\mu \phi^0 - \phi^0 \partial_\mu H) + \\
 & M (\frac{1}{c_w} Z_\mu^0 \partial_\mu \phi^0 + W_\mu^+ \partial_\mu \phi^- + W_\mu^- \partial_\mu \phi^+) - ig \frac{s_w^2}{c_w} M Z_\mu^0 (W_\mu^+ \phi^- - W_\mu^- \phi^+) + ig_{s_w} M A_\mu (W_\mu^+ \phi^- - \\
 & W_\mu^- \phi^+) - ig \frac{1-2c_w^2}{2c_w} Z_\mu^0 (\phi^+ \partial_\mu \phi^- - \phi^- \partial_\mu \phi^+) + ig_{s_w} A_\mu (\phi^+ \partial_\mu \phi^- - \phi^- \partial_\mu \phi^+) - \\
 & \frac{1}{4}g^2 W_\mu^+ W_\mu^- (H^2 + (\phi^0)^2 + 2\phi^+ \phi^-) - \frac{1}{8}g^2 \frac{1}{c_w} Z_\mu^0 Z_\mu^0 (H^2 + (\phi^0)^2 + 2(2s_w^2 - 1)^2 \phi^+ \phi^-) - \\
 & \frac{1}{2}g^2 \frac{s_w^2}{c_w} Z_\mu^0 \phi^0 (W_\mu^+ \phi^- + W_\mu^- \phi^+) - \frac{1}{2}ig^2 \frac{s_w^2}{c_w} Z_\mu^0 H (W_\mu^+ \phi^- - W_\mu^- \phi^+) + \frac{1}{2}g^2 s_w A_\mu \phi^0 (W_\mu^+ \phi^- + \\
 & W_\mu^- \phi^+) + \frac{1}{2}ig^2 s_w A_\mu H (W_\mu^+ \phi^- - W_\mu^- \phi^+) - g^2 \frac{s_w}{c_w} (2c_w^2 - 1) Z_\mu^0 A_\mu \phi^+ \phi^- - \\
 & g^2 s_w^2 A_\mu A_\nu \phi^+ \phi^- + \frac{1}{2}ig_s \lambda_{ij}^a (\bar{q}_i^c \gamma^\mu q_j^c) g_\mu^a - \bar{e}^\lambda (\gamma^\mu + m_e^\lambda) e^\lambda - \bar{\nu}^\lambda (\gamma^\mu + m_\nu^\lambda) \nu^\lambda - \bar{u}_j^\lambda (\gamma^\mu + \\
 & m_u^\lambda) u_j^\lambda - \bar{d}_j^\lambda (\gamma^\mu + m_d^\lambda) d_j^\lambda + ig_{s_w} A_\mu (-\bar{e}^\lambda \gamma^\mu e^\lambda + \frac{2}{3}(\bar{u}_j^\lambda \gamma^\mu u_j^\lambda) - \frac{1}{3}(\bar{d}_j^\lambda \gamma^\mu d_j^\lambda)) + \\
 & \frac{ig}{4c_w} Z_\mu^0 \{ (\bar{\nu}^\lambda \gamma^\mu (1 + \gamma^5) \nu^\lambda) + (\bar{e}^\lambda \gamma^\mu (4s_w^2 - 1 - \gamma^5) e^\lambda) + (\bar{d}_j^\lambda \gamma^\mu (\frac{1}{3}s_w^2 - 1 - \gamma^5) d_j^\lambda) + \\
 & (\bar{u}_j^\lambda \gamma^\mu (1 - \frac{8}{3}s_w^2 + \gamma^5) u_j^\lambda) \} + \frac{ig}{2\sqrt{2}} W_\mu^+ ((\bar{\nu}^\lambda \gamma^\mu (1 + \gamma^5) U^{lep}{}_{\lambda\kappa} e^\kappa) + (\bar{u}_j^\lambda \gamma^\mu (1 + \gamma^5) C_{\lambda\kappa} d_j^\kappa)) + \\
 & \frac{ig}{2\sqrt{2}} W_\mu^- ((\bar{e}^\kappa U^{lep}{}_{\kappa\lambda} \gamma^\mu (1 + \gamma^5) \nu^\lambda) + (\bar{d}_j^\kappa C_{\kappa\lambda} \gamma^\mu (1 + \gamma^5) u_j^\lambda)) + \\
 & \frac{ig}{2M\sqrt{2}} \phi^+ (-m_e^\kappa (\bar{\nu}^\lambda U^{lep}{}_{\lambda\kappa} (1 - \gamma^5) e^\kappa) + m_\nu^\lambda (\bar{\nu}^\lambda U^{lep}{}_{\lambda\kappa} (1 + \gamma^5) e^\kappa) + \\
 & \frac{ig}{2M\sqrt{2}} \phi^- (m_e^\lambda (\bar{e}^\lambda U^{lep}{}_{\lambda\kappa} (1 + \gamma^5) \nu^\kappa) - m_\nu^\kappa (\bar{e}^\lambda U^{lep}{}_{\lambda\kappa} (1 - \gamma^5) \nu^\kappa) - \frac{g}{2} \frac{m_\lambda^2}{M} H (\bar{\nu}^\lambda \nu^\lambda) - \\
 & \frac{g}{2} \frac{m_\lambda^2}{M} H (\bar{e}^\lambda e^\lambda) + \frac{ig}{2} \frac{m_\lambda^2}{M} \phi^0 (\bar{\nu}^\lambda \gamma^5 \nu^\lambda) - \frac{ig}{2} \frac{m_\lambda^2}{M} \phi^0 (\bar{e}^\lambda \gamma^5 e^\lambda) - \frac{1}{4} \bar{\nu}_\lambda M_{\lambda\kappa}^R (1 - \gamma_5) \hat{\nu}_\kappa - \\
 & \frac{1}{4} \bar{\nu}_\lambda M_{\lambda\kappa}^R (1 - \gamma_5) \hat{\nu}_\kappa + \frac{ig}{2M\sqrt{2}} \phi^+ (-m_d^\kappa (\bar{u}_j^\lambda C_{\lambda\kappa} (1 - \gamma^5) d_j^\kappa) + m_u^\lambda (\bar{u}_j^\lambda C_{\lambda\kappa} (1 + \gamma^5) d_j^\kappa)) + \\
 & \frac{ig}{2M\sqrt{2}} \phi^- (m_d^\lambda (\bar{d}_j^\kappa C_{\lambda\kappa}^\dagger (1 + \gamma^5) u_j^\kappa) - m_u^\kappa (\bar{d}_j^\kappa C_{\lambda\kappa}^\dagger (1 - \gamma^5) u_j^\kappa) - \frac{g}{2} \frac{m_\lambda^2}{M} H (\bar{u}_j^\lambda u_j^\lambda) - \\
 & \frac{g}{2} \frac{m_\lambda^2}{M} H (\bar{d}_j^\lambda d_j^\lambda) + \frac{ig}{2} \frac{m_\lambda^2}{M} \phi^0 (\bar{u}_j^\lambda \gamma^5 u_j^\lambda) - \frac{ig}{2} \frac{m_\lambda^2}{M} \phi^0 (\bar{d}_j^\lambda \gamma^5 d_j^\lambda) + G^a \partial^2 G^a + g_s f^{abc} \partial_\mu \bar{G}^a G^b g_\mu^c + \\
 & \bar{X}^+ (\partial^2 - M^2) X^+ + \bar{X}^- (\partial^2 - M^2) X^- + \bar{X}^0 (\partial^2 - \frac{M^2}{c_w^2}) X^0 + \bar{Y} \partial^2 Y + ig_{c_w} W_\mu^+ (\partial_\mu \bar{X}^0 X^- - \\
 & \partial_\mu \bar{X}^+ X^0) + ig_{s_w} W_\mu^+ (\partial_\mu \bar{Y} X^- - \partial_\mu \bar{X}^+ Y) + ig_{c_w} W_\mu^- (\partial_\mu \bar{X}^- X^0 - \\
 & \partial_\mu \bar{X}^0 X^+) + ig_{s_w} W_\mu^- (\partial_\mu \bar{X}^- Y - \partial_\mu \bar{Y} X^+) + ig_{c_w} Z_\mu^0 (\partial_\mu \bar{X}^+ X^+ - \\
 & \partial_\mu \bar{X}^- X^-) + ig_{s_w} A_\mu (\partial_\mu \bar{X}^+ X^+ - \\
 & \partial_\mu \bar{X}^- X^-) - \frac{1}{2}gM (\bar{X}^+ X^+ H + \bar{X}^- X^- H + \frac{1}{c_w} \bar{X}^0 X^0 H) + \frac{1-2c_w^2}{2c_w} igM (\bar{X}^+ X^0 \phi^+ - \bar{X}^- X^0 \phi^-) + \\
 & \frac{1}{2c_w} igM (\bar{X}^0 X^- \phi^+ - \bar{X}^0 X^+ \phi^-) + igM s_w (\bar{X}^0 X^- \phi^+ - \bar{X}^0 X^+ \phi^-) + \\
 & \frac{1}{2}igM (\bar{X}^+ X^+ \phi^0 - \bar{X}^- X^- \phi^0) .
 \end{aligned}$$

[Diagrammatica, Lecture Notes, M. Veltman]



# Every family gets own energy range

## Family is determined by QM

- quark content
- isospin

### Example 1:

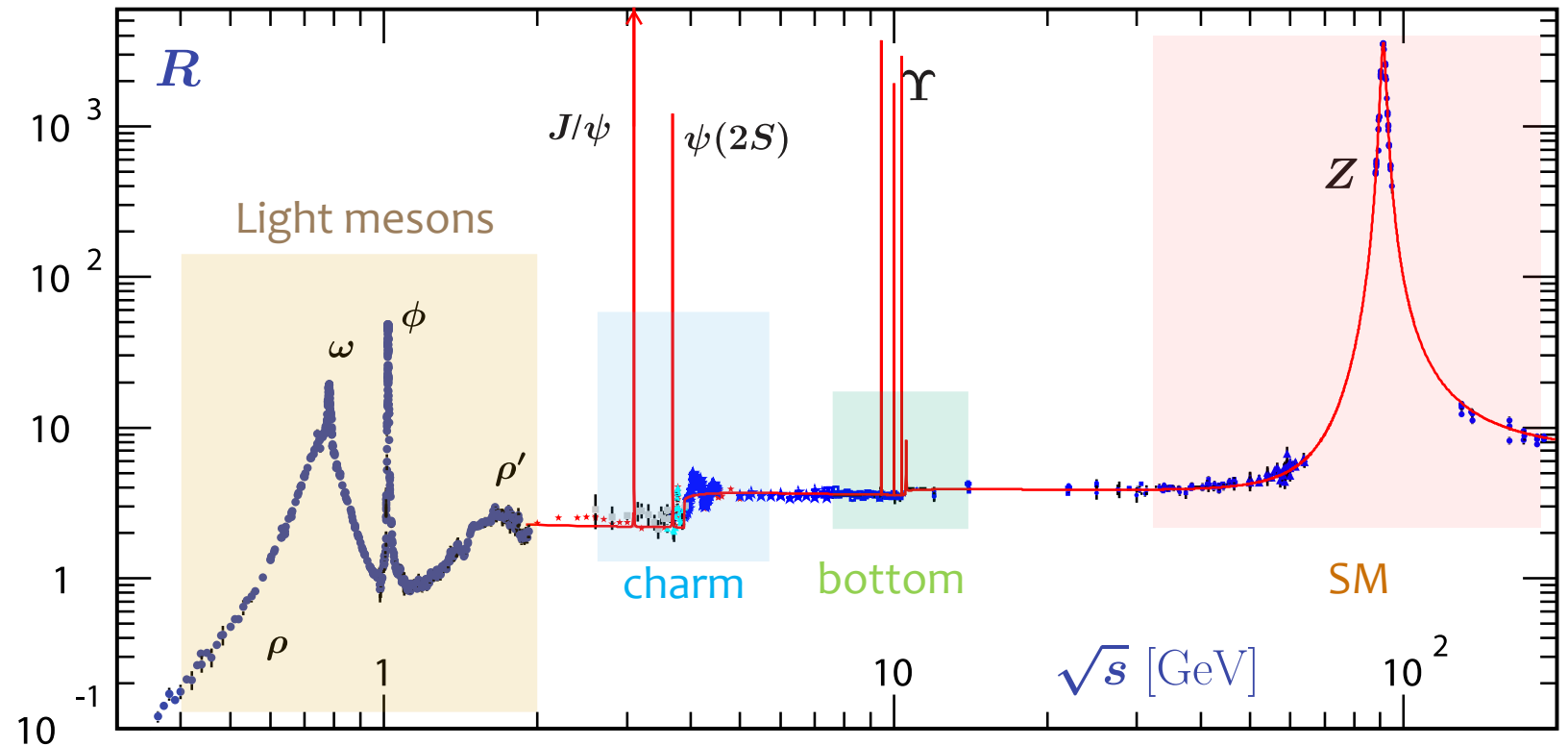
$udc$  with  $I=0 \Rightarrow \Lambda_c^+$

