COSMIC RAY PHYSICS: DIFFUSION AND ACCELERATION

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LECTURE I

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OUTLINE

- **The SNR-CR connection**
  - ***Why SNRs?***
  - ***Propagation in the Galaxy***

- **Diffusive motion**
  - ***Motion of particles in a perturbed magnetic field***

- **From diffusion to energy gain**
  - ***Second order Fermi acceleration***

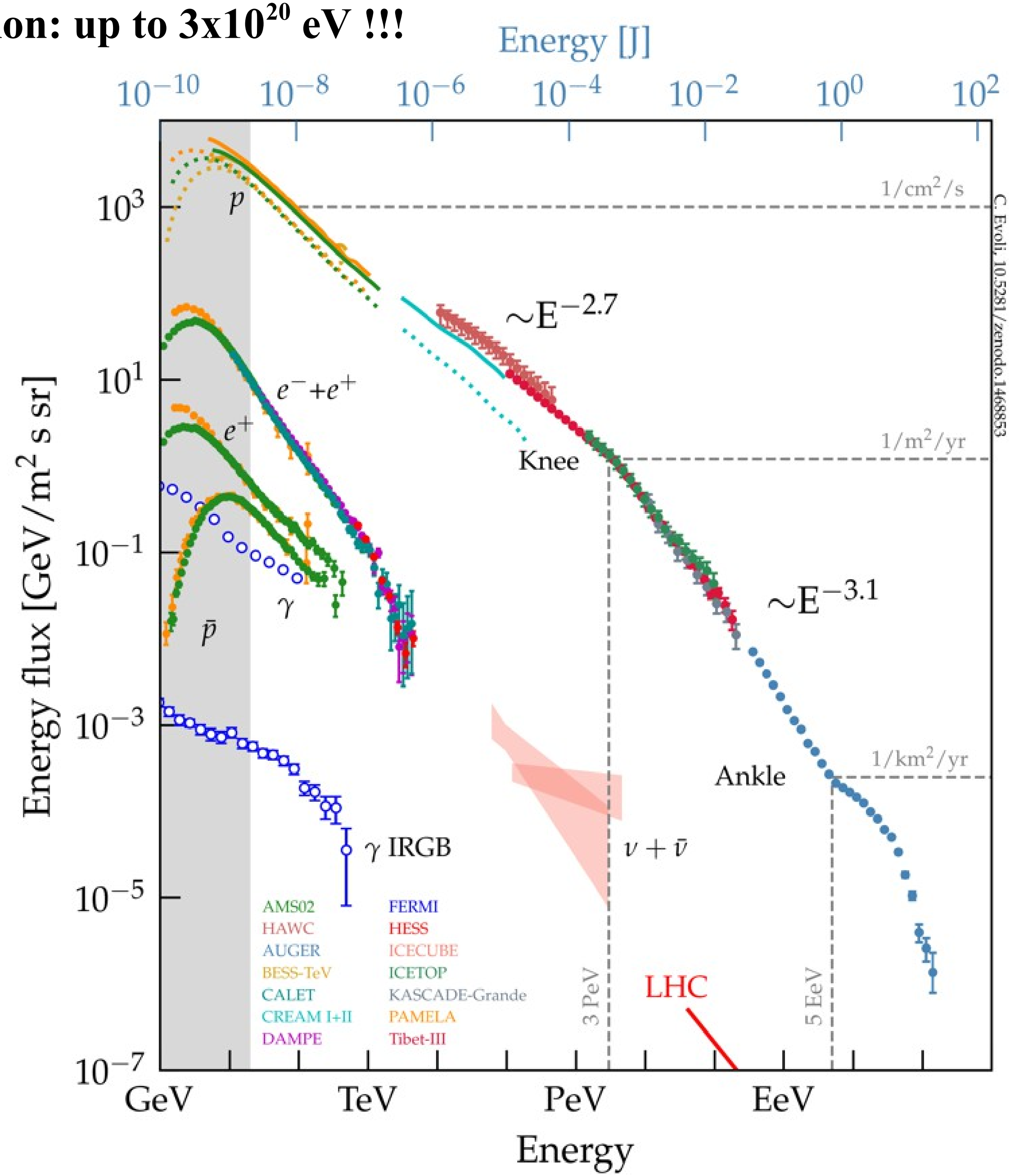
THE SNR-CR CONNECTION

Quick view on CR spectrum

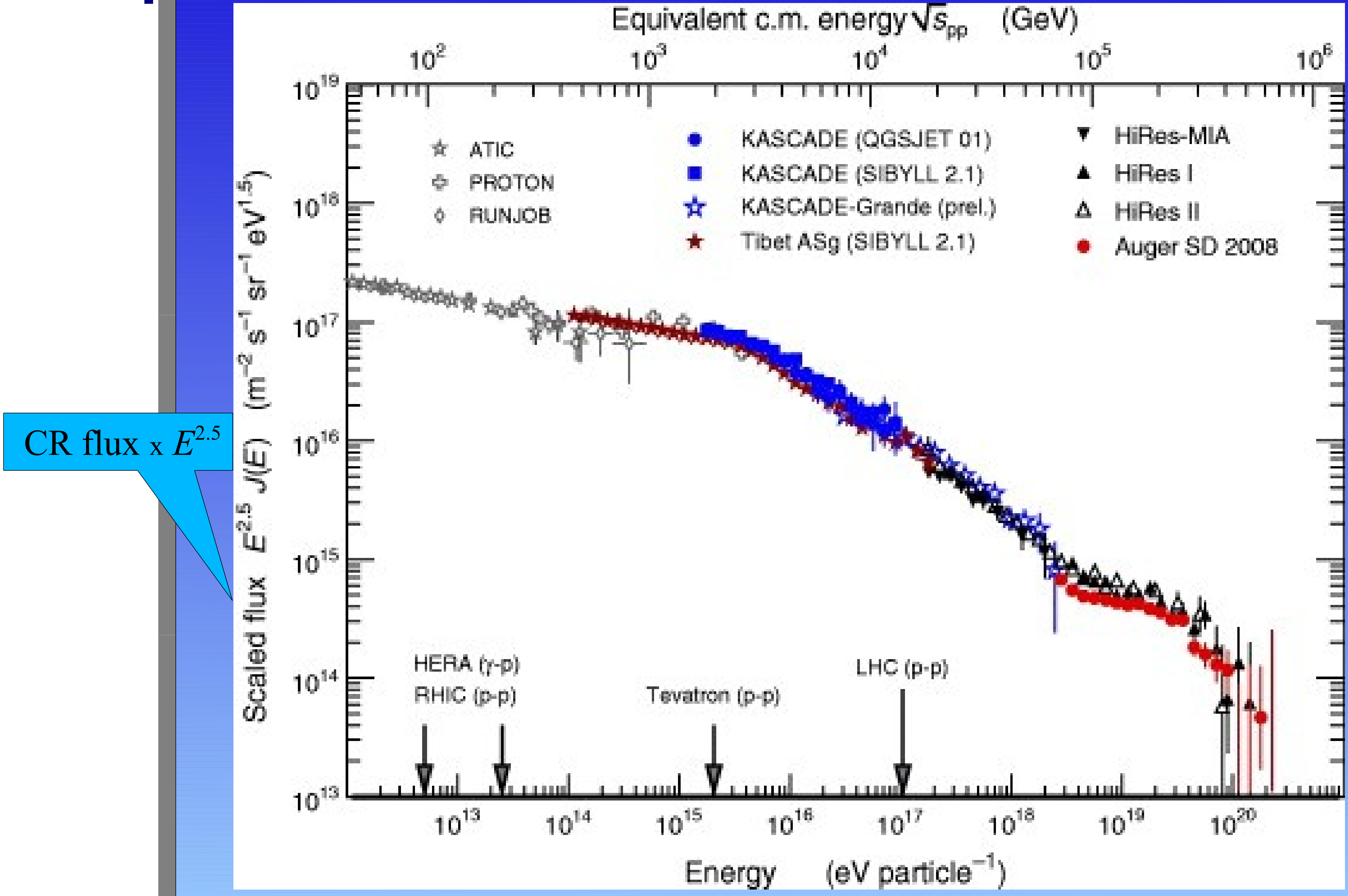
Incredible energy extension: up to $3x10^{20}$ eV !!!

CRs energy density compared with other components:

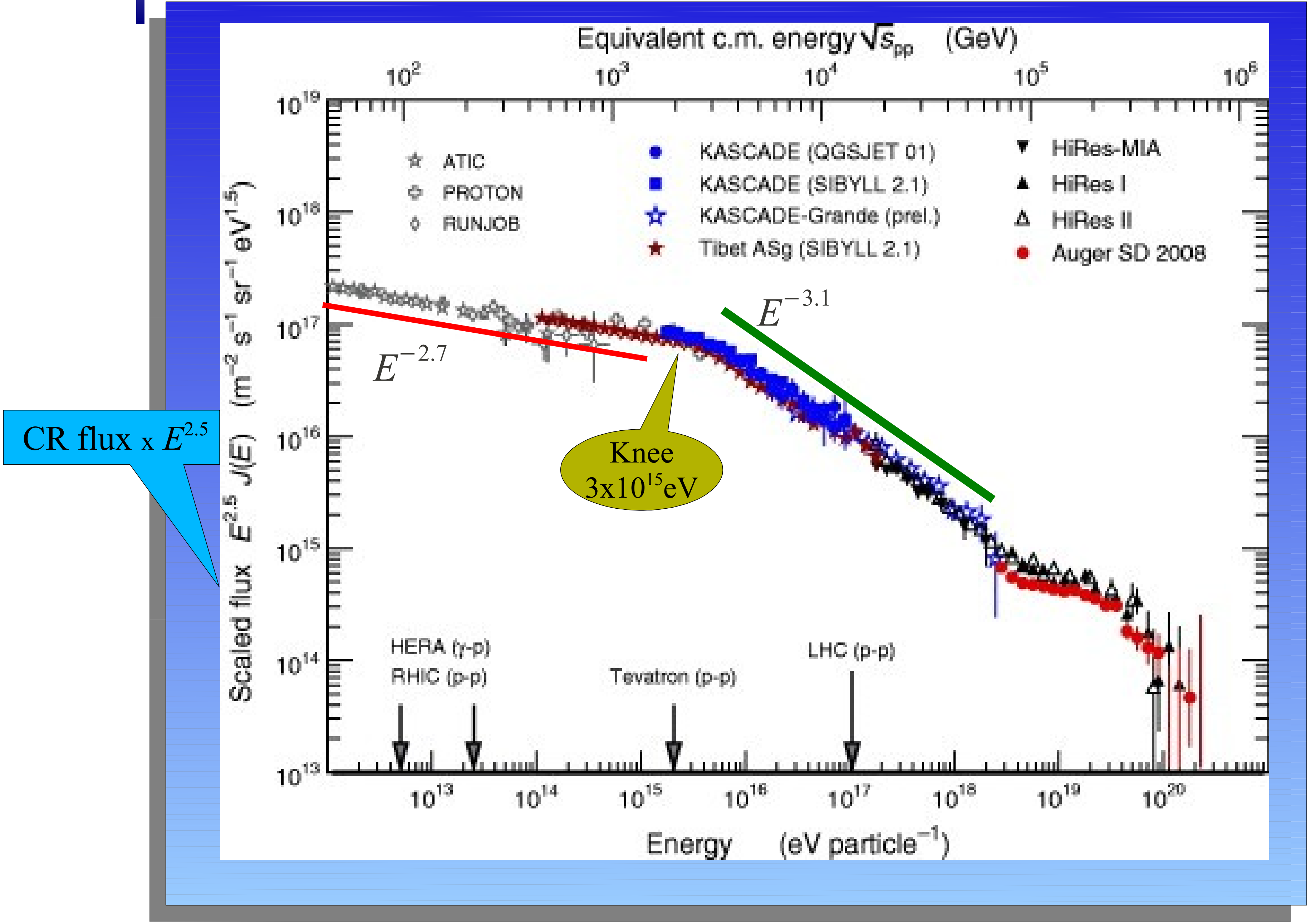
ENERGY DENSITY IN THE GALAXY	eV/cm^3
Magnetic field ($B^2/8\pi$)	~ 0.5
Gas motion ($Mv^2/2$)	~ 0.5
Starlight	~ 0.5
CMB (2.7 K)	~ 0.5
CRs	~ 0.5



