PARTICLE ACCELERATION IN SUPERNOVA REMNANTS

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LECTURE II

4th graduate school on Plasma-Astroparticle Physics

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OUTLINE

- **From diffusion to energy gain**
    - ***The nature of collision-less shocks***
    - ***First order Fermi acceleration***

- **Maximum energy in Diffusive Shock Acceleration (DSA)**

- **Self-generation of magnetic waves**
    - ***Resonant (streaming) instability***
    - ***Non-resonant (Bell) instability***

- **The injection problem**

- **Application to SNR shocks**
    - ***Radiative processes***
    - ***The role of scattering centres***
    - ***SNR in non homogeneous medium***

Derivation of the transport equation

We look for a transport equation for the CR distribution function $f(t,\vec{x},\vec{p}) \overset{\text{def}}{=} \frac{dN}{d^3x\, d^3p}$

Because we can neglect collisions, we can use the Liouville theorem:

$$\begin{cases} \dfrac{Df}{Dt} = \dfrac{\partial f}{\partial t} + \vec{u}\cdot\nabla_x f + \dot{\vec{p}}\cdot\nabla_p f = 0 & \text{Vlasov equation} \\ \dot{\vec{p}} = q\dfrac{\vec{u}}{c}\wedge\vec{B} & \text{Lorentz force} \end{cases}$$

[See sec. 4.3 in Vietri, “Foundation of High energy Astrophysics”]

$$\Rightarrow \quad \frac{\partial f}{\partial t} + \vec{u}\cdot\nabla f = \nabla\left[D_{xx}\nabla f\right] + \frac{p}{3}(\nabla\cdot\vec{u})\frac{\partial f}{\partial p} + Q(z,p)$$

$$D_{xx} = \frac{1}{3}\frac{r_L c}{k_{res} F(k_{res})}$$

Shock acceleration through the distribution function

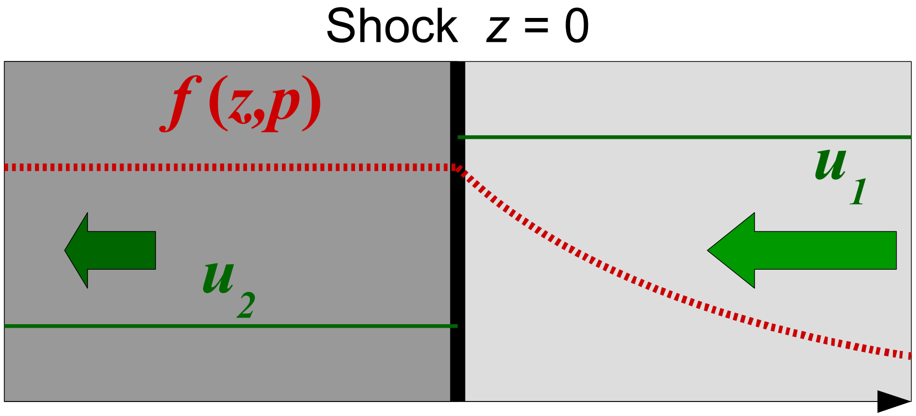
$$\frac{\partial f}{\partial t} + \vec{u}\cdot\nabla f = \nabla\left[D\nabla f\right] + \frac{p}{3}(\nabla\cdot\vec{u})\frac{\partial f}{\partial p} + Q(z,p)$$

Stationary 1D system

$$\cancel{\frac{\partial}{\partial t}}$$

$$\nabla_x \rightarrow \partial_z$$

$$u(z) = u_2 + (u_1 - u_2)\theta(z)$$



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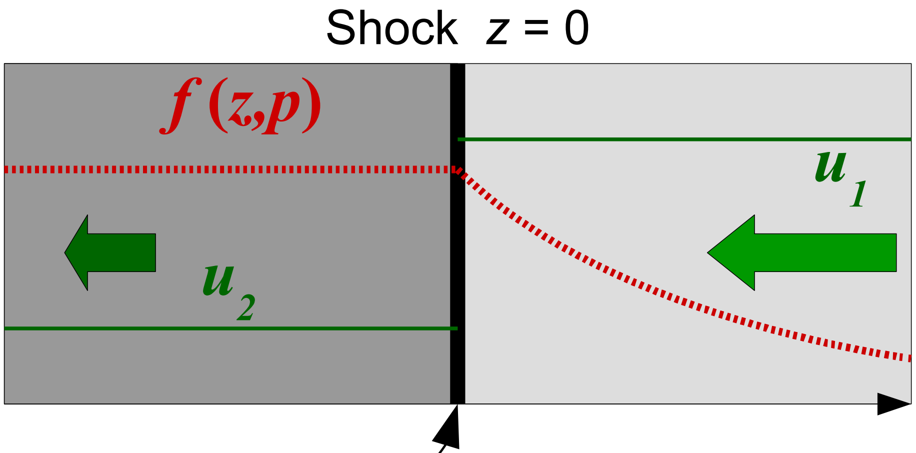

$$u\frac{\partial f}{\partial z} = \frac{\partial}{\partial z}\left[D\frac{\partial f}{\partial z}\right] + \frac{p}{3}\frac{du}{dz}\frac{\partial f}{\partial p} + Q_0(p)\delta(z)$$

Shock acceleration through the distribution function

$$\frac{\partial f}{\partial t} + \vec{u}\cdot\nabla f = \nabla\left[D\nabla f\right] + \frac{p}{3}(\nabla\cdot\vec{u})\frac{\partial f}{\partial p} + Q(z,p)$$

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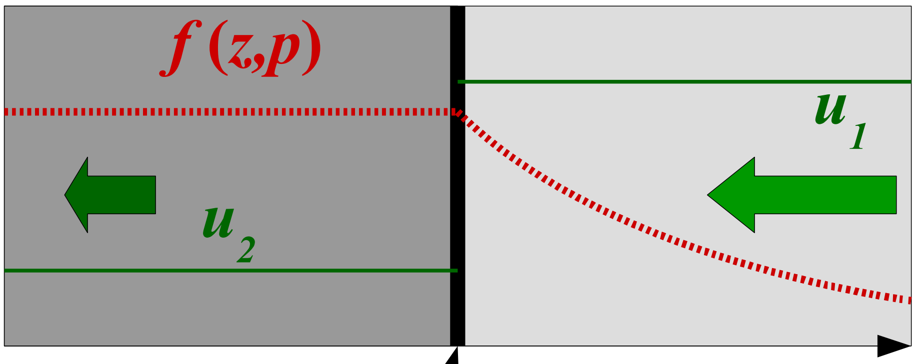

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Integration across the shock discontinuity

$$0 = -\left[D\frac{\partial f}{\partial z}\right]_{0^+} + \frac{u_1-u_2}{3}p\frac{\partial f_0}{\partial p} + Q_0(p)$$

Integration from 0^+ to ∞

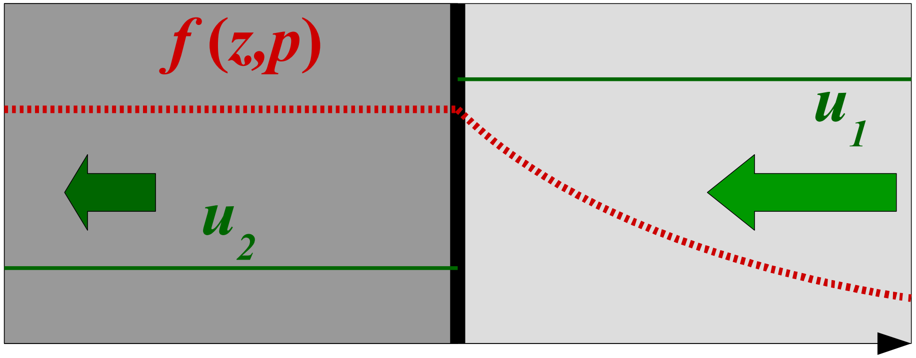
$$u_1(f_\infty - f_0) = -\left[D\frac{\partial f}{\partial z}\right]_{0^+}$$

Shock acceleration through the distribution function

Full transport equation for CR distribution at the shock position

$$p\frac{\partial f_0}{\partial p} = \frac{3u_1}{u_1 - u_2}\left[\left(f_0 - f_\infty\right) - \frac{Q_0(p)}{u_1}\right]$$

Shock $z = 0$



Shock acceleration through the distribution function

Full transport equation for CR distribution at the shock position

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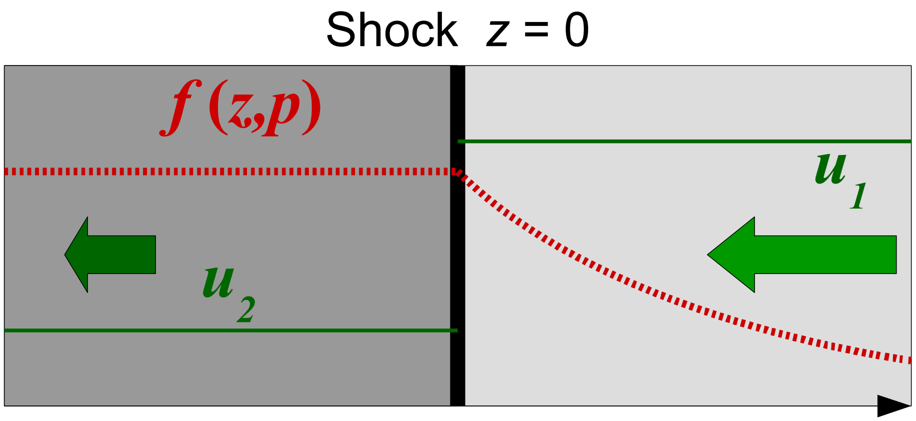


Spectral slope

$$s = \frac{3u_1}{u_1-u_2} \xrightarrow[u_1\to 4u_2]{} 4$$

Solution at the shock

$$f_0(p) = s\,p^{-s}\int_{p_0}^{p} dp'\,p'^{\,s-1}\left[\frac{Q_0(p)}{u_1}+f_\infty\right] \longrightarrow \boxed{f_0 \propto \left(\frac{p}{p_0}\right)^{-s}}$$

$$\uparrow$$

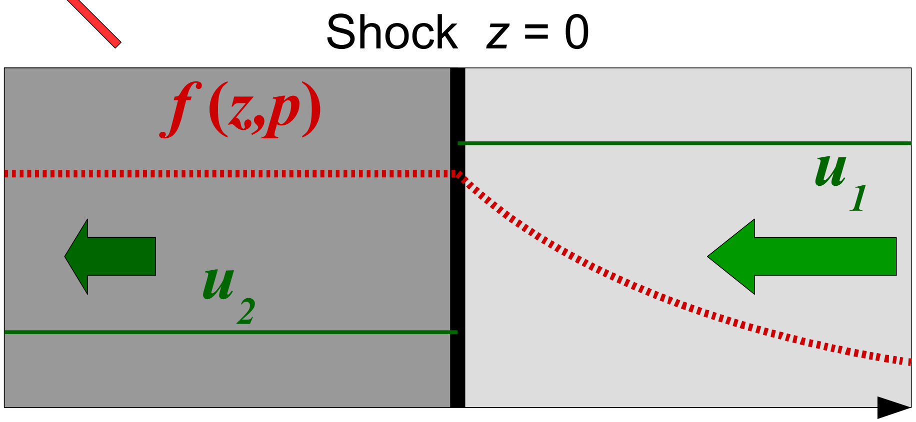
$$\begin{cases} Q_0 = k_{inj}\;\delta(p-p_0)\\ f_\infty = 0\end{cases}$$

Power-law in momentum!!!

Shock acceleration through the distribution function

Solution upstream and downstream of the shock

$$u\frac{\partial f}{\partial z} = \frac{\partial}{\partial z}\left[D\frac{\partial f}{\partial z}\right] + \cancel{\frac{p}{3}\frac{du}{dz}\frac{\partial f}{\partial p}} + \cancel{Q_0(p)\delta(z)}$$



Downstream

$$f(z,p) = f_0(p)$$

Upstream

$$f_{up}(z,p) = f_0(p)e^{-uz/D}$$

The typical diffusion length upstream is D/u

From 2nd order to 1st order Fermi acceleration

In the '70s many people realized that the Fermi mechanism give a totally different result if applied to shocks (Skilling, 1975; Axford et al., 1977; Krymskii, 1977; Bell, 1978; Blandford and Ostriker, 1978)

WHAT IS A SHOCK?

THE NATURE OF COLLISIONLESS SHOCKS

What is a shock?

A shock is a discontinuity solution of the fluid equations where a supersonic fluid becomes subsonic (i.e. the entropy increases)

$$[\rho u]_1 = [\rho u]_2$$

$$[\rho u^2+P]_1 = [\rho u^2+P]_2$$

$$\left[\frac{1}{2}\rho u^3+\frac{\gamma}{\gamma-1}P u\right]_1 = \left[\frac{1}{2}\rho u^3+\frac{\gamma}{\gamma-1}P u\right]_2$$

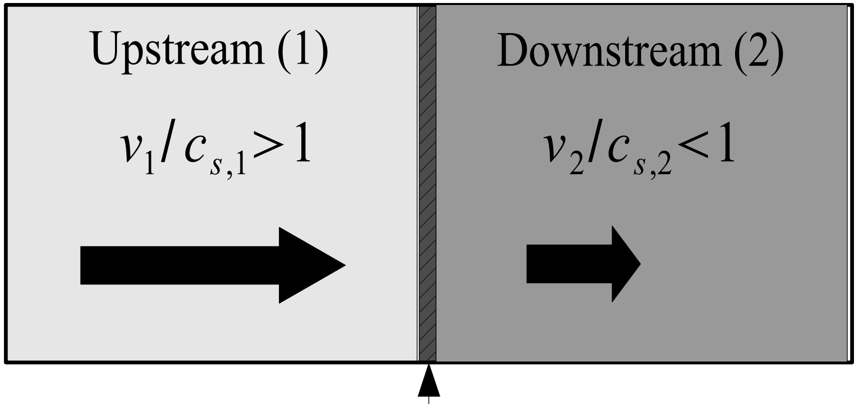
$$\frac{P_2}{P_1} = 1+\frac{2\gamma}{\gamma+1}\left(M_1^2-1\right) \rightarrow \frac{2\gamma M_1^2}{\gamma+1}$$

$$\frac{\rho_2}{\rho_1} = \frac{(\gamma+1)M_1^2}{(\gamma-1)M_1^2+2} \rightarrow \frac{\gamma+1}{\gamma-1} \overset{\text{def}}{=} r$$

$$\frac{T_2}{T_1} = \frac{P_2}{P_1}\frac{\rho_1}{\rho_2}$$

$M_1 \gg 1$

Shock reference frame



Upstream (1)

$v_1/c_{s,1}>1$

Downstream (2)

$v_2/c_{s,2}<1$

$$M \overset{\text{def}}{=} v/c_s$$

Shock transition ~ λ

What is a shock?

A shock is a discontinuity solution of the fluid equations where a supersonic fluid becomes subsonic (i.e. the entropy increases)

$$\begin{aligned} \left[\rho u\right]_1 &= \left[\rho u\right]_2 \\ \left[\rho u^2+P\right]_1 &= \left[\rho u^2+P\right]_2 \\ \left[\frac{1}{2}\rho u^3+\frac{\gamma}{\gamma-1}P u\right]_1 &= \left[\frac{1}{2}\rho u^3+\frac{\gamma}{\gamma-1}P u\right]_2 \end{aligned}$$

$$\begin{aligned} \frac{P_2}{P_1} &= 1+\frac{2\gamma}{\gamma+1}\left(M_1^2-1\right) \rightarrow \frac{2\gamma M_1^2}{\gamma+1} \\ \frac{\rho_2}{\rho_1} &= \frac{\left(\gamma+1\right)M_1^2}{\left(\gamma-1\right)M_1^2+2} \rightarrow \frac{\gamma+1}{\gamma-1} \overset{\text{def}}{=} r \\ \frac{T_2}{T_1} &= \frac{P_2}{P_1}\frac{\rho_1}{\rho_2} \end{aligned}$$

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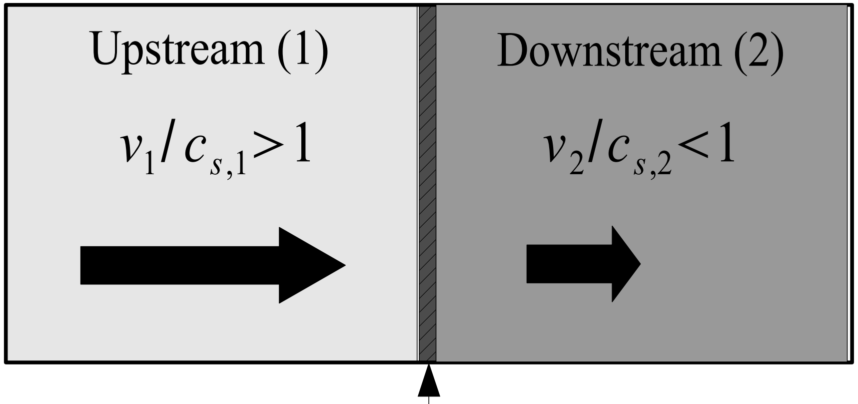

Upstream (1)

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Shock transition ~ λ

Caveats:
1) What produces the transition?
2) Does the fluid equations describe correctly astrophysical plasmas?

Collisionless Shocks Physics

What produce the shock transition?

$$\lambda_{mfp} \sim \frac{1}{n\sigma} = \begin{cases} \dfrac{1}{N_A \rho_{air} \left(2\pi a_0^2\right)} \sim \ 10^{-5}\, cm \\[2em] \dfrac{1}{n_{ISM}\, \sigma_{Coul}} \ > \ 1\, pc \end{cases}$$

Collisions in air

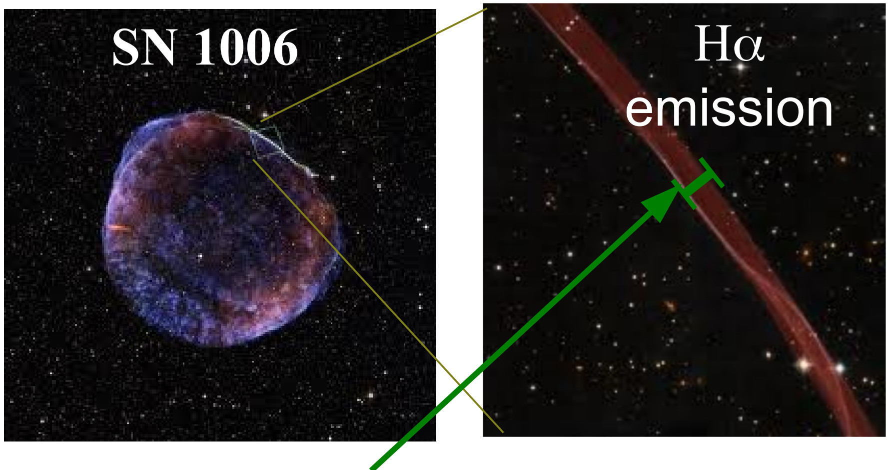
$N_A \sim 6 \times 10^{23}$

$\rho_{air} \sim 1\, kg/m^3$

Collisions in the ISM

But observationally
(from Balmer emission):

$\lambda_{sh} \ll 10^{15}\, cm = \ 3 \times 10^{-4}\, pc$



Thickness ~ 10" → 3×10^{17} cm
Hα resolution ~ 0.7" → 2×10^{16} cm

Collisionless Shocks Physics

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Length-scale for EM processes:

$\omega_{pe} = elctron\ plasma\ frequency$

$\omega_{pi} = ion\ plasma\ frequency$

Electron skin depth $$\frac{c}{\omega_{pe}} = \left(\frac{m_e c^2}{4\pi n_e e^2} \right)^{1/2} = \; 5.3\times10^5\, n_e^{-1/2}\, cm$$

Ion skin depth $$\frac{c}{\omega_{pi}} = \left(\frac{m_i c^2}{4\pi n_i e^2} \right)^{1/2} = \; 2.3\times10^7\, n_i^{-1/2}\, cm$$

p's Larmor radius $$r_L(v_{sh}) = \frac{m_p v_{sh} c}{eB} = \; 10^{10} \left(\frac{v_{sh}}{3000\, km/s} \right) \left(\frac{B}{3\mu G} \right)^{-1} cm$$

Shock thickness between these two length-scales

Electro-magnetic instability

The shock transition is mediated by electromagnetic interactions.

