Measurement of the inelastic pp cross-section at LHCb

– a look behind the curtain of JHEP 06 (2018) 100 –

Michael Schmelling – MPI for Nuclear Physics

Outline

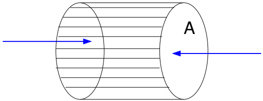
- Introduction
- The LHCb experiment
- Cross-section measurement
- Fiducial cross-section
- Extrapolation to full phase space
- Summary

Matthäus Merian, 1647

1 Introduction

❖ cross-section σ_X and probability p for X to happen in a scattering process

$$p = \frac{\sigma_X}{A}$$



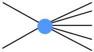
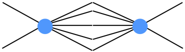
- A: (effective) transverse area of the collision zone
- connection between a number $0 \leqslant p \leqslant 1$ and σ_X with dimension of area

- fundamental observables in particle physics:
  - elastic cross-section σ_{el}: same particles in final and initial state
  - inelastic cross-section σ_{inel}: creation of additional particles
  - total cross-section σ_{tot}

$$\sigma_{\mathrm{tot}} = \sigma_{\mathrm{el}} + \sigma_{\mathrm{inel}}$$

The optical theorem

❖ relation between total cross-section and forward scattering amplitude

$\sigma_{tot} \sim \sum |$  $|^2 = \sum$  $= A(0)$

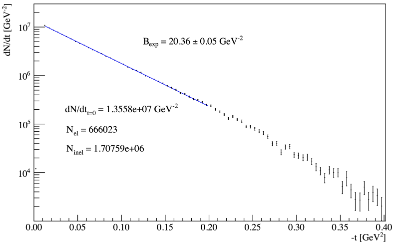
- proportionality between amplitude for no scattering and total cross-section
- can be probed by elastic scattering in the limit $t \to 0$

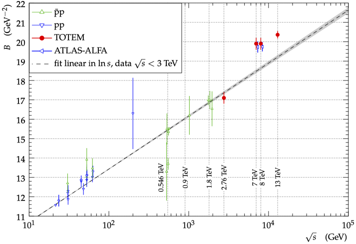
$$|A(0)|^2 \sim \sigma_{\mathrm{tot}}^2 = \frac{16\pi(\hbar c)^2}{1+\rho^2} \left.\frac{d\sigma_{\mathrm{el}}}{d|t|}\right|_{t=0} \quad \text{with} \quad \rho = \frac{\mathrm{Re}\,A(0)}{\mathrm{Im}\,A(0)} = 0.10\ldots0.14$$

► measure elastic pp scattering as a function of t and extrapolate to $t = 0$

the TOTEM experiment ➜

13 TeV pp collisions





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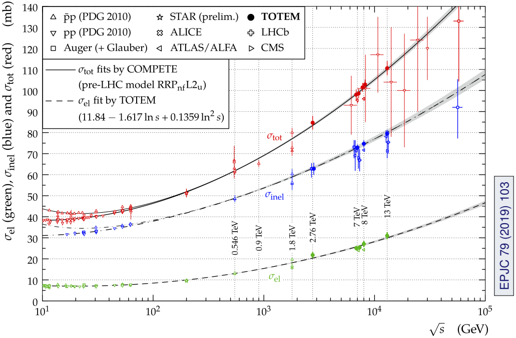
- $d\sigma_{el}/d|t|$ well described by a simple exponential
- suspicious jump in slope paremeter around 3 TeV