$udc$  with  $I=1 \Rightarrow \Sigma_c^+$

### Example 2:

$uud$  - proton,  $\sim 1\text{GeV}$

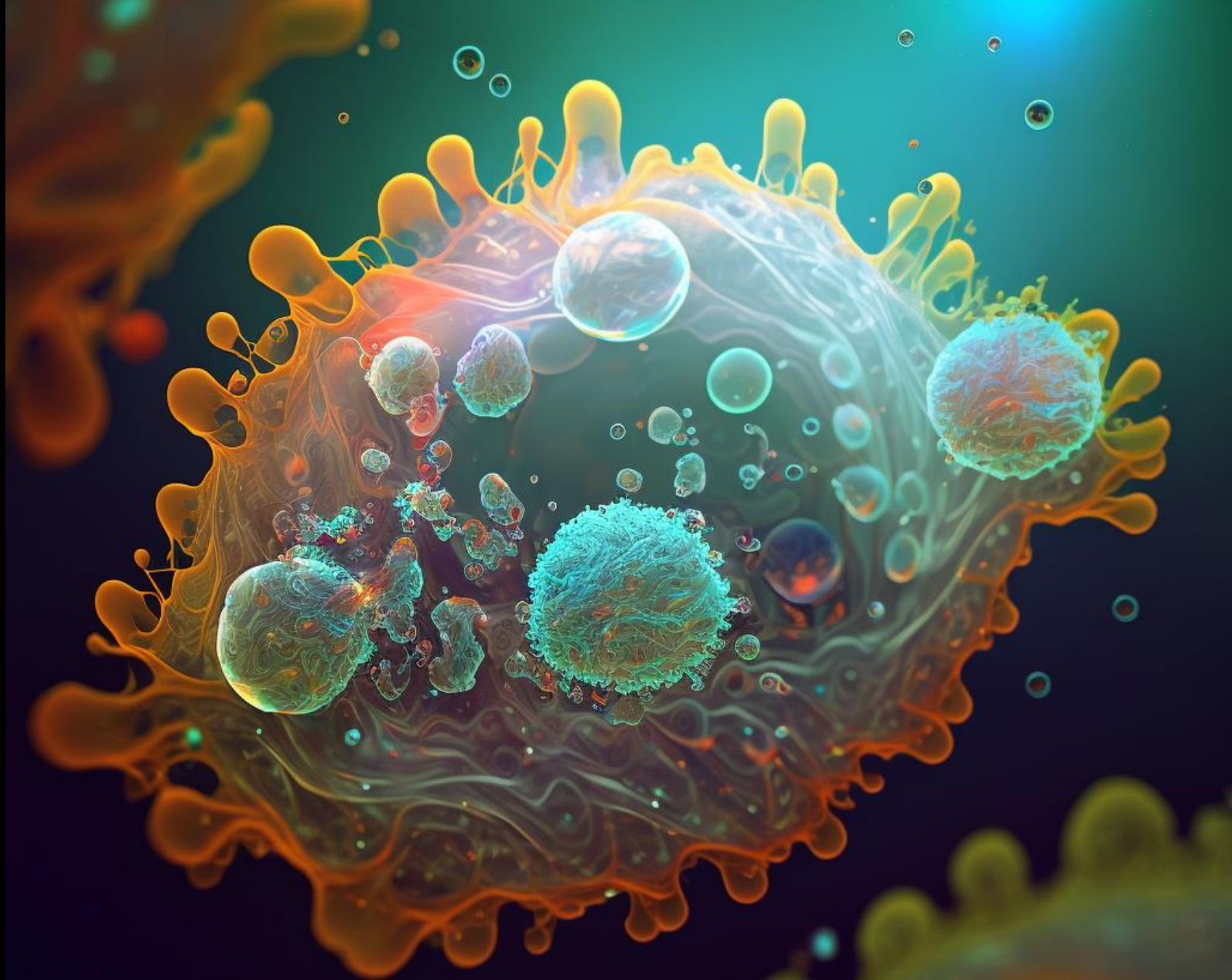
$uudc\bar{c}$  - pentaquark,  $\sim 4.3\text{GeV}$



$$R = \frac{\sigma^{(0)}(e^+e^- \rightarrow \text{hadrons})}{\sigma(e^+e^- \rightarrow \mu^+\mu^-)}$$



# Hadronic Excitations

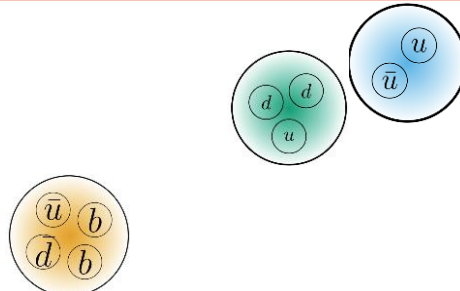


[Midjourney 2023, MM] Highly defined macrophotography of  
a weird unseen quantum world<sup>9+</sup> images

# Possible configurations of hadrons

**Conventional Quark Model:**  $(q\bar{q}, qqq)$

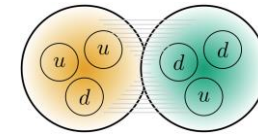
**Bigger Quark Model**  $(q\bar{q}q\bar{q}, qqqq\bar{q}, \dots)$



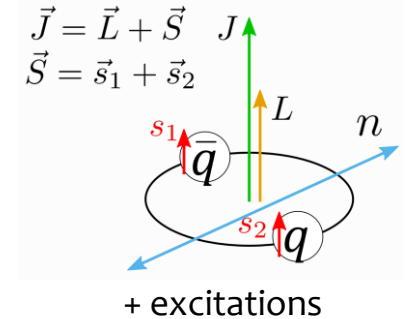
**Conventional Hadronic Molecules = Nuclei:**  $(qqq)(qqq)$

**Heavy-Flavor Hadronic Molecules:**  $(Qqq)(Qqq), (Q\bar{q})(Qqq), \dots$

**Admixed Molecules:**  $q\bar{q} \rightarrow (q\bar{q})(q\bar{q})$

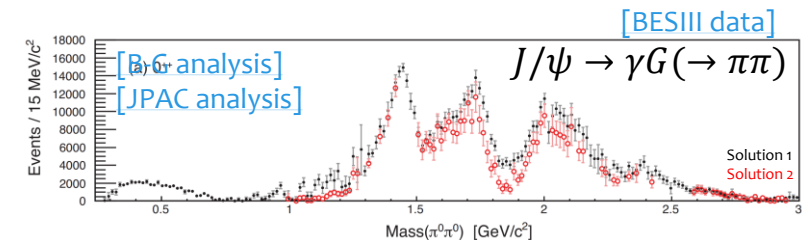
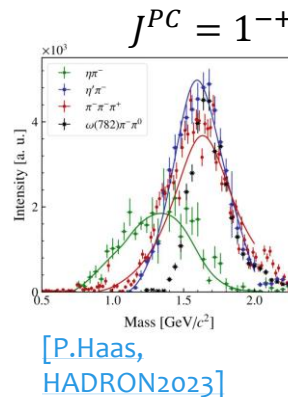
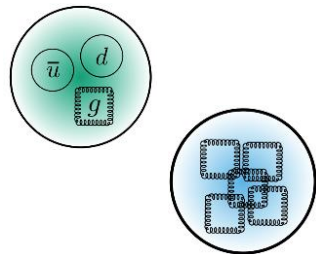


+ nuclei chart



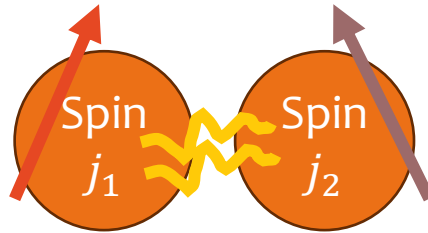
**Hybrids:**  $q \sim g \sim \bar{q}$

**Glueballs:**  $g \sim g$



# Reminder on spin algebra

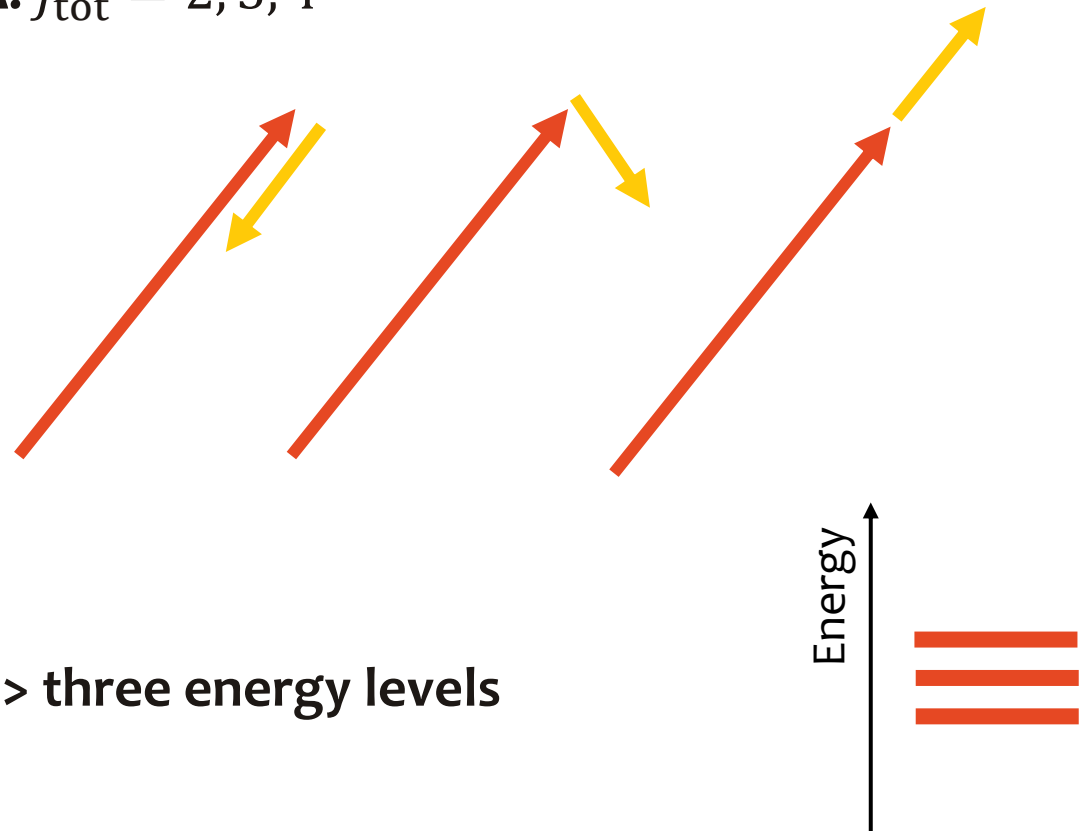
Consider an interacting system of spins



Q: what is an energy spectrum?

Say,  $j_1 = 1$  and  $j_2 = 3$

A:  $j_{\text{tot}} = 2, 3, 4$

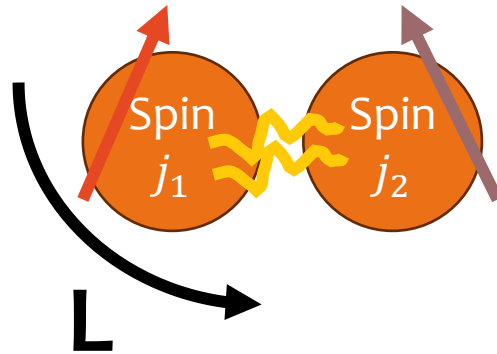


=> three energy levels



# Adding orbital excitation

The system can be excited radially



$$j_1 \times j_2$$

$$3/2 \times 1/2$$

1 2 [S-wave]

(0 1 2) (1 2 3) [P]

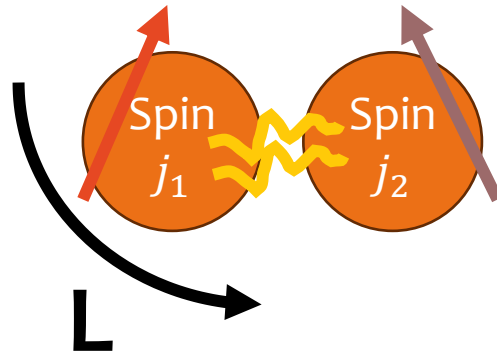
(1 2 3) (0 1 2 3 4) [D]

(2 3 4) (1 2 3 4 5) [F]

Q: what are multiplicities of L-wave multiplets?

# Add parity

Interaction conserves parity



$$j_1 \times j_2$$

$$3/2^+ \times 1/2^-$$

1- 2- [S-wave]

(0+ 1+ 2+) (1+ 2+ 3+) [P]

(1- 2- 3-) (0- 1- 2- 3- 4-) [D]

(2+ 3+ 4+) (1- 2- 3- 4- 5-) [F]

Q: what are parities of different configurations?

Say,  $3/2^+ \times 1/2^-$

# Radial excitations and orbital excitations

Hydrogen atom:

$n$  – principle quantum number

$l$  – orbital quantum number  
( $l < n$ )

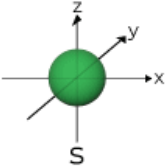
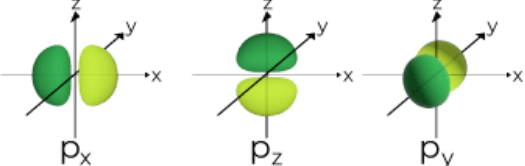
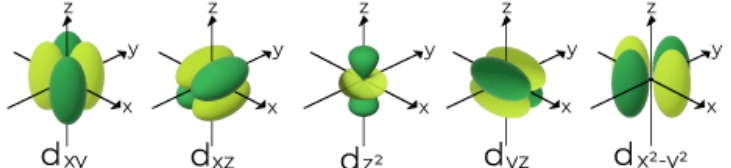
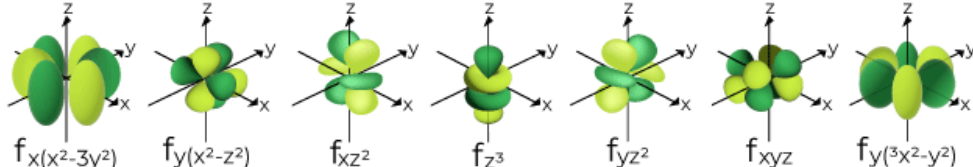
The spectrum is

...