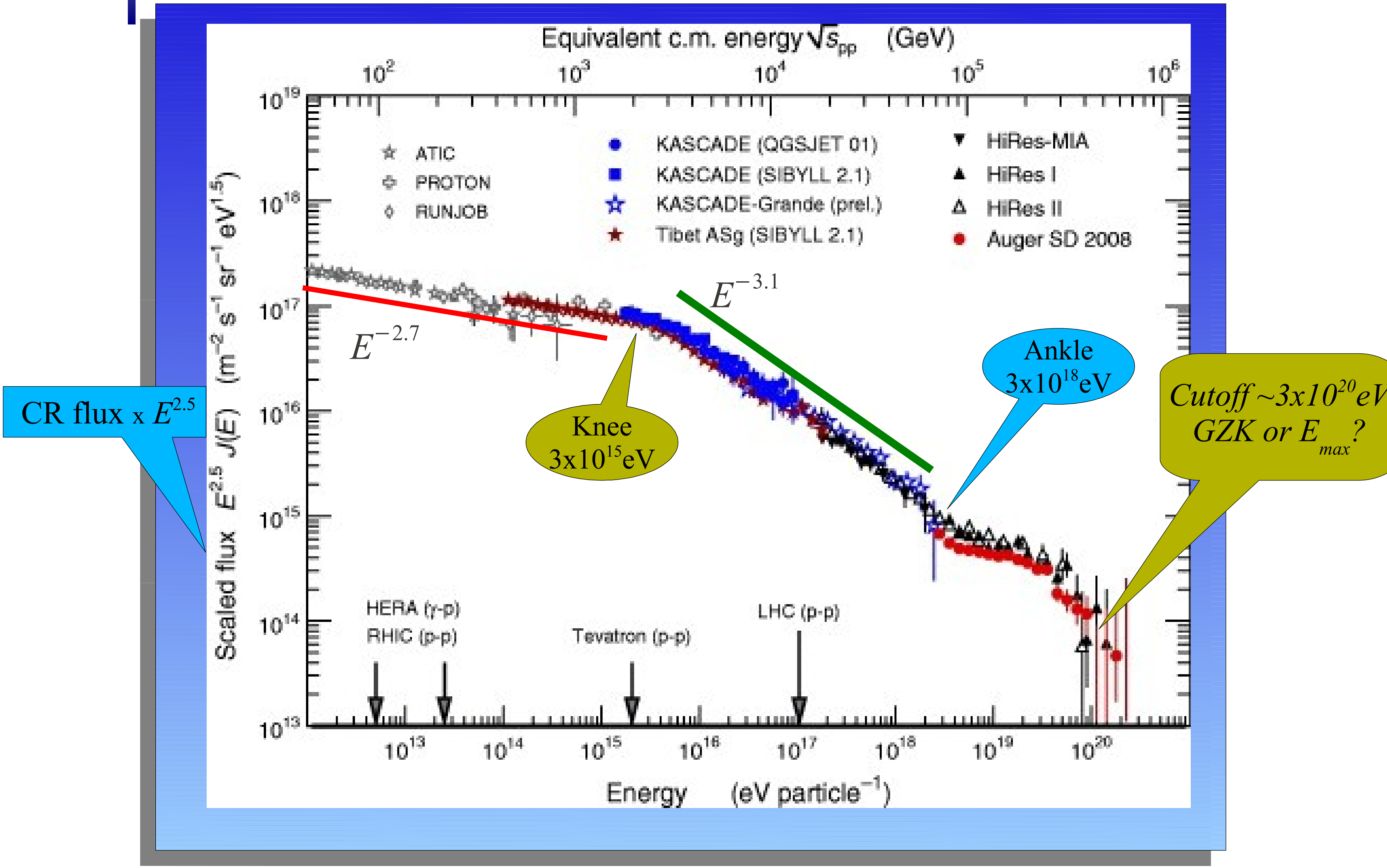
Quick view on CR spectrum



Quick view on CR spectrum



Quick view on CR spectrum



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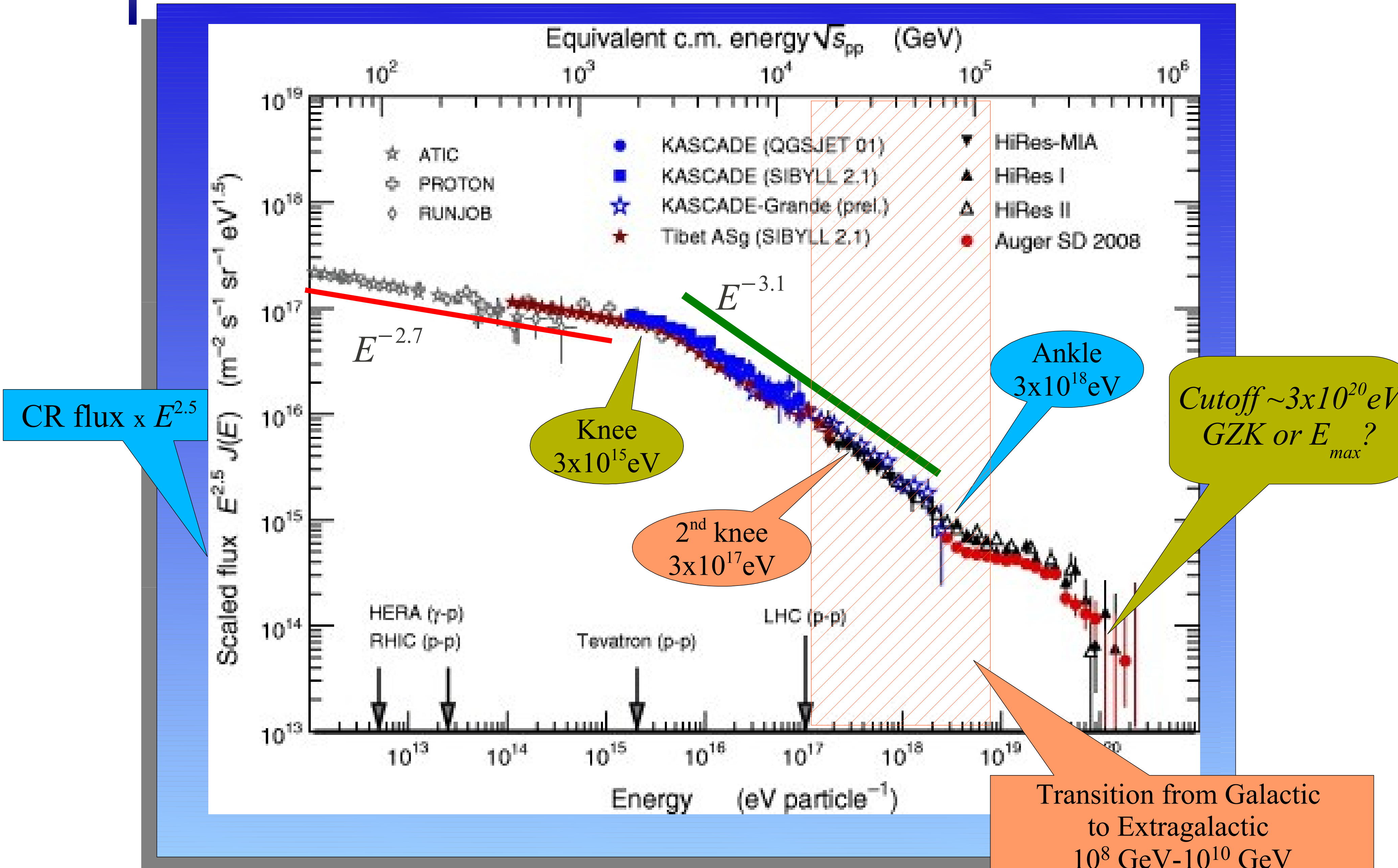
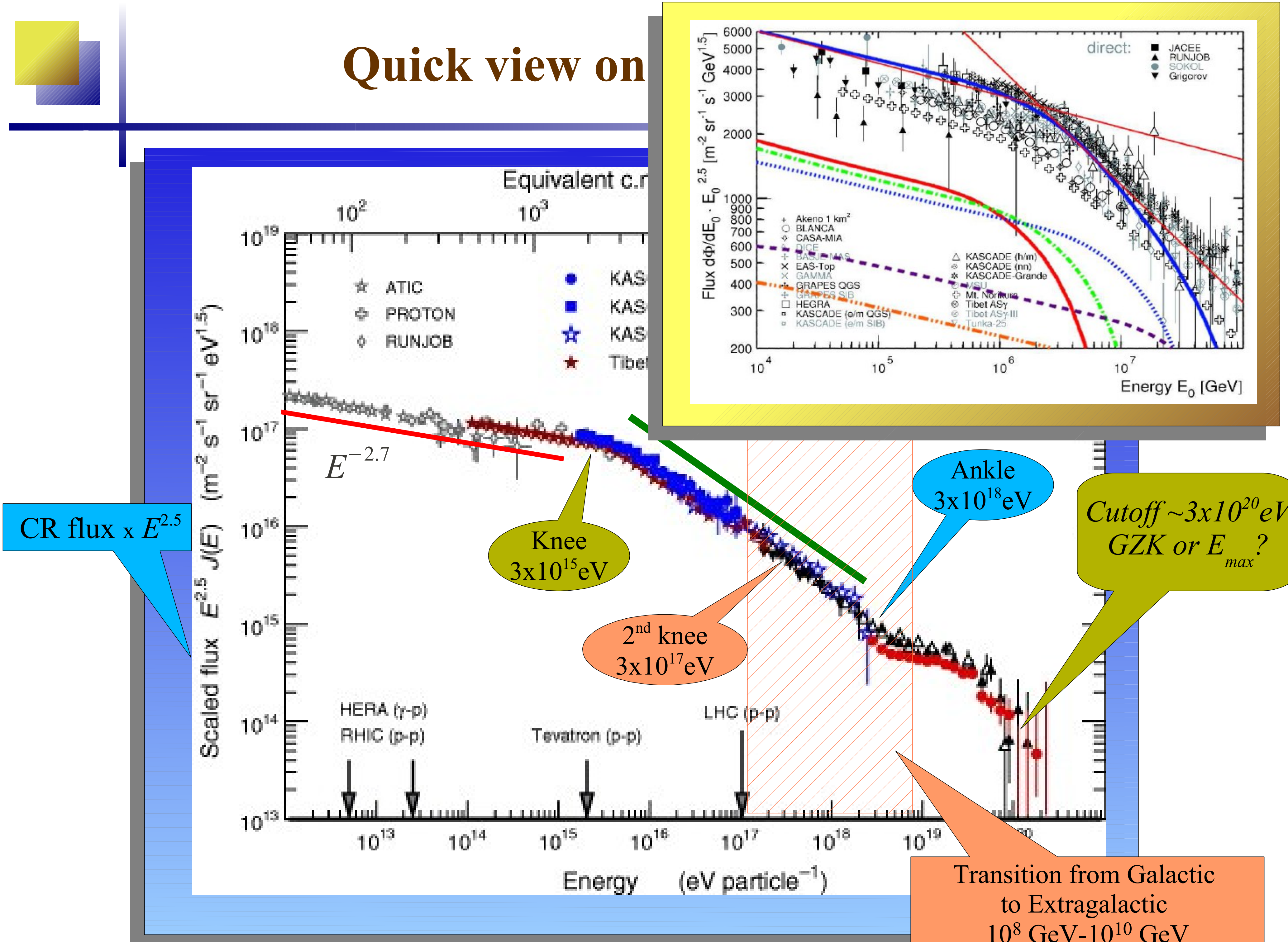
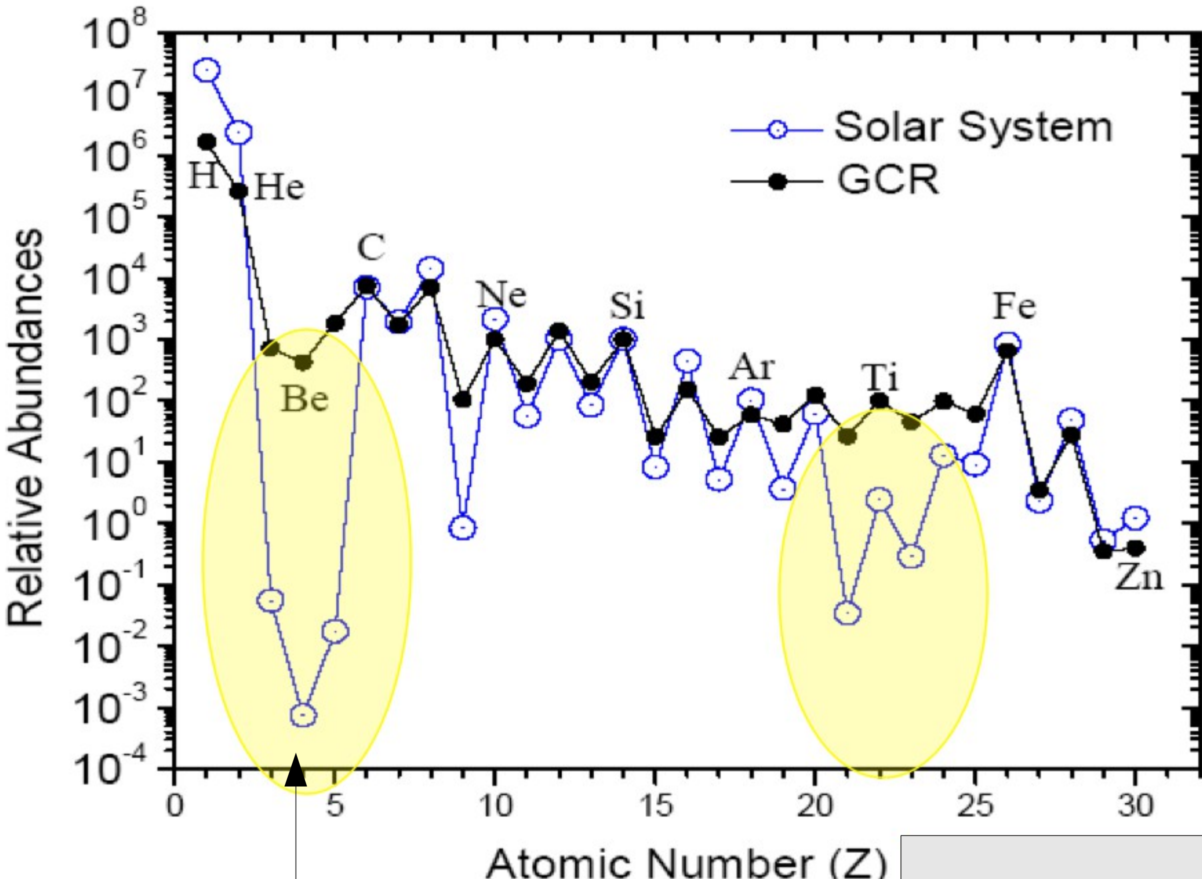


Quick view on



The chemical composition of CRs



LiBeB

The relative abundances of some elements is larger than in the Solar composition

Those "secondary" elements are produced by primary CRs by spallation

→ **Primary CRs should propagate in the Galaxy for a time comparable with the interaction time**

$$\tau_{interaction} \approx \frac{1}{n_{gas}\, c\, \sigma_{spal}} \approx few\ Myr$$

$$\sigma_{sp} \simeq 45\, A^{0.7}\ \ mbarn$$

Propagation time of CRs

Assuming that cosmic rays propagate simply gyrating along magnetic field lines, than:

$$\tau_{DISC} = \frac{300\ \mathrm{pc}}{(1/3)c} \approx 3000\ \mathrm{years}$$

Propagation time in the vertical direction of the disk

$$\tau_{GAL} = \frac{15\ \mathrm{kpc}}{(1/3)c} \approx 150{,}000\ \mathrm{years}$$

Propagation time in the Galactic plane

$$\tau_{HALO} = \frac{3\ \mathrm{kpc}}{(1/3)c} \approx 30{,}000\ \mathrm{years}$$

Propagation time in the Galactic magnetic Halo

$$\ll \tau_{interaction}$$

All these time scales are extremely short compared with the residence time

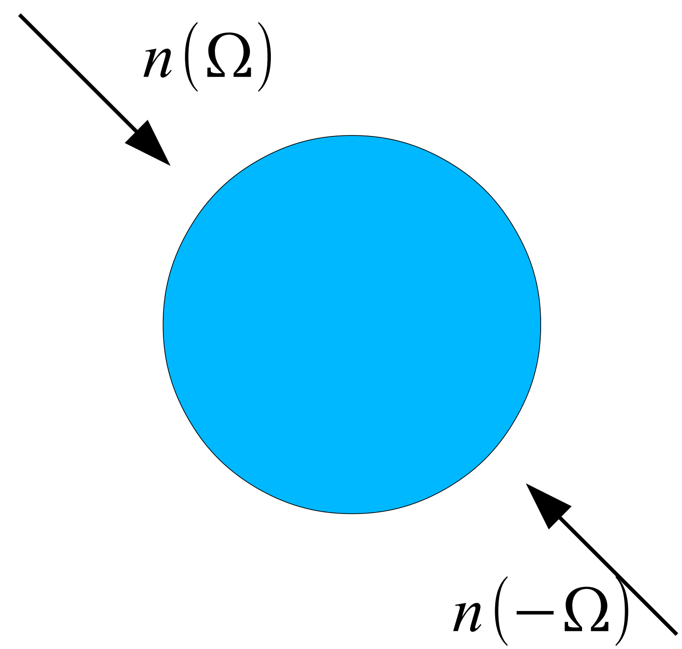
→ CRs have to diffuse in the Galaxy

Anisotropy

The second argument supporting the diffusion scenario is the anisotropy of arrival direction of CR to the Earth

→ The location of sources is lost and cannot be identified measuring the arrival directions