Collisions have no role

→ the Mach number does not properly describe the shock properties

Alvénic Mach number is more appropriate:

$$M_A = \frac{v_{sh}}{v_A}; \qquad v_A = \frac{B}{\sqrt{4\pi\rho}} \approx 2 B_{\mu G} \left(\frac{n}{cm^{-3}}\right)^{-1/2} km/s$$

Alfvén waves are a combination of electromagnetic-hydromagnetic waves

Analogy with waves on a string: $v = \sqrt{T/\mu}; \quad T \rightarrow B^2/4\pi, \quad \mu \rightarrow \rho$

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Analogy with waves on a string: $v = \sqrt{T/\mu}; \quad T \to B^2/4\pi, \quad \mu \to \rho$

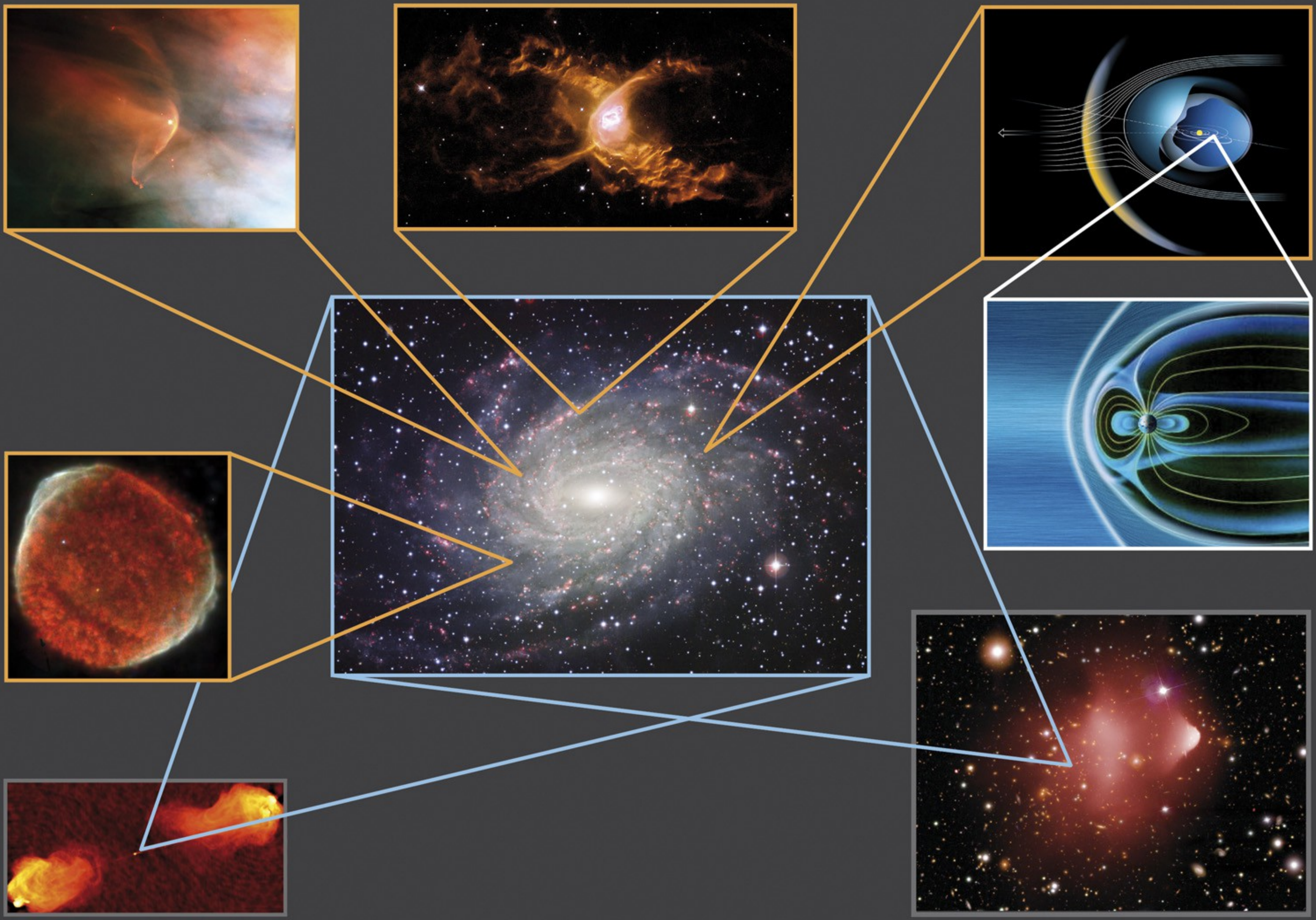
Collisionless shocks require $M_A > 1$

Which instability is responsible for the shock transition?

- Two stream instability
- Weibel instability
- Oblique instability
- Filamentation
- ...

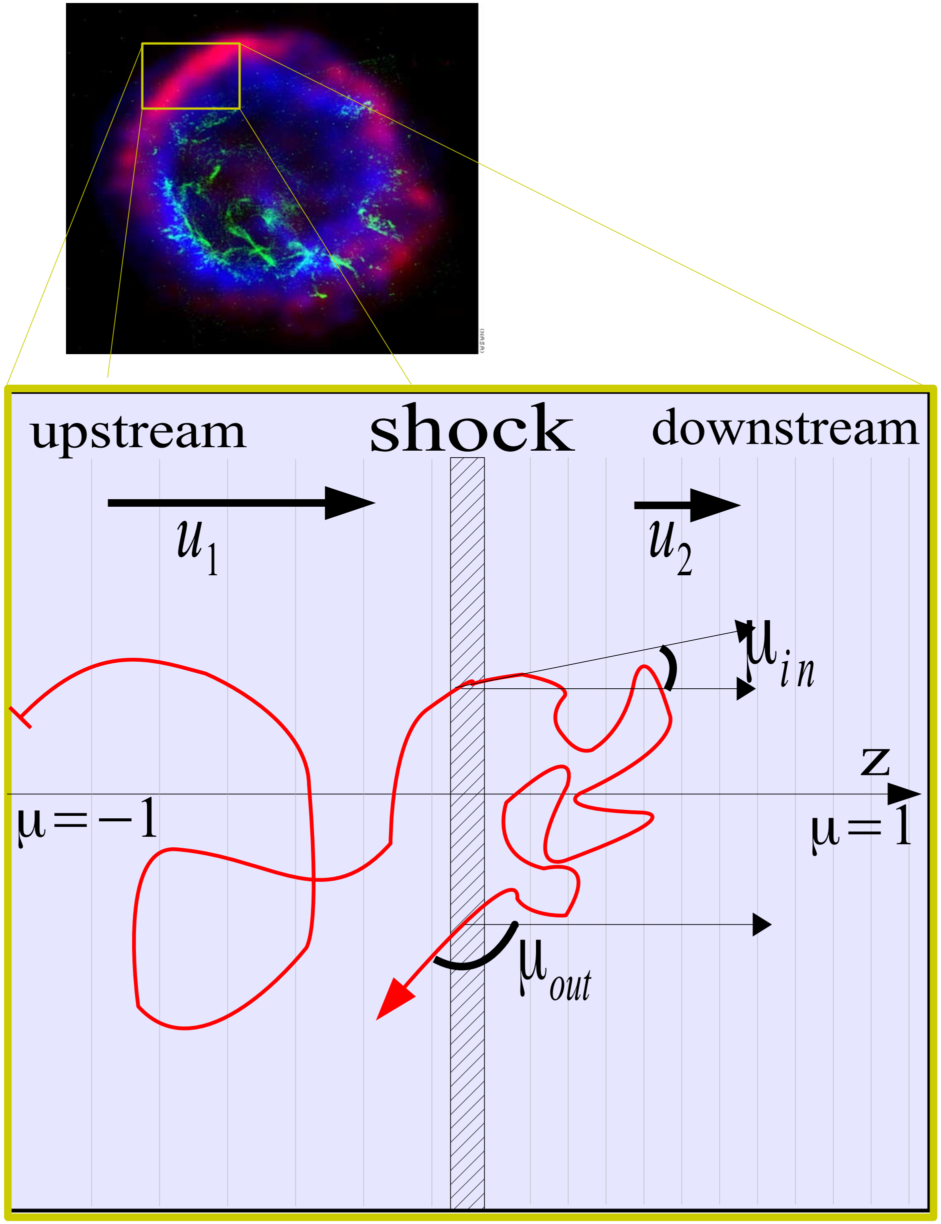
The relative importance depends on the initial conditions of the plasma

SHOCKS ARE EVERYWHERE...



HOW DO SHOCKS ACCELERATE PARTICLES?

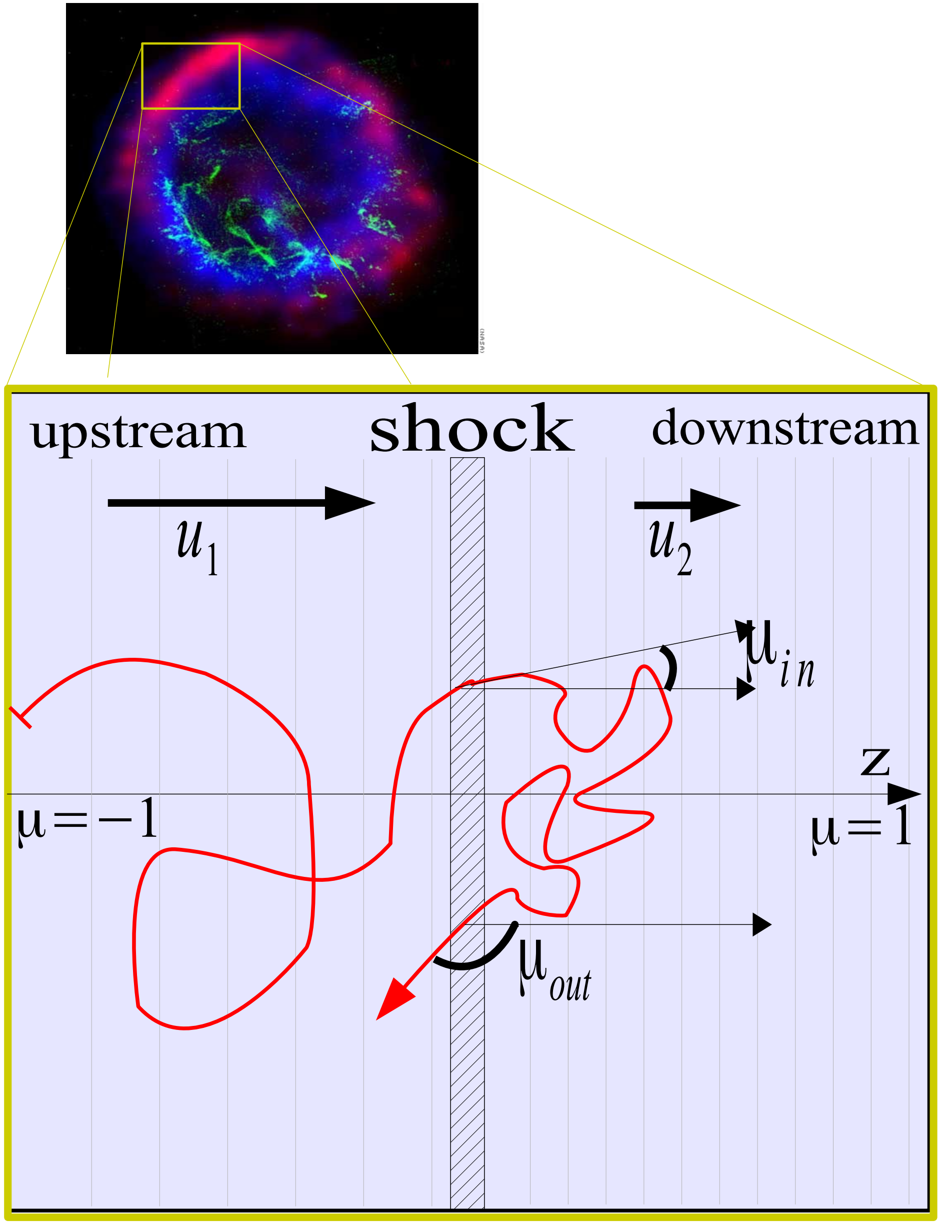
ACCELERATION AT SHOCK WAVES: THE TEST-PARTICLE APPROACH



ENERGY GAIN

$$E_2=\frac{(1-\beta_{rel}\mu_{in})(1+\beta_{rel}\mu'_{out})}{1-\beta^2_{rel}}\;E_1$$

ACCELERATION AT SHOCK WAVES: THE TEST-PARTICLE APPROACH



ENERGY GAIN

$$E_2 = \frac{(1-\beta_{rel}\mu_{in})(1+\beta_{rel}\mu'_{out})}{1-\beta_{rel}^2} \quad E_1$$

Averaging over $0 < \mu_{in} < 1$ and $-1 < \mu_{out} < 0$:

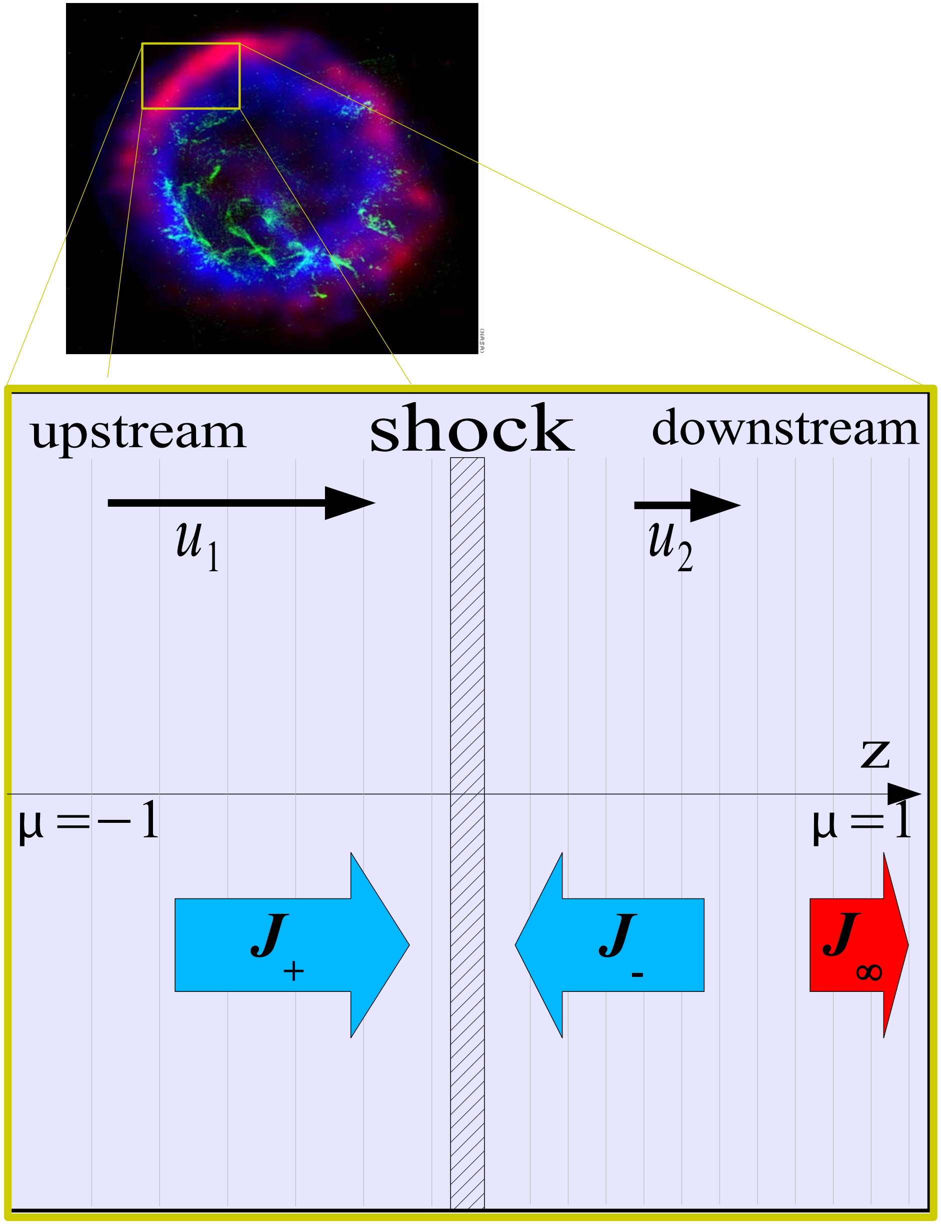
$$\left\langle \frac{\Delta E}{E} \right\rangle = \frac{1+\frac{4}{3}\beta_{rel}+\frac{4}{9}\beta_{rel}^2}{1-\beta_{rel}^2} - 1 \simeq \frac{4}{3}\beta_{rel}$$

The energy gain is now 1st order in V_{sh} because in each cycle

upstream → downstream → upstream

the particle can only gain energy

ACCELERATION AT SHOCK WAVES: PARTICLE SPECTRUM



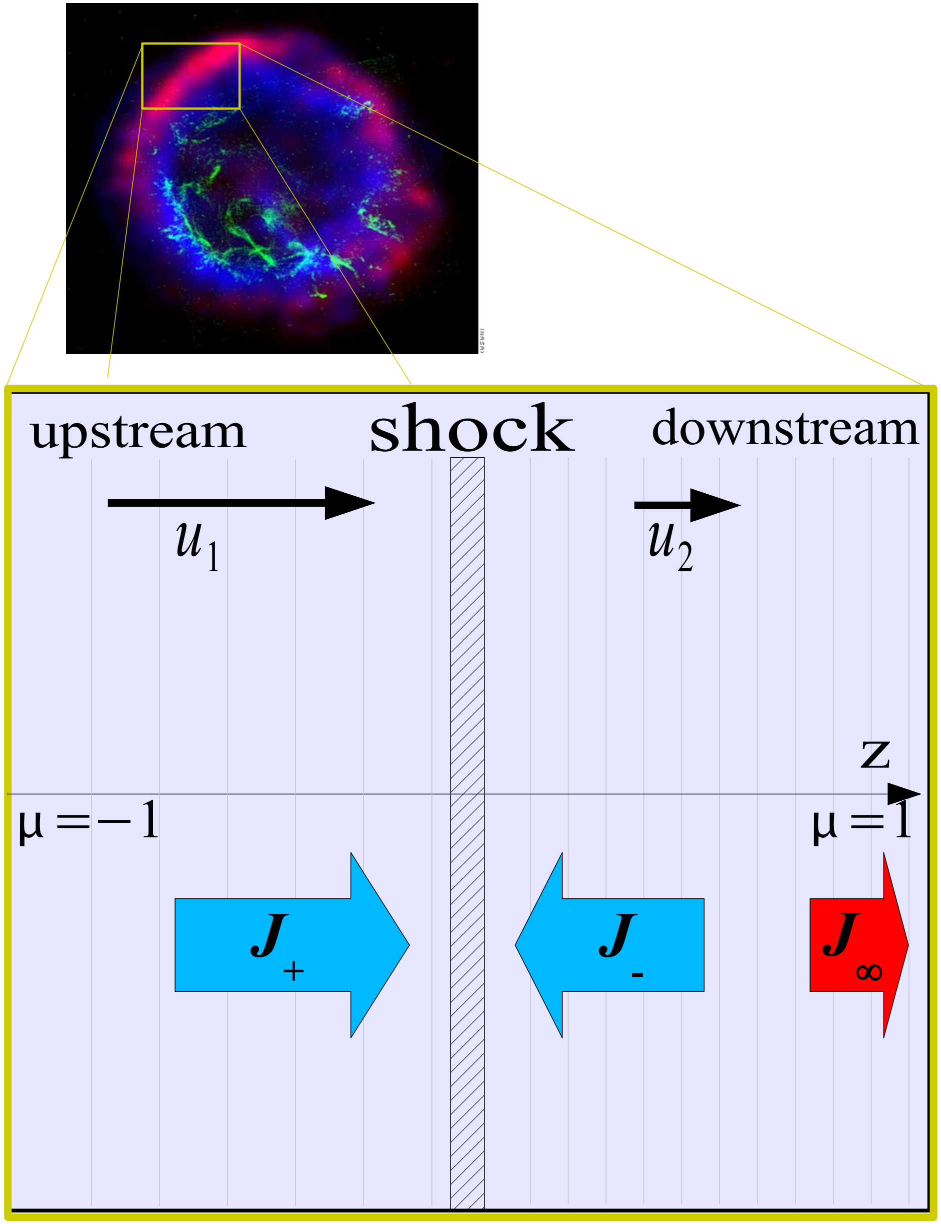
$$J_\infty = n u_2$$

$$J_- = \int \frac{d\Omega}{4\pi} n c \cos(\theta) = \frac{nc}{4}$$

$$P_{esc} = \frac{J_\infty}{J_+} = \frac{J_\infty}{J_\infty + J_-} \approx 4\frac{u_2}{c}$$