1S

2S 1P

3S 2P 1D

<b>S</b> orbital		$n = 1, 2, 3, \dots 7$ $l = 0$ $m = 0$
<b>p</b> orbital		$n = 2, 3, \dots 6$ $l = 1$ $m = 0, \pm 1$
<b>d</b> orbital		$n = 3, 4, \text{and } 5$ $l = 2$ $m = 0, \pm 1, \pm 2$
<b>f</b> orbital		$n = 4$ $l = 3$ $m = 0, \pm 1, \pm 2, \pm 3$

[<https://www.geeksforgeeks.org/quantum-numbers/>]

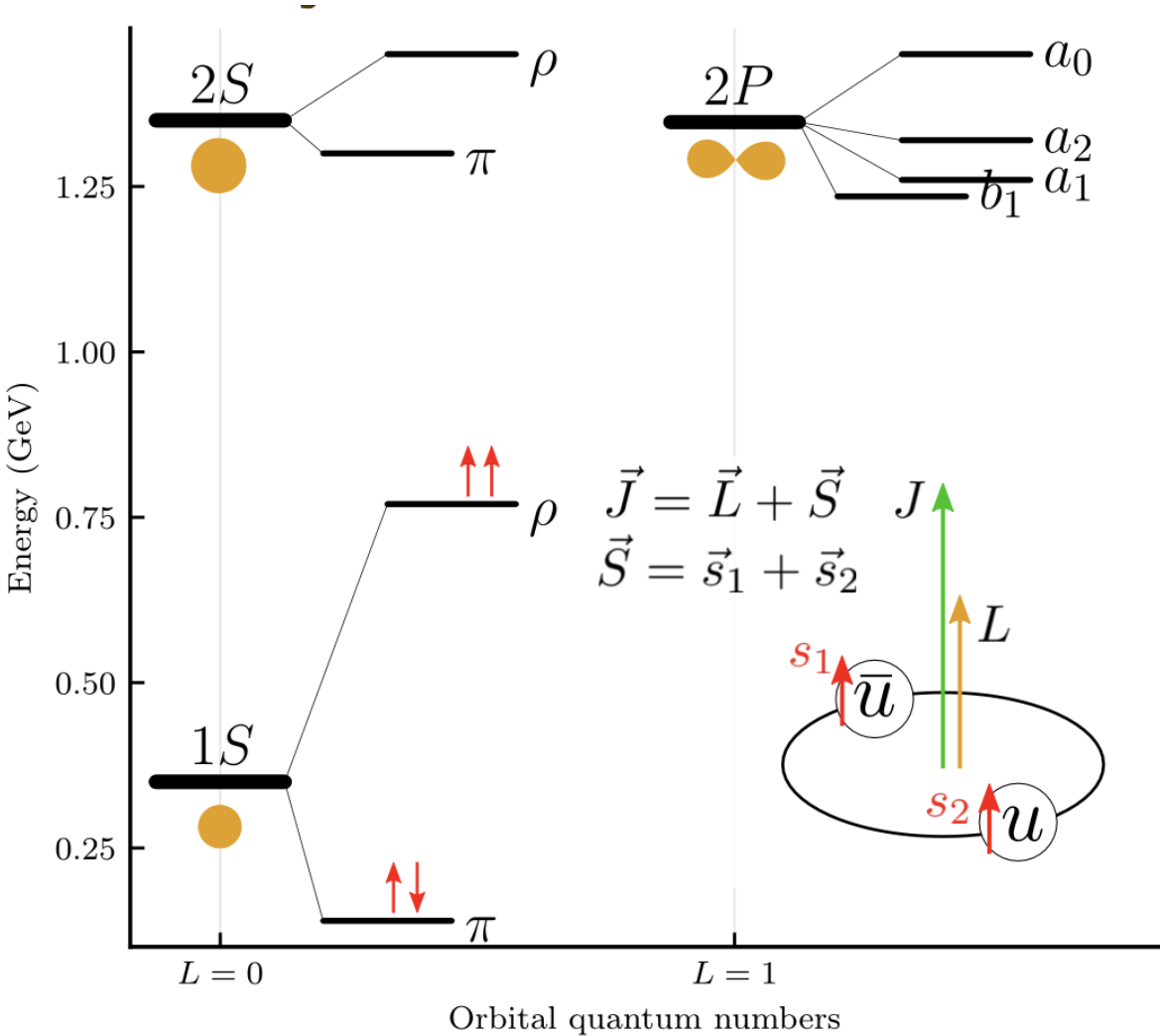


# Meson excitations

Same for all conventional mesons,  $q\bar{q}$

$1/2^+ \times 1/2^- \mid 0^- \ 1^- \text{ [S-wave]}$

=> doublets( $nS$ ), and  
quadruplets ( $nP, nD, nF\dots: 3+1$ )



# Exercise 2: baryon spectrum

Determine spin and parity of the excited states

$\Lambda_c^+$  ( $\Lambda_b^0$ ) spectrum:

$Q\ 1/2^+ \times [11] 0^+$



$\Omega_c^0$  ( $\Omega_b^-$  or  $\Sigma_{c/b}$ ) spectrum:

$Q\ 1/2^+ \times [11] 1^+$



# Charmonium in Quark model

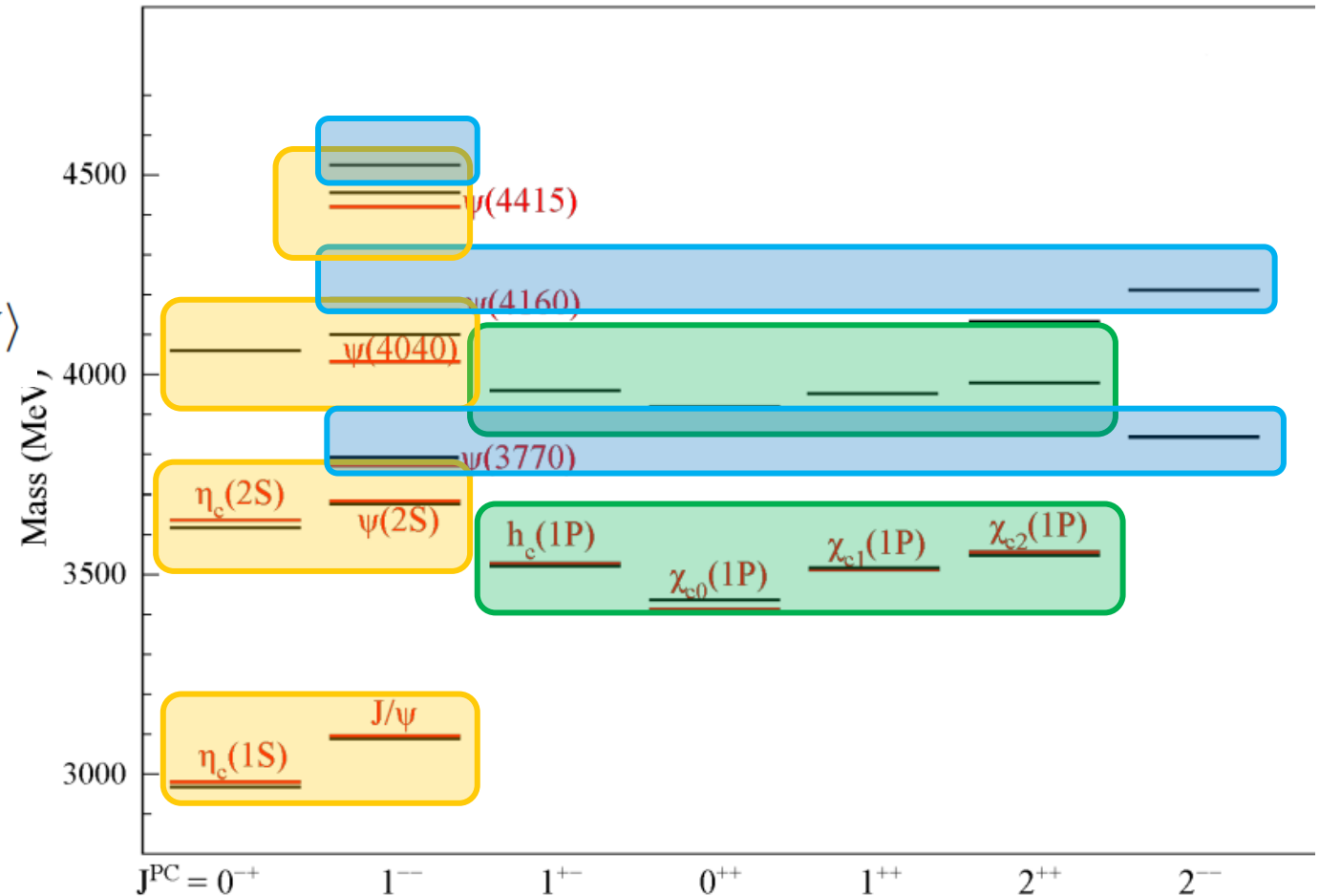
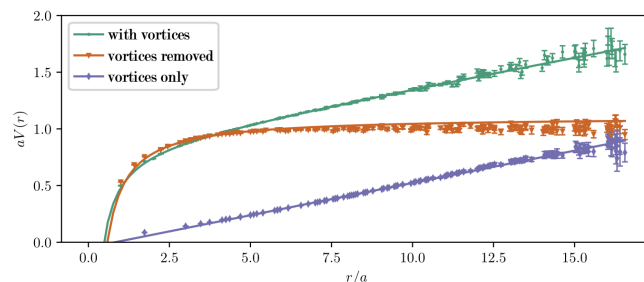
## Relativistic Quark Model

[Gottfrey-Isgur, PRD32, 1985]

$$\left( \sqrt{m_1^2 + p^2} + \sqrt{m_2^2 + p^2} + V \right) |\Psi\rangle = E|\Psi\rangle$$

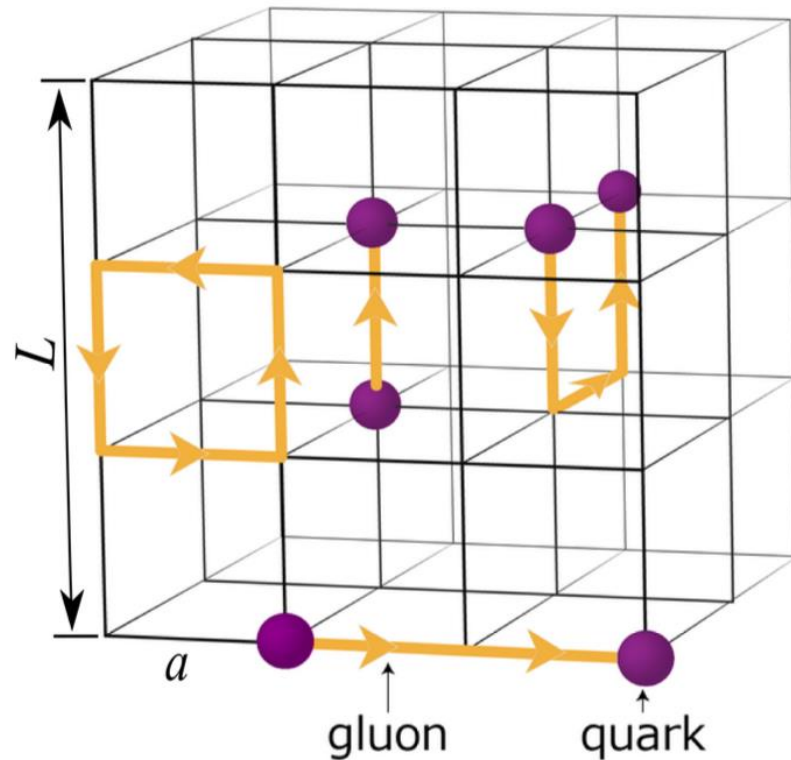
### potential $V$ :

One-gluon exchange +  
linear confinement +  
relativistic effects





# Lattice QCD



## First-principle computation tool

- start from **QCD Lagrangian**

$$\mathcal{L} = \bar{\psi} (i\gamma^\mu D_\mu - m) \psi - \frac{1}{4} G_{\mu\nu}^a G_a^{\mu\nu}$$

- compute **expectation value** for an operator

$$\langle \mathcal{O} \rangle = \frac{1}{Z} \int \mathcal{D}U \mathcal{D}\psi \mathcal{D}\bar{\psi} \mathcal{O} e^{-S_{QCD}}$$

- integrate **fermion fields** analytically, **gluon** numerically (importance sampling, average over ensemble)