In fact the anisotropy is very small



OPERATIVE DEFINITION

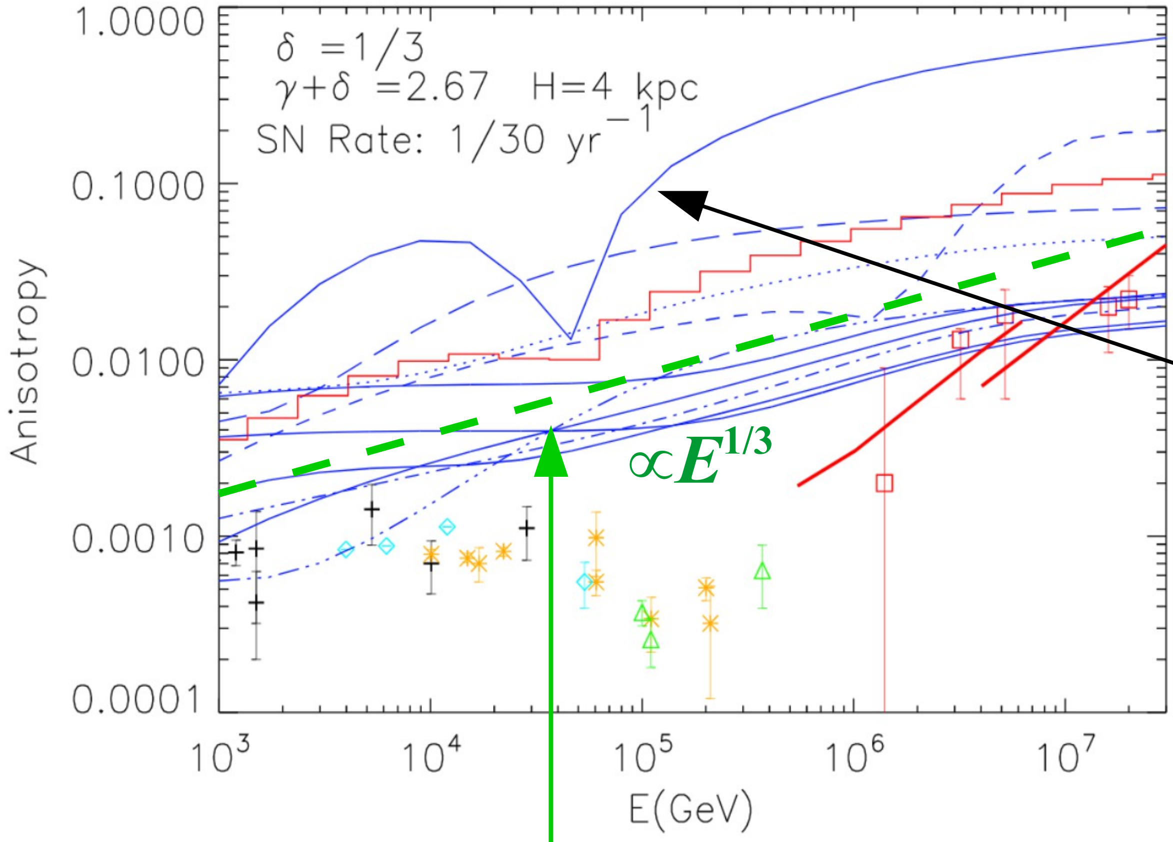
$$\delta \stackrel{\text{def}}{=} \frac{n(\Omega)-n(-\Omega)}{n(\Omega)+n(-\Omega)} \simeq 10^{-3}$$

THEORETICAL DEFINITION

$$\delta \stackrel{\text{def}}{=} \frac{\textit{diffusive flux}}{\textit{ballistic flux}} = \frac{D\nabla n}{c/3\,n}$$

Anisotropy

[Amato & Blasi, 2012]



Anisotropy produced by different realization of randomly distributed sources in the Galaxy

Single local sources can produce bumps in the anisotropy but gives higher amplitude

$$\delta = \frac{3D}{c}\frac{\nabla n}{n}$$

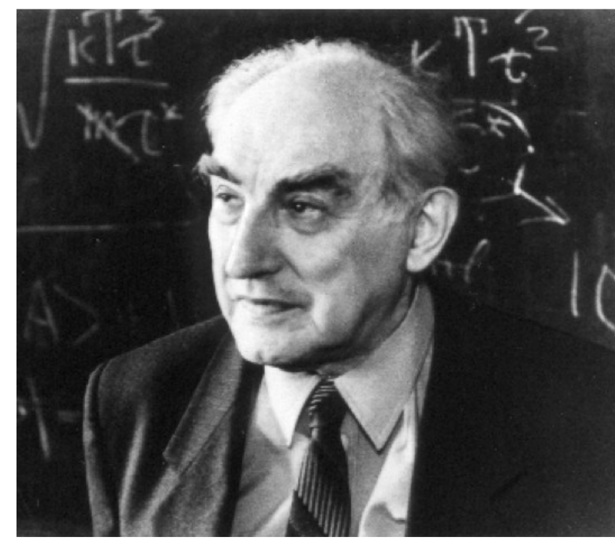
Local D assumed to be equal to the average Galactic D obtained from B/C (see later)

$$D(E) = 3\times 10^{28}\left(\frac{E}{GeV}\right)^{1/3}$$

Origin of Galactic CRs

Zwicky & Baade were the first to mention SNRs as sources of CRs (1934) but arguing against them because CRs where thought to be extragalactic

Vitali Lazarevich Ginzburg made the argument for SNRs as sources of galactic CR in the 60's in a more quantitative form.



The SNR hypothesis

Why supernova remnant are so popular?

1) Enough power to sustain the CR flux (~10% of kinetic energy)

$$W_{CR} \sim \frac{U_{CR} V_{CR}}{\tau_{res}} \approx 10^{40} \frac{erg}{s}$$

$$W_{SN} \sim R_{SN} \; E_{SN} \approx 3 \cdot 10^{41} \frac{erg}{s} \quad \Rightarrow \quad \frac{W_{CR}}{W_{SN}} \approx 0.03 \div 0.3$$

2) Spatial distribution of SNRs compatible with CR distribution (inferred from diffuse gamma-ray emission)

3) Enough sources to explain anisotropy [SN rate ~ (1-3)/100 yr]

4) Observations show the presence of non thermal particles

5) A well developed theory for particle acceleration (DSA) predicting a power law spectrum

Short hints on MHD

Basic equations of MHD

Fluid equations

Mass conservation

$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \vec{v}) = 0$$

Momentum conservation

$$\frac{\partial \vec{v}}{\partial t} + \vec{v} \cdot \nabla \vec{v} = -\frac{\nabla P}{\rho} + \frac{\vec{j} \times \vec{B}}{\rho c}$$

Entropy variation

$$T \frac{D s}{D t} = \frac{j^2}{\rho \sigma}$$

Maxwell equations

Electric field

$$\nabla \cdot \vec{E} = 4 \pi \rho = 0 ; \qquad \nabla \times \vec{E} = -\frac{1}{c} \frac{\partial \vec{B}}{\partial t}$$

Magnetic field

$$\nabla \cdot \vec{B} = 0 ; \qquad \nabla \times \vec{B} = -\frac{4 \pi}{c} \vec{j}$$

Ohm's law

$$\vec{j} = \sigma \vec{E}\,' = \sigma \left(E + \frac{1}{c} \vec{v} \times \vec{B} \right)$$

Lab frame ($\vec{E}\,'$)

Fluid frame ($E + \frac{1}{c} \vec{v} \times \vec{B}$)

Limit of ideal MHD

$$\frac{\partial \vec{B}}{\partial t} = -c\,\nabla\times\vec{E} = -c\,\nabla\times\left[\frac{\vec{j}}{\sigma} - \frac{\vec{v}\times\vec{B}}{c}\right] = \frac{c^2}{4\pi\sigma}\nabla^2\vec{B} + \nabla\times(\vec{v}\times\vec{B})$$

Timescale for magnetic dissipation

$$\sigma \simeq 7\times10^7\ \frac{T^{3/2}}{\ln\Lambda}\ s^{-1}$$

$$\tau_{diss} = \frac{4\pi\sigma L^2}{c^2} \approx 10^{11}\left(\frac{L}{1\,AU}\right)^2\left(\frac{T}{10^4\,K}\right)^{3/2} yr$$

$$\sigma \to \infty$$

$$\vec{E} = -\frac{1}{c}\vec{v}\times\vec{B}$$

$$\frac{\partial \vec{B}}{\partial t} = \nabla\times(\vec{v}\times\vec{B})$$

Limit of ideal MHD

Fluid equations

Mass conservation

$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \vec{v}) = 0$$

Momentum conservation

$$\frac{\partial \vec{v}}{\partial t} + \vec{v} \cdot \nabla \vec{v} = -\frac{\nabla P}{\rho} + \frac{1}{4\pi\rho} (\nabla \times \vec{B}) \times \vec{B}$$

Entropy variation $T \frac{Ds}{Dt} = 0$ ⇒ Adiabatic system $P \rho^{-\gamma} = const$

Maxwell equations

Electric field

$$\nabla \cdot \vec{E} = 0; \qquad \frac{\partial \vec{B}}{\partial t} = \nabla \times (\vec{v} \times \vec{B})$$

Magnetic field

$$\nabla \cdot \vec{B} = 0; \qquad \nabla \times \vec{B} = -\frac{4\pi}{c} \vec{j}$$

Waves in ideal MHD

We apply the technique of small perturbations

1) Assume a small perturbation of a stationary system

$$\rho=\rho_0+\delta\rho\,; \qquad \vec{v}=\delta\vec{v}\,; \qquad p=p_0+\delta p\,; \qquad \vec{B}=B_0\hat{z}+\delta\vec{B}$$

2) Plug the perturbations in the ideal MHD equations and retain the first order terms

$$\begin{cases} \dfrac{\partial}{\partial t}\dfrac{\delta\rho}{\rho_0}=\nabla\cdot\delta\vec{v} & \textbf{mass conservation} \\[2ex] \dfrac{\partial\,\delta\vec{v}}{\partial t}= -c_2^2\nabla\dfrac{\delta\rho}{\rho} + \dfrac{\left(\nabla\times\delta\vec{B}\right)\times\vec{B}_0}{4\pi\rho_0} & \textbf{momentum conservation} \\[2ex] \dfrac{\partial}{\partial t}\delta\vec{B}= B_0\dfrac{\partial}{\partial z}\delta v-B_0\hat{z}\nabla\cdot\delta\vec{v} & \textbf{Faraday equation} \end{cases}$$

3) Assume a sinusoidal variation of the perturbations

$$\frac{\delta\rho}{\rho_0}= r\,e^{i(\omega t-\vec{k}\cdot\vec{x})}; \qquad \delta\vec{v}= \vec{V}\,e^{i(\omega t-\vec{k}\cdot\vec{x})}; \qquad \frac{\delta\vec{B}}{B_0}= \vec{b}\,e^{i(\omega t-\vec{k}\cdot\vec{x})};$$

Waves in ideal MHD

We get a linear homogeneous system

$$\begin{cases} \omega r = \vec{k} \cdot \vec{V} \\ \omega \vec{V} = c_s^2 r \vec{k} - \dfrac{(\vec{k} \wedge \vec{b}) \wedge \hat{z} B_0^2}{4\pi \rho_0} \\ \omega \vec{b} = -k_z \vec{V} + \hat{z} \vec{k} \cdot \vec{V} \end{cases}$$