Escaping probability

ACCELERATION AT SHOCK WAVES: PARTICLE SPECTRUM



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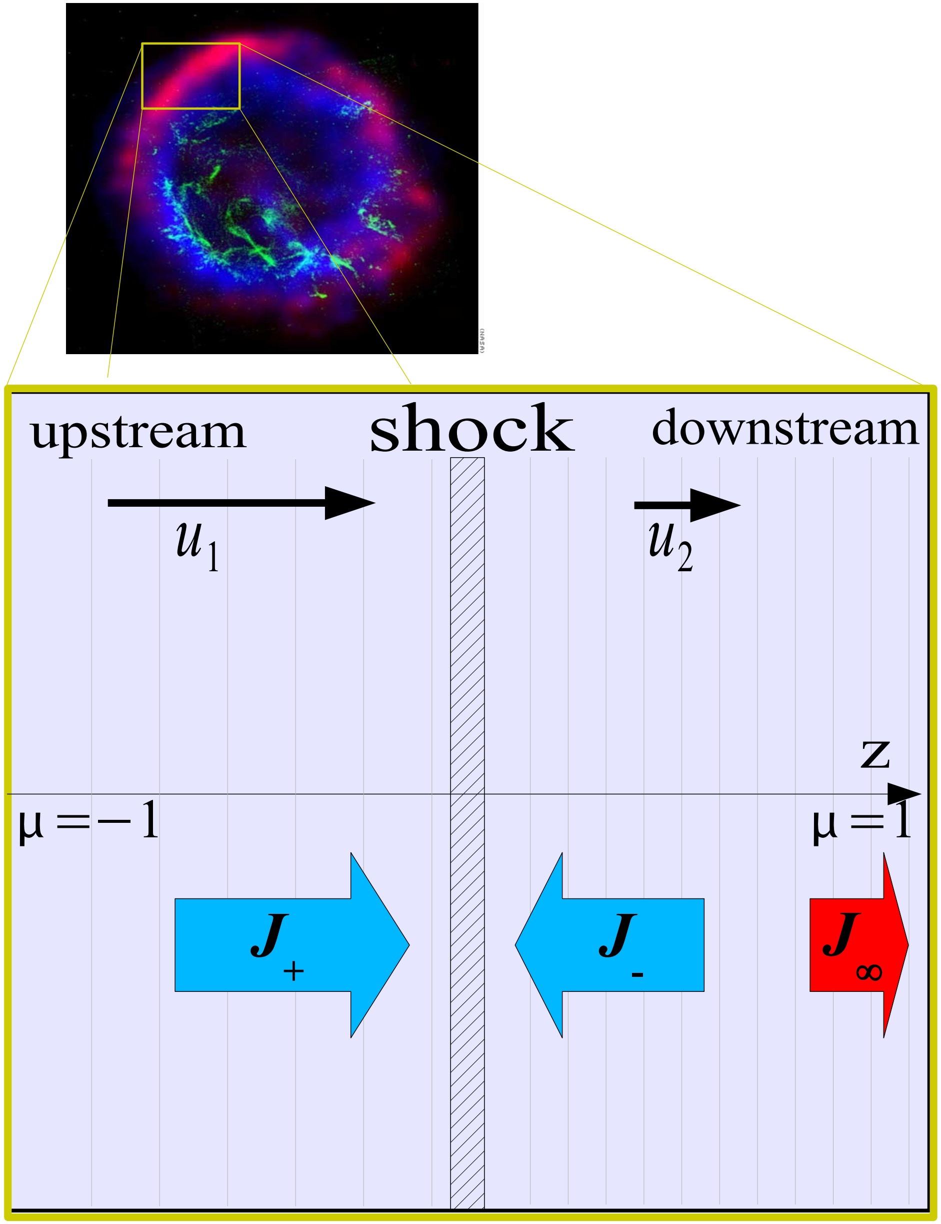
Escaping probability

Energy after *k* interactions:

$$E_k = E_0 (1+\xi)^k \rightarrow k = \frac{\ln(E/E_0)}{\ln(1+\xi)}$$

ACCELERATION AT SHOCK WAVES: PARTICLE SPECTRUM



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Escaping probability

Energy after k interactions:

$$E_k = E_0(1+\xi)^k \rightarrow k = \frac{\ln(E/E_0)}{\ln(1+\xi)}$$

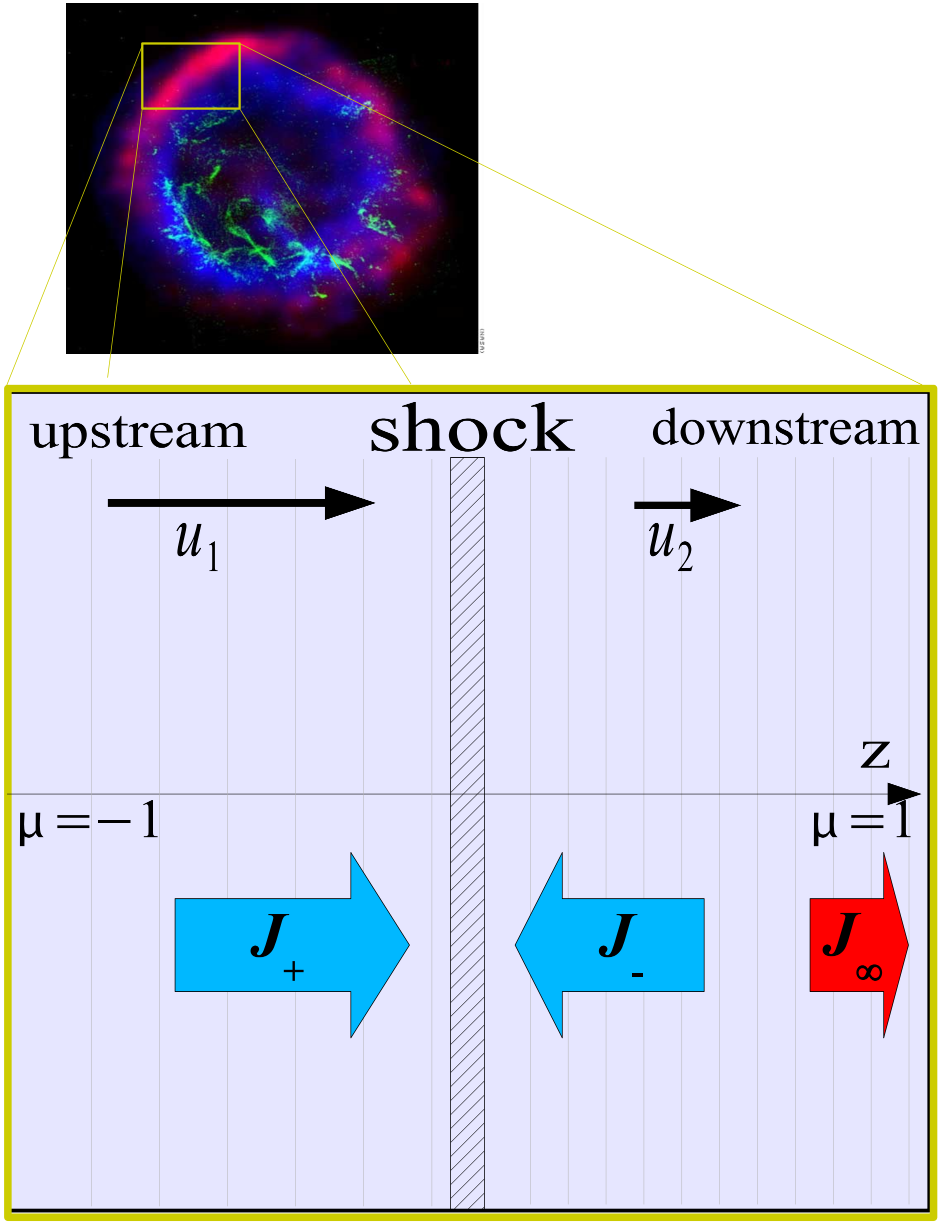
The number of particles with energy > E is:

$$N(>E) \propto \sum_{i=k}^{\infty}(1-P_{\mathrm{esc}})^i = \frac{(1-P_{\mathrm{esc}})^k}{P_{\mathrm{esc}}} = \frac{1}{P_{\mathrm{esc}}}\left(\frac{E}{E_0}\right)^{-\delta}$$

$$\delta = -\frac{\ln(1-P_{esc})}{\ln(1+\xi)} \simeq \frac{P_{esc}}{\xi}$$

Both independent on energy

ACCELERATION AT SHOCK WAVES: PARTICLE SPECTRUM



Differential energy spectrum:

$$\boxed{f(E) \equiv \frac{dN}{dE} \propto E^{-\alpha}}$$

Slope:

$$\begin{aligned}\alpha &= 1+\delta \simeq 1+\frac{P_{esc}}{\xi} \\ &= 1+\frac{4\mathrm{u}_2/c}{4(u_1-u_2)/3\mathrm{c}} \\ &= \frac{r+2}{r-1} \to 2\end{aligned}$$

For strong shocks and monoatomic gas: $r \equiv \frac{u_1}{u_2} \to 4$

Spectrum in momentum *p*:

$$4\pi p^2 dp\, f(p) = f(E)dE$$

$$\boxed{f(E) \propto E^{-2} \to f(p) \propto p^{-4}}$$

REMARKS

Important points:

1) The particle spectrum obtained from the 1st order Fermi acceleration is independent from the scattering properties

2) A power law spectrum is the consequence of P_{esc} and $\Delta E/E$ being independent on the initial energy

3) The spectrum $f(E) \sim E^{-2}$ is valid for strong shocks ($r \rightarrow 4$)

What depends on the scattering properties is the maximum achievable energy

MAXIMUM ENERGY

MAXIMUM ENERGY

Is it possible to accelerate protons up to the *knee* ?

The maximum energy is obtained comparing the acceleration time with the age of the accelerator and the energy losses

$$\boxed{t_{acc} = min\left[t_{loss}, T_{age}\right]} \Rightarrow E_{max}$$

Energy losses are usually negligible for protons but are important for electrons

Acceleration time: $t_{\mathrm{acc}} = \frac{t_{\mathrm{cycle}}}{\Delta E/E}$

Time for one cycle upstream → downstream → upstream: $t_{cycle} = \tau_{diff,1} + \tau_{diff,2}$

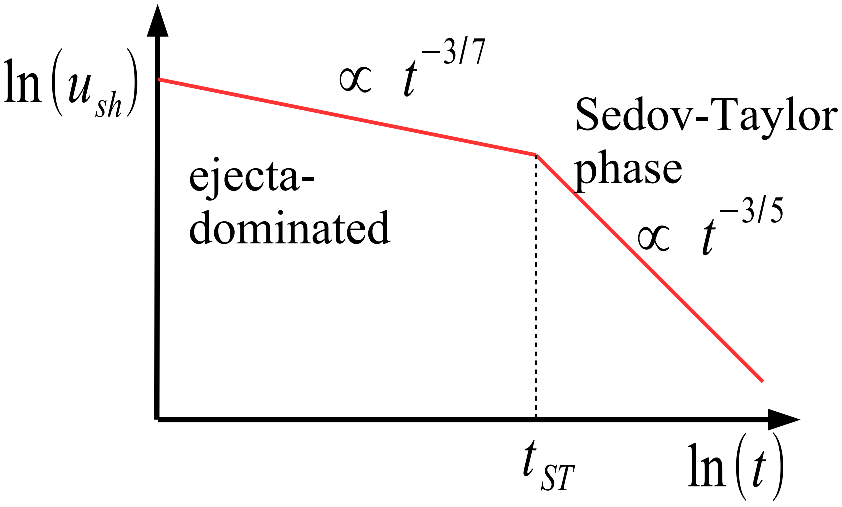
Equating the particle injected from downstream with the particles upstream:

$$\frac{nc}{4}\Sigma\tau_{\mathrm{diff},1} = n\Sigma\frac{D_1}{u_1} \Rightarrow \tau_{diff,1} = \frac{4D_1}{c\,u_1} \quad \wedge \quad \tau_{diff,2} = \frac{4D_2}{c\,u_2}$$

$$\frac{\Delta E}{E} = \frac{4}{3}\frac{u_1 - u_2}{c} \Rightarrow \boxed{t_{\mathrm{acc}} = \frac{t_{\mathrm{cycle}}}{\Delta E/E} = \frac{3}{u_1 - u_2}\left(\frac{D_1}{u_1} + \frac{D_2}{u_2}\right) \approx 8\,\frac{D_1}{u_{\mathrm{sh}}^2}}$$

MAXIMUM ENERGY

Maximum energy can increase only during the ejecta dominated phase of the SNRs because $u_{\text{sh}} \sim const$



Shock radius: $\begin{cases} R_{sh}(t) \propto t^{4/7} & \text{Ejecta-dominated} \\ R_{sh}(t) \propto t^{2/5} & \text{Sedov-Taylor} \end{cases}$

But particles diffuse ahead of the shock: $d \propto \sqrt{D\,t}$

→ **during the ST phase the highest energy particles cannot be caught by the shock and escape towards upstream**

Estimate of the beginning of the Sedov-Taylor phase:

$$\begin{cases} t_{ST} = R_{ST} / u_{sh} \\ E_{SN} = \dfrac{1}{2} M_{ej} u_{sh}^2 \\ M_{ej} = \dfrac{4\pi}{3} \rho_{ISM} R_{ST}^3 \end{cases} \Longrightarrow \quad t_{\mathrm{ST}} \approx 50 \left(\frac{M_{\mathrm{ej}}}{M_\odot}\right)^{\frac{5}{6}} \left(\frac{E_{\mathrm{SN}}}{10^{51}\mathrm{erg}}\right)^{-\frac{1}{2}} \left(\frac{n_{\mathrm{ISM}}}{\mathrm{cm}^{-3}}\right)^{-\frac{1}{3}} \mathrm{yr},$$

MAXIMUM ENERGY

Maximum energy can be obtained by equating the acceleration time with the end of the ejecta dominated phase:

$$t_{ST} \approx 50 \left(\frac{M_{ej}}{M_\odot}\right)^{\frac{5}{6}} \left(\frac{E_{SN}}{10^{51}\mathrm{erg}}\right)^{-\frac{1}{2}} \left(\frac{n_{ISM}}{\mathrm{cm}^{-3}}\right)^{-\frac{1}{3}} \mathrm{yr},$$

$$t_{acc} \simeq 8 \frac{D_1}{u_{sh}^2}$$

Using the diffusion coefficient from quasi-linear theory:

$$D = \frac{1}{3} \frac{r_L v}{k_{res} F(k_{res})}$$

$$t_{acc} = t_{ST} \Rightarrow \boxed{E_{max} = 50 Z k F(k) \left(\frac{B_0}{\mu G}\right) \left(\frac{M_{ej}}{M_{sun}}\right)^{(-1/6)} \left(\frac{E_{SN}}{10^{51} erg}\right)^{1/2} \left(\frac{n_{ISM}}{cm^{-3}}\right)^{-1/3} TeV}$$

$E_{\max}$ is weakly dependent on the ejecta mass and ISM density

High energies, up to PeV, can be achieved only if $\mathcal{F}(k) >> 1$.

Using the interstellar turbulence

Turbulence is injected at a scale comparable with the size of SNR (or super-bubbles) and than cascades at smaller scales.

Injection scale: $k_0 = 1/L_0 \approx (10\, pc)^{-1}$

Kolmogorov turbulence: $k\, F(k) = \frac{2}{3}\eta_B \left(\frac{k}{k_0}\right)^{-2/3}$

Resonant scale: $k_{res}(E) = \frac{1}{r_L(E)} = 1 \times \left(\frac{E}{10^{15}\, eV}\right)^{-1} \left(\frac{B_0}{1\,\mu G}\right) pc^{-1}$

$$\Rightarrow \quad k_{res} F(k_{res}) = \frac{2}{3}\eta_B \left(\frac{k_{res}}{k_0}\right)^{-2/3} \approx 10^{-2} \left(\frac{\eta_B}{0.1}\right)^{2/3} \left(\frac{E}{10^{15}\, eV}\right)^{2/3} B_{\mu G}^{-2/3}$$

$$\Rightarrow \quad \boxed{E_{max} \approx 1\, MeV}$$

We are missing 12 orders of magnitude!
Interstellar turbulence is not enough

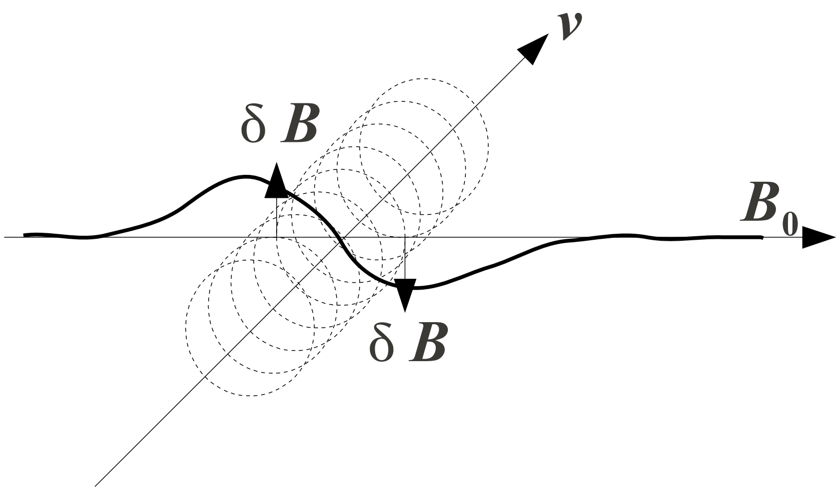
Magnetic field needs to be amplified

SELF-GENERATION OF WAVES

WHO GENERATES WAVES?