status ➜

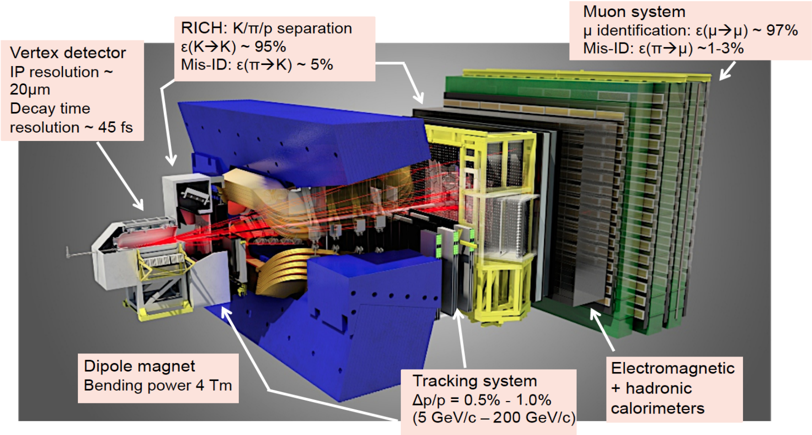
Compilation of cross-section results



- highest energies probed by cosmic-ray experiments
- overall consistent picture
- some tension at high energies?

- best accuracy at LHC energies: elastic scattering and optical theorem
- complementary: measurement of inelastic particle-production cross-section

2 The LHCb experiment (Run1&2)



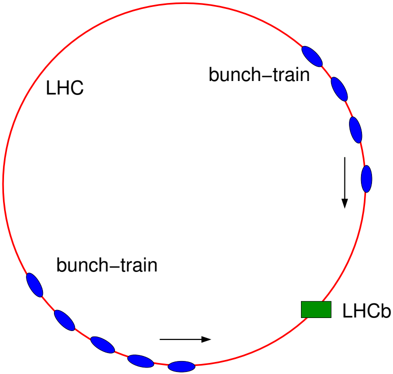
JINST 3 (2008) S08005, IJMPA 30 (2015) 1530022

installation in the cavern

20 m long, 10 m high

- ➔ 22 countries
- ➔ 98 institutes
- ➔ 1641 members

Experimental setup



- counter-propagating bunch trains
- 25 or 50 ns bunch separation
- bunch-crossing types
  - beam-beam
  - beam-empty
  - empty-beam
  - empty-empty
- data for cross-section analysis
  - 13 TeV pp collisions
  - 691 million events
  - leading bunch crossings
  - nobias triggers
  - known luminosities

Pileup

- randomly varying number of interactions per beam-beam bunch-crossing
  - assume a bunch with n protons to collide with another bunch with N protons
  - ε: probability for two protons to interact and be removed from the beams

- combinatorics
  - probability P_0 that the bunches pass each other without interaction

$$P_0 = (1-\varepsilon)^{nN} = q^{nN} \quad \text{with} \quad q = 1-\varepsilon$$

  - probability for $k > 0$ interactions

$$P_k = q^{nN} \prod_{m=1}^{k} \frac{q}{1-q^m}(q^{m-1-n}-1)(q^{m-1-N}-1) \approx (1-\varepsilon)^{nN} \prod_{m=1}^{k} \frac{\varepsilon n N}{m}$$

the approximation requires $\varepsilon n, \varepsilon N \ll 1$ and $n, N \gg k$

❖ LHC conditions

- particles per bunch $n \approx N = O(10^{11})$
- gaussian bunch profiles with typical transverse width $\sigma = 40\,\mu\text{m}$
- inelastic cross-section around $\sigma_{\text{inel}} = 60\,\text{mb}$

$$\varepsilon = \sigma_{\text{inel}} \int dx\, dy\, \rho_1(x,y)\rho_2(x,y) = \frac{\sigma_{\text{inel}}}{4\pi\sigma^2} \approx 3 \cdot 10^{-22}$$

► the approximation holds:

$$P_k = e^{-\mu}\frac{\mu^k}{k!} \quad \text{with} \quad \mu = \varepsilon\, n\, N$$

► the number of simultaneous interactions follows a Poisson distribution

3 Cross-section measurement

- working definition of "inelastic pp cross-section"

  pp interaction with at least one promptly produced massive particle

- experimental problem: finite acceptance and pileup
  - particles are seen only in a limited pseudorapidity range
  - particles are seen only for momenta above a certain threshold
  - the LHCb tracking system detects only charged particles
  - multiple interactions per event
- measurement strategy
  - measure a fiducial cross-section that is accessible by the detector
  - extrapolate the result to the full phase space

implementation ➜

Definitions

- **fiducial cross-section**

  pp cross-section for the production of at least one prompt long-lived charged particle with $p > 2\,\text{GeV}$ and $2 \leqslant \eta < 5$