Sound speed: $c_s^2 = \gamma \dfrac{p_0}{\rho_0}$

Alfvén speed: $v_A = \dfrac{B_0}{\sqrt{4\pi \rho_0}}$

The solution are found from

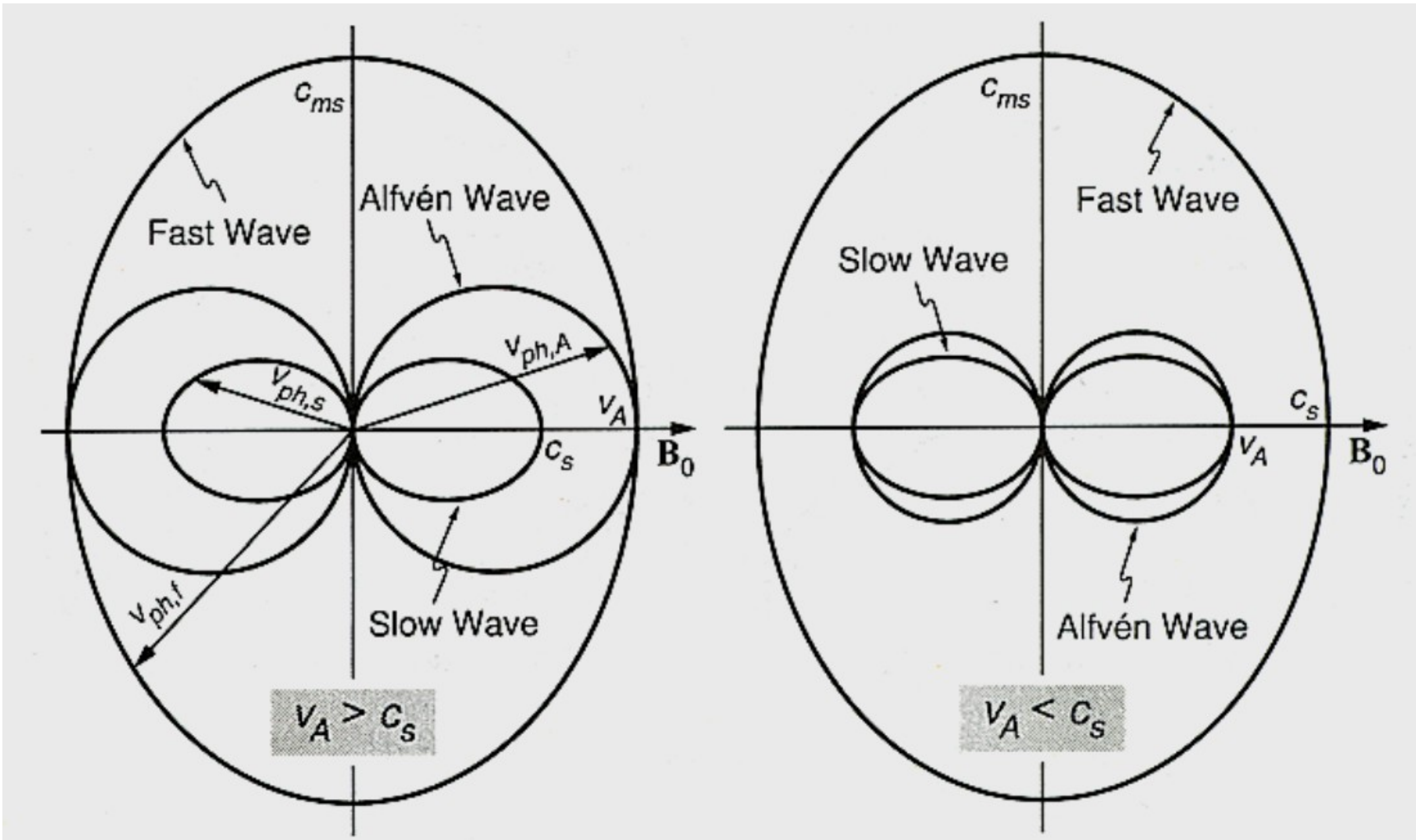
$$\det \begin{pmatrix} \omega^2 - v_A^2 k_\parallel^2 & 0 & 0 \\ 0 & \omega^2 - v_A^2 k_\parallel^2 - (c_s^2 + v_A^2)\, k_\perp^2 & -c_s^2 k_\parallel k_\perp \\ 0 & -c_s^2 k_\parallel k_\perp & \omega^2 - c_s^2 k_\parallel^2 \end{pmatrix} = 0$$

Three different solutions

$$\omega^2 = v_A^2 k_\parallel^2$$ → Alfvén waves

$$\omega^2 = \frac{k_\parallel^2}{2} \left[v_a^2 + c_s^2 \pm \sqrt{(v_a^2 + c_s^2)^2 - 4 v_a^2 c_s^2 \cos\theta} \right]$$ → Magnetosonic waves (fast and slow)

Phase-velocity polar diagram for MHD waves

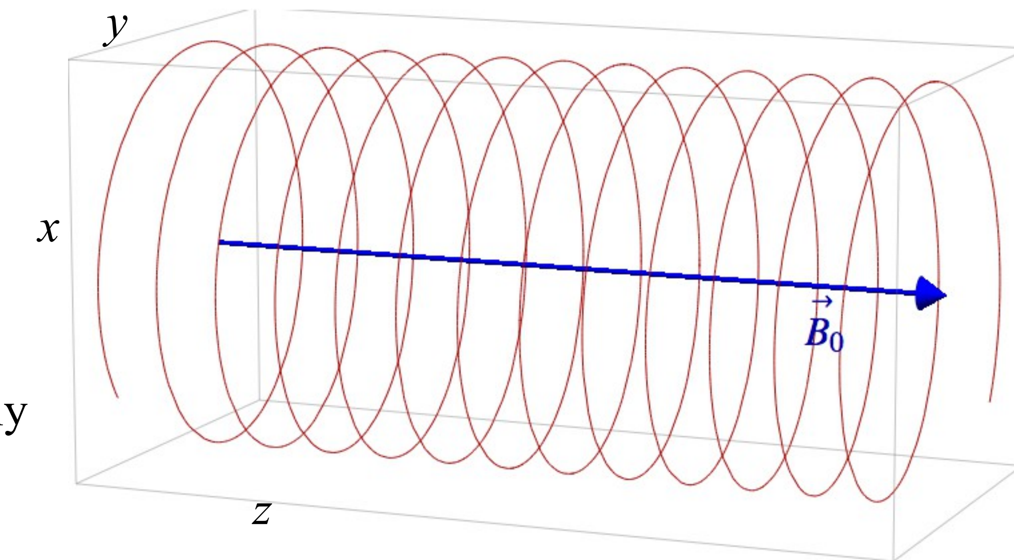


$$\omega^2 = v_A^2 k_\parallel^2$$ → Alfvén waves: mainly propagate along B_0

$$\omega^2 = \frac{k_\parallel^2}{2}\left[v_a^2 + c_s^2 \pm \sqrt{(v_a^2 + c_s^2)^2 - 4 v_a^2 c_s^2 \cos\theta}\right]$$ → Magnetosonic waves (fast and slow)

DIFFUSIVE MOTION

MOTION IN A REGULAR FIELD



Equation of motion

$$\frac{\mathrm{d}\mathbf{p}}{\mathrm{d}t} = \frac{q}{c}(\mathbf{v} \times \mathbf{B}_0)$$

Electric field is usually shortcut in a plasma

$$m\gamma\frac{\mathrm{d}\mathbf{v}}{\mathrm{d}t} = \frac{q}{c}(\mathbf{v} \times \mathbf{B}_0) \implies \begin{cases} m\gamma\dfrac{\mathrm{d}v_x}{\mathrm{d}t} = \dfrac{q}{c}v_y B_0 \\ m\gamma\dfrac{\mathrm{d}v_y}{\mathrm{d}t} = -\dfrac{q}{c}v_x B_0 \\ m\gamma\dfrac{\mathrm{d}v_z}{\mathrm{d}t} = 0 \end{cases}$$

Making the second derivative:

$$\frac{\mathrm{d}^2 v_x}{\mathrm{d}t^2} = -\left(\frac{qB_0}{m\gamma c}\right)^2 v_x = -\Omega^2 v_x$$

Larmor frequency

$$\Omega = \frac{q\,B_0}{mc\,\gamma}$$

Solution

$$\begin{cases} v_x(t) = v_{0x}\cos(\Omega t) \\ v_y(t) = -v_{0y}\sin(\Omega t) \\ v_z(t) = v_{0z} \end{cases}$$

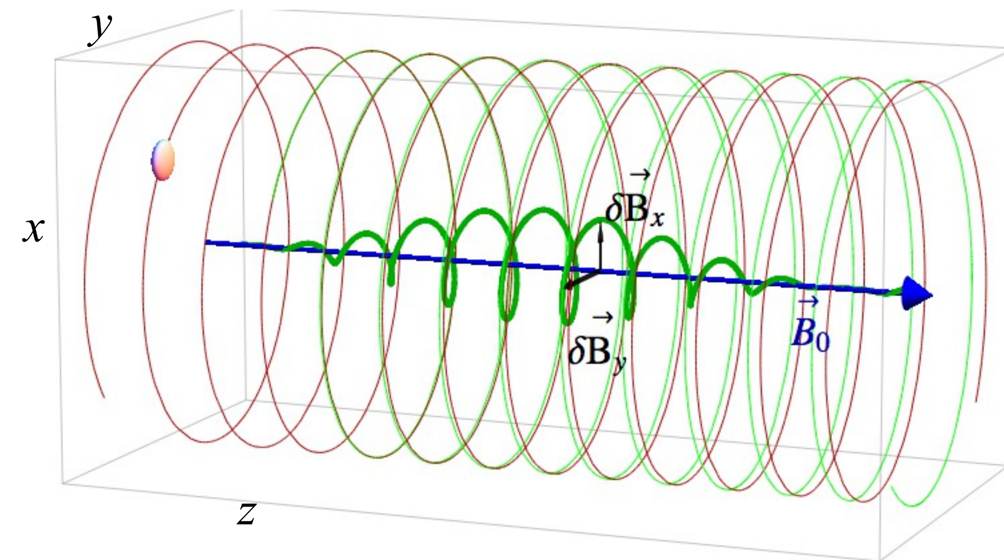
MOTION IN PRESENCE OF IRREGULARITIES

Assuming a small sinusoidal perturbation due to Alfven waves

$\delta B \perp B_0 \quad ; \qquad \delta B \ll B_0$

$$\begin{aligned} \delta B_x &= \delta B_k \cos(kz - \omega t + \phi); \\ \delta B_y &= \mp \delta B_k \sin(kz - \omega t + \phi) \end{aligned}$$

Right/left polarized waves



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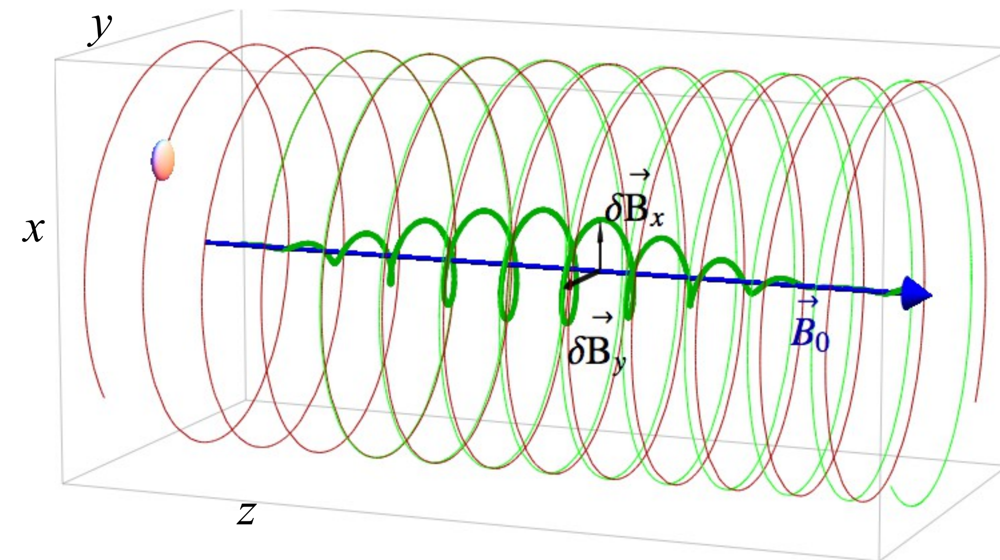