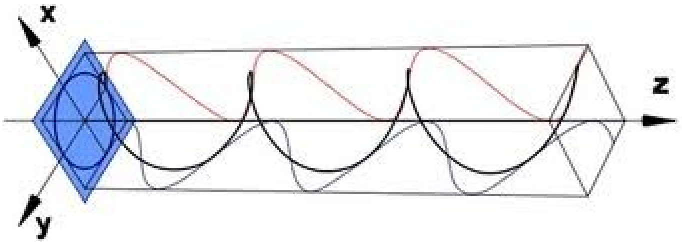
WAVES MAY BE GENERATED BY DIFFERENT SOURCES (e. g. SN EXPLOSION) BUT THERE IS A MORE INTERESTING AND PHYSICALLY IMPORTANT PHENOMENON:
SELF - GENERATION VIA RESONAT INSTABILITY

[e.g. Skilling (1975), Bell & Lucek (2001), Amato & Blasi (2006)]



Charged particles moving transverse to the magnetic field line produce a variable magnetic field $\delta\boldsymbol{B}$ which perturbs $\boldsymbol{B}_0$ producing an Alfvén wave.
→ Alfvén waves, in turn, scatter particles

The effect of scatter is to isotropize CRs.

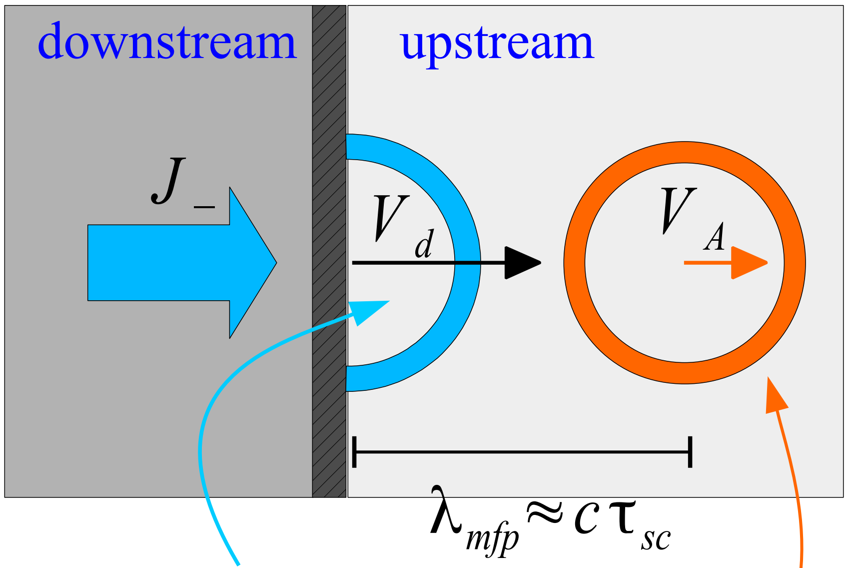


Generated Alfvén waves are circularly polarized

SELF GENERATION OF WAVES: RESONANT INSTABILITY

WAVES MAY BE GENERATED BY DIFFERENT SOURCES (e. g. SN EXPLOSION) BUT THERE IS A MORE INTERESTING AND PHYSICALLY IMPORTANT PHENOMENON:
SELF - GENERATION VIA RESONAT INSTABILITY

[e.g. Skilling (1975), Bell & Lucek (2001), Amato & Blasi (2006)]



After the shock the distribution is anisotropic

The distribution becomes isotropic after one mean free path and moves at the same speed of the waves

Assume particles are drifting with $v_d > v_A$ and are isotropyzed on a time-scale τ_{sc}:

$$\tau_{sc} \simeq \frac{1}{k\,F(k)\,\Omega}$$

Initial momentum **final momentum**

$$n_{CR}\, m\, \gamma_{CR}\, v_d \longrightarrow n_{CR}\, m\, \gamma_{CR}\, v_A$$

The momentum lost by particles is:

$$\frac{dP_{CR}}{dt} = \frac{P_2 - P_1}{\tau_{sc}} = \frac{n_{CR}\, m\, \gamma_{CR}\,(v_d - v_A)}{\tau_{sc}}$$

SELF GENERATION OF WAVES: RESONANT INSTABILITY

The momentum lost is transferred to waves $$\frac{dP_{CR}}{dt}=\frac{n_{CR}m\gamma_{CR}(v_d-v_A)}{\tau_{sc}}$$

Transport equation for waves: $$\frac{dP_W}{dt}\approx\frac{\Gamma_W}{v_A}\frac{\delta B^2}{8\pi};$$

Growth rate

Equating momentum lost by CR and momentum gain by waves $$\frac{dP_W}{dt}=\frac{dP_{CR}}{dt}\longrightarrow\Gamma_W=\frac{n_{CR}}{n_{gas}}\Omega_{cyc}\left(\frac{v_D-v_A}{v_A}\right)$$

$$\Omega_{cyc}=\gamma\Omega$$

For $n_{CR}=10^{-10}$ cm$^{-3,}$ $n_{gas}=0.1$ cm^{-3} and $B_0=1\mu$G, and assuming $v_d=2\ v_A$, one finds:

$v_A = 10$ km/s
$v_D = v_{sh}/4 \sim 1000$ km/s
$\Omega_{cyc} = 10^{-2}$ s^{-1}

$$\longrightarrow\Gamma_W\simeq\frac{n_{CR}}{n_{gas}}\Omega_{cyc}\left(\frac{v_D}{v_A}\right)\approx\ 10^{-3}\quad yr^{-1}$$

VERY RAPID GROWTH

HOW MUCH THE SELF-GENERATED TURBULENCE CAN GROW?

Turbulence can grow for at most one advection time

$$t_{adv} = D_1/u_{sh}^2$$

Equating the grow time with the advection time we get the maximum level of turbulence at the shock:

$$t_{adv} = t_{grow} = 1/\Gamma_W$$

⇒ $$F_0(k) = \frac{\pi}{2}\frac{\xi_{CR}}{\ln\left(p_{max}/m_p c\right)}\frac{u_{sh}}{v_A} \approx 10$$

$$\left\{\begin{array}{l} \xi_{CR} = P_{CR}/\left(\rho\, u_{sh}^2\right) \sim 0.1 \\ u_{sh} \sim 5000\, km/s \\ v_A \sim 10\, km/s \\ p_{max} \sim 10^5\, GeV \end{array}\right.$$

The condition $F(k) >> 1$ violates the quasi-linear theory used to derive the growth time.

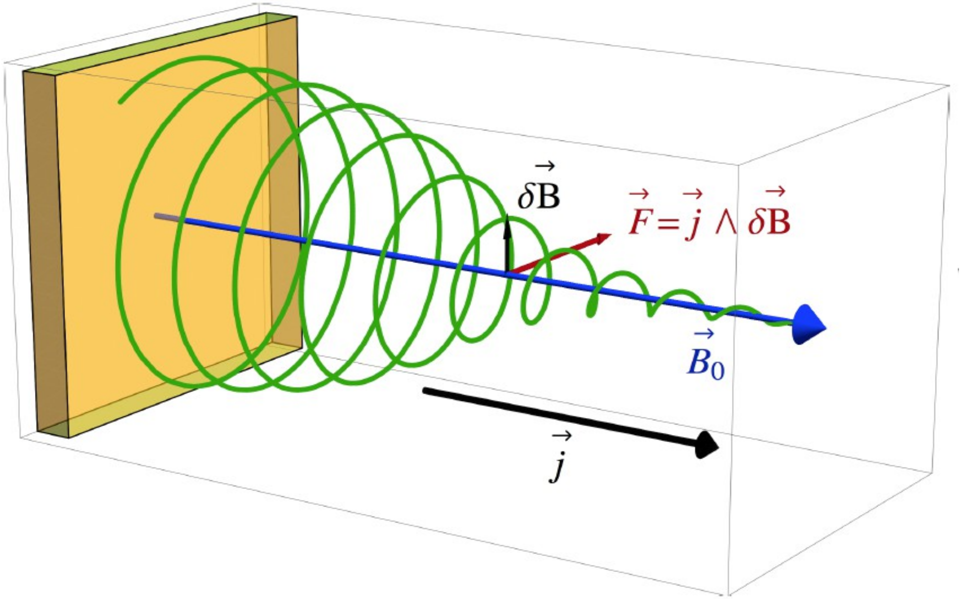
A more realistic estimate including the modification to the <u>dispersion relation</u> induced by CRs gives:

$$F_0(k) = \left(\frac{\pi}{6}\frac{\xi_{CR}}{\ln\left(p_{max}/m_p c\right)}\frac{c}{u_{sh}}\right)^{1/2} \leq 1$$

Resonant-amplification can produce $\delta B \sim B_0$

⇒ $$\boxed{E_{max} \approx 10^{13} - 10^{14}\, eV}$$

NON-RESONANT AMPLIFICATION



There are other possibility to amplify the magnetic field.

The most invoked one is the non-resonant Bell instability **[Bell, A.R. (2004)]**

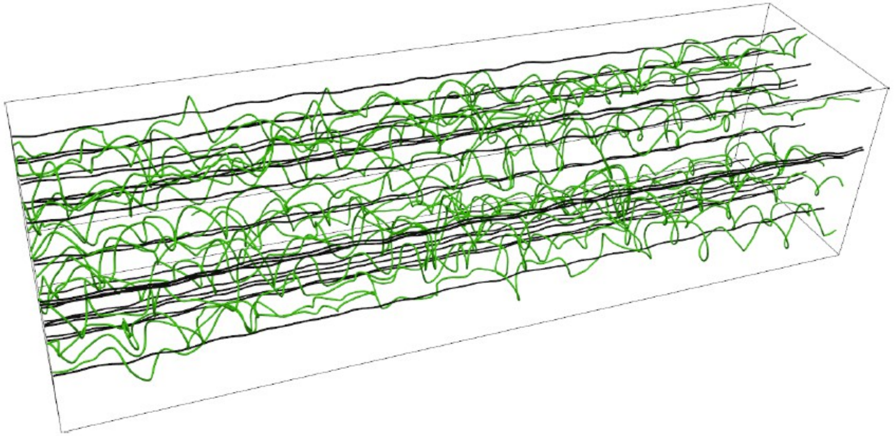
This instability is excited by the force

$$\vec{j}_{CR} \times \delta \vec{B}$$

where the current is due to escaping particles upstream.
It amplifies almost purely growing waves with wave-numbers much greater than the inverse particle gyroradius.

➔ works for very high shock velocity (initial phase of SNR expansion)

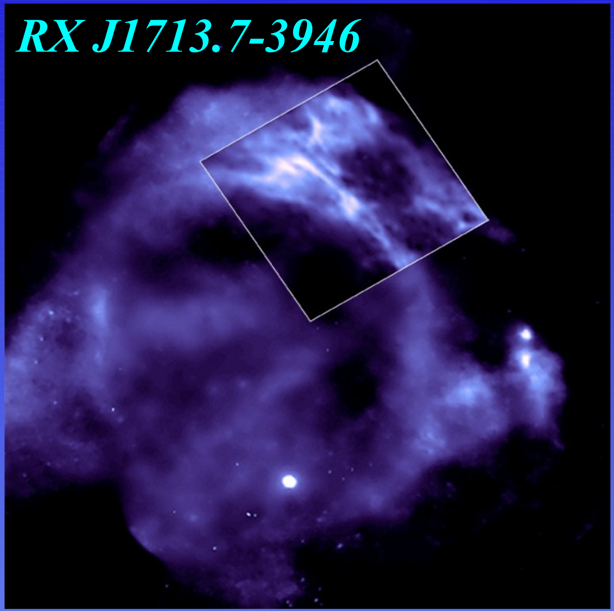
We can have $\frac{\delta B}{B_0} > 10$ if $\xi_{CR} = \frac{P_{CR}}{\rho u_{sh}^2} > 0.1$



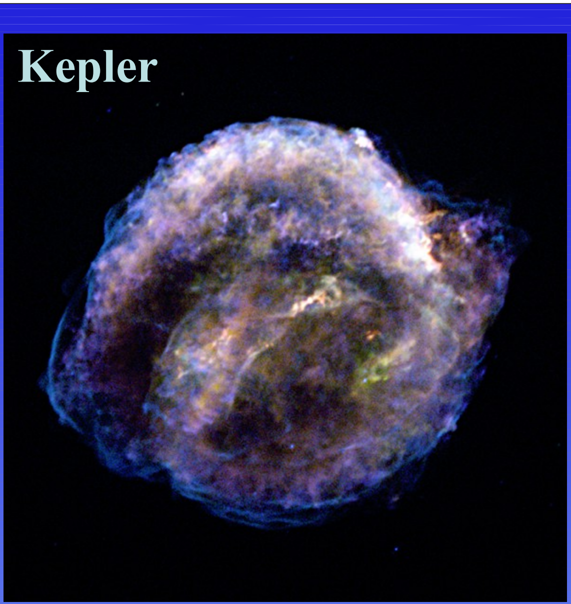
Simulation from Reville & Bell (2013)

DO WE SEE MAGNETIC FIELD AMPLIFICATION?

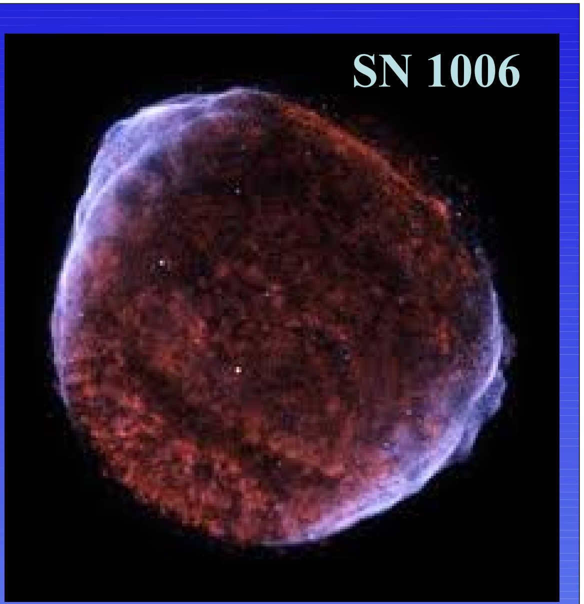
Galactic SNRs in X-rays



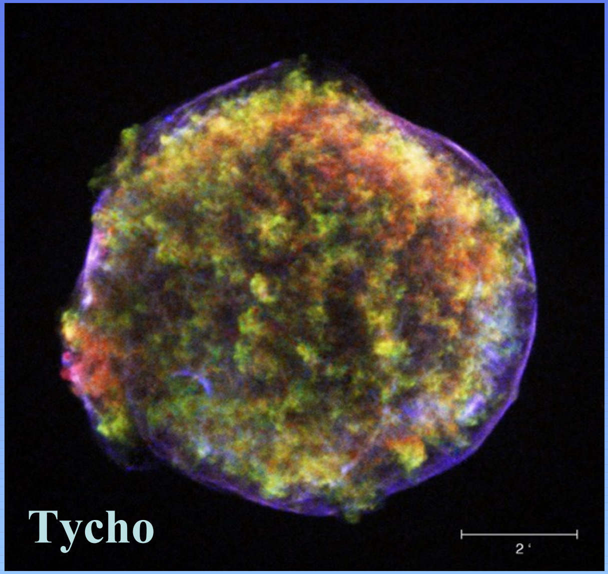
RX J1713.7-3946



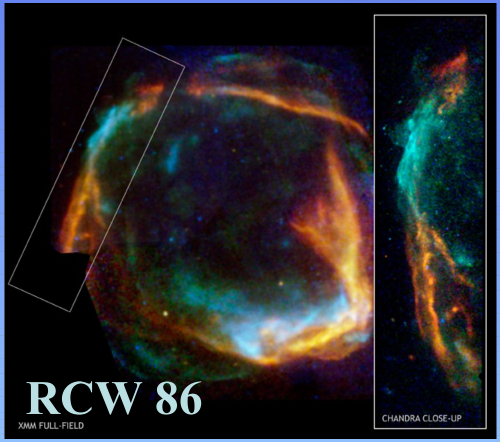
Kepler



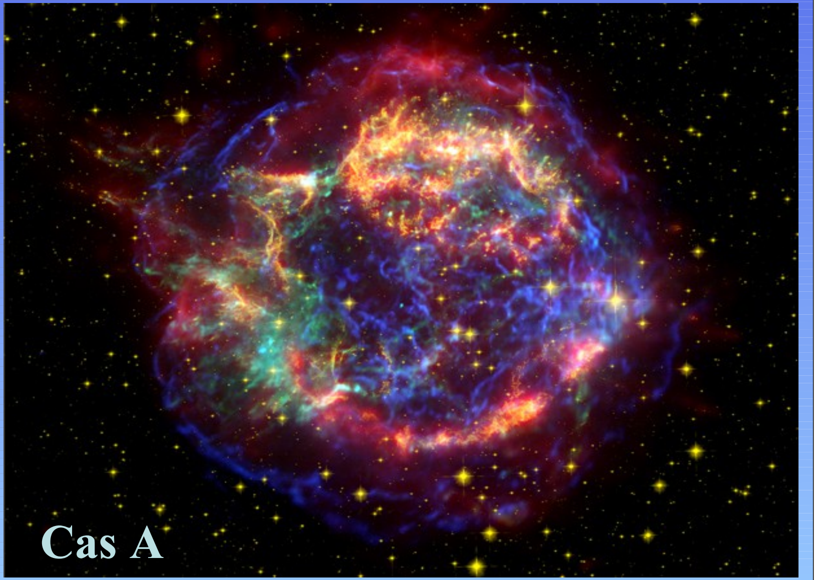
SN 1006



Tycho



RCW 86



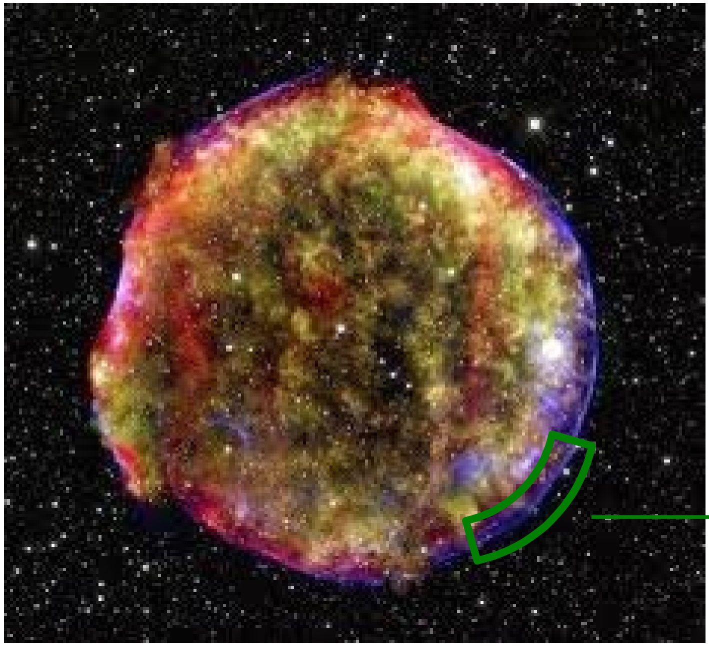
Cas A

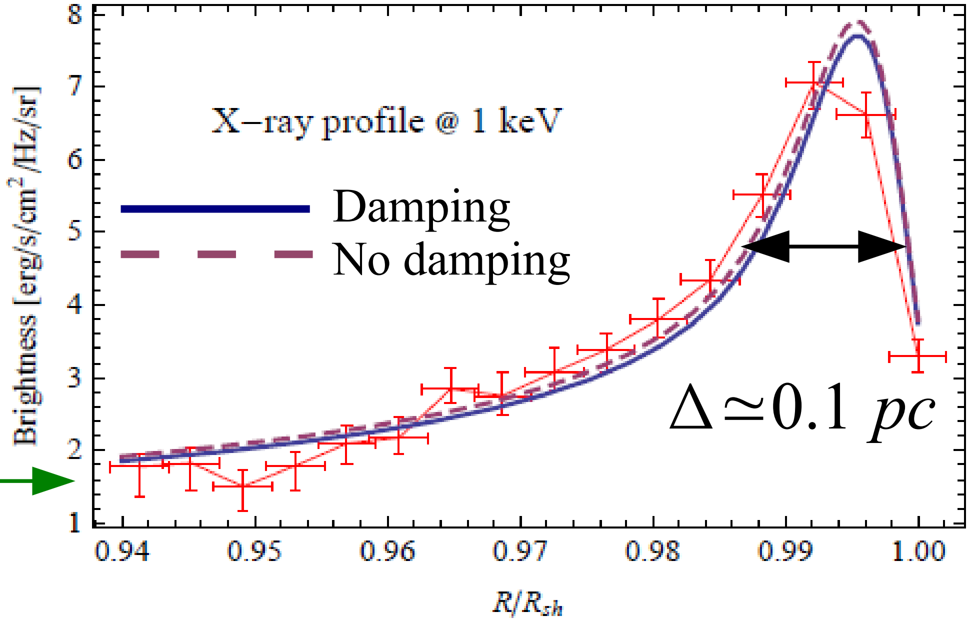
Evidences for magnetic field amplification

