Problems

Problem 1

- <u>Particle acceleration at the LHC and in the Cosmos</u>

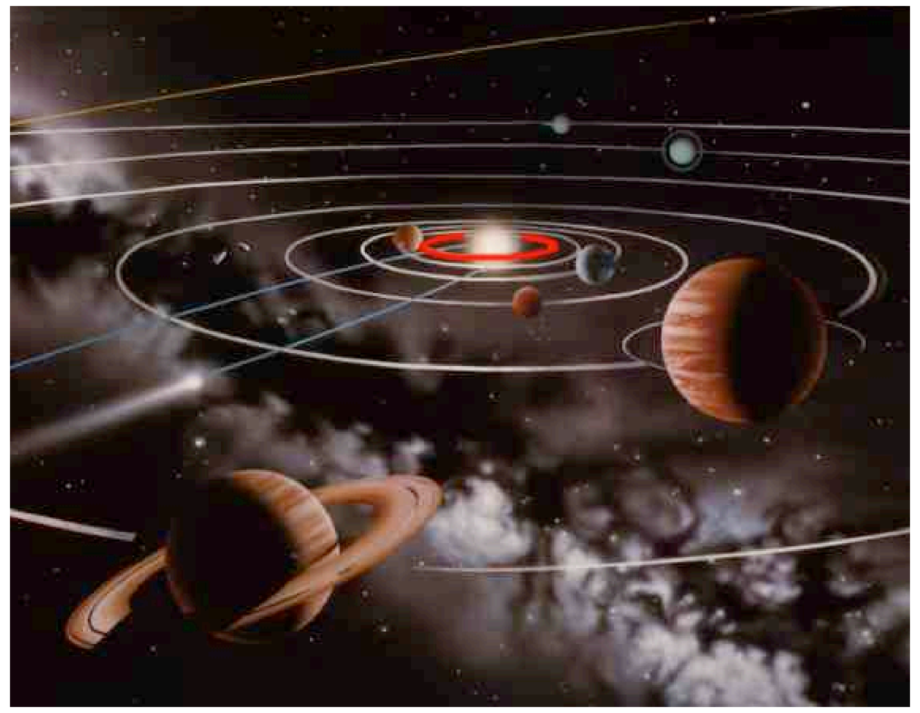
Comic accelerators, such as active galactic nuclei (AGNs), accelerate particles to energies of up to 10^{20} eV = 100 EeV. The most powerful human-made particle accelerator, the Large Hadron Collider (LHC), reaches proton energies of $7 \cdot 10^{12}$ eV = 7 TeV.

a) Derive the equation for the center-of-mass energy for a collider and feed-target experiment using the relativistic energy and momentum conservation laws.

b) Calculate the center-of-mass energy of two colliding portions at the LHC.

c) Calculate the center-of-mass energy of a cosmic proton with energy 10^{20} eV colliding with a proton at rest in the atmosphere.

d) What energy is required for a cosmic proton in order to reach a center-of-mass energy of 14 TeV in the interaction from problem c).



Problem 2

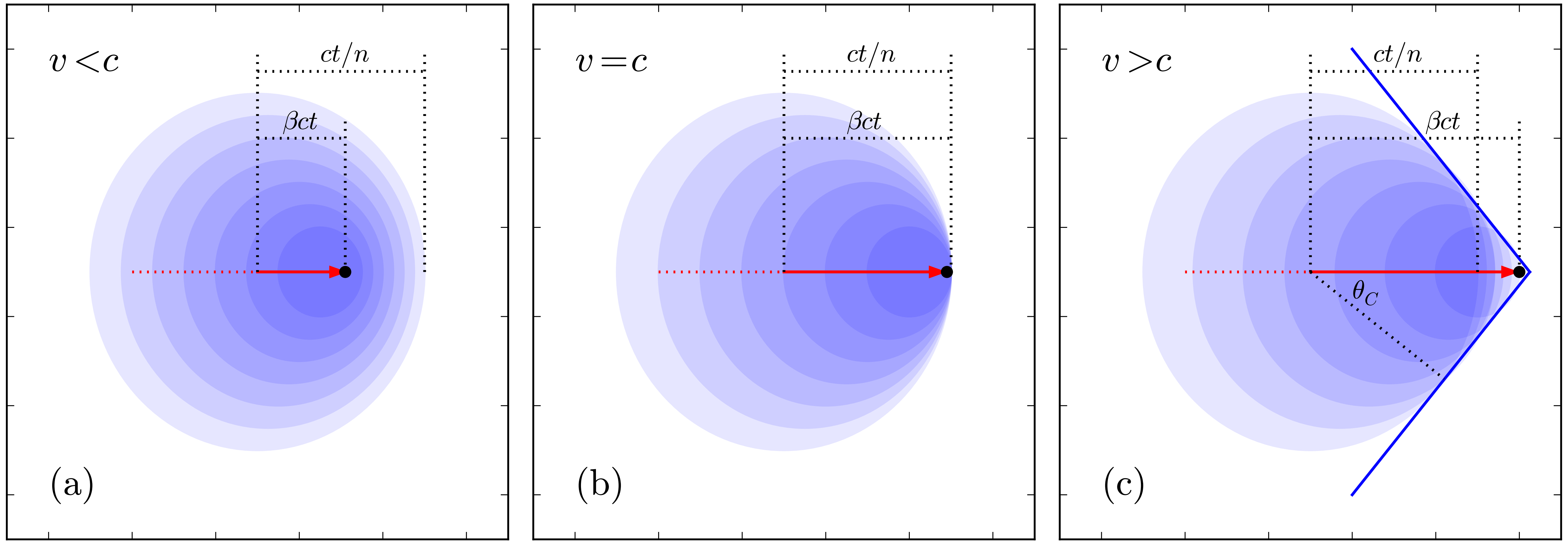
- Particle detection with Cherenkov radiation