- **long-lived particle**

  particle with average proper lifetime $\tau > 30\,\text{ps}$

  - at LHCb such particles travel macroscopic distances of O(10) cm

- **prompt particle**

  particle without long-lived ancestors

  - particles created directly in the interaction
  - particles created in decay chains of short lived particles

comparison with alternatives ➜

Discussion

- long-lived-prompt (L)

  all particles with a proper lifetime $\tau > \tau_0$ that have no ancestors with $\tau > \tau_0$

  - definition based on particle properties – not on how the event evolved
  - insensitive to detector and independent on Lorentz frame

- final-state (F)

  all particles that did not decay until the end of the simulation

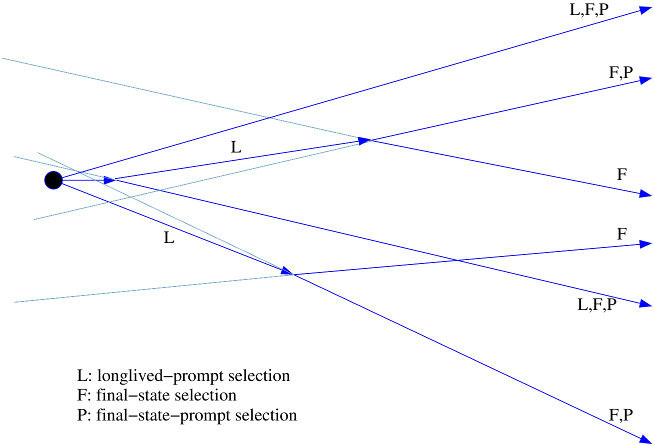
  - depends on how long the simulation is being followed
  - depends on whether secondary interactions are modelled
  - extra random component since e.g. K_S^0 may or may not decay

- final-state-prompt (P)

  all final-state particles with impact parameter $D < D_{\mathtt{cut}}$ at the primary vertex

  - impact-parameter requirement can be matched to experimental resolution
  - same caveats as final-state selection

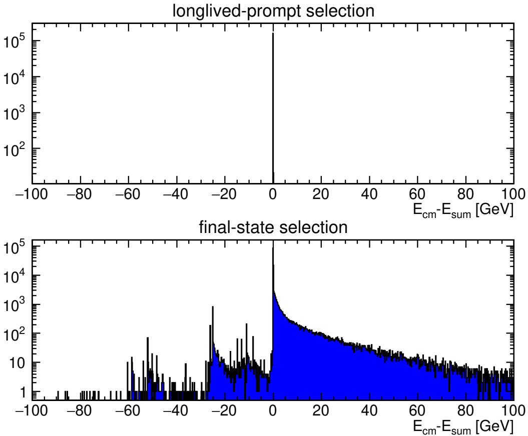
illustration:



L: longlived−prompt selection
F: final−state selection
P: final−state−prompt selection

quantitative comparisons ➜

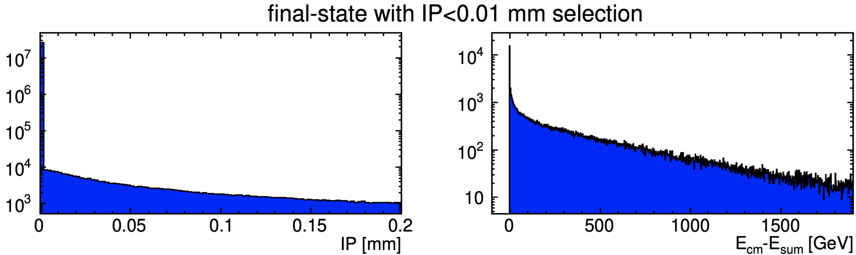
❖ long-lived-prompt vs final-state: total energy



total energy of collision in selected configuration

approximate energy conservation; some losses; occasional double counting

❖ test of energy-conservation for final-state-prompt selection



❖ finding: long-lived-prompt selection works best

- Lorentz invariant since based on particle properties only
- easily defined in generator-level-only studies, also for 4-momentum-only models