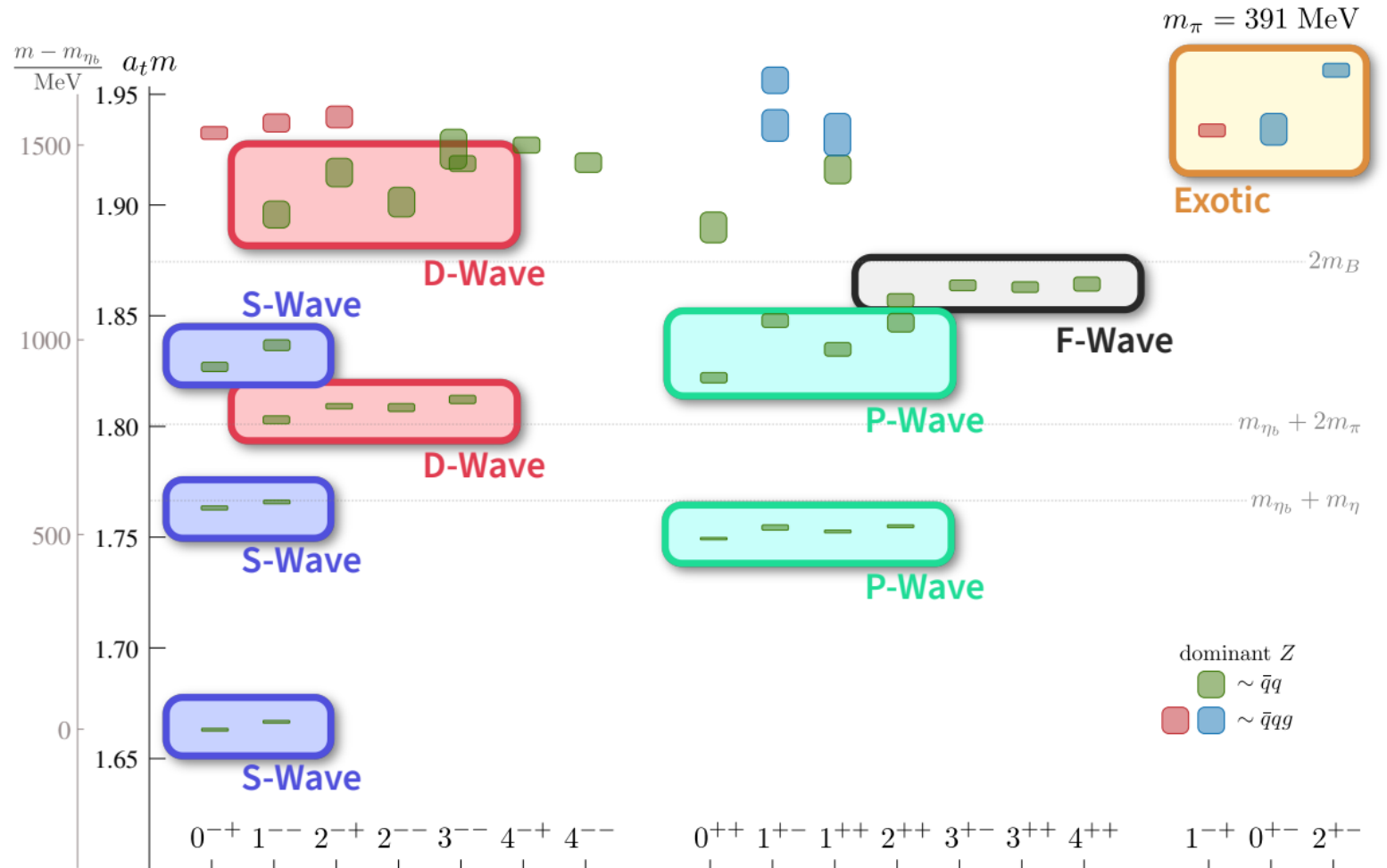
- using Euclidian time find energy,  $E_0$

$$e^{iE_0 t} \rightarrow e^{-E_0 T}$$

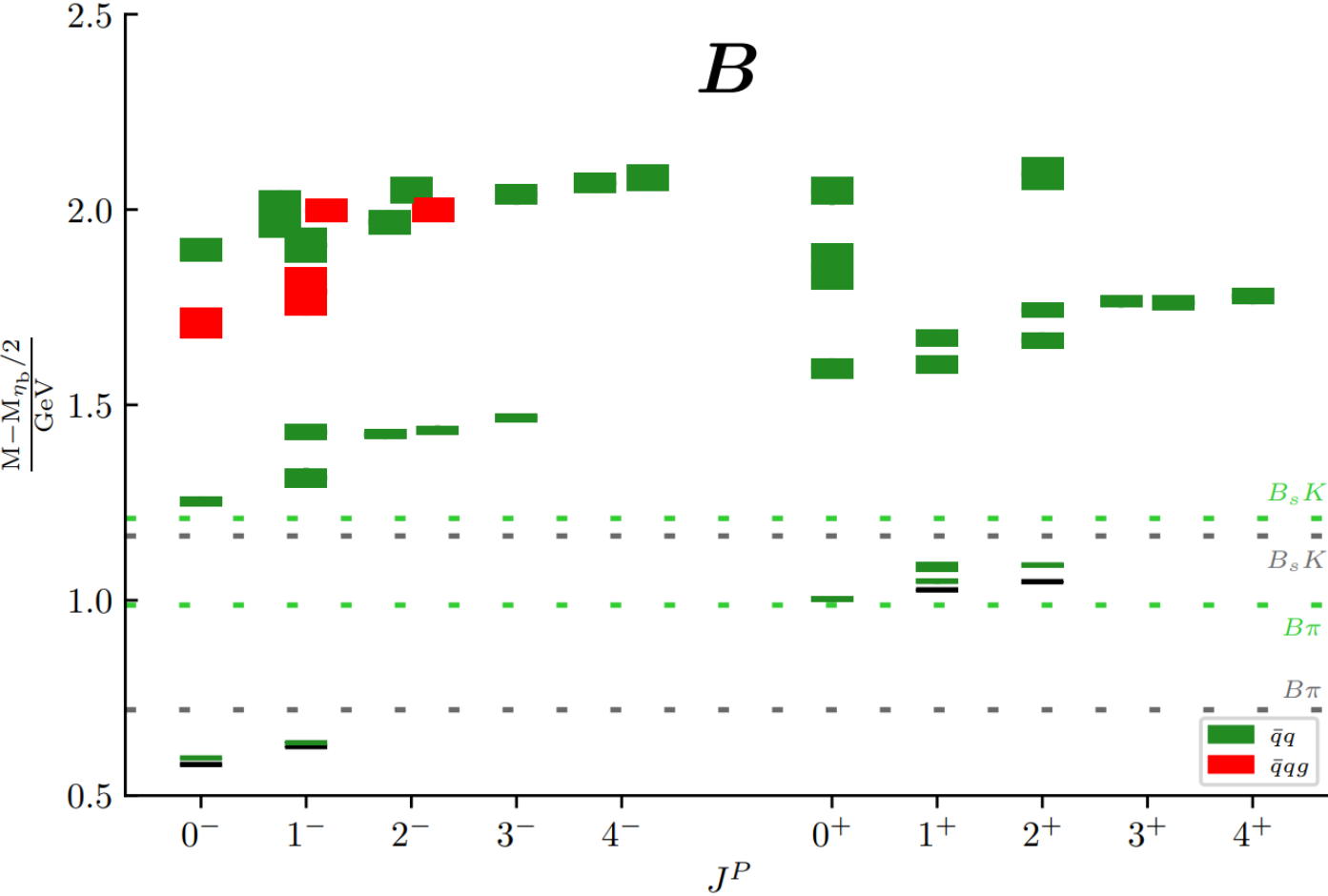
# Bottomonium spectrum

[HadSpec, JHEP 02 (2021) 214]

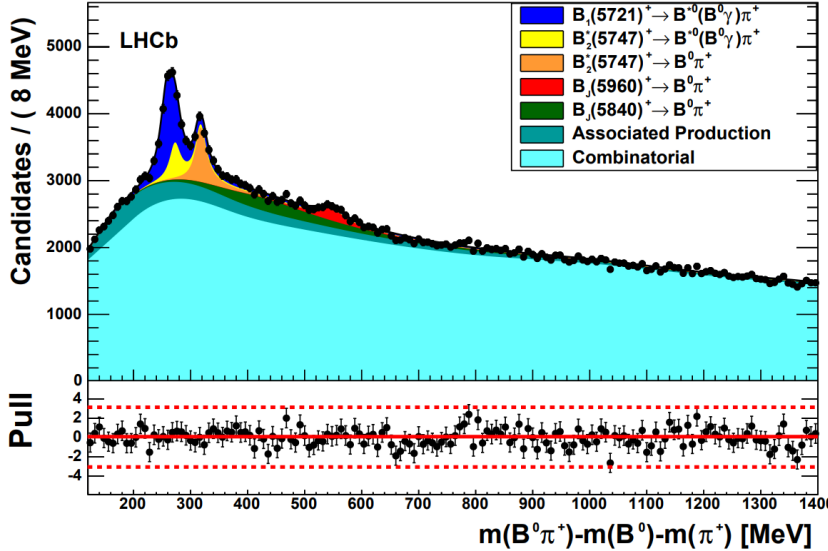
Always same pattern



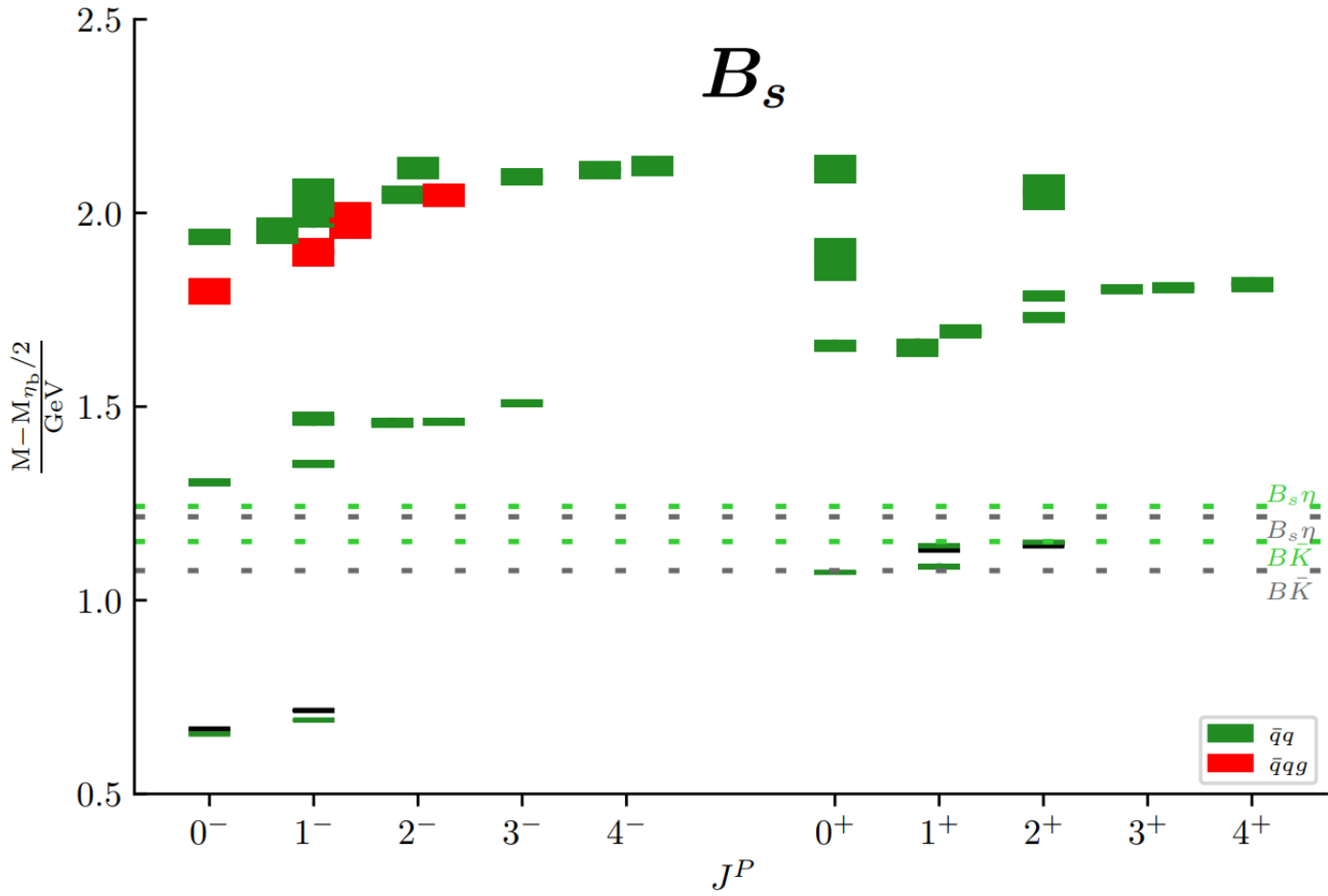
# Excitations spectrum of B mesons



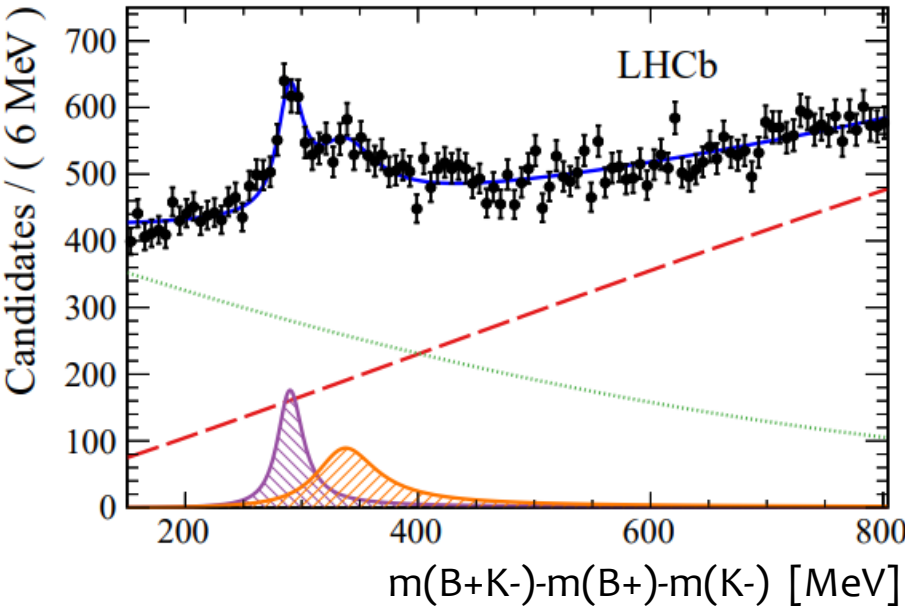
[LHCb, JHEP 1504 (2015) 024]