A common methods identify different particles and to measure their energy is the detection via Cherenkov radiation which has been discussed in the lecture.

a) Explain the physical principle of the Cherenkov effect.

b) Derive the formula for the Cherenkov angle using the Huygens principle, as described in the lecture.

c) What is the minimum energy for an electron and muon to produce Cherenkov radiation in air (refractive index $n = 1.0003$) and in water ($n = 1.333$)?



Problem 3

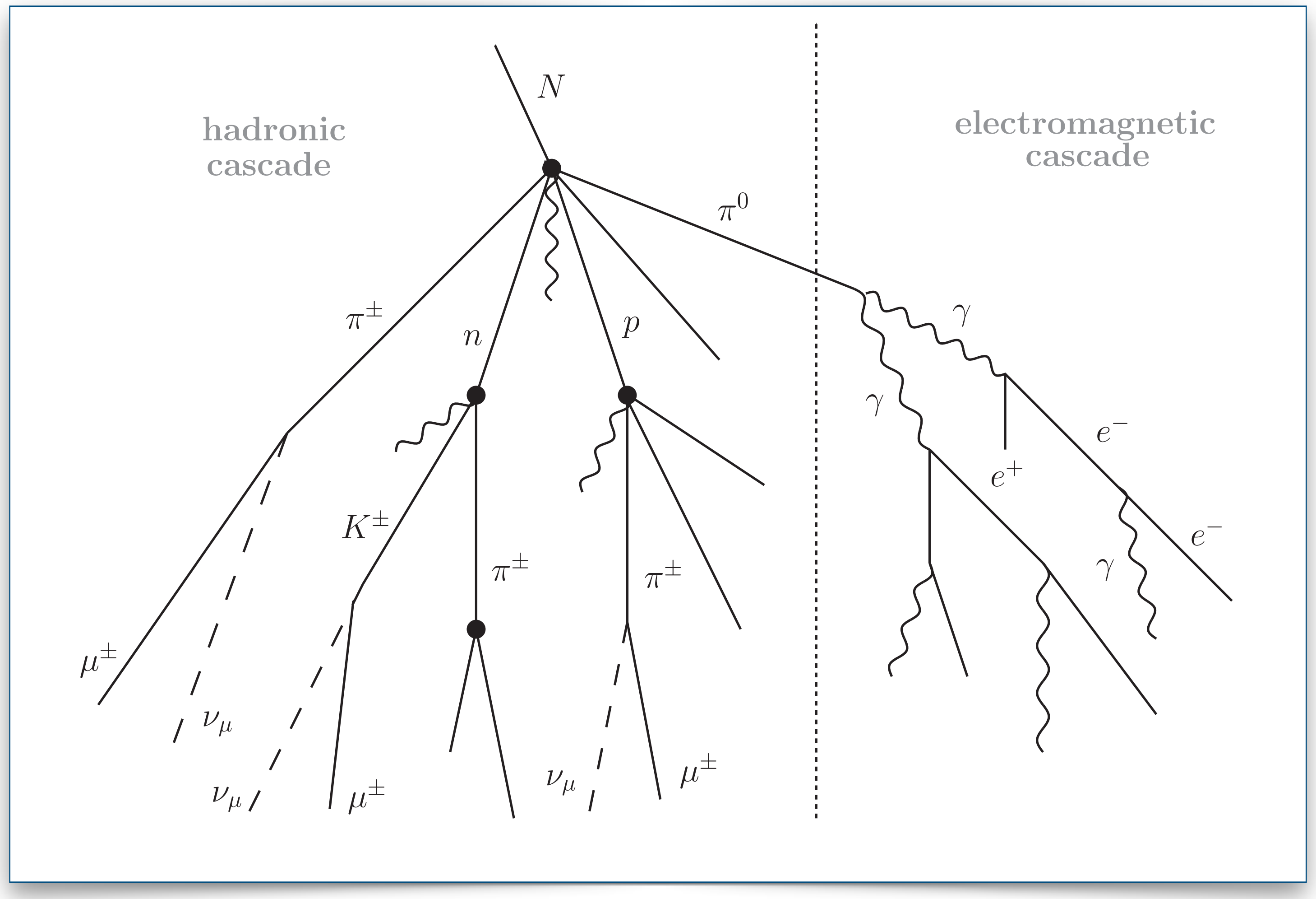
- <u>Air shower development</u>

The development of the electromagnetic part of an air shower is dominated by Bremsstrahlung and pair production.The simple Heitler model assumes that an electron with energy E_0 emits an photon of energy $E_0/2$ after one radiation length