4 Fiducial cross-section

❖ relation between interaction count N and integrated luminosity L

$$\langle N \rangle = \sigma\, L \qquad \rightarrow \qquad \hat{\sigma} = \frac{N_{\mathrm{obs}}}{L}$$

- L: known integrated luminosity
  - given by $L = N_{\mathrm{ref}}/\sigma_{\mathrm{ref}}$ with reference cross-section σ_{ref} ("lumi counter") from Van-der-Meer scan and event count N_{ref} from zero-counting method
- N_{obs}: number of interactions satisfying fiducial requirements
  - estimated from experimental proxy of the fiducial cross-section by correcting for detector effects

counting interactions →

The zero-counting method for unbiased events – step by step

- fiducial particle: prompt long-lived charged particle with $p > 2\,\mathrm{GeV}$ and $2 \leqslant \eta < 5$
- fiducial interaction: interaction with at least one fiducial particle
- the number of interactions per event is a Poisson random variable with average μ
- the total number of events is N_{evt}, the total number of interactions is N_{int}

$$N_{\mathrm{int}} = N_{\mathrm{evt}}\,\mu$$

- the Poisson average μ is related to the probability p_0 to observe an empty event

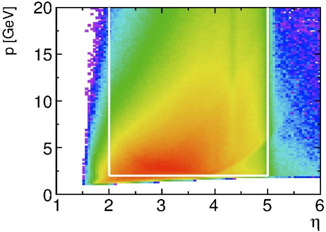
$$p_0 = e^{-\mu} \quad \text{and thus} \quad \mu = -\ln p_0$$

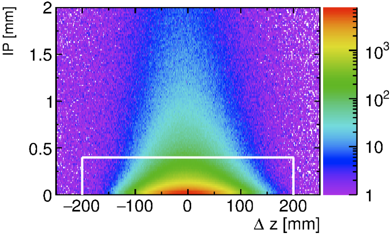
- with N_0 the number of events without fiducial particle the fiducial cross-section is

$$\sigma_{\mathrm{acc}} = -\frac{N_{\mathrm{evt}}}{L} \ln \frac{N_0}{N_{\mathrm{evt}}}$$

Experimental realisation

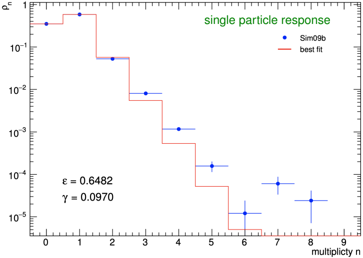
- problem: prompt long-lived particles is NOT what is measured
  - experimental information: charged tracks
  - define fiducial tracks as a proxy for fiducial particles
    - tracks measured in VELO and main tracker with fake-track p-value $p_{\mathrm{gh}} < 0.3$
    - measured momentum $p > 2\,\mathrm{GeV}$ and pseudorapidity $2 \leqslant \eta < 5$
    - estimated point of origin close to the average interaction point





❖ measurement: estimate p_0 from data

- account for the fact that fiducial track multiplicity $\neq$ fiducial particle multiplicity
- assumption: measurements of fiducial particles are independent
  - good approximation for low multiplicity events
  - deviations at large multiplicities do not affect the empty-event count
  - start from detector response to a single fiducial particle



- dominant: efficiency ε
  - 0-track probability $\rho_0 = 1 - \varepsilon$
  - 1-track probability $\rho_1 = \varepsilon$
- generalisation: fake track parameter γ
  - 0-track probability $\rho_0 = 1 - \varepsilon$
  - 1-track probability $\rho_1 = \varepsilon(1-\gamma)$
  - 2-track probability $\rho_2 = \varepsilon(1-\gamma)\gamma$
  - 3-track probability $\rho_3 = \varepsilon(1-\gamma)\gamma^2 \ldots$