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$$\delta B_x = \delta B_k \cos(kz - \omega t + \phi);$$
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Right/left polarized waves

Equation of motion

$$m\gamma \frac{\mathrm{d}\mathbf{v}}{\mathrm{d}t} = \frac{q}{c}\mathbf{v} \times (\mathbf{B}_0 + \delta\mathbf{B}) \implies \begin{cases} m\gamma \frac{\mathrm{d}v_x}{\mathrm{d}t} = \frac{q}{c}(v_y B_0 - v_z \delta B_y) \\ m\gamma \frac{\mathrm{d}v_y}{\mathrm{d}t} = -\frac{q}{c}(v_x B_0 - v_z \delta B_x) \\ m\gamma \frac{\mathrm{d}v_z}{\mathrm{d}t} = \frac{q}{c}(v_x \delta B_y - v_y \delta B_x) \end{cases}$$

B_0 changes only *x* and *y* components of the momentum

δB changes only z component of the momentum

Solution for v_z

$$v_0 \frac{\mathrm{d}\mu}{\mathrm{d}t} = \frac{q\delta B}{m\gamma c} v_0 (1-\mu^2)^{1/2} \left[\cos(kz - \omega t + \varphi)\cos(\Omega t) \mp \sin(kz - \omega t + \varphi)\sin(\Omega t)\right]$$

MOTION IN PRESENCE OF IRREGULARITIES

Solution for v_z

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The wave motion is much smaller than the particle motion

$$\frac{kz}{\omega t} = \frac{k v_0 \mu t}{k v_A t} = \mu \frac{v_0}{v_A} \gg 1 \quad \Rightarrow \quad \frac{\mathrm{d}\mu}{\mathrm{d}t} = \frac{q\delta B}{m\gamma c}(1-\mu^2)^{1/2} \cos\left(\Omega t \mp kz \mp \varphi\right)$$

MOTION IN PRESENCE OF IRREGULARITIES

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Average displacement for a time Δt:

$$\langle \Delta\mu \rangle = \frac{1}{\Delta t}\int_0^{\Delta t} \mathrm{d}t \left(\frac{\mathrm{d}\mu}{\mathrm{d}t}\right) = 0$$

MOTION IN PRESENCE OF IRREGULARITIES

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Average displacement for a time Δt: $$\langle \Delta\mu \rangle = \frac{1}{\Delta t}\int_0^{\Delta t} \mathrm{d}t \left(\frac{\mathrm{d}\mu}{\mathrm{d}t}\right) = 0$$

Computing the diffusion coefficient: $$D_{\mu\mu} := \frac{1}{2}\left\langle \frac{\Delta\mu\Delta\mu}{\Delta t} \right\rangle$$

Step 1:
time average $$\langle \Delta\mu\Delta\mu \rangle = \left(\frac{q\delta B}{m\gamma c}\right)^2 (1-\mu^2) \int_0^{\Delta t} \mathrm{d}t \int_0^{\Delta t} \mathrm{d}t' \cos\left(\Omega t \mp k v_0 \mu t \mp \varphi\right) \cos\left(\Omega t' \mp k v_0 \mu t' \mp \varphi\right)$$

Step 2:
Phase average $$\langle \Delta\mu\Delta\mu \rangle_\varphi = \frac{1}{2\pi}\int_0^{2\pi} \mathrm{d}\varphi \, \langle \Delta\mu\Delta\mu \rangle = \left(\frac{q\delta B}{m\gamma c}\right)^2 (1-\mu^2) 2\pi \Delta t \delta(\Omega \mp v_0 k \mu)$$

MANY WAVES

Particles are scattered only by resonating waves

Final result:

$$\langle \Delta\mu\Delta\mu \rangle_{\varphi} = \left(\frac{q\delta B}{m\gamma c}\right)^2 (1-\mu^2)\frac{2\pi\Delta t}{v_0\mu}\delta\left(k \mp \frac{\Omega}{v_0\mu}\right)$$

Resonant wave-number

$$k_{res} = \frac{\Omega}{v_0\mu} \approx \frac{1}{r_L}$$

MANY WAVES

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Resonant wave-number

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In general magnetic waves are distributed in a large *k* range

We introduce the power spectrum: $$\frac{\delta B^2(k)}{B_0^2} =: \mathcal{F}(k)\mathrm{d}k$$

And the logarithmic power spectrum: $P(k) = k\,F(k)$

Step 3: averaging over a power spectrum of waves $$D_{\mu\mu} := \frac{1}{2}\left\langle \frac{\Delta\mu\Delta\mu}{\Delta t} \right\rangle$$

$$D_{\mu\mu} = \Omega(1-\mu^2)\int \mathrm{d}k\,\mathcal{F}(k)\pi k_{\mathrm{res}}\delta(k \mp k_{\mathrm{res}}) = \pi\mathcal{F}(k_{\mathrm{res}})k_{\mathrm{res}}\Omega(1-\mu^2)$$

MANY WAVES

1) PARTICLES DIFFUSE IN ANGLE

Diffusion coefficient in angle: $D_{\theta\theta} = \frac{1}{2}\left\langle \frac{\Delta\theta\,\Delta\theta}{\Delta t} \right\rangle = \frac{D_{\mu\mu}}{\sin^2\theta} = \pi\,\Omega\,P(k_{res})$

Time needed for a particle to change direction by π $\Delta t = \frac{\pi}{D_{\theta\theta}} = \frac{1}{\Omega\,P(k)} \sim \frac{T_{gyration}}{P(k)} \gg T_{gyration}$

usually $P(k) << 1$

2) PARTICLES DIFFUSE IN SPACE

Spatial diffusion coefficient

$$D_{zz} = \frac{1}{3} v\,\lambda_{mfp} = \frac{1}{3} v\,(v\,\Delta t) \approx \frac{v^2}{3\,\Omega\,P(k_{res})} = \frac{1}{3}\frac{r_L v}{P(k_{res})}$$

Bohm diffusion $D_{Bohm} = \frac{1}{3} r_L v$

The most efficient scattering happens for $P=1 \Rightarrow \delta B \sim B_0$

PARTICLE SCATTERING

Summarizing

- Each time a resonance occurs, the particle change pitch angle by $\Delta\theta \sim \delta B/B_0$ with a random sign

- The resonance occurs only with right-hand polarized waves if positive charged particles move to the right (and *vice versa*)

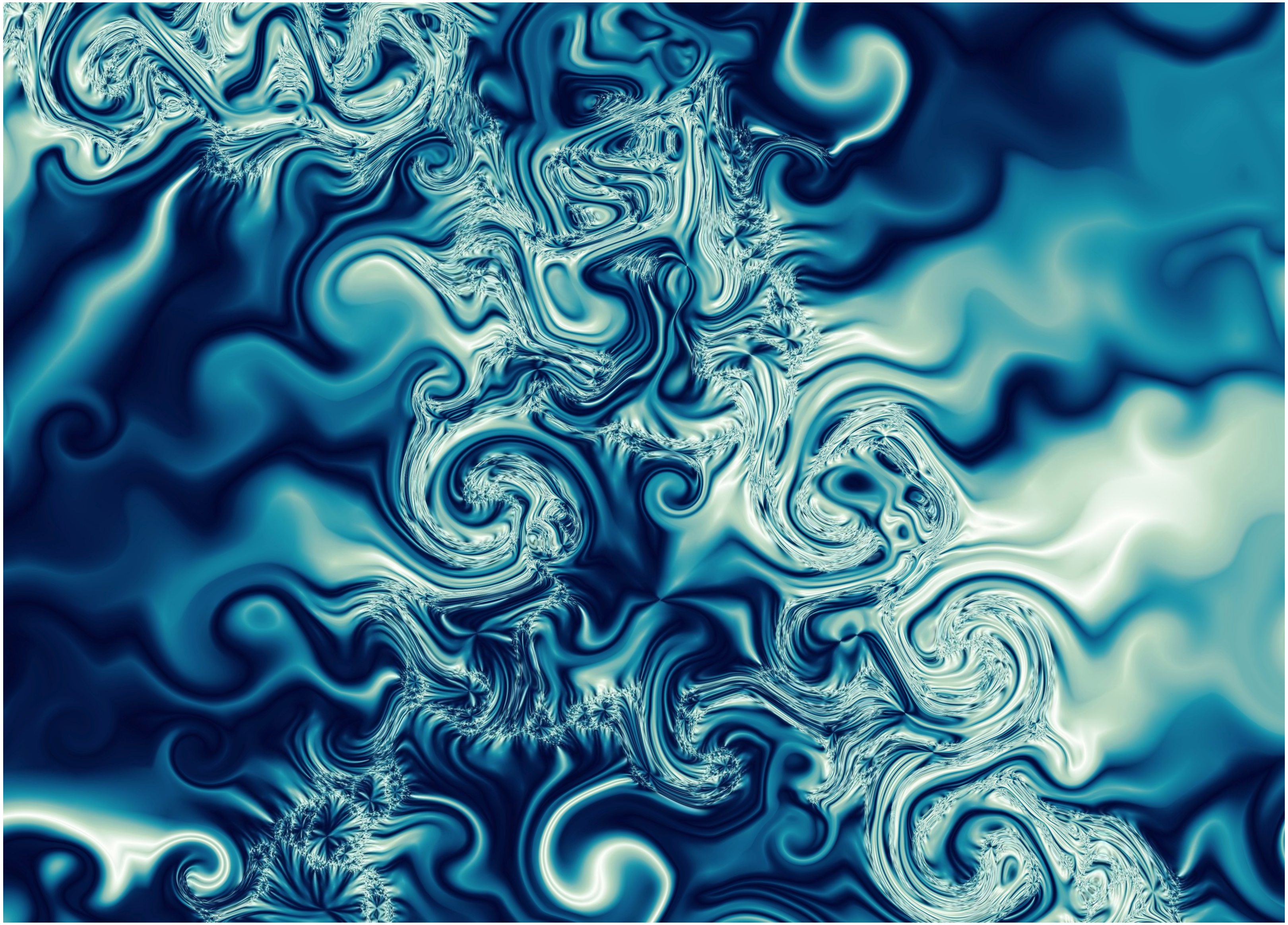
- The resonant condition tells us that:

  - $r_L \ll \lambda$ particles surf adiabatically

  - $r_L \gg \lambda$ particles do not feel the wave

Where do waves come from?