X_0. After having traveled another X_0, the photon produces a e^+/e^- pair with each particle carrying $E_0/4$. This process continues until a critical energy ξ_C is reached where no new particles can be produced.

a) After which distance X_{max} does the shower development reach its maximum? How many particles, N_{max}, have been produced at X_{max}? Make a sketch of the behavior of E, N, and X during the shower development.

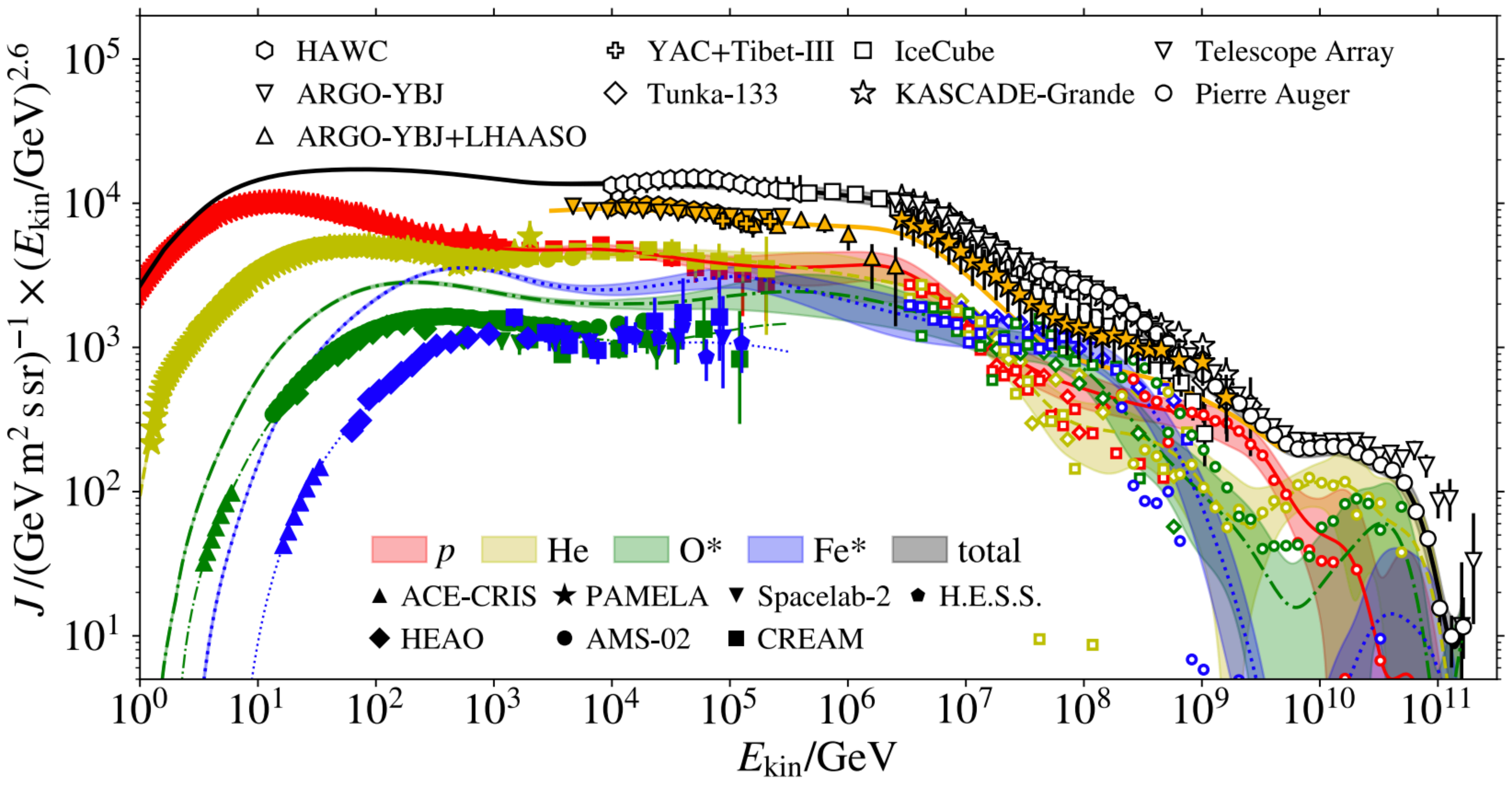
b) For an electromagnetic shower in air the critical energy is about $\xi_C = 100\,\text{MeV}$. What is the definition of ξ_C? What is the length of the shower initiated by an electron with energy 10^{12} eV and 10^{20} eV, respectively, assuming that the shower maximum, X_{max}, is reached at about 2/3 of the shower length.