Thin non-thermal X-ray filaments provide evidence for magnetic field amplification

[Hwang el al(2002); Bamba et al (2005)]

Chandra X-ray map.
Data for the green sector are from Cassam-Chenaï et al (2007)





Assuming Bohm diffusion:

$$\begin{cases} D = r_L c/3 \propto E B^{-1} \\ \tau_{syn} = \dfrac{E}{dE/dt} = \dfrac{3 m_e c^2}{4 \sigma_T c \gamma \beta^2 U_B} \propto E^{-1} B^{-2} \end{cases}$$

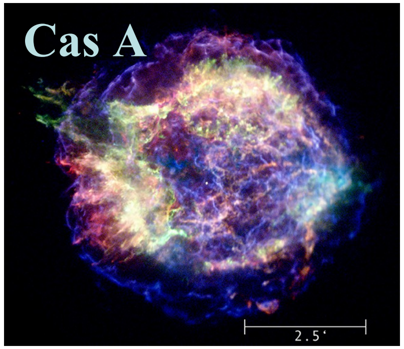
X-ray thickness = Synchrotron loss-length

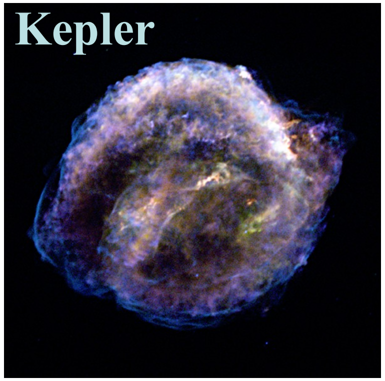
$$\Delta \simeq \sqrt{D \tau_{syn}} \propto B^{-3/2}$$

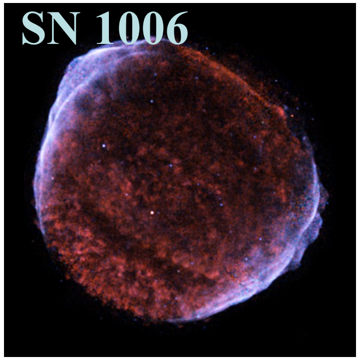
***B*~200-300 μG**

Evidences for magnetic field amplification

Thin X-ray rims (~0.1 pc) are observed in almost all young SNRs







Magnetic pressure downstream can reach ~few% of total pressure

SNR	**B_{down} (μG)**	**$B^2_{down/}/(8\pi p)$ [%]**
Cas A	**250-390**	**3.2-3.6**
Kepler	**210-340**	**2.3-2.5**
Tycho	**240-530**	**1.8-3.1**
SN1006	**90-110**	**4.0-4.2**
RCW 86	**75-145**	**1.5-3.8**

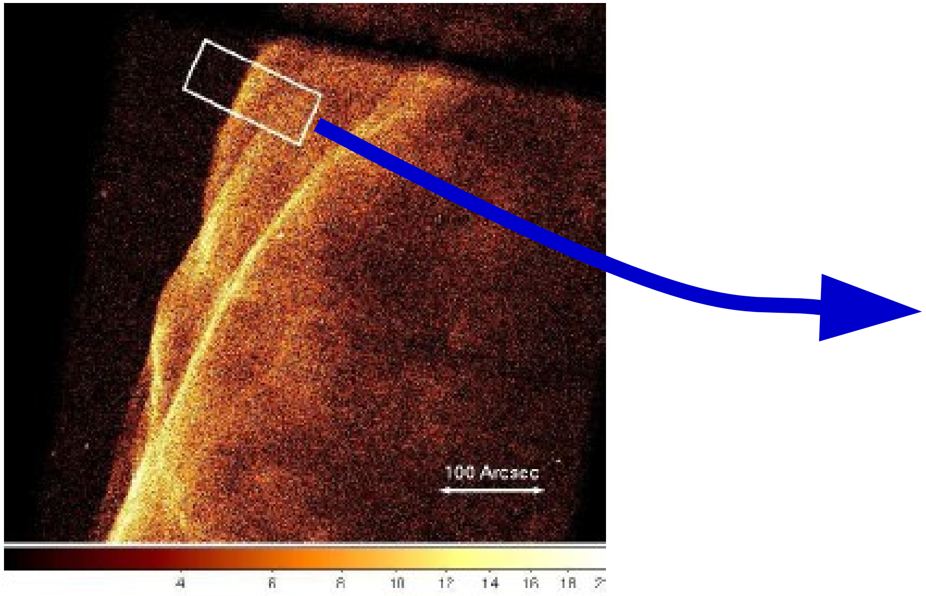
Where is the magnetic field amplified?

DOWNSTREAM: MHD instabilities (shear-like)

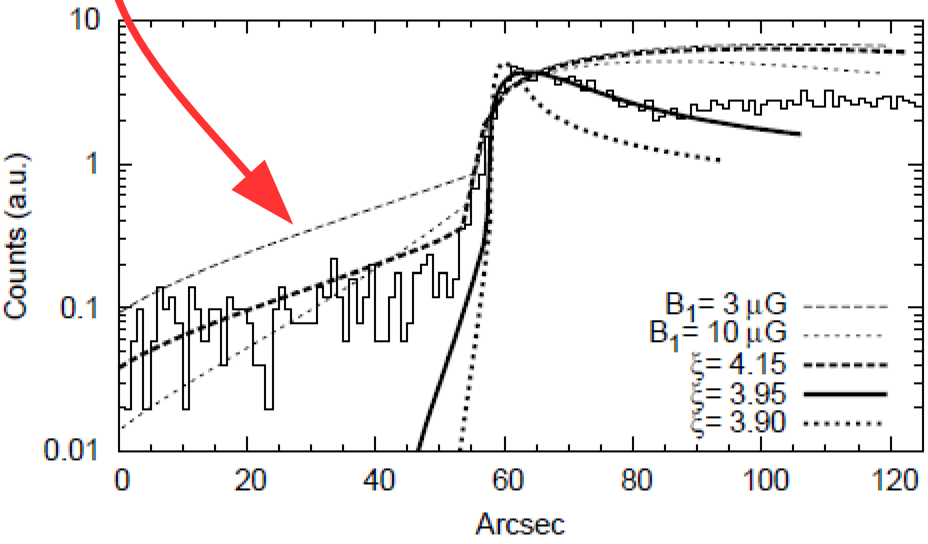
UPSTREAM: only through instabilities driven by CRs (Streaming, Bell)

BUT we need amplification upstream
of the shock to reach high energies

**Low magnetic field upstream
produces a more extended emission
NOT OBSERVED!**

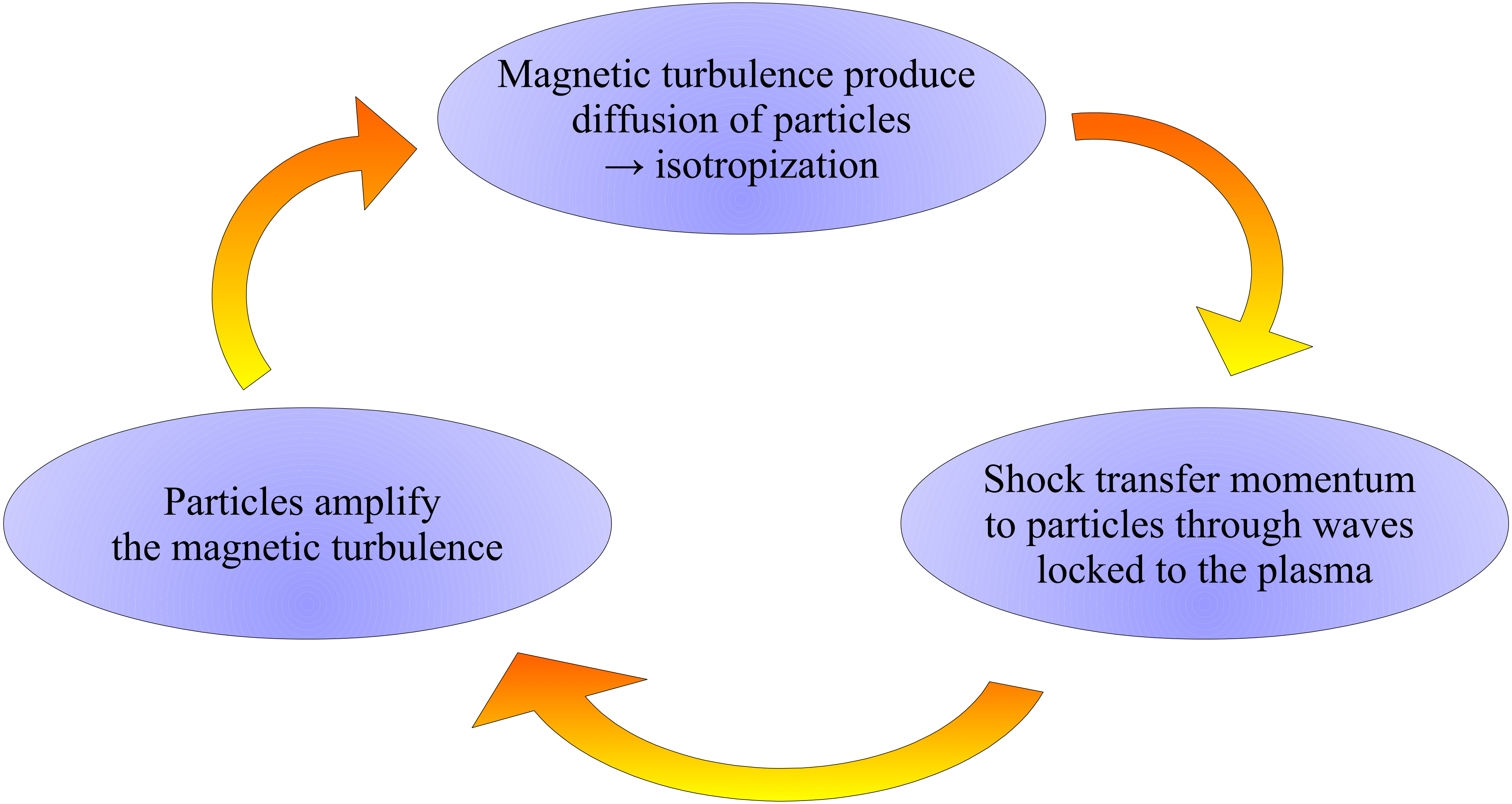


SN1006 in X-rays (*Chandra*)



[from G.M., Amato, Blasi, 2009, MNRAS]

THE ESSENCE OF NON-LINEARITY



Magnetic turbulence produce diffusion of particles → isotropization

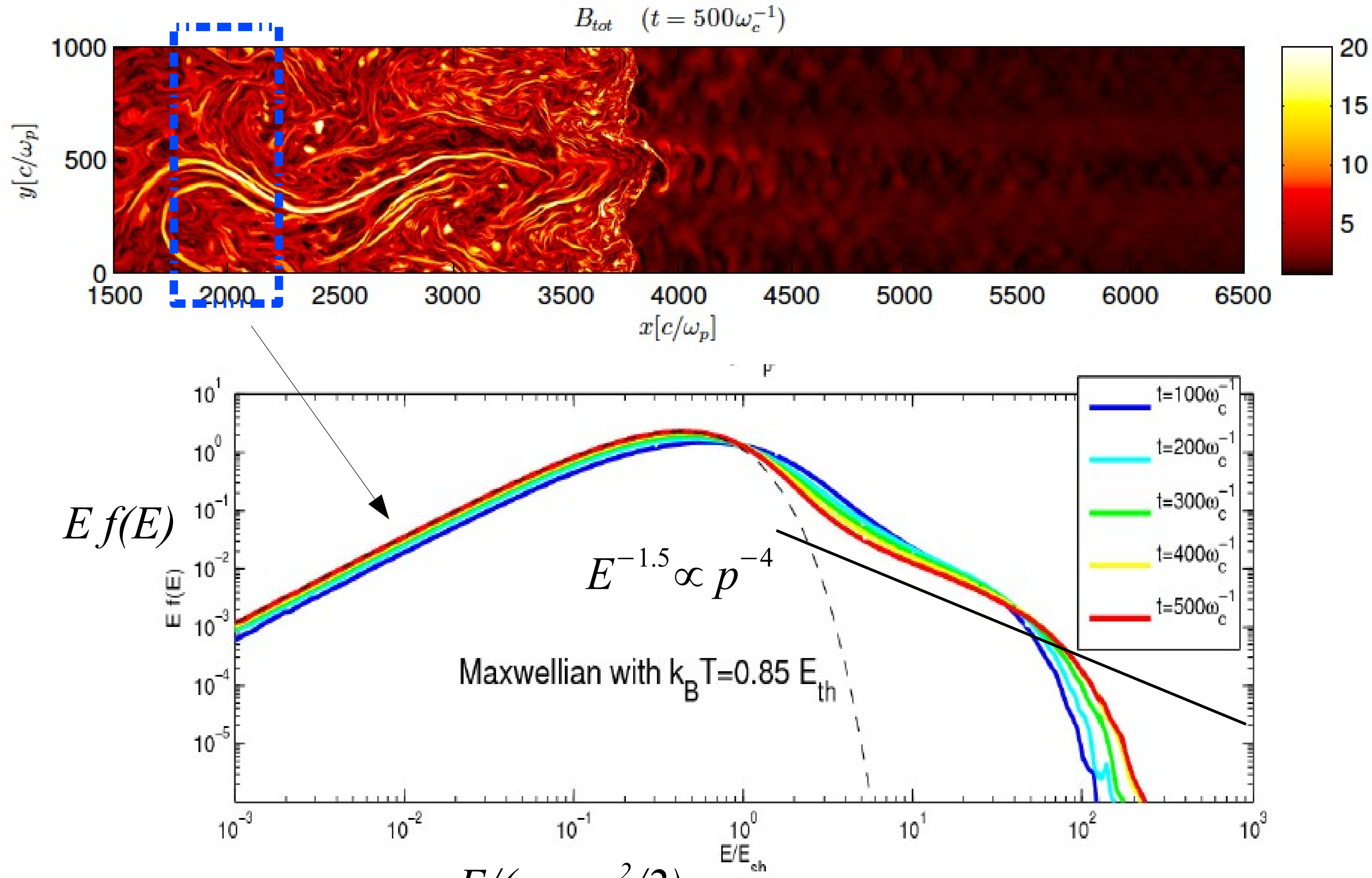
Shock transfer momentum to particles through waves locked to the plasma

Particles amplify the magnetic turbulence

PARTICLE INJECTION

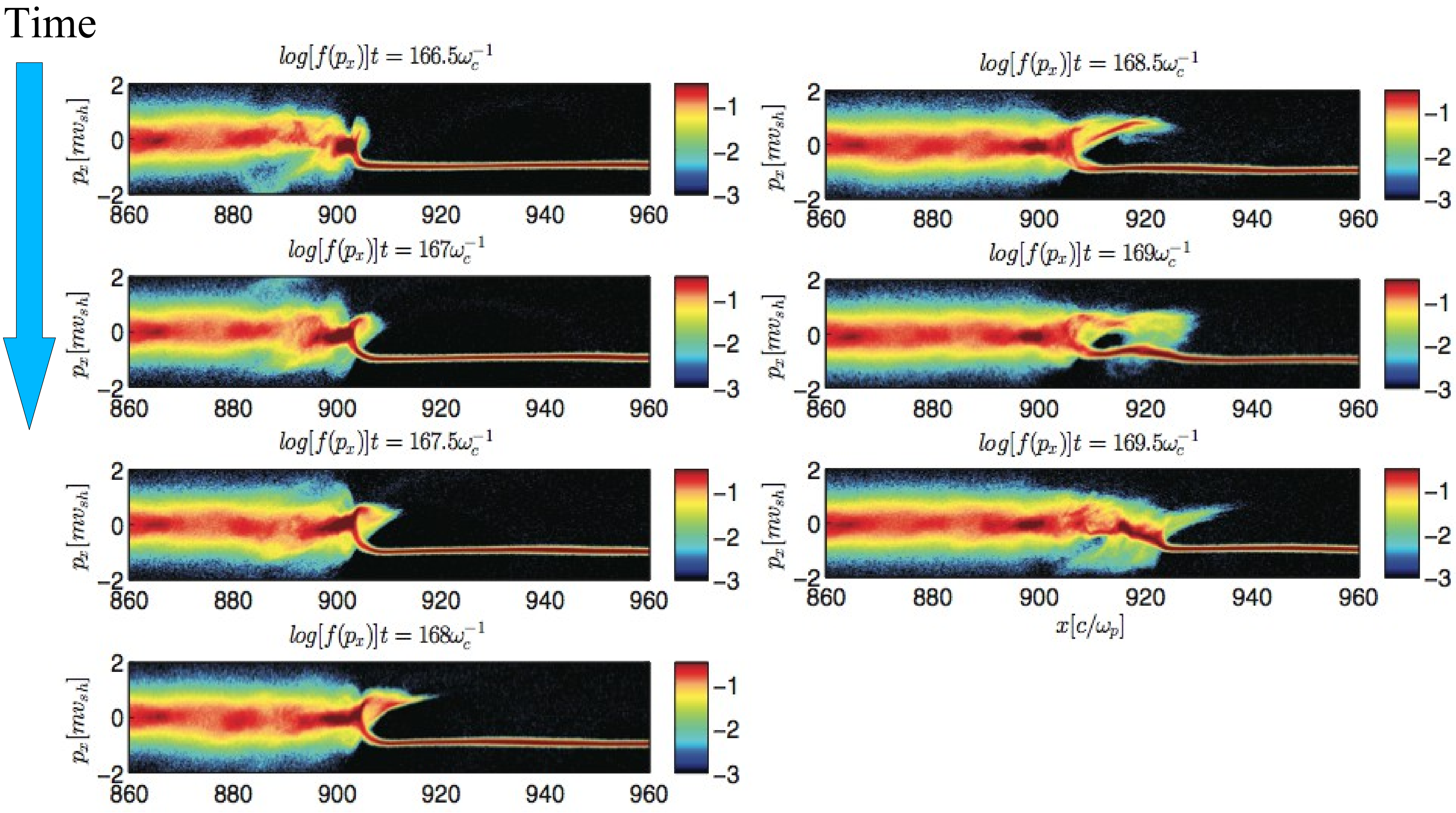
SIMULATIONS: results for the spectrum

D. Caprioli & A. Spitkovsky, 2013



Shock reformation

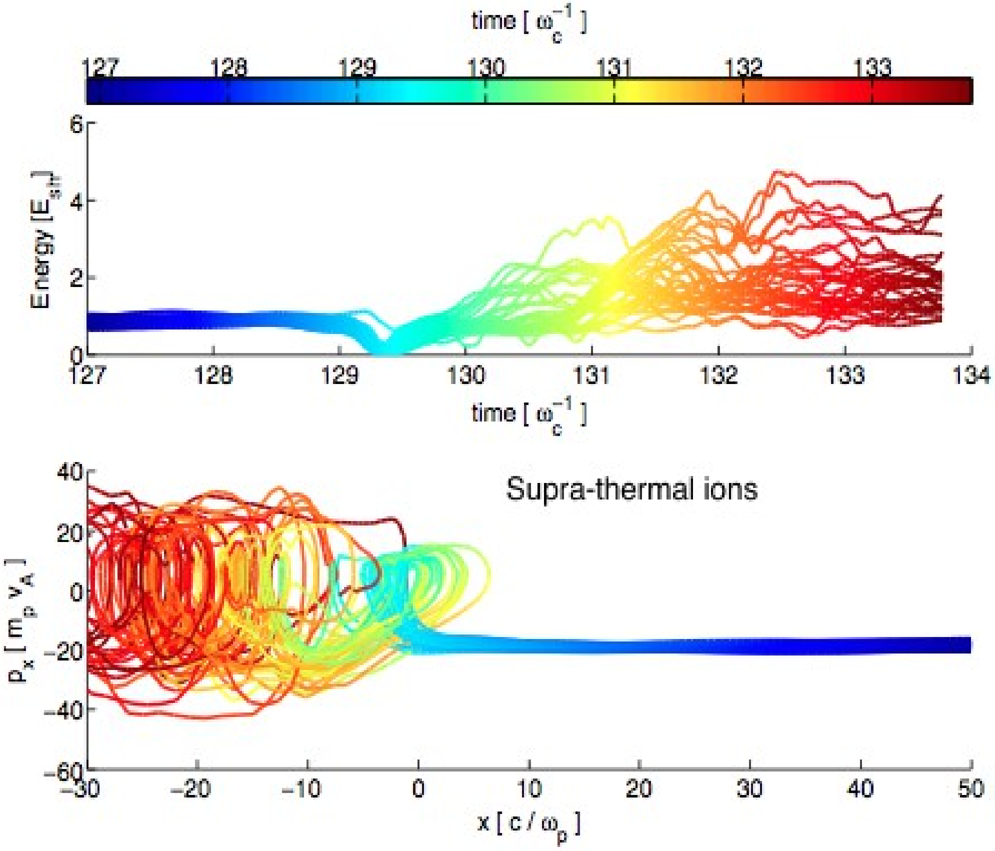
From Caprioli, Pop & Spitkovsky (2015)