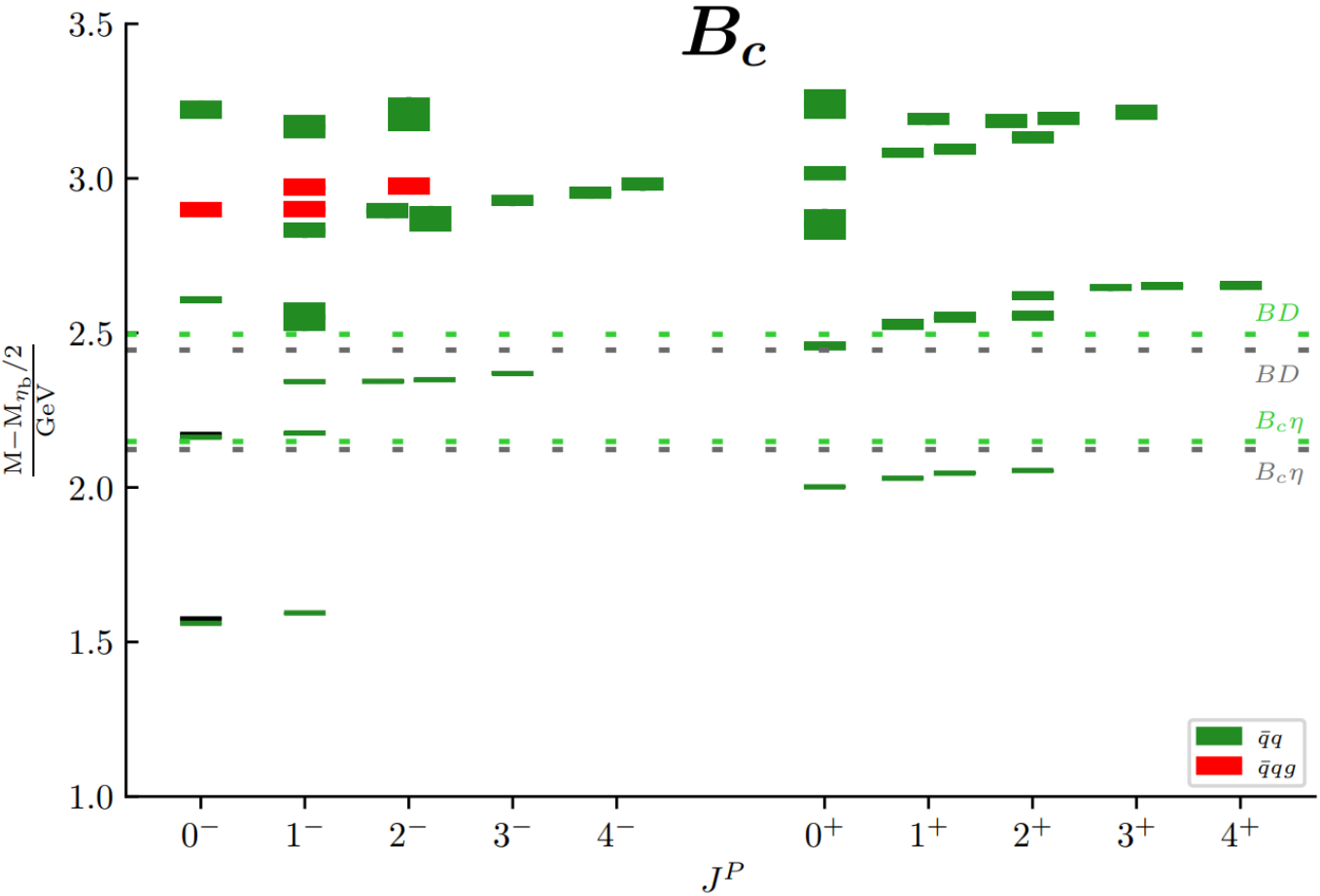
# Excitations spectrum of Bs mesons



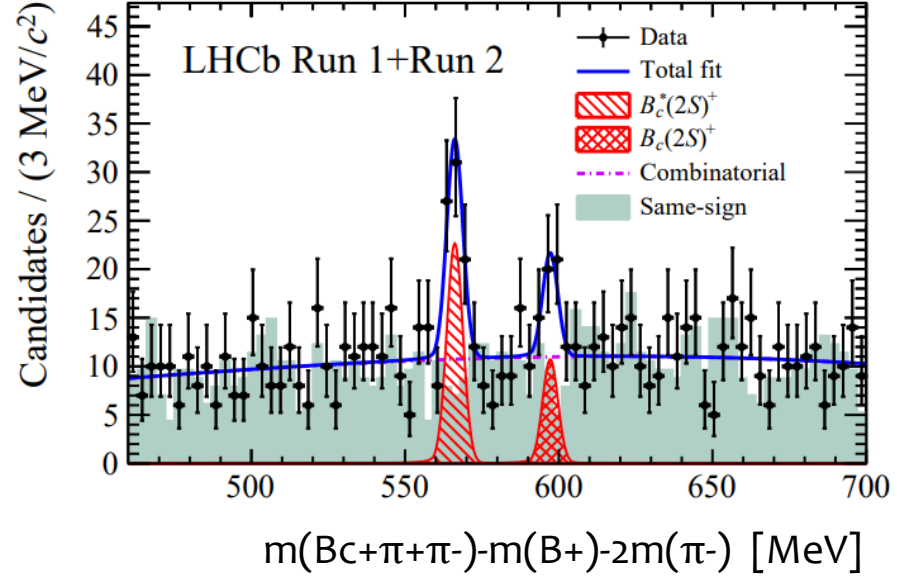
[LHCb, Eur. Phys. J. C81 (2021) 601]



# Excitations spectrum of Bc mesons



[LHCb, Phys.Rev.Lett. 122 (2019) 23, 232001]





# Hadron scattering

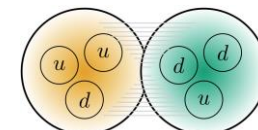
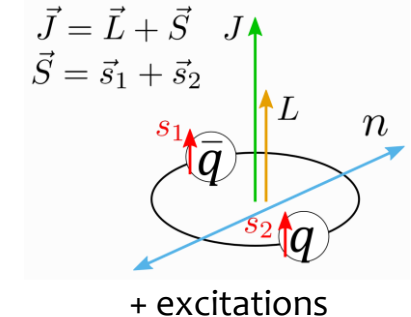
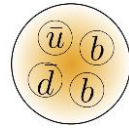
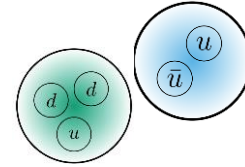


[Midjourney 2023, MM] collision of elementary particles, explosion with cut fruits going to all direction

# Possible configurations of hadrons

Conventional Quark Model:  $(q\bar{q}, qqq)$

Bigger Quark Model  $(q\bar{q}q\bar{q}, qqqq\bar{q}, \dots)$

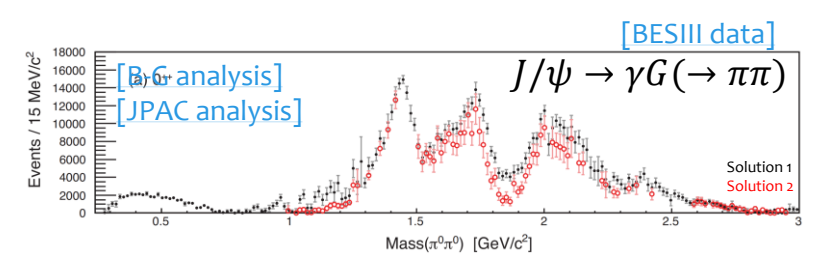
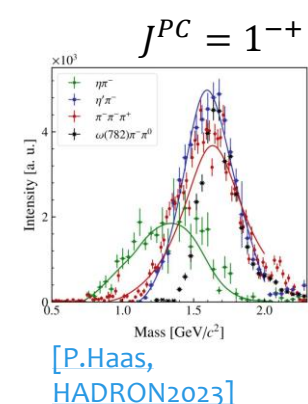
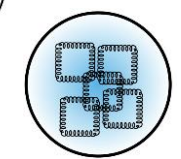
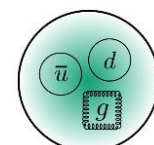


+ nuclei chart

**Conventional Hadronic Molecules = Nuclei:**  $(qqq)(qqq)$   
**Heavy-Flavor Hadronic Molecules:**  $(Qqq)(Qqq), (Q\bar{q})(Qqq), \dots$   
**Admixed Molecules:**  $q\bar{q} \rightarrow (q\bar{q})(q\bar{q})$

Hybrids:  $q \sim g \sim \bar{q}$

Glueballs:  $g \sim g$

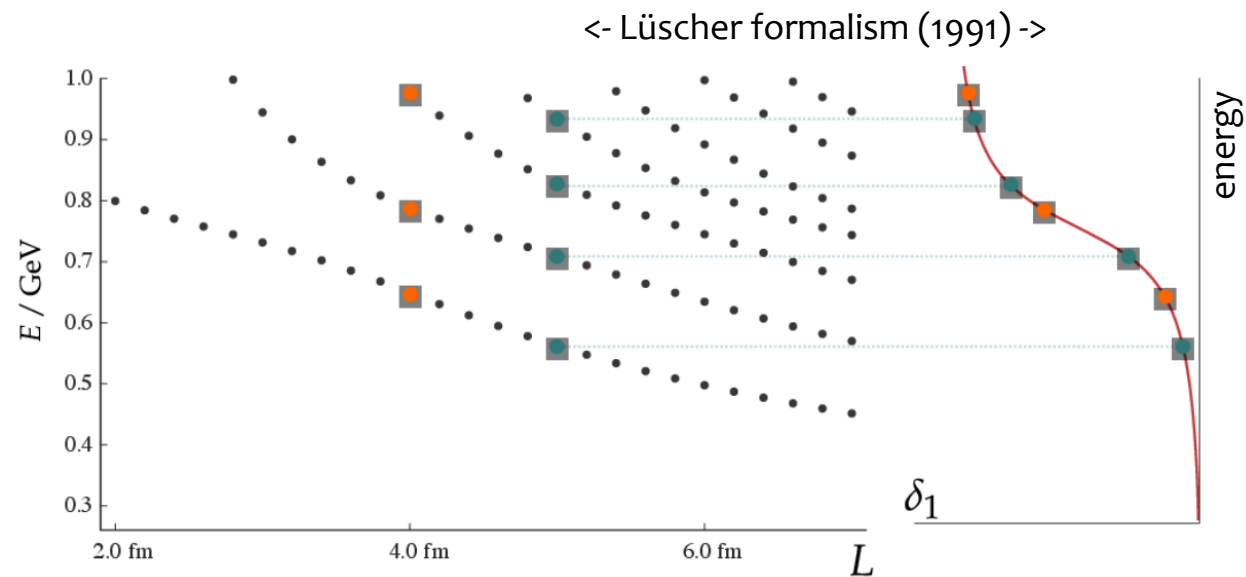
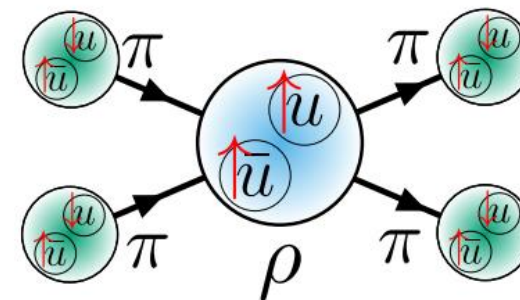


# QCD states as resonances

Most of hadrons can decay  $P_0 \rightarrow P_1, P_2$

- "particle" – a genuine QCD states
- "continuum" – spectrum of  $P_1, P_2$

- No rest energy for the "particle" – continuum spectrum
- width  $\sim 1/\text{lifetime}$
- One can compute the same on lattice:  
< pi pi > correlation





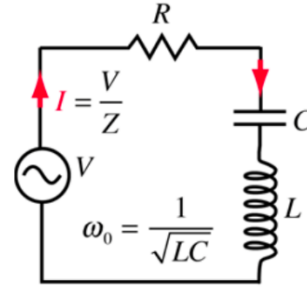
# Resonances

A resonance is a common phenomenon across fields (peak, phase motion, pole)

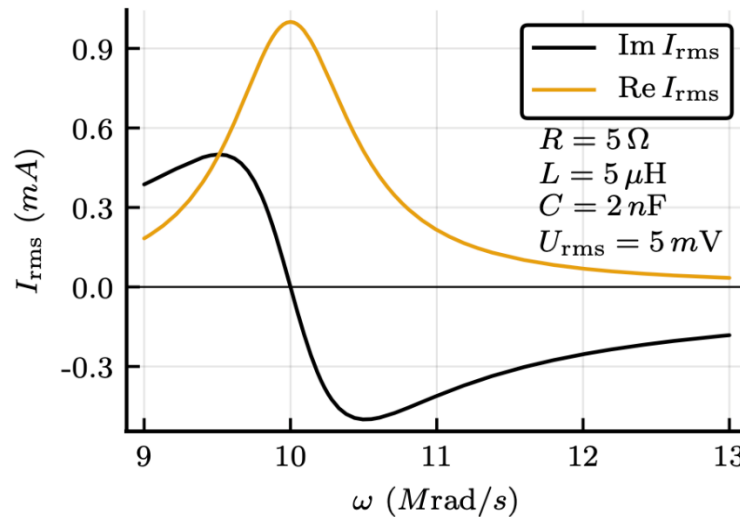
QCD excited states appear as **resonances** in scattering / decays

Hence often called "resonances"

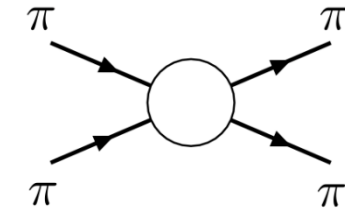
## Electric resonance



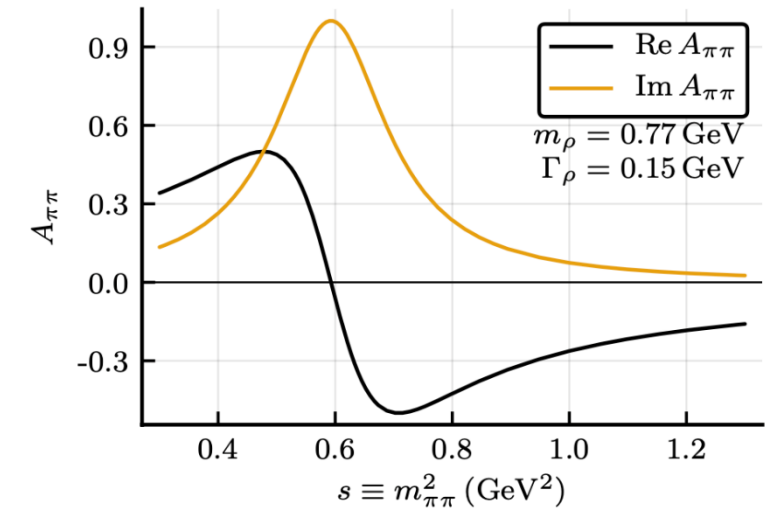
$$I_{\text{rms}} = \frac{U}{R + iL\omega - \frac{i}{C\omega}}$$