c) The atmospheric depth x (in units of g/cm^2) depends on the altitude h (in km) above sea level and $x = X\exp(-h/H)$, where $H = 6.5\,\text{km}$ and $X = 1030\,\text{g/cm}^2$ is the total atmospheric depth. At which altitude produce the electrons from problem b) a shower maximum X_{max}, assuming that their radiation length is $X_0 = 37\,\text{g/cm}^2$?



Problem 4

- <u>Cosmic ray flux</u>



The flux of cosmic rays is given by

$$\varphi(E) = \frac{d\Phi}{dE_0} = \frac{dN}{dt dA d\Omega dE_0}$$

The differential flux of cosmic rays follows a power-law of the form $\varphi(E) \propto E_0^{-\gamma}$ where $\gamma = 3$ is the special index.

a) Show that the integrated energy spectrum follows a power-law with spectral index $(\gamma - 1)$.

b) In order to challenge the Pierre Auger Observatory we want to build a detector which observes at least 10 cosmic ray events with minimum energies of $10^{20}\,\mathrm{eV}$ per year. How large is the required area assuming that for a minimum energy of $10^{15}\,\mathrm{eV}$ the (integrated) flux is 1 particle per year per square meter?

c) The composition of cosmic rays is still uncertain at the highest energies. At low energies the flux is dominated by protons and helium and heavier elements become important towards higher energies. At which energy is the fraction of proton and helium nuclei equal assuming a spectral index of $\gamma_{\mathrm{H}} = 2.77$ and $\gamma_{\mathrm{He}} = 2.64$, respectively? Assume that at an energy of $\hat{E} = 10^{12}\,\mathrm{eV}$ the corresponding fluxes are $\varphi(\hat{E}) = 11.5 \cdot 10^{-2}\,\mathrm{m^{-1}\,s^{-1}\,sr^{-1}\,TeV^{-1}}$ and $\varphi(\hat{E}) = 7.19 \cdot 10^{-2}\,\mathrm{m^{-1}\,s^{-1}\,sr^{-1}\,TeV^{-1}}$.

Problem 5

- Muons is extensive air showers

In addition to hadrons and the electromagnetic component, EAS produce a large number of muons which can be measured at the ground.

a) How are muons produced in EAS?

b) What is the minimum energy for a muon that is produced at an altitude of 20 km to reach the ground (sea level)? (the muon lifetime is $2.2\,\mu$s)

c) The mean energy loss of relativistic muons with energy E that traverse a dense medium is given by $-dE/dX = \alpha + \beta \cdot E$ where $\alpha = 2.5\,\mathrm{MeV\,cm^2/g}$ is the energy loss due to ionization. The second term describes the radiative energy losses. Calculate the mean distance a muon with energy 10 TeV can travel in rock, assuming a density of $\alpha = 3\,\mathrm{g/cm^3}$ (assume that the critical energy at which the losses due to ionization and radiative losses become equal is $\alpha = 1\,\mathrm{TeV}$).