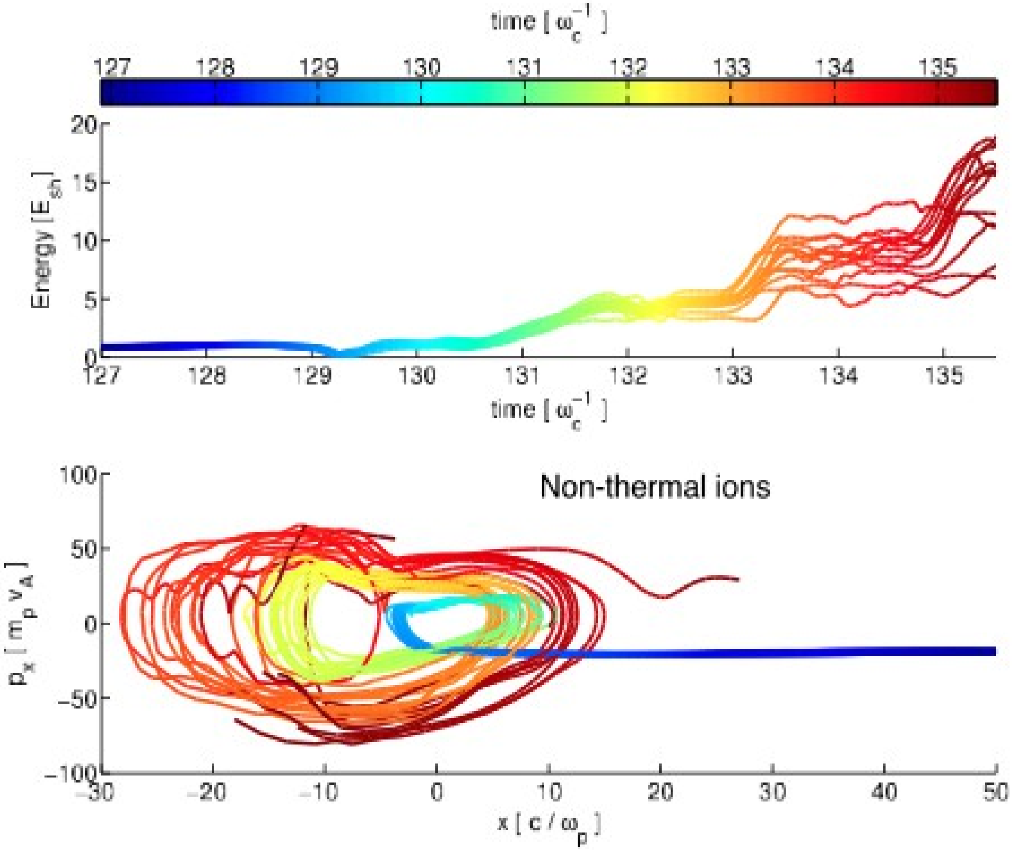
Shock reformation

From Caprioli, Pop & Spitkovsky (2015)

Supra-thermal ions

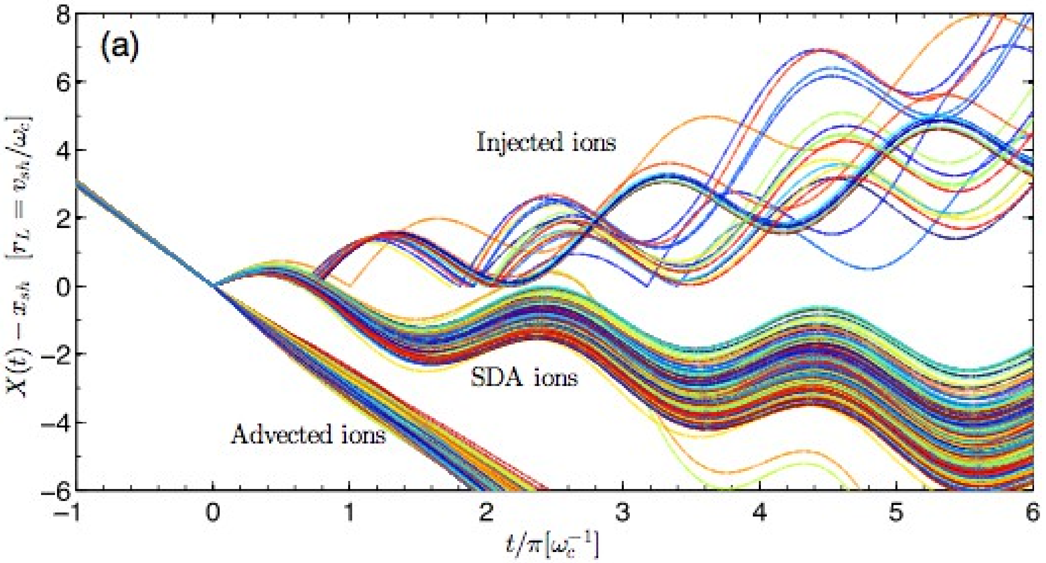


Non-thermal ions

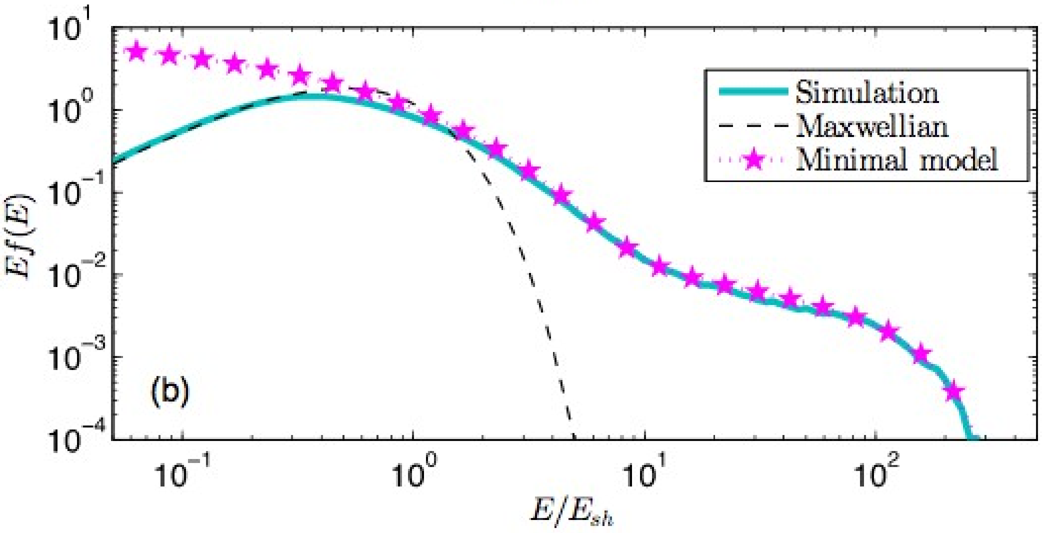


Ion injection

From Caprioli, Pop & Spitkovsky (2015)

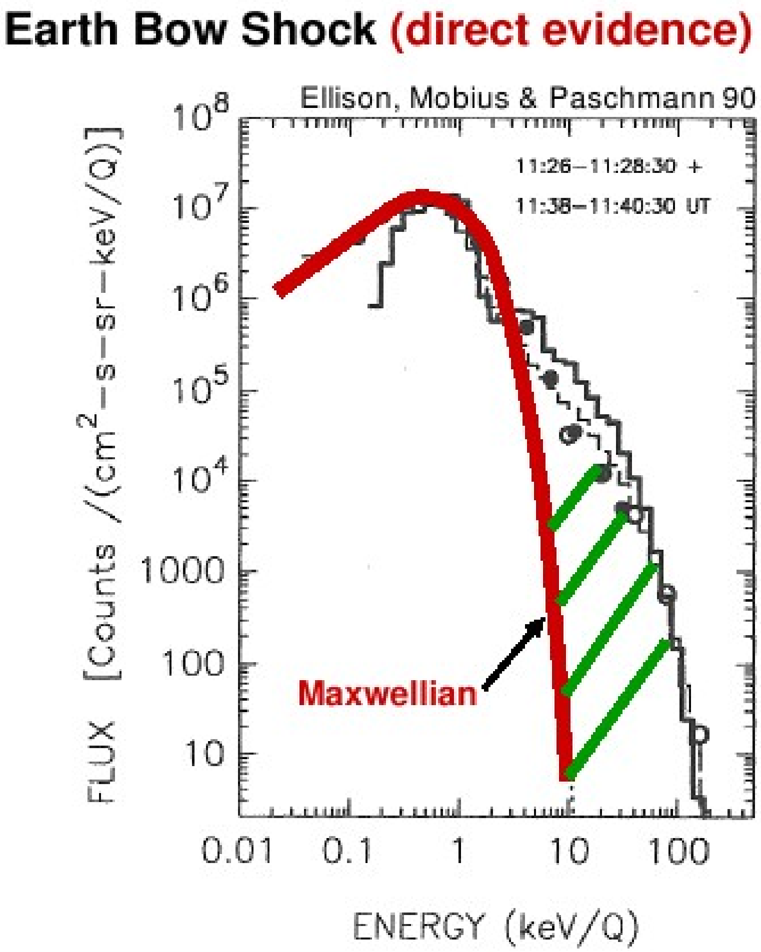


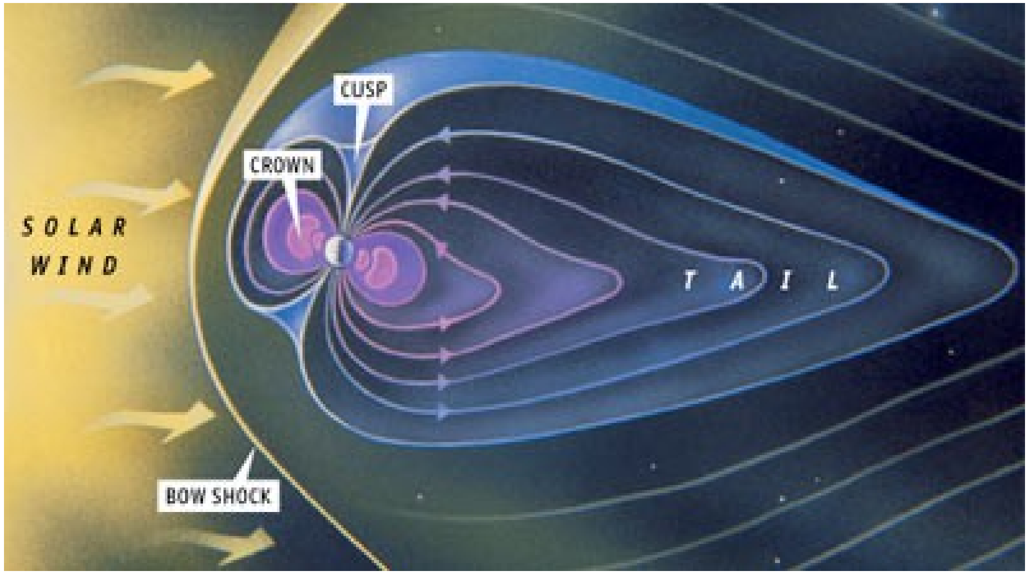
Trajectories of test-particles impinging at random times on a periodically reforming shock with $M = 10$ and $\theta = 45°$



Post-shock ion spectrum for a parallel shock with M = 20. The minimal model perfectly matches the spectrum obtained in simulations

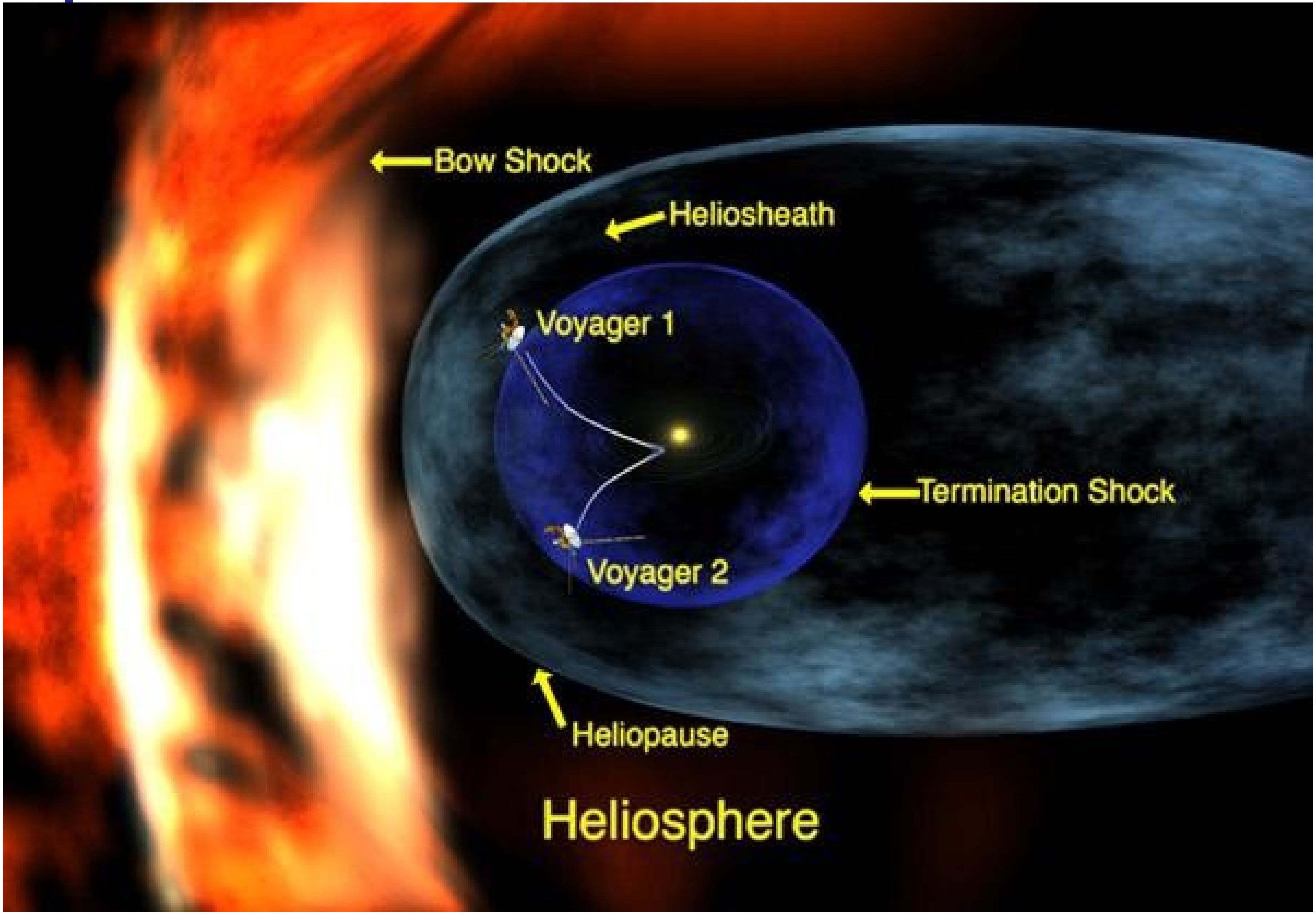
Evidence for efficient shock acceleration in Earth bow shock