## Hadronic resonance



$$A_{\pi\pi} = \frac{m\Gamma}{m^2 - s - im\Gamma}$$



# Hadronic amplitude

Probability density function is a square of amplitude summed over spin projections

$$I(s) = \sum_{\text{spin}} |A(s)|^2$$

$A(s)$  is a complex function of energy,  $s = E^2$

Example of a resonance amplitude

$$A(s) = \frac{N(s)}{m^2 - s - ig^2\rho(s)}$$

$N(s)$  is reaction dependent (B-decays / e+e-),  
denominator is universal

**Imaginary part** is something we control well:

1. I do not know how this thing decays / decay threshold is far away

$$ig^2\rho(s) = m\Gamma \text{ (const)}$$

2. The only relevant continuum channel is the one I consider

$$ig^2\rho(s)$$

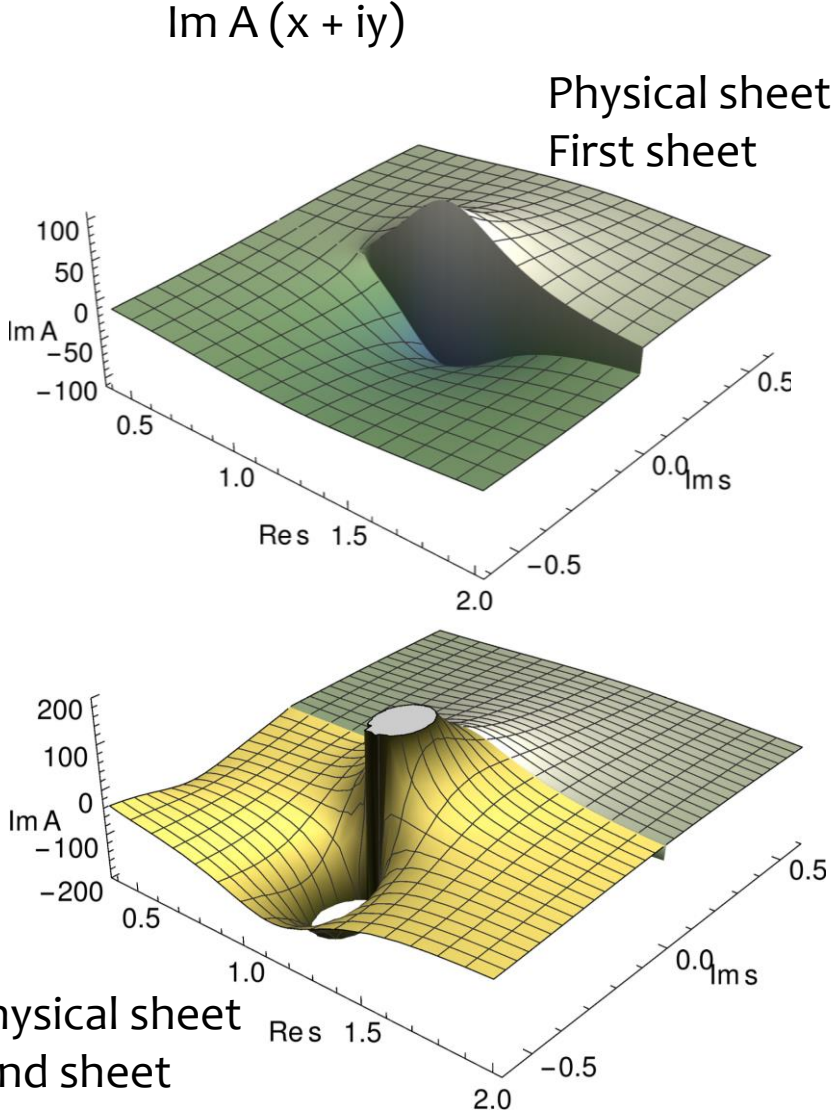
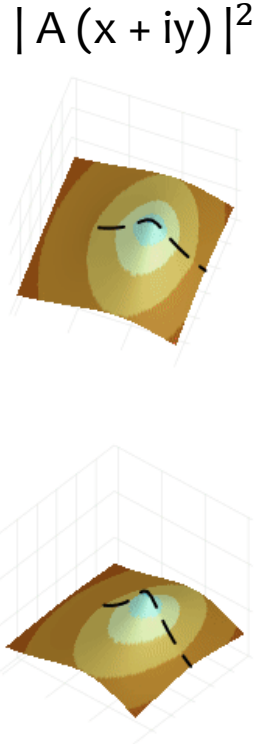
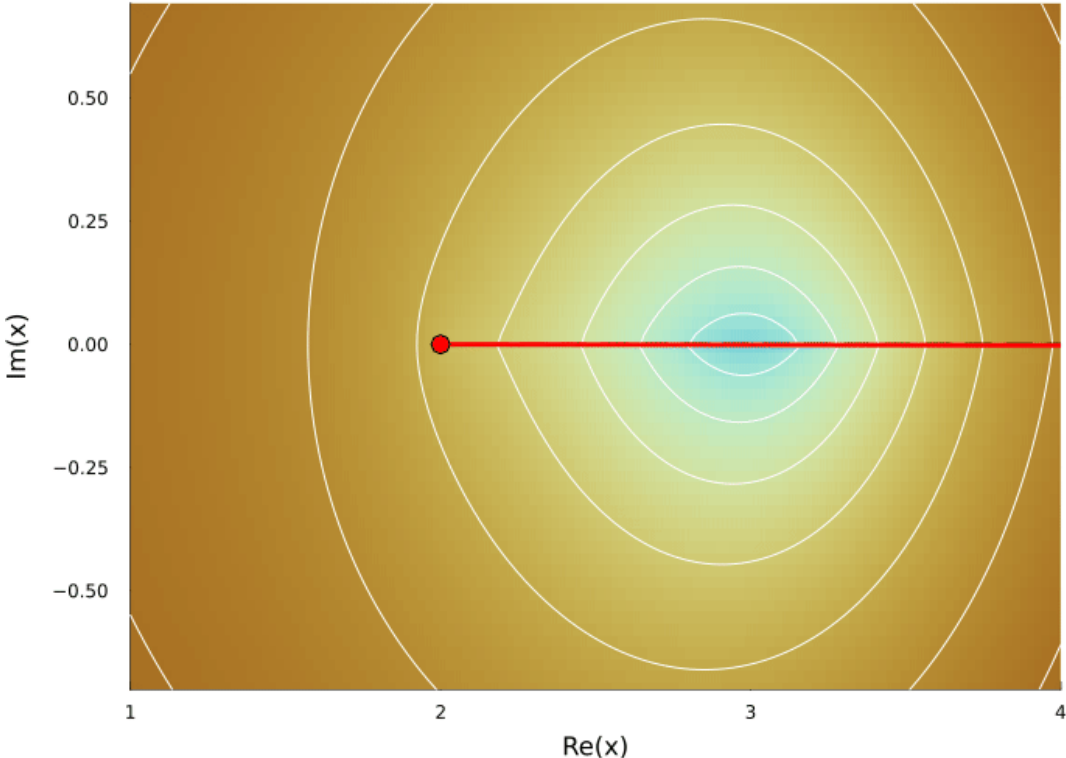
3. there are multiple channels to consider

$$i(g_1^2\rho_1 + g_2^2\rho_2 + \dots)$$

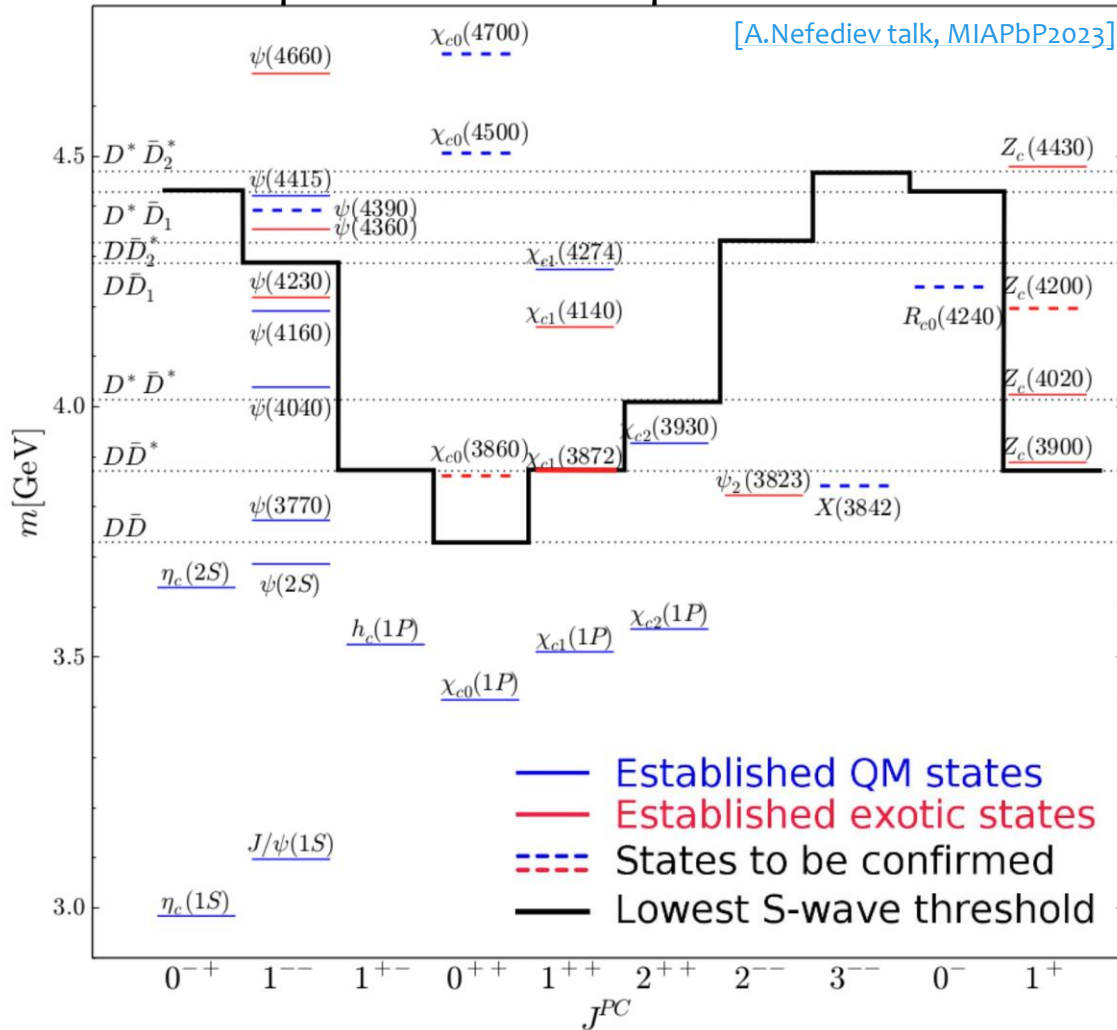


# First and second Riemann sheets

- Amplitude  $A$  is a complex function of  $E = x+iy$
- $\text{Im}(1/A) \sim \text{phase sp.} \sim \text{sqrt}(\text{kin. energy})$
- sqrt branch point – forms two sheets



## An example: charmonium spectrum



# QM states and thresholds

Most of hadrons are not isolated:  
near hadron-hadron threshold,

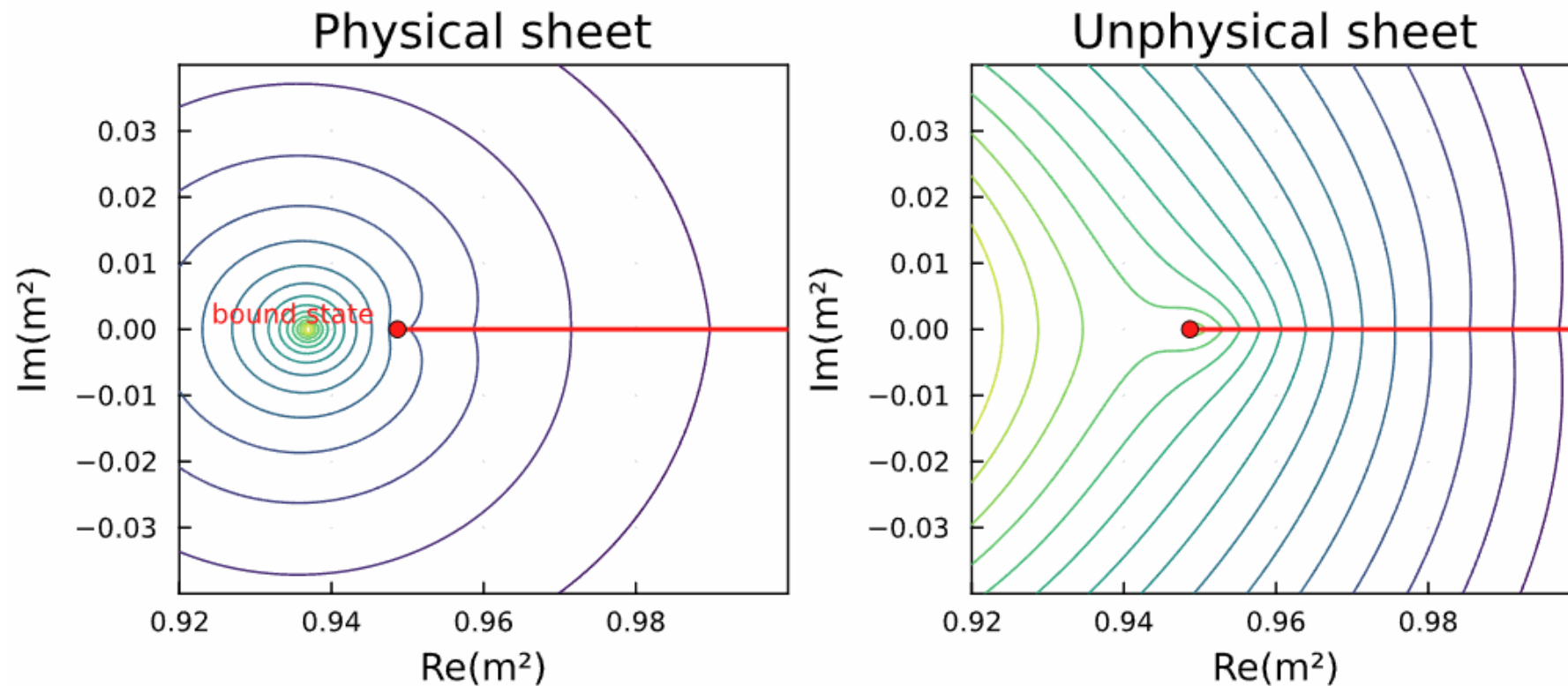
e.g.  $q\bar{q} \rightarrow (q\bar{q})(q\bar{q})$ ,

hadronic states are coupled to hadron-hadron continuum

Molecule component:

a part of the state wave function is  $(q\bar{q})(q\bar{q})$

# How molecule is often a good model



Transition: **bound state**  $\rightarrow$  **virtual state**  $\rightarrow$  **resonance**. No fundamental difference  
The state is mostly **molecular** in vicinity of the threshold