Kolmogorov theory of turbulence

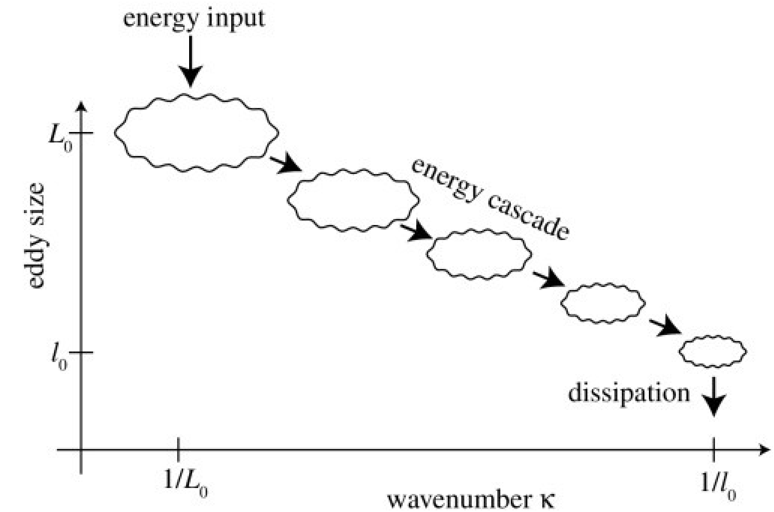


Kolmogorov theory of turbulence

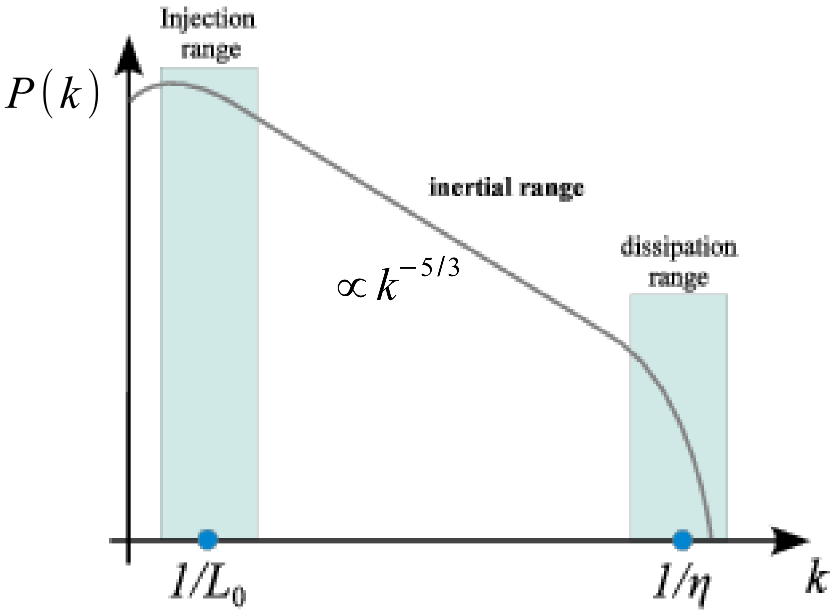
Turbulence in isotropic uniform 3D fluids

Dissipation scale: $[\lambda]=L$
Viscosity: $[\nu]=L^2T^{-1}$
Dissipation rate: $[\epsilon]=L^2T^{-3}$

$$\Rightarrow \quad \lambda_{diss}\propto\left(\frac{\nu^3}{\epsilon}\right)^{1/4}$$



Power spectrum: $F(k)\,dk=\frac{dU}{\rho} \quad \Rightarrow \quad [F]=L^3T^{-2}$



In the inertial range F should not depend on viscosity

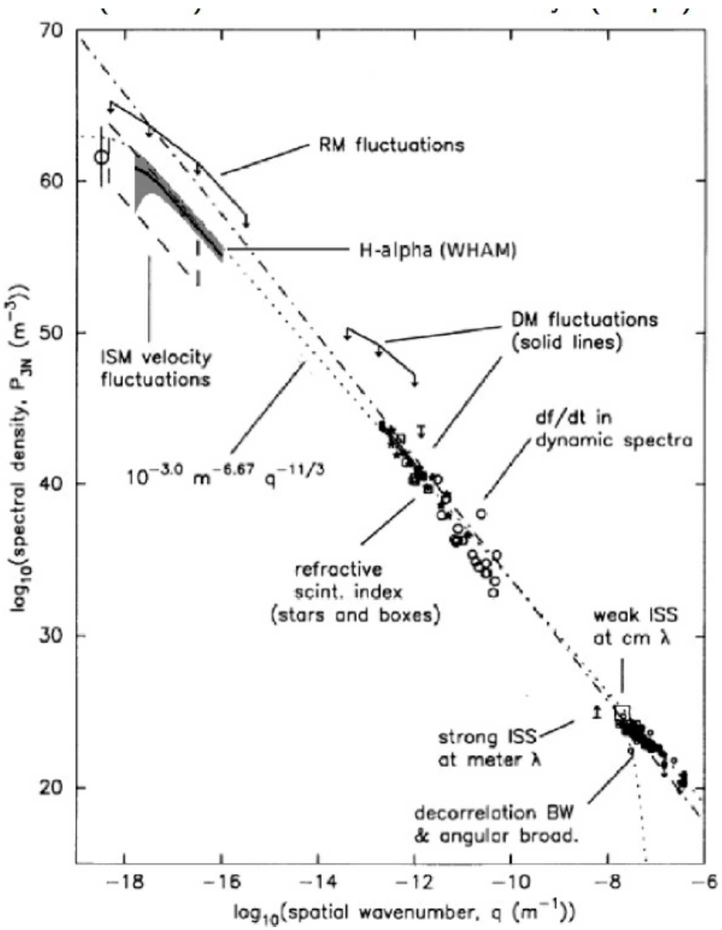
$$F=F(k,\epsilon)$$

From dimensional analysis:

$$\Rightarrow \quad F(k)\propto \epsilon^{2/3}\ k^{-5/3}$$

The interstellar turbulence

A simplified model for turbulence



Electron density fluctuation in the ISM
[Armstrong et al. 1995, ApJ 443, 209]

- Electron density fluctuation follow Kolmogorov spectrum: $\delta n \propto k^{-5/3}$
- Magnetic turbulence has a Kolmogorov spectrum $k^{-5/3}$ (density is a passive tracer so it has the same spectrum: $\delta n \propto \delta B^2$):

$$F(k) = \frac{\langle \delta B(k) \rangle^2}{B_0^2} \propto \left(\frac{k}{k_0} \right)^{-5/3}$$

- Turbulence is stirred by SNe at a typical scale $L_0 = 1/k_0 \sim$ 10-100 pc
- Fluctuation of velocity and magnetic field are assumed to be Alfvénic

Diffusion from interstellar turbulence

The main origin of turbulence are thought to be SN explosion.

Turbulence is injected at a scale comparable with the size of SNR (or super-bubbles) and than cascades at smaller scales.

Power injected at: $k_0 = 1/L_0 \approx (10\,pc)^{-1}$

Kolmogorov cascade: $F(k) = \frac{2}{3}\frac{\eta_B}{k_0}\left(\frac{k}{k_0}\right)^{-5/3}$

Energy density of turbulence (from observation): $\eta_B = \int_{k_0}^{\infty} F(k)\,dk = \frac{\delta B_{tot}^2}{B_0^2} \sim 0.01 - 0.1$

$$D_{zz}(p) = \frac{v\,r_L}{3}\frac{1}{k_{res}F(k_{res})} = \frac{c}{2\,\eta_B} r_L^{1/3} L_c^{2/3}$$

$$\sim 3\times 10^{28} \left(\frac{p\,c}{GeV}\right)^{1/3}\left(\frac{\eta_B}{0.1}\right)^{-1}\left(\frac{B_0}{3\,\mu G}\right)^{1/3}\left(\frac{L_c}{10\,pc}\right)^{2/3} cm^2\,s^{-1}$$

FROM DIFFUSION TO ENERGY GAIN

Basic concepts

- ALL ACCELERATION MECHANISMS ARE ELECTROMAGNETIC IN NATURE

- MAGNETIC FIELDS DO NOT MAKE WORK ON CHARGED PARTICLES!

- WE NEED ELECTRIC FIELDS

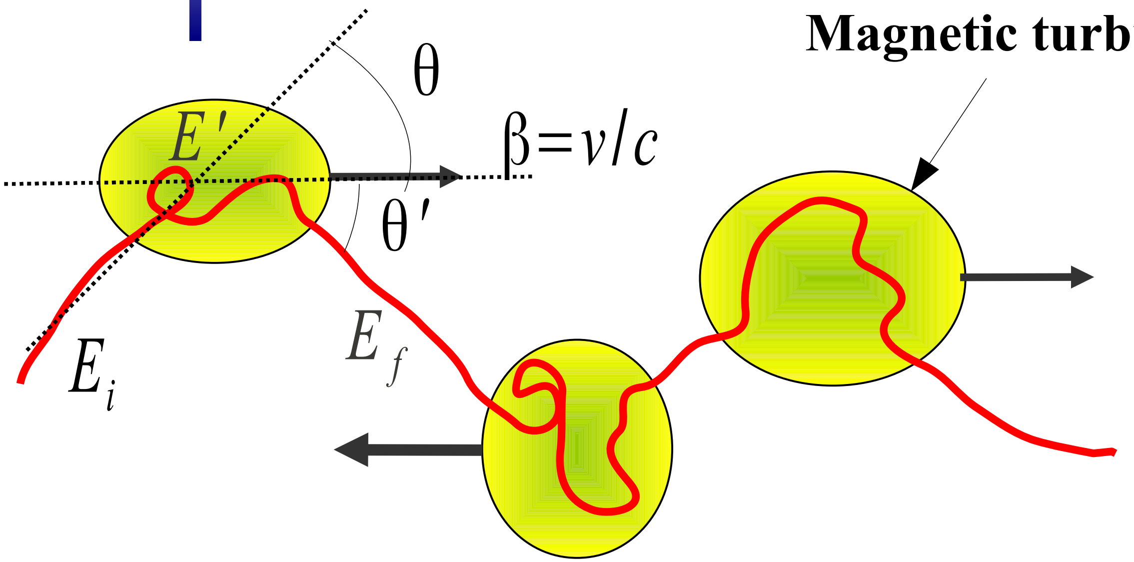
- BUT FOR THE MAJORITY OF ASTROPHYSICAL THE CONDUCTIVITY □ ∞, HENCE <E> = 0

- THE MAJORITY OF ACCELERATION PROCESS ARE STOCHASTIC

<u>STOCHASTIC ACCELERATION</u>

$$\langle \vec{E} \rangle = 0 \quad \langle \vec{E}^2 \rangle \neq 0$$

A quick look to 2nd order Fermi acceleration (Fermi, 1949)