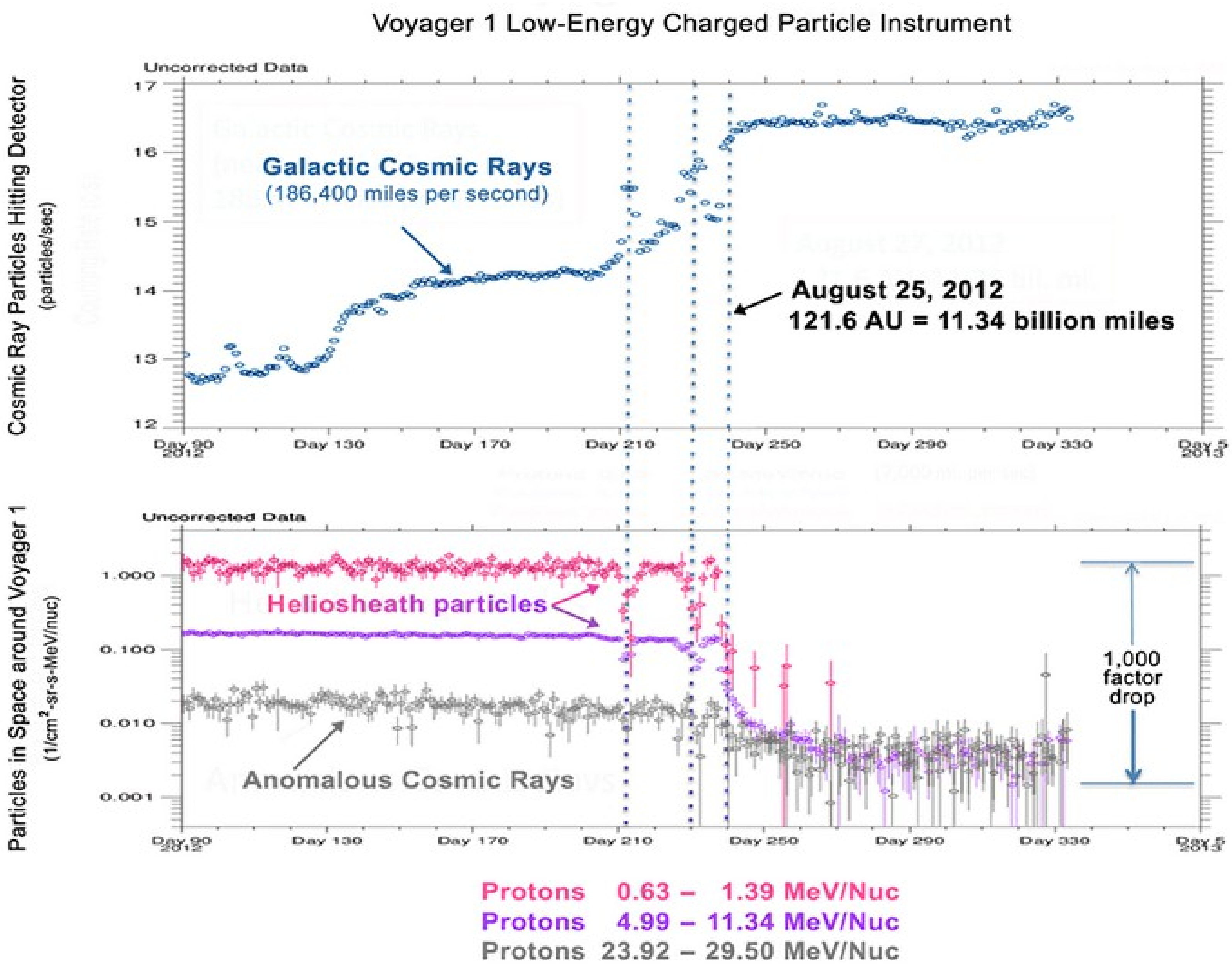


>25% of energy flux in superthermal ions

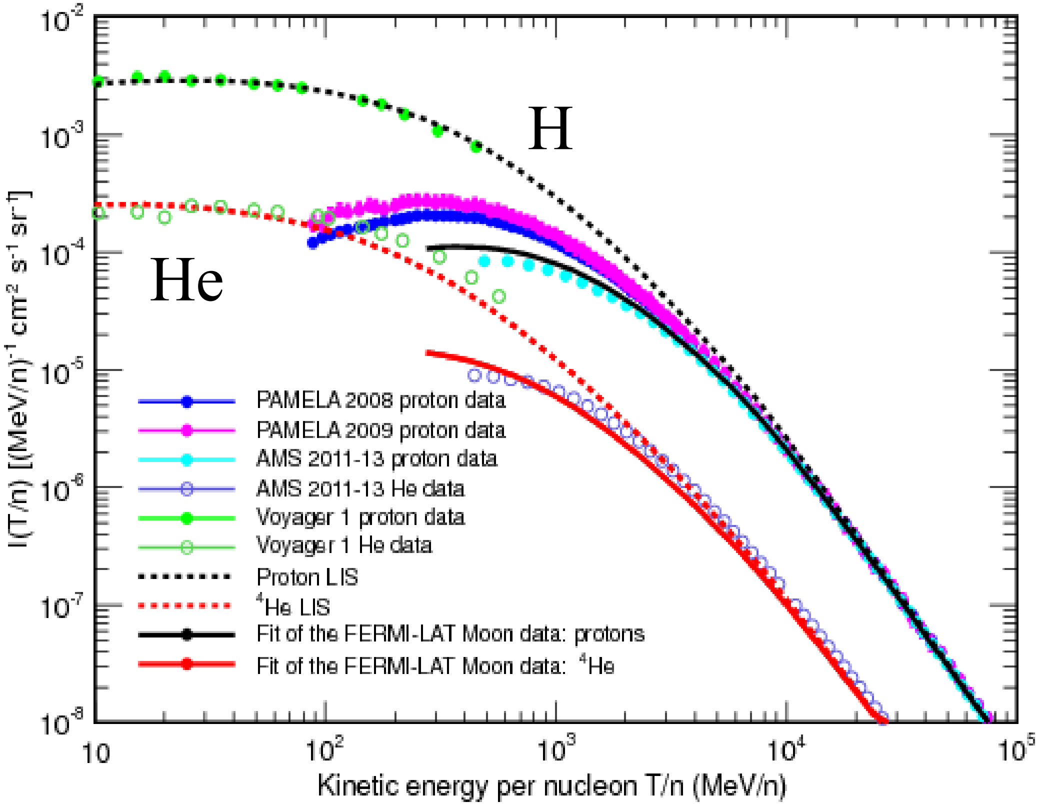
Solar wind termination shock



The CR spectrum seen by Voyager



The CR spectrum seen by Voyager



Comparison between CR proton and He spectra observed locally (PAMELA and AMS) and the spectrum observed by Voyager in the Heliopause (shocked ISM).

The difference below ~ 20 GeV/n is due to the solar modulation.

NON-THERMAL EMISSION

EM radiation from accelerated particles

Radiative processes relevant for Galactic CR physics:

- **Leptons**
  - Synchrotron emission $e^{\pm} + B \rightarrow e^{\pm} + \gamma$
  - Bremsstrahlung $e^{\pm} + Nucl. \rightarrow e^{\pm} + \gamma$
  - Inverse Compton $e^{\pm} + \gamma_{bg} \rightarrow e^{\pm} + \gamma$

- **Hadrons**
  - Pion production: $p_{CR} + p_{gas} \rightarrow p_{CR} + p_{gas} + \begin{cases} \pi^{0} \rightarrow \gamma\,\gamma \\ \pi^{\pm} \rightarrow \mu^{\pm}\,\nu_{\mu}\left(\bar{\nu}_{\mu}\right) \end{cases}$

$$\hookrightarrow e^{\pm} + \nu_{e}\left(\bar{\nu}_{e}\right)$$

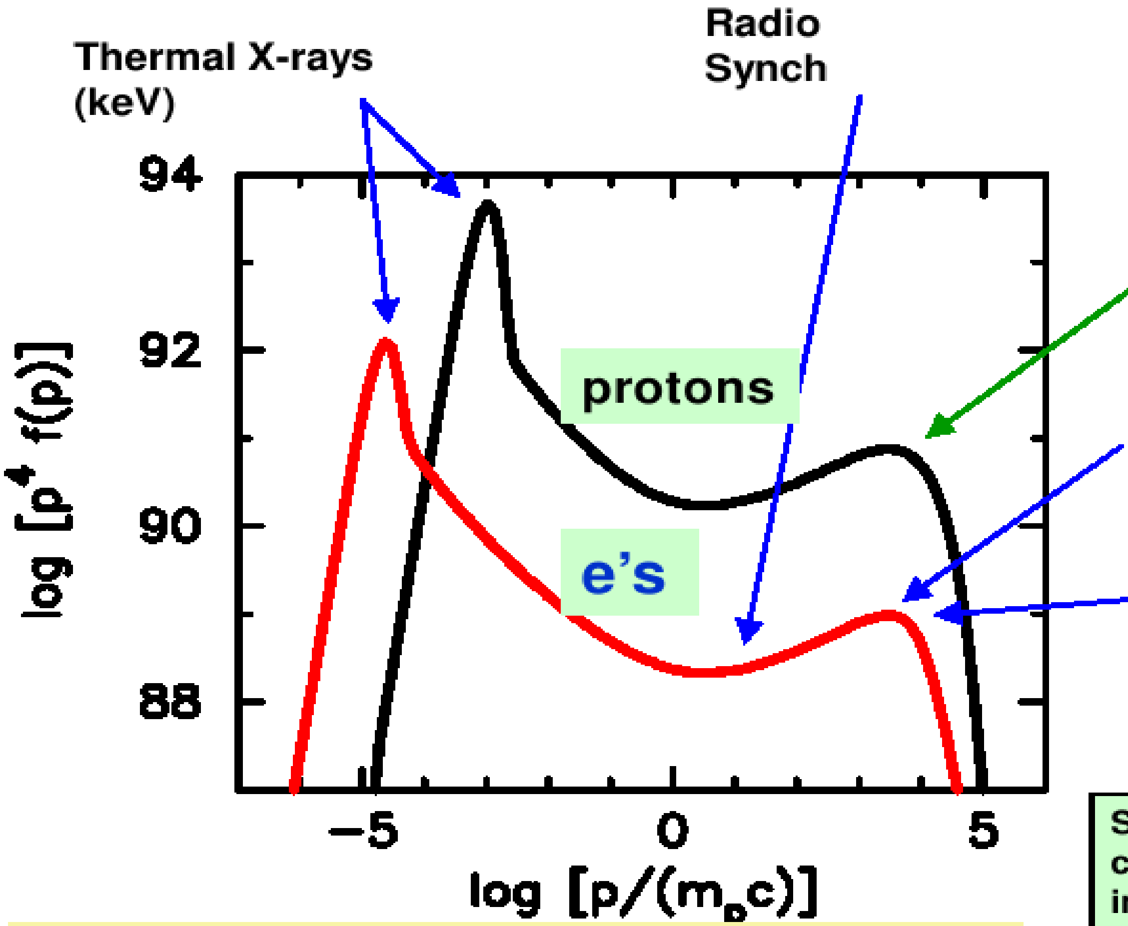
See:

- Rybicki & Lightman (1985)

- G. Ghisellini “Radiative process in Astrophysics” (2012)

EM radiation from accelerated particles

Electron and Proton distributions from efficient (nonlinear) diffusive shock acceleration



Thermal X-rays (keV)

Radio Synch

Pion-decay (GeV-TeV) only emission from protons

compete at GeV-TeV energies

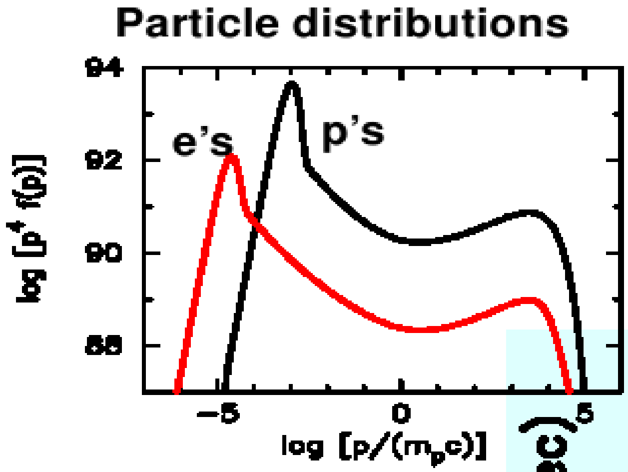
X-ray Synch (keV X-rays)

Inverse Compton (GeV-TeV γ-rays) from electrons

Spectra calculated with semi-analytic model of Blasi, Gabici & Vannoni 2005

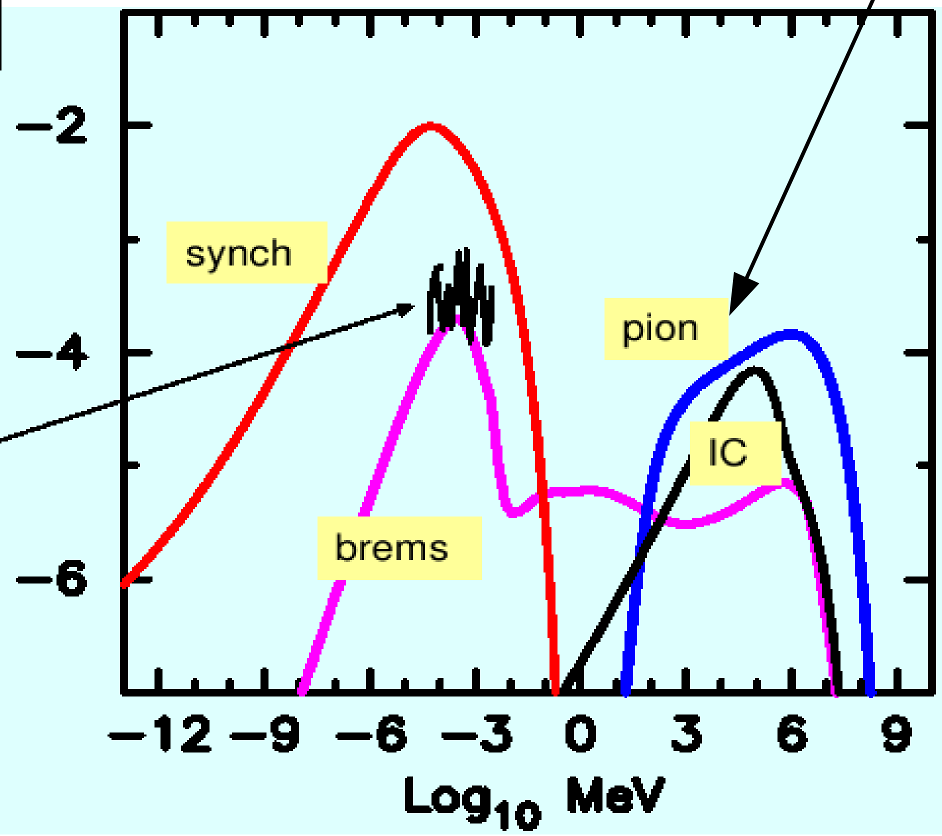
Several free parameters required to characterize particle spectra, including B-field, e/p ratio, diffusion coefficient

EM radiation from accelerated particles



continuum emission

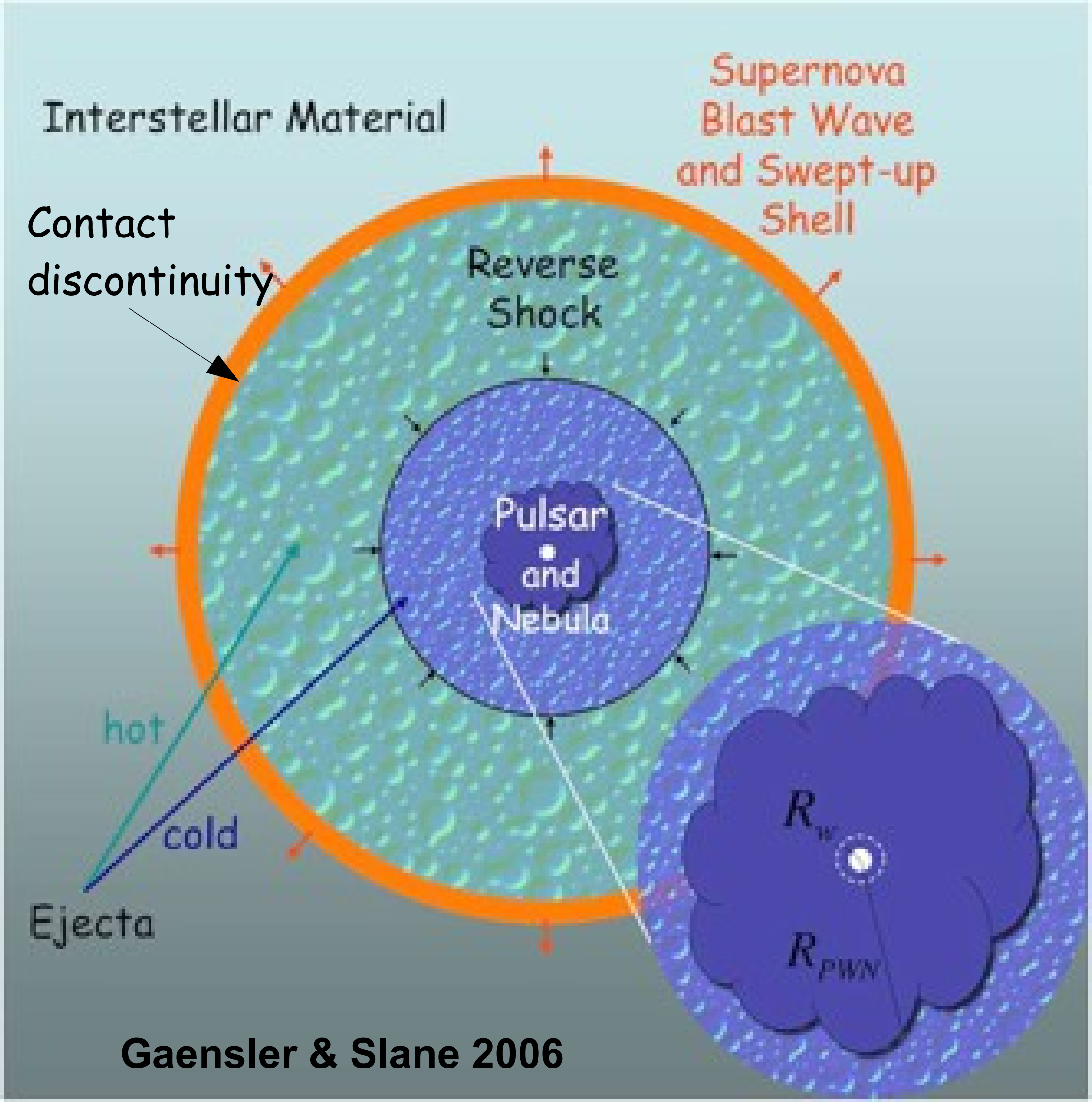
Pion decay and IC are competitive mechanisms



Hadronic models	**Leptonic models**
↓	↓
Large B >~ 100 μG	**Low B** ~ 10 μG

APPLICATION TO ISOLATED SNRs

SNR structure



Gaensler & Slane 2006

SNR structure

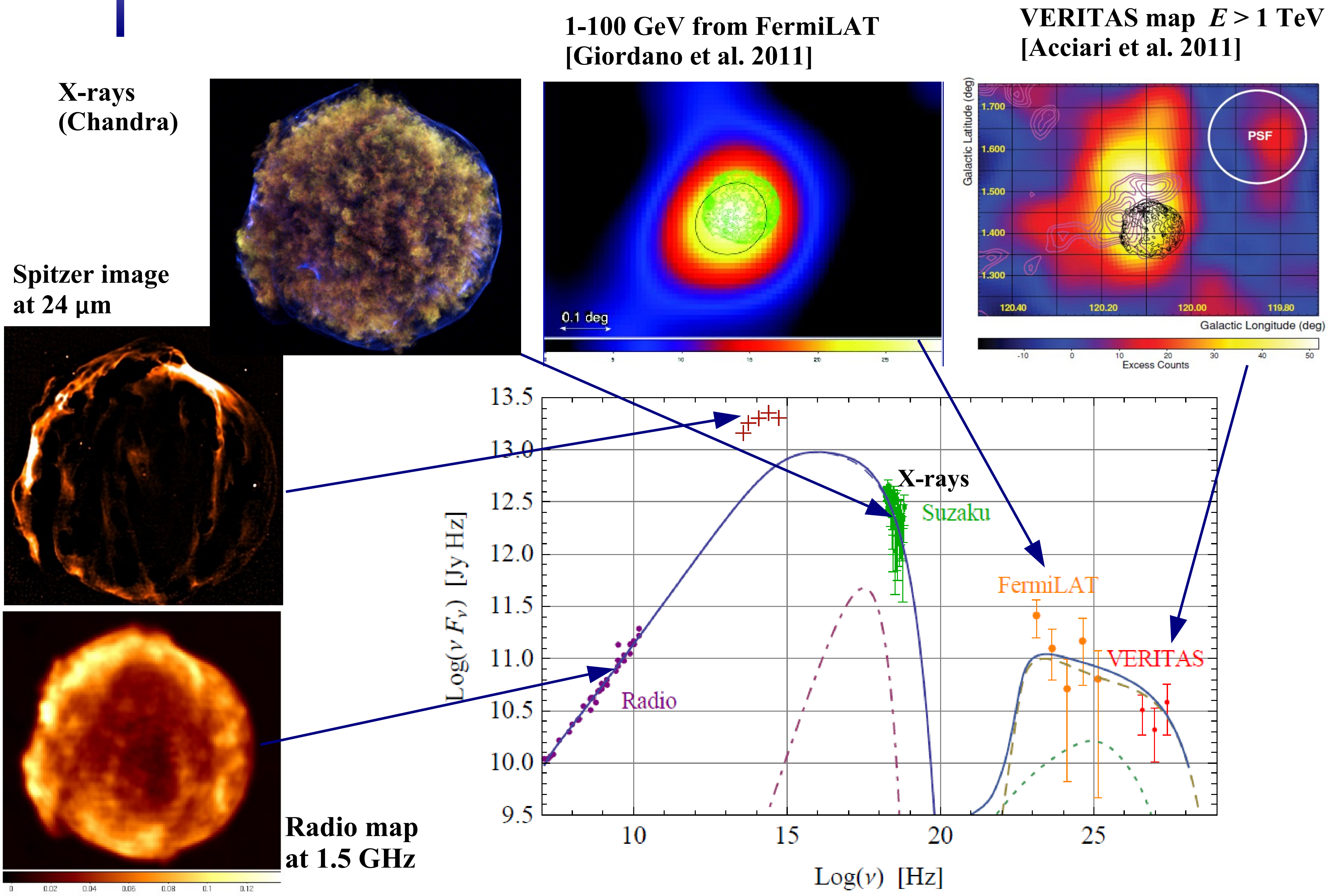
- ISM
- Forward shock
- Shocked ISM
- Contact discontinuity
- Shocked ejecta
- Reverse shock
- Unshocked ejecta

For core-collapse SNR

- PWN
- Termination shock
- Pulsar wind
- Pulsar

WHERE NON-THERMAL PARTICLE ARE PRODUCED?