[\[GitHub/mmikhasenko\]](https://github.com/mmikhasenko)



# Experimental stand

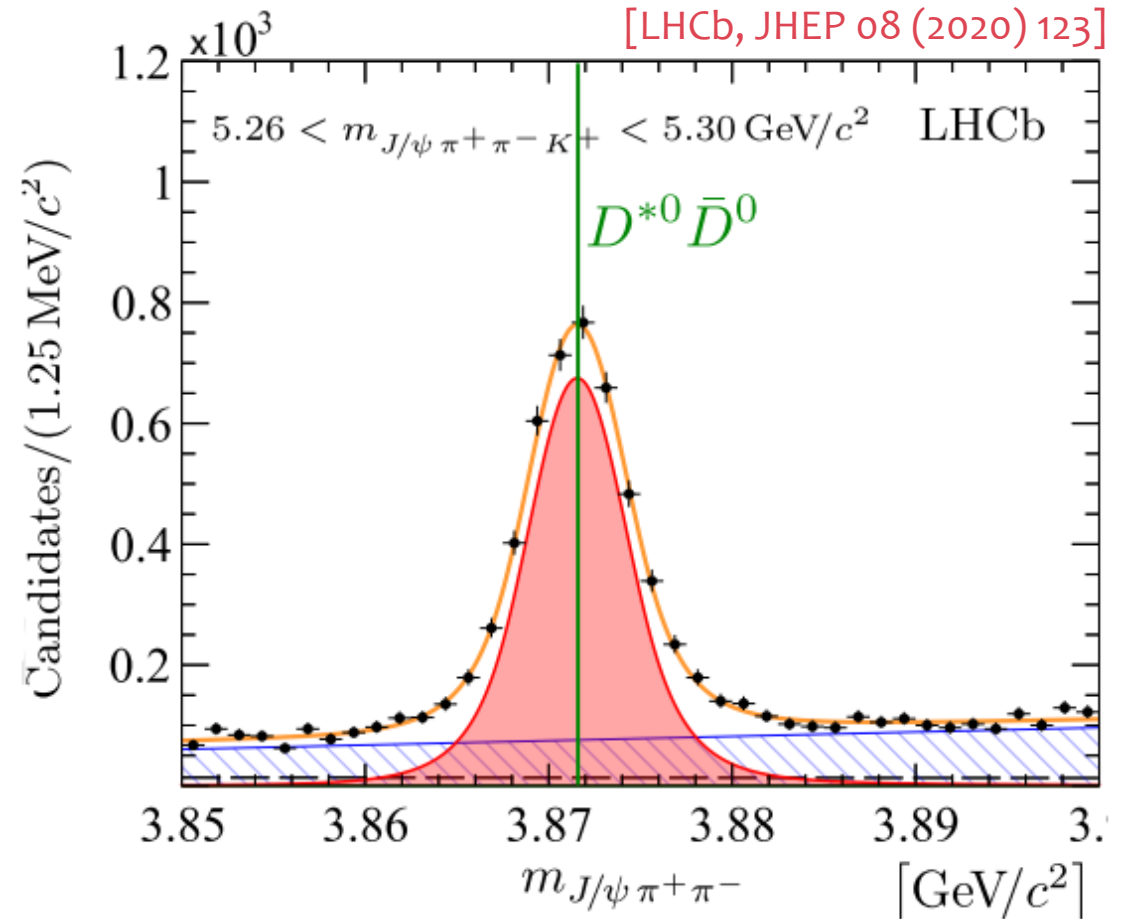
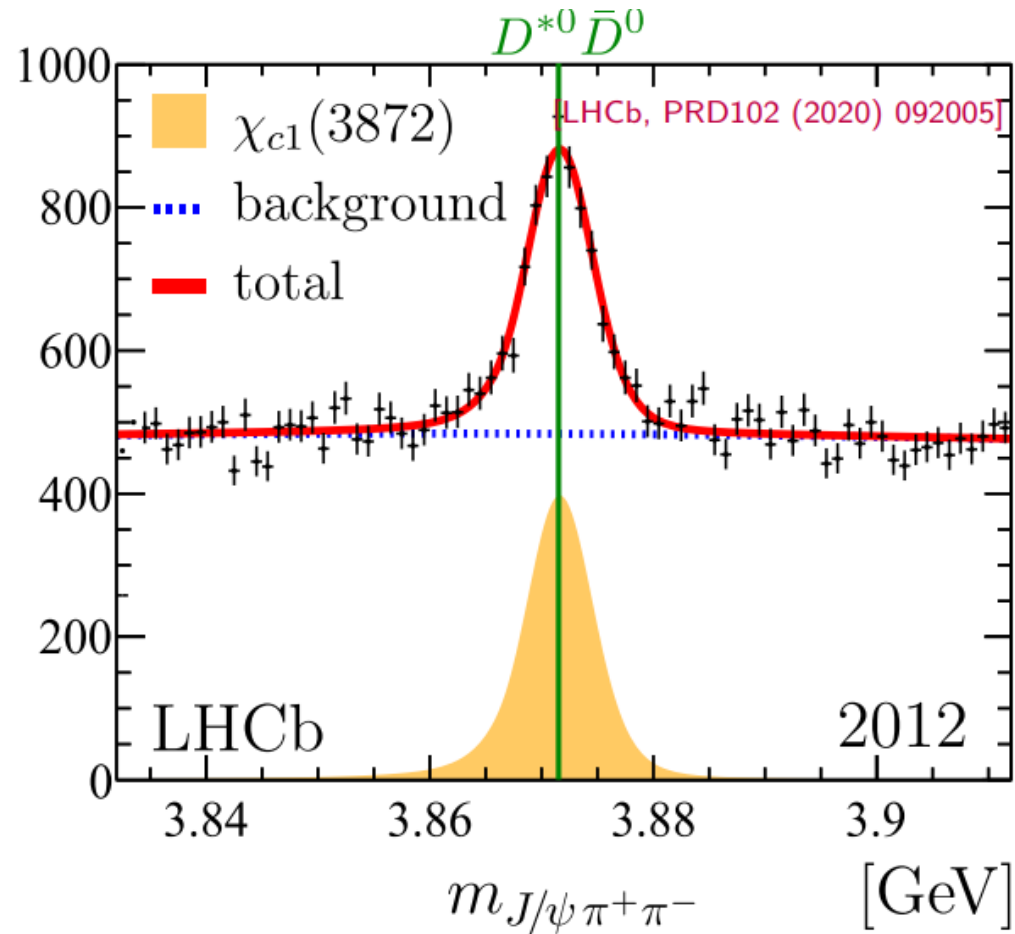


[Midjourney 2023, MM] 19th century photograph of happy smiling children playing with a **collider**

# $\chi_{c1}(3872)$ is right at the $D^0 D^{*0}$ threshold

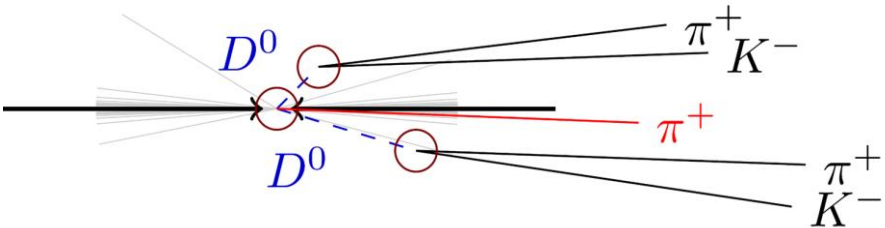
Prompt production ( $pp \rightarrow \chi_{c1} X$ )

From B-decays ( $B^+ \rightarrow \chi_{c1} K^+$ )





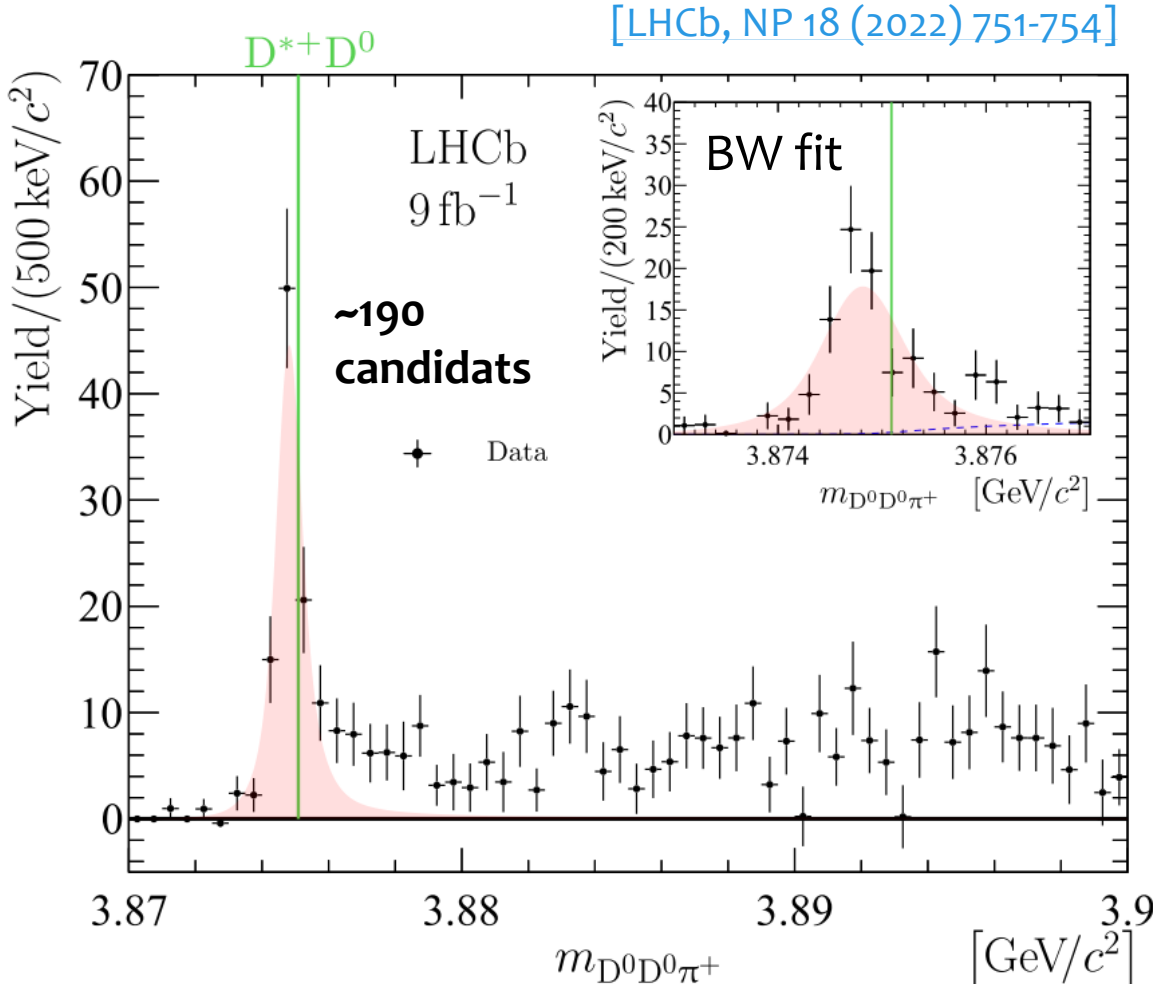
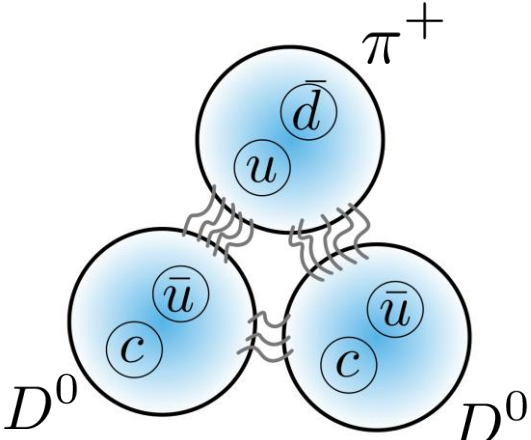
# Doubly-charm tetraquark $T_{cc}^+$ right at the $D^0 D^{*+}$ threshold