Probability Generating Functions to handle multiplicity distributions

- PGF definition: sum of probabilities ρ_n weighted with powers of an auxiliary variable x

$$F_\rho(x) = \sum_{n=0}^{\infty} \rho_n x^n$$

- the individual probabilities are obtained by differentiation

$$\rho_n = \frac{1}{n!} \left. \frac{d^n F_\rho(x)}{dx^n} \right|_{x=0} \quad \text{in particular} \quad \rho_0 = F_\rho(0)$$

- when convolving two distributions p_n and q_n the PGFs multiply

$$(p \otimes q)_n = \sum_{i=0}^{\infty} \sum_{j=0}^{\infty} p_i q_j \delta_{i+j,n} \quad \rightarrow \quad F_{p\otimes q}(x) = F_p(x)\, F_q(x)$$

application ➜

Relation between true multiplicity p_n and observed multiplicity q_n

- single particle response PGF given finite efficiency ε and fake track parameter γ

$$g(x) = \sum_{n=0}^{\infty} \rho_n x^n = (1-\varepsilon)\, x^0 + \frac{\varepsilon(1-\gamma)}{\gamma} \sum_{n=1}^{\infty} (\gamma\, x)^n = \frac{1-\varepsilon+\varepsilon\, x-\gamma\, x}{1-\gamma\, x}$$

- exploit PGF behaviour under convolutions to relate p_n and q_n

$$F_q(x) = \sum_{n=0}^{\infty} p_n g^n(x) = F_p(g(x)) \quad \rightarrow \quad F_p(x) = F_q(g^{-1}(x)) = \sum_{n=0}^{\infty} q_n (g^{-1}(x))^n$$

- general result: corrected p_0 from the observed multiplicity distribution q_n

$$p_0 = \sum_{n=0}^{\infty} q_n\, \alpha^n \quad \text{with} \quad \alpha = g^{-1}(0) \overset{\text{model}}{\rightarrow} \frac{\varepsilon-1}{\varepsilon-\gamma}$$

➜ a single unfolding parameter α to correct for detector effects!

Determination of the unfolding parameter α

- strategy: constrain a multiplicity-matched weighted MC estimate by data
  - extract an initial α^{sim} from p_0 and q_n in fully simulated events
  - use the model dependent $\alpha_m = (\varepsilon - 1)/(\varepsilon - \gamma)$ to tie α^{sim} to data

$$\alpha = \alpha^{\mathrm{sim}} \frac{\alpha_m}{\alpha_m^{\mathrm{sim}}}$$

    - $\varepsilon^{\mathrm{sim}}$ and γ^{sim} for α_m^{sim} from simulation
    - corrected efficiency ε from tabulated correction factors
    - estimate of true γ by correcting the MC estimate

$$\gamma = \gamma^{\mathrm{sim}} \frac{f_{\mathrm{gh}}}{f_{\mathrm{gh}}^{\mathrm{sim}}}$$

    f_{gh}: fraction of tracks rejected by the p_{gh}-cut, assumed to be proportional to γ
  - take difference between initial and corrected estimate as systematic uncertainty

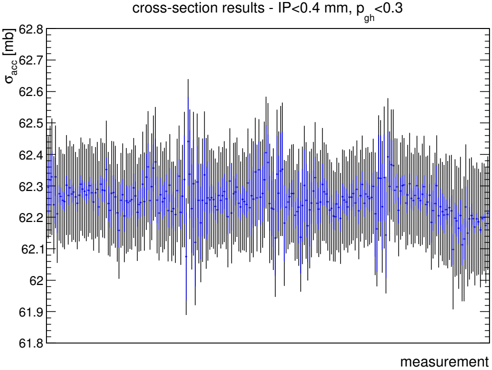
result: $\alpha = -0.60 \pm 0.10$

Configurations used in the measurement

- 243 statistically independent measurements
  - all leading bunch crossings from 49 runs in 8 LHC fills
  - both magnet polarities
  - pileup values in the range $\mu \in [0.4, 1.4]$
  - interaction counts determined in 8-second time slices to follow luminosity variations
  - statistical biases due to non-linearity of ln-function corrected
  - background subtraction from data in be and eb events
  - variations of selection cuts
- sanity check