$$E' = \gamma E_i (1-\beta\mu)$$

$$E_f' = E_i' = E'$$

$$E_f = \gamma E' (1+\beta\mu')$$

$$\rightarrow E_f = \gamma^2 E_i (1-\beta\mu)(1+\beta\mu')$$

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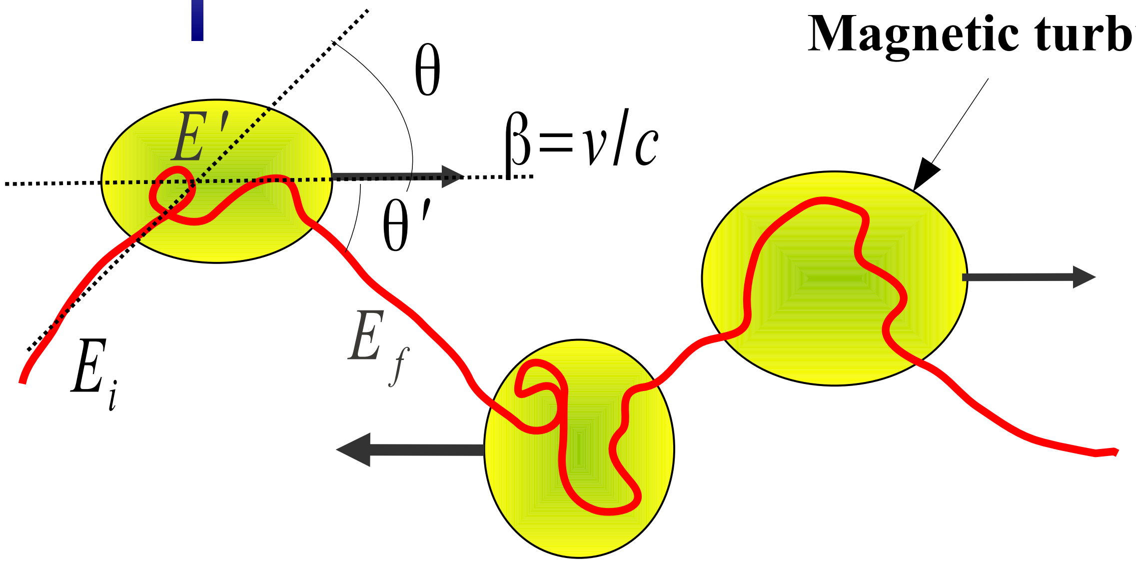

$$E' = \gamma E_i (1 - \beta\mu)$$
$$E_f' = E_i' = E'$$
$$E_f = \gamma E' (1 + \beta\mu')$$
$$\rightarrow E_f = \gamma^2 E_i (1 - \beta\mu)(1 + \beta\mu')$$

$$\left\langle \frac{\Delta E}{E} \right\rangle_{\mu'} = \int_{-1}^{1} \frac{E_f - E_i}{E_i} d\mu' = 2\left[\gamma^2 (1 - \beta\mu) - 1\right]$$

Assuming isotropy in the cloud's reference frame

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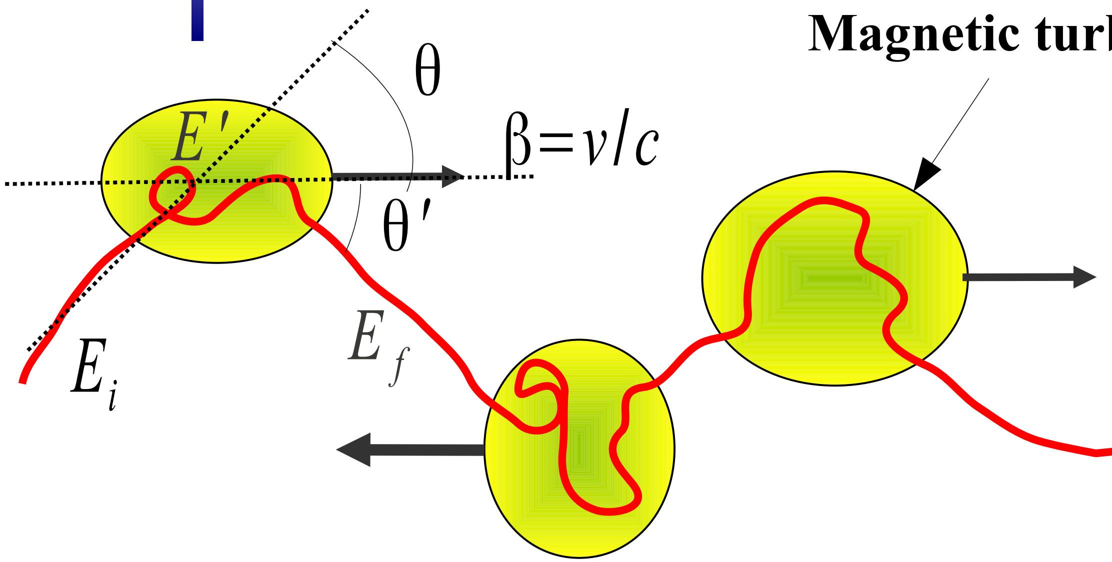

$$E' = \gamma E_i (1-\beta\mu)$$
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Assuming isotropy in the cloud's reference frame

$$\left\langle \frac{\Delta E}{E} \right\rangle_{\mu'\mu} = \int_{-1}^{1} d\mu \underbrace{\frac{1}{2}(1-\beta\mu)} \; 2\left[\gamma^2 (1-\beta\mu) - 1\right] = \frac{4}{3}\beta^2$$

Losses and gains are present but do not compensate exactly

Particle flux incident of the cloud $\beta_{rel} = (c - v\mu)/c$

A quick look to 2nd order Fermi acceleration (Fermi, 1949)

- IF MAGNETIC FIELD DOES NOT MAKE WORK, WHO ENERGIZE PARTICLES?

A quick look to 2nd order Fermi acceleration (Fermi, 1949)

- IF MAGNETIC FIELD DOES NOT MAKE WORK, WHO ENERGIZE PARTICLES?

TIME VARING MAGNETIC FIELD ELECTRIC FIELD

$$\frac{\partial \delta \vec{B}}{\partial t} = -\nabla \times \delta \vec{E}$$

- THE INDUCED ELECTRIC FIELD ENERGIEZES THE PARTICLES

- THE SCATTERING PRODUCES A MOMENTUM TRANSFER, BUT TO WHAT?

A quick look to 2nd order Fermi acceleration (Fermi, 1949)

$$\left\langle \frac{\Delta E}{E} \right\rangle \propto \left(\frac{v}{c} \right)^2$$

- THE ENERGY GAIN IS ONLY PROPORTIONAL TO $(v/c)^2$ AND TYPICALLY $v \sim v_A \sim 10^{-4}\, c$
- THE PREDICTED SPECTRUM STRONGLY DEPENDS ON DETAILS LIKE THE CLOUDS DISTRIBUTION IN THE GALAXY AND THEIR VOLUME FILLING FACTOR
  - IT IS DIFFICULT TO EXPLAIN THE OBSERVED SPECTRUM $E^{-2.7}$
  - THE MAXIMUM ENERGY IS AT MOST ~10 GeV

Exercise on the 2nd order Fermi acceleration

EXERCISE.

Estimate the II order Fermi acceleration of 1 GeV particles in the Galaxy, assuming that the magnetic turbulence power spectrum follows a Kolmogorov distribution:

$$k\,F(k) \simeq (k\,L_c)^{-2/3} = \left(\frac{L_c}{r_L}\right)^{-2/3}$$

with L_c ~10 pc (coherence scale) and the average magnetic field in the Galaxy is B_0= 3 μG.
Assume the residence time in the Galaxy to be ~ 100 Myr and the average Galactic density is 1 proton/cm^3.

Exercise on the 2nd order Fermi acceleration

EXERCISE: estimate the II order Fermi acceleration of 1 GeV particle in the Galaxy

Total energy gain

$$\frac{E_f}{E_0} = \left(1+\frac{\Delta E}{E}\right)^n = \left(1+\frac{4}{3}\left(\frac{v_A}{c}\right)^2\right)^n$$

Number of interactions

$$n=\frac{\tau_{res}}{\Delta t}$$

Time of a single interaction

$$\Delta t \approx \frac{1}{D_{\theta\theta}} = \frac{1}{\Omega k_{res} F(k_{res})}$$

Kolmogorov power spectrum of turbulence in the Galaxy

$$k F(k) \simeq (k L_c)^{-2/3} = \left(\frac{L_c}{r_L}\right)^{-2/3}$$

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Kolmogorov power spectrum of turbulence in the Galaxy

$$k F(k) \simeq (k L_c)^{-2/3} = \left(\frac{L_c}{r_L}\right)^{-2/3}$$

$$L_c \sim 10 \div 100\, pc$$

$$r_L = \frac{pc}{eB} = 10^{-6} E_{GeV} B_{\mu G}^{-1} \quad pc$$

Exercise on the 2nd order Fermi acceleration

EXERCISE: estimate the II order Fermi acceleration of 1 GeV particle in the Galaxy

Total energy gain

$$\frac{E_f}{E_0} = \left(1+\frac{\Delta E}{E}\right)^n = \left(1+\frac{4}{3}\left(\frac{v_A}{c}\right)^2\right)^n \approx 2$$

Number of interactions

$$n = \frac{\tau_{res}}{\Delta t} \approx \frac{100\ Myr}{0.1\ yr} \approx 10^9$$

$$v_A = \frac{B}{\sqrt{4\pi\rho}} = 6.5 \frac{B}{3\mu G}\left(\frac{\rho}{1\ cm^{-3}}\right)^{-1/2} \frac{km}{s}$$

Time of a single interaction

$$\Delta t \approx \frac{1}{D_{\theta\theta}} = \frac{1}{\Omega k_{res} F(k_{res})} = \frac{r_L}{c}\left(\frac{L_c}{r_L}\right)^{2/3} \approx 0.1\ yr\ E_{GeV}^{1/3} B_{\mu G}^{-1/3} L_{10pc}^{2/3}$$

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