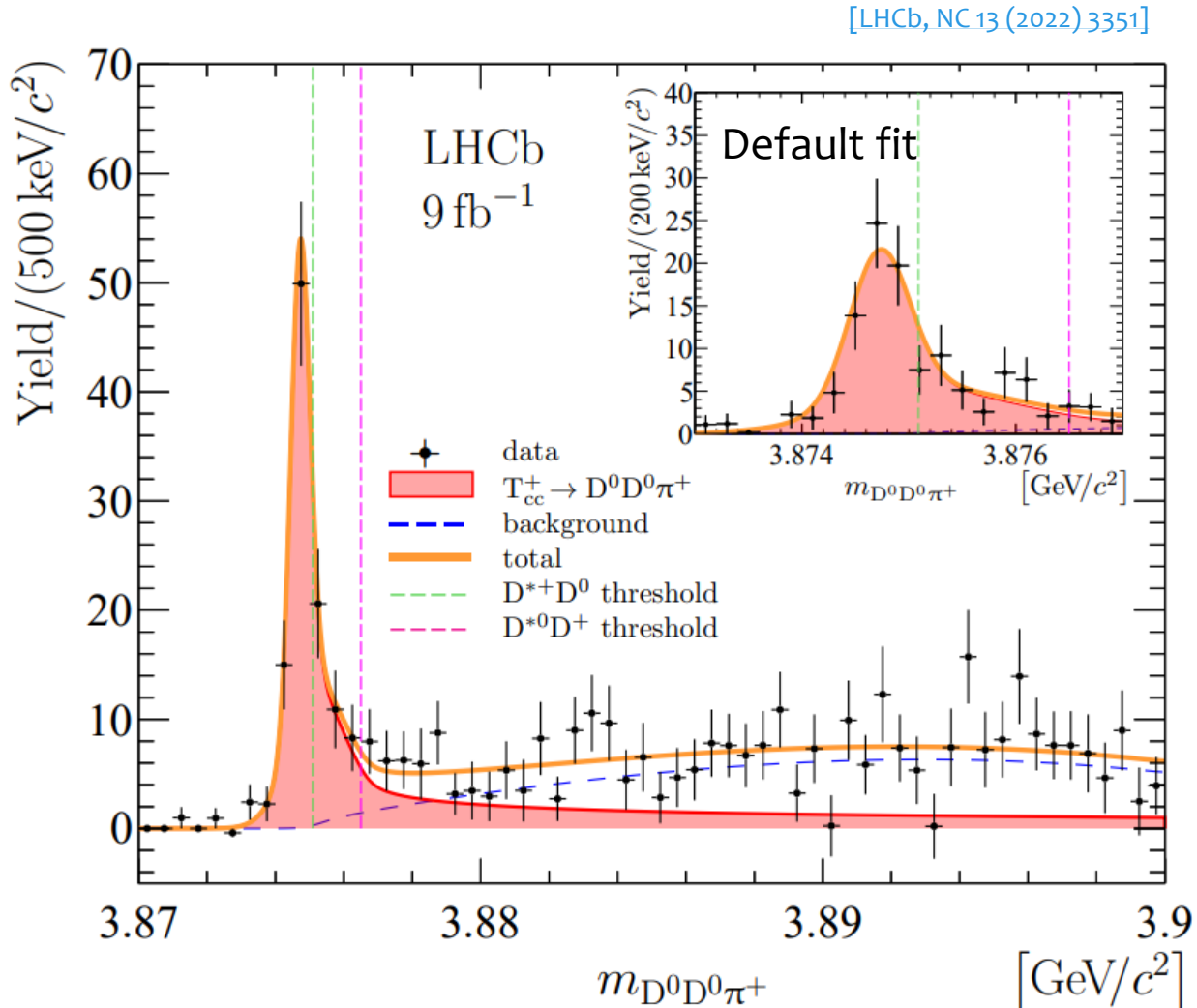
Peak in  $D^0 D^0 \pi^+$  just below  $D^{*+} D^0$  threshold

Extremely narrow,  $\sim 300\text{keV}$   
(resolution)

Needs to be treated as  
three-body effect



# Studies of the doubly-charm tetraquark $T_{cc}^+$



QN: isoscalar ( $I = 0$ ), axial ( $J^{PC} = 1^{++}$ )

Coupled channel model

$$D^{*+}D^0 + D^{*0}D^+$$

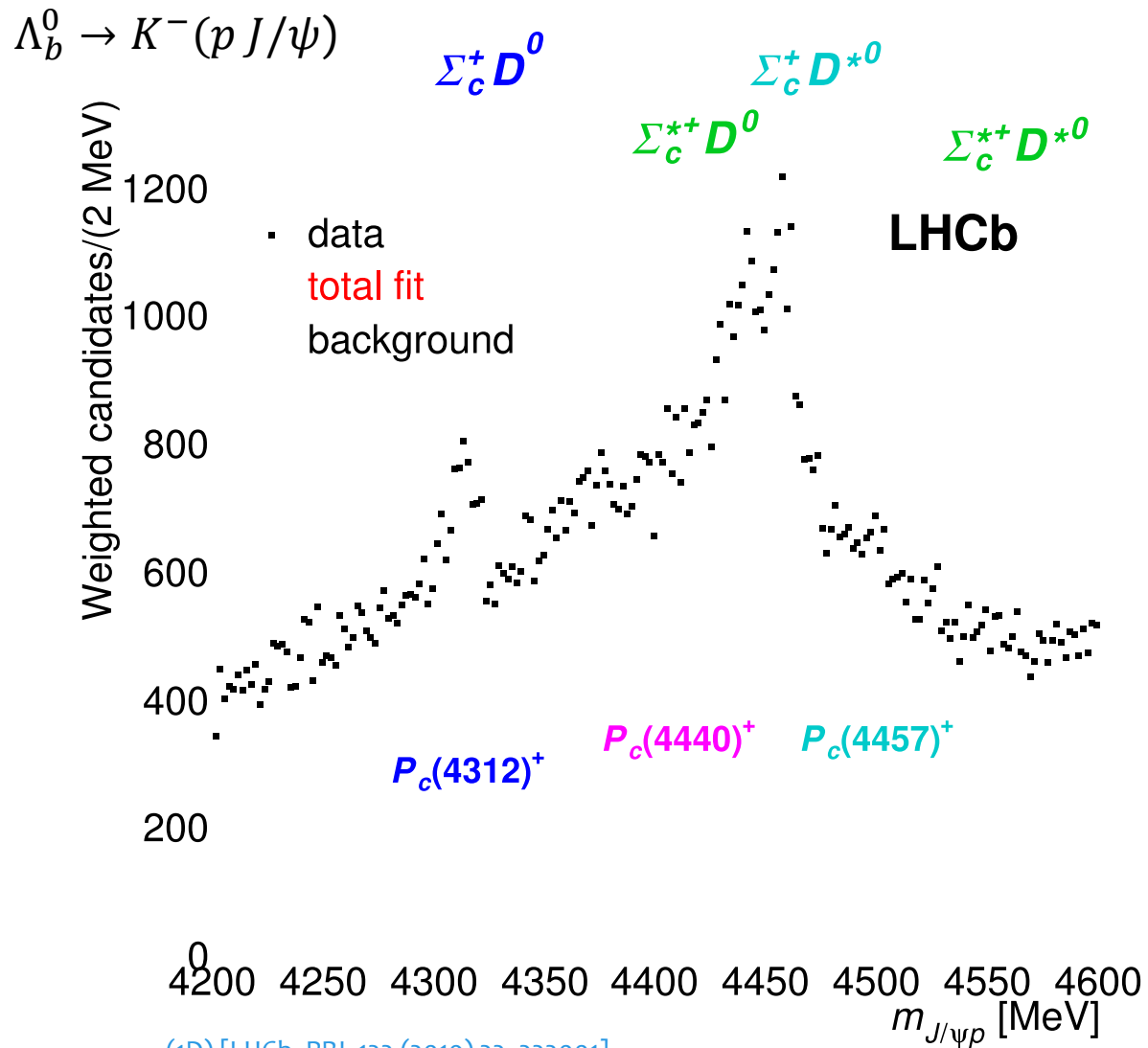
$$\rightarrow \{D^0D^0\pi^+, D^0D^+\pi^0, D^0D^+\gamma\}$$



Yields pole parameters:

❖ Binding energy:  $-360 \pm 40_{-0}^{+4}$  keV

❖ Width:  $48 \pm 2_{-14}^{+0}$  keV



(1D) [LHCb, PRL 122 (2019) 22, 222001]

(AmAn) [LHCb, PRL 115 (2015), 072001]

# Pentaquarks $P_{c\bar{c}}^+$

Near threshold

Multiplicity matches threshold spin algebra

QM states are complex and unknown

# Strange Pentaquark $P_{c\bar{c}s}^+$

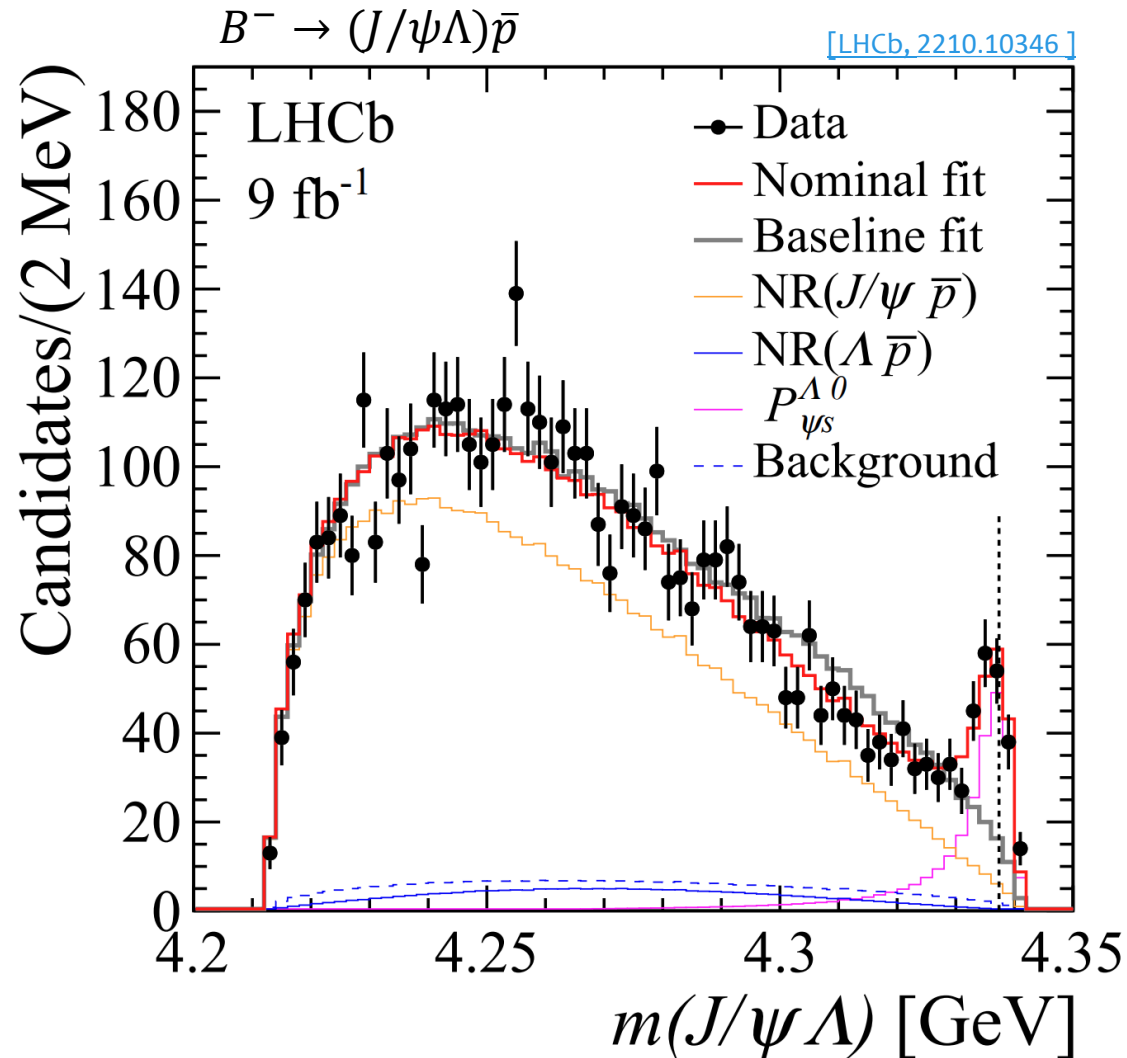
Prominent peak near  $\Xi_c \bar{D}$  threshold

❖  $0.8 \pm 0.7$  MeV above  $\Xi_c^+ D^-$

❖  $2.9 \pm 0.7$  MeV above  $\Xi_c^0 \bar{D}^0$

$J^P = 1/2^-$  is preferred

Aligned with  $\Xi_c^+ D^-$  molecule



# Challenges for near future

1. Observation of  $T_{cb}^0$  (Run-III / IV),  $T_{bb}^-$  (future) ~ 1 phdaway
2. Continuum effects for open-flavor mesons (DJ, DsJ, BJ, BsJ) ~ 1 phdaway
3. Charm/bottom baryons: diquark excitations, continuum ~ few phdaway
4. Genuine QCD pentaquarks, seed for hadronic molecules? ~ few phdaway
5. Double -  $J/\psi$  spectrum ~ 1 phdaway
6. Light hybrids / glueballs with LHCb ~ 1 phdaway
7. Observing three-body hadronic effects: triangle singularity ~ 1 phdaway
8. ....



# Open questions on exotic hadrons

Having significant molecular contribution for hadronic state is fine.

- We do not understand effect how the continuum acts.  
Why it sets some states right to the threshold? [ $\chi_{c1}(3872) \rightarrow D^0 D^{*0}, T_{cc}^+ \rightarrow D^0 D^{*0}$ ]
- Does one always need a genuine QCD seed? – (extra numerous states wrt QM)  
From nuclear physics – “No” (plenty of atoms and isotops)  
 $P_{c\bar{c}}^+$  ?
- Other configurations: hybrids, glueballs?

# Join pentaquarks and tetraquarks investigation