  ➜ ignore detector effect and use q_0 instead of p_0: $\approx$ 3% larger cross-section

Results



- 243 independent measurements
- RMS scatter of results 0.1 %
- $\alpha = -0.60 \pm 0.10$
- systematics due to α: 0.25 %
  - 0.18 % simulation
  - 0.15 % efficiency
  - 0.12 % fake tracks
- 4% luminosity uncertainty
  - not shown

blue: statistical uncertainties, black: total uncertainties

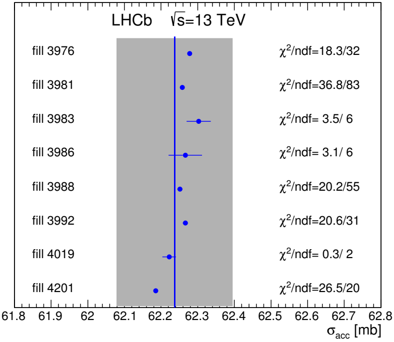
❖ cross-section combination

- uncorrelated statistical errors
- fully correlated systematics
- weights $\propto$ integrated luminosity

$$\sigma_{\mathrm{acc}} = 62.237 \pm 0.002 \pm 0.158\ \mathrm{mb}$$

❖ LHC-fill averages

- internally consistent within statistics
- significant fill-to-fill variations
- parametrise 243 residuals from global average by linear combination of: average position and extension of luminous region, background level, B-field orientation, pileup



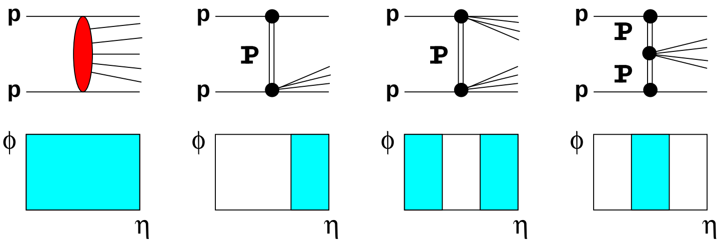
- ► scatter explained by $\langle y \rangle$ and $\sigma(y)$ of luminous region and background level
- ► additional systematics of 0.05 %

fiducial cross-section: $\sigma_{\mathrm{acc}} = 62.2 \pm 0.2\,\mathrm{mb}$

5 Extrapolation to full phase space

- generator level analysis following LHCb-PAPER-2014-057
  - event generators represent knowledge from 60 years of hadronic physics
  - the inelastic cross-section can be approximated by an incoherent sum

$$\sigma_{\text{inel}} = \sum_{X} \sigma_X \qquad \text{with} \qquad X \in \{\text{ND}, \text{SDA}, \text{SDB}, \text{DD}\}$$



non−diffractive (ND) | *single−diffractive (SD)* | *double−diffractive (DD)* | *double pomeron exchange (DPE)*

- study 32 LHC proton-proton tunes that come with PYTHIA 8.230:
  - cross-section fractions f_X
  - fractions v_X contributing to the fiducial cross-section
  - mean fiducial multiplicity $n_{\mathrm{ch},X}$ of events contributing to the fiducial cross-section

	f_X	v_X	$n_{\mathrm{ch},X}$
non-diffractive	0.720 ± 0.012	0.9963 ± 0.0005	17.94 ± 1.45
single diffractive(A)	0.083 ± 0.003	0.7154 ± 0.0051	8.11 ± 0.52
single diffractive(B)	0.083 ± 0.003	0.3411 ± 0.0077	7.83 ± 0.44
double diffractive	0.114 ± 0.006	0.6263 ± 0.0049	6.15 ± 0.31