Tycho's SNR (Type Ia SNR)



X-rays (Chandra)

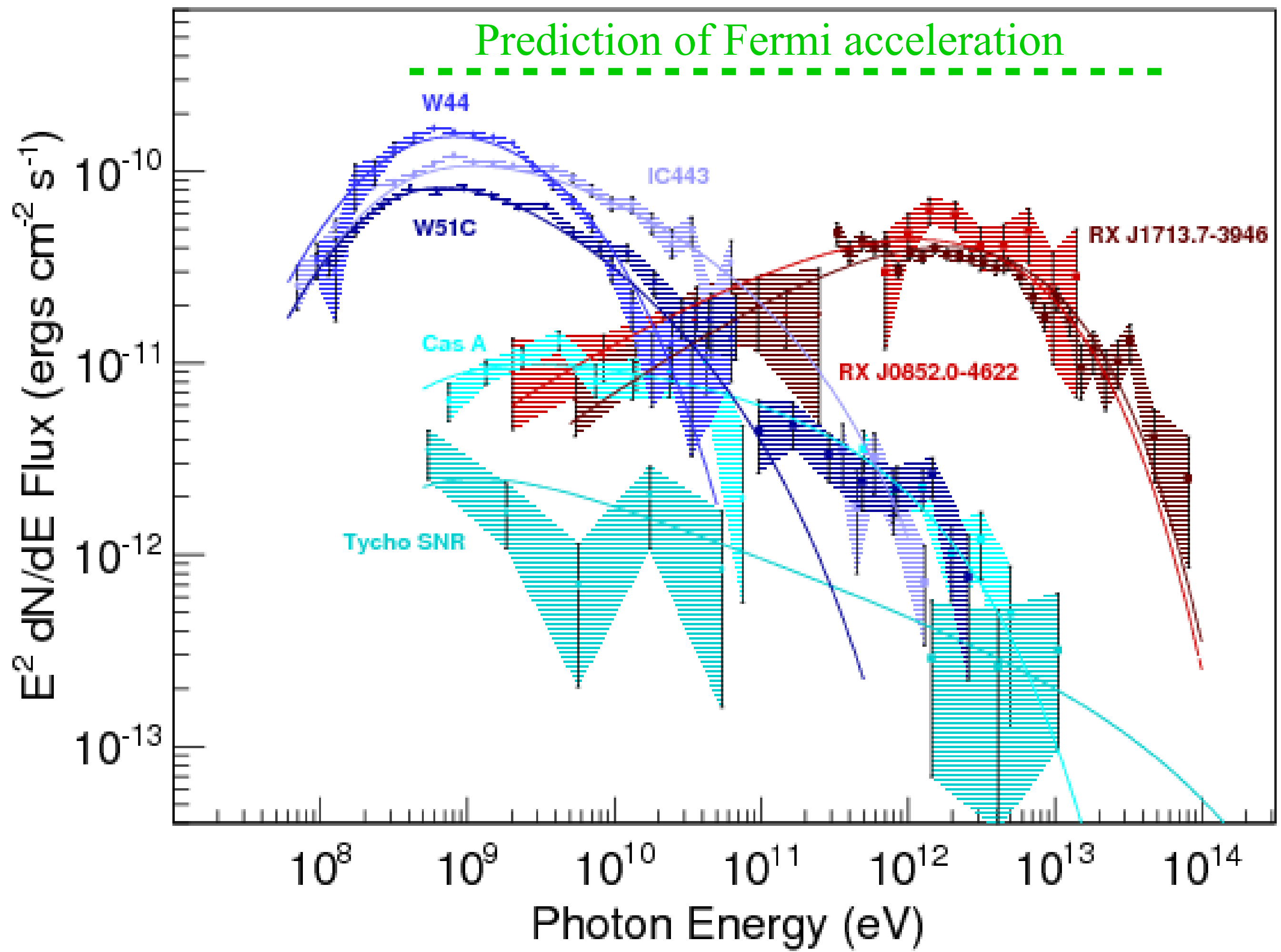
Spitzer image at 24 μm

Radio map at 1.5 GHz

1-100 GeV from FermiLAT [Giordano et al. 2011]

VERITAS map $E > 1$ TeV [Acciari et al. 2011]

Gamma-rays from SNRs



[S. Funk, Ann.Rev.Nucl.Part.Sci. 65 (2015)]

NOTE: this general trend could be an artifact of the enviromental conditions

Middle-aged SNRs (~20.000 yrs)

- hadronic emission
- steep spectra ~E^{-3}
- E_{max} < 1 TeV

Young SNRs (~2000 yr)

- Hadronic/leptonic?
- Hard spectra
- E_{max} ~ 10-100 TeV

Very young SNRs (~300 yr)

- hadronic
- steep spectra ~$E^{-2.3}$
- E_{max} ~ 10-100 TeV

Not enough to explain the Knee at ~ PeV

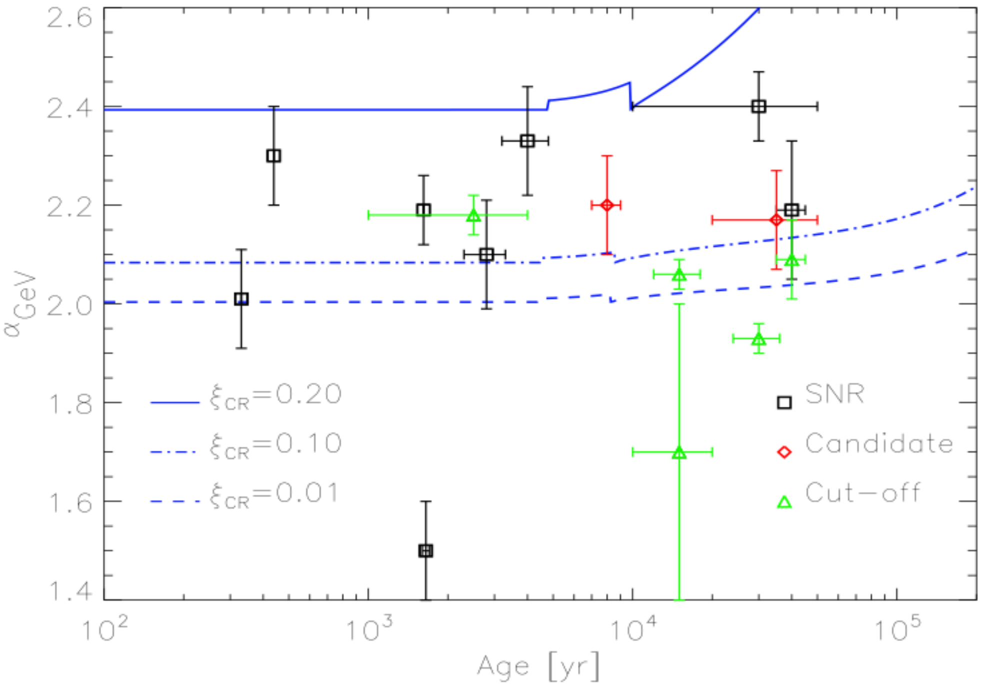
Slope of gamma-ray emission of SNRs

In many observed SNRs the slope is steeper than E^{-2}
→ difficult to explain theoretically

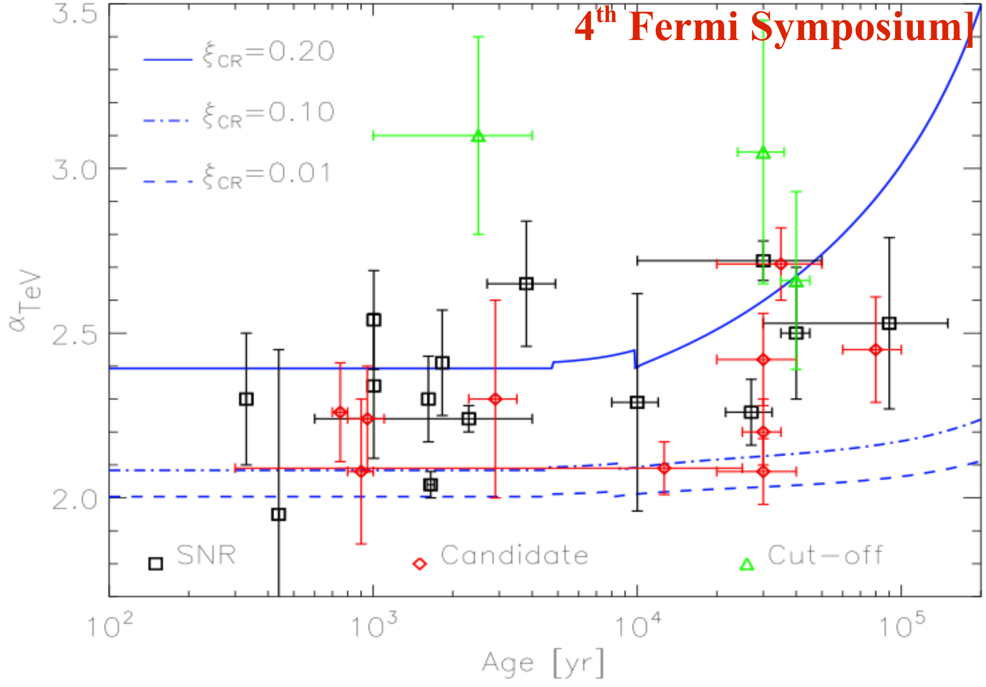
If the γ-ray spectrum is hadronic ($\pi^0 \rightarrow \gamma\gamma$) the slope is the same as the proton spectrum

If the γ-ray spectrum is leptonic (IC) the spectrum is harder

$$f_e(E) \propto E^{-s} \rightarrow \phi_\gamma \propto E^{-(s-1)/2}$$



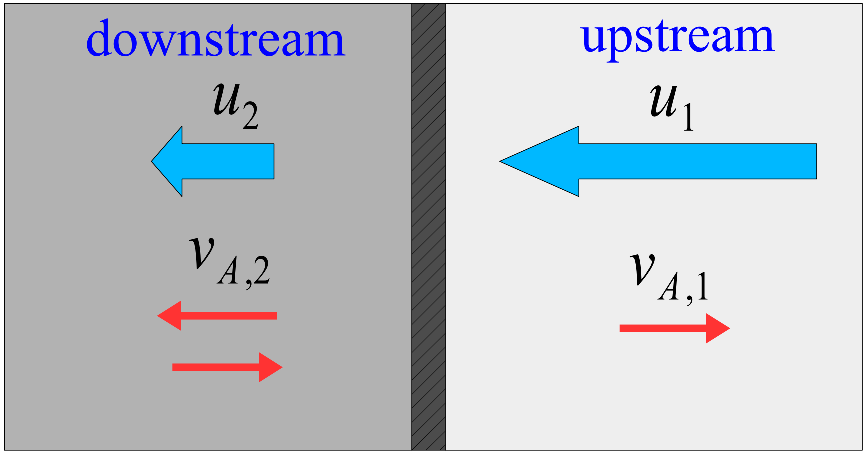
[plot from D. Caprioli (2012) 4th Fermi Symposium]



The role of scattering centers in presence of strong magnetic amplification

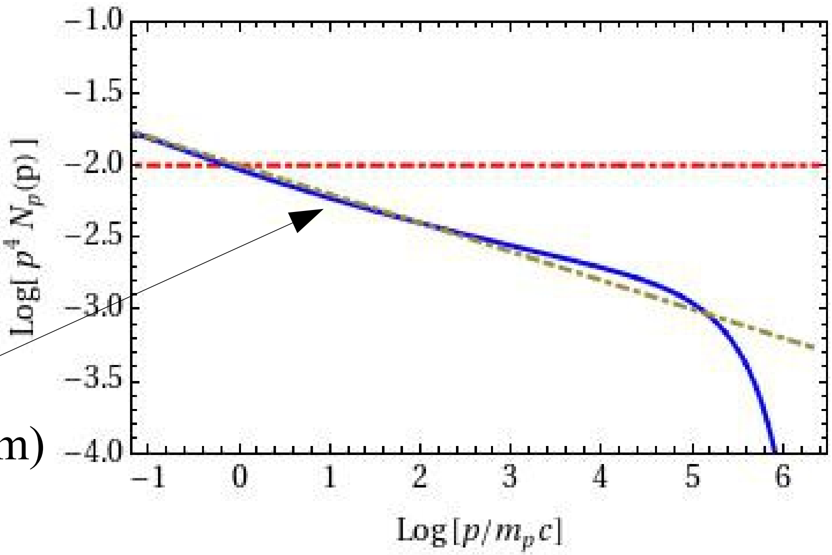
When the magnetic field is strongly amplified the Alfvén speed can become a non negligible fraction of the shock speed. In this case the effective compression ratio is:

$$r = \frac{u_1 - v_{A,1}}{u_2 \pm v_{A,2}} \simeq \begin{cases} \dfrac{u_1 - v_{A,1}}{u_2} \\ \dfrac{u_1}{u_2 + v_{A,2}} \end{cases}$$



If we consider only the modification upstream, $v_{A,2} \approx 0$, in the case of Tycho:

$$v_{A,1} = \frac{B_1}{\sqrt{4\pi\rho_1}} \approx 0.15\, V_{sh} \rightarrow s = \frac{r+2}{r-1} \simeq 2.2$$



(4.2 in momentum)

Modelling the multi-wavelength spectrum of Tycho

[G.M. & D. Caprioli, 2012]

Simultaneous fit of multi-wavelength spectrum with non-linear DSA model

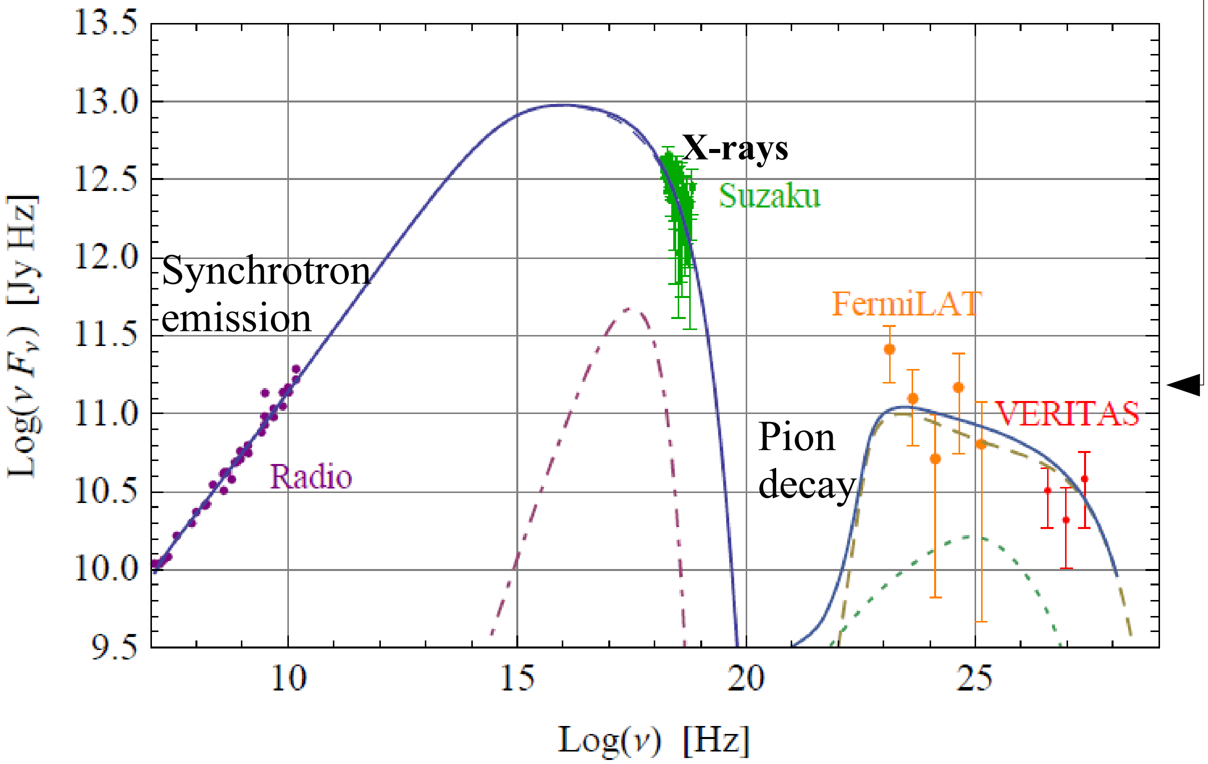
1) Maximum energy of ions $E_{max} = 470\,TeV$

2) Non-thermal spectrum $N(E) \propto E^{-2.3}$

3) Amplified magnetic field $\delta B_2 \approx 300\,\mu G$

4) TOTAL CRs ENERGY $\boldsymbol{\epsilon_{CR} = 12\%\, E_{SN}}$

Results for the Tycho's remnant



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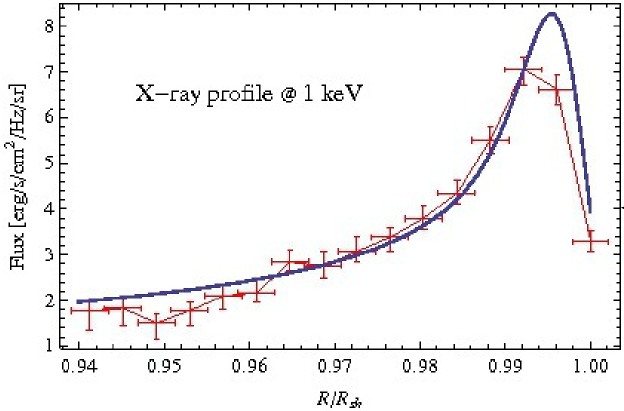
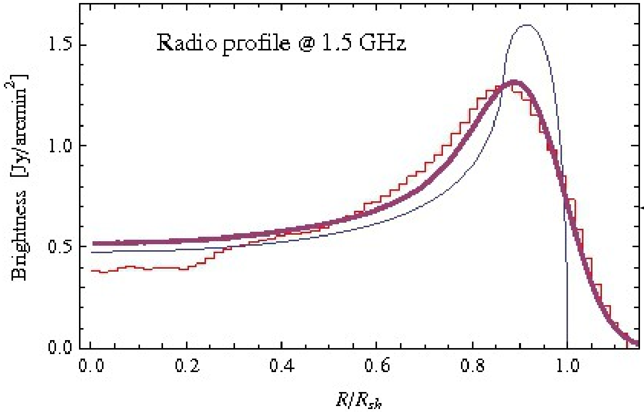
4) TOTAL CRs ENERGY $\epsilon_{CR} = 12\%\ E_{SN}$





Results for the Tycho's remnant