- variance of PYTHIA fractions f_X much smaller than in EPJC 73 (2013) 2456 (ALICE)
- use only v_X and $n_{\mathrm{ch},X}$ from PYTHIA - don't use the f_X

❖ constructing the cross-section extrapolation factor

$$s = \frac{\text{total}}{\text{visible}} = \frac{\sum_X \sigma_X}{\sum_X \sigma_X \, v_x} = \frac{1}{\sum_X f_X \, v_X}$$

- determine the distribution of s for all sets $f_X > 0$ satisfying

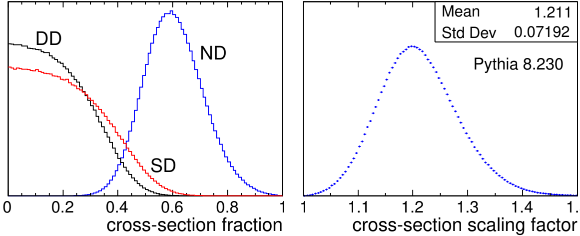
$$\sum_X f_X = 1 \qquad \text{and} \qquad \sum_X f_X \, n_{\text{ch},X} = n_{\text{ch}} = 13.9 \pm 0.9$$

  - 13.9 ± 0.9 is the mean fiducial multiplicity per interaction in data
  - inferred from fiducial multiplicity 15.2 in the LHCb simulation, which fits the data, taking into account an about 3% larger efficiency in data, and an about 5% smaller fraction of tracks associated to true particles

❖ implementation

- uniform sampling of f_X in the constrained phase space $\sum_X f_X = 1$
- for every set of f_X calculate n_{ch} and s
- accept s with weight $w = \exp(-(n_{\mathrm{ch}} - 13.9)^2/2 \cdot 0.9^2)$

❖ result



total inelastic cross-section: $\sigma_{\mathrm{inel}} = 75.4 \pm 3.0_{\mathrm{exp}} \pm 4.5_{\mathrm{extr}}$ mb

6 Summary

❖ fiducial cross-section at $\sqrt{s} = 13\,\text{TeV}$

inelastic pp interactions with $\geqslant 1$ prompt (produced directly or from short lived ancestors), long-lived ($\tau > 30\,\text{ps}$) charged particles with $p > 2\,\text{GeV}$ and $2 \leqslant \eta < 5$

$$\sigma_{\text{acc}} = 62.2 \pm 0.2 \pm 2.5_{\text{lumi}}\,\text{mb}$$

- intrinsic precision of the measurement 0.25 %
- dominant 4 % uncertainty from luminosity

❖ total inelastic pp cross-section at $\sqrt{s} = 13\,\text{TeV}$

$$\sigma_{\text{inel}} = 75.4 \pm 3.0_{\text{exp}} \pm 4.5_{\text{extr}}\,\text{mb}$$

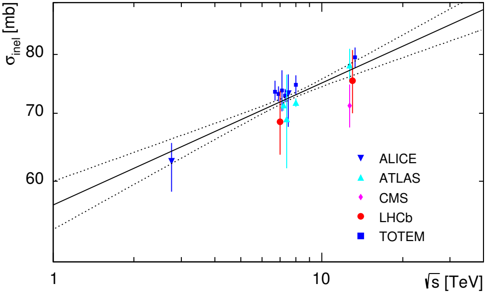
❖ updated (inoficial) results with improved 2021 luminosity calibration

$$\sigma_{\text{acc}} = 61.6 \pm 0.2 \pm 1.2_{\text{lumi}}\,\text{mb}$$

$$\sigma_{\text{inel}} = 74.6 \pm 1.2_{\text{exp}} \pm 4.4_{\text{extr}}\,\text{mb}$$

LHC cross-section results

❖ measurements at 2.76, 7, 8,and 13 TeV



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consistent picture: $\sigma_{\mathrm{inel}} = 56.9 \left(\frac{\sqrt{s}}{\mathrm{TeV}} \right)^{0.12}$ mb with $\chi^2/\mathrm{NDF} = 13.2/13$