Astroparticle Physics

- Selected Topics -

4th Graduate School on Plasma-Astroparticle Physics

Dennis Soldin
Karlsruhe Institute of Technology

Karlsruher Institut für Technologie

Astroparticle Physics

Air Shower Physics

- Selected Topics -

4th Graduate School on Plasma-Astroparticle Physics

Dennis Soldin
Karlsruhe Institute of Technology

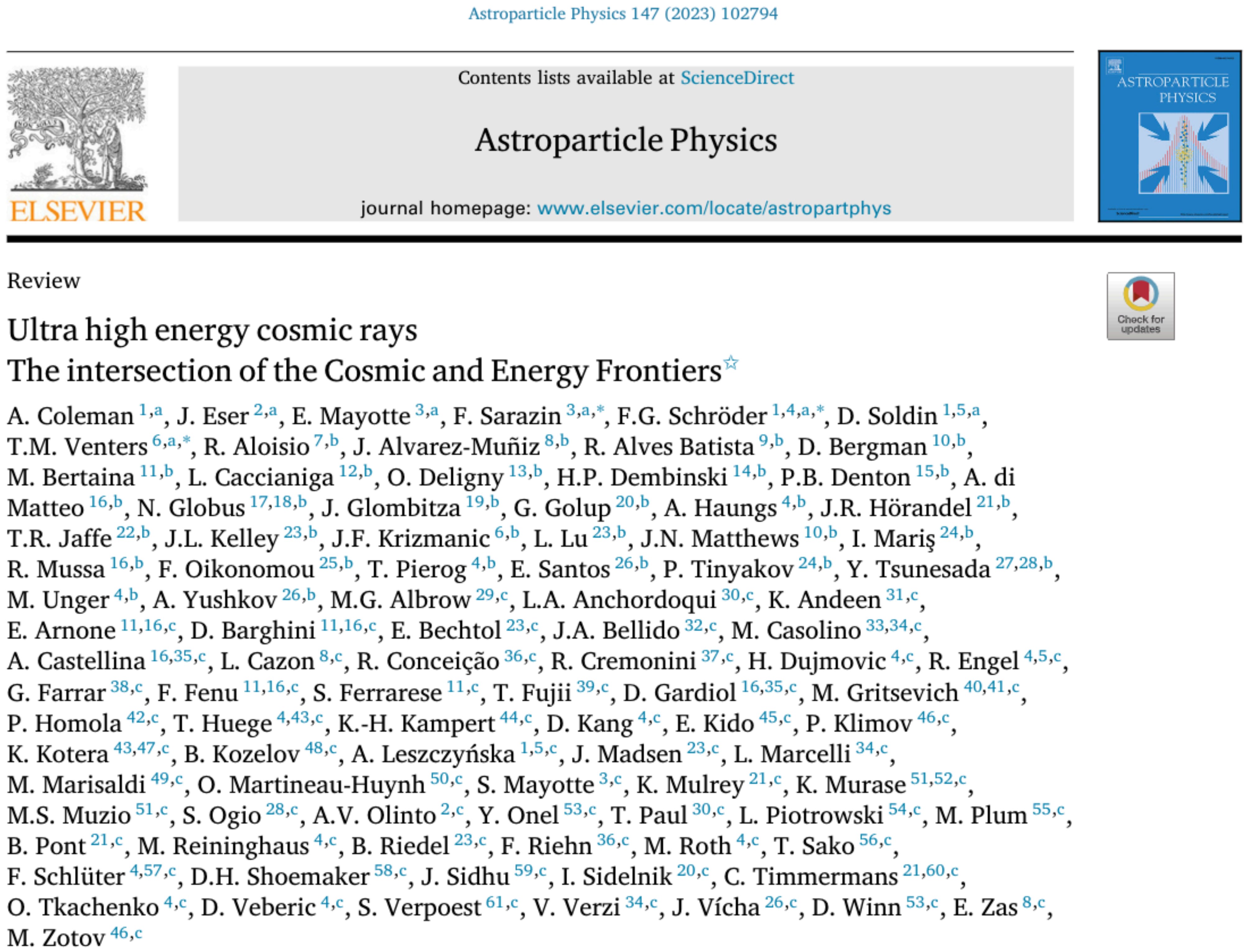
Karlsruher Institut für Technologie

Disclaimer I

- Unfortunately, 2 lectures are way too less time for a complete overview of the field of astroparticle physics, even with a focus on "particle physics"…
- We also want to have exercises to solve some problems and time for discussions
- Thus, this lecture will be highly biased, we will mainly talk about the selected topics:
  - High-energy cosmic rays and extensive air showers
  - Indirect detection of cosmic rays and recent results
  - ~~Astrophysical neutrinos~~
- We will not talk about (or only touch):
  - Low-energy cosmic rays, neutrinos, gamma rays, astrophysical sources, acceleration and propagation of cosmic rays, Dark Matter or exotic particle physics scenarios, …

Disclaimer II

- Everything discussed in the following and everything beyond about "Cosmic Rays and Particle Physics" can be found in the textbook by Tom Gaisser et al. (read it!)
- Comprehensive review of recent results (read it!)


Astroparticle Physics 147 (2023) 102794

Contents lists available at ScienceDirect

Astroparticle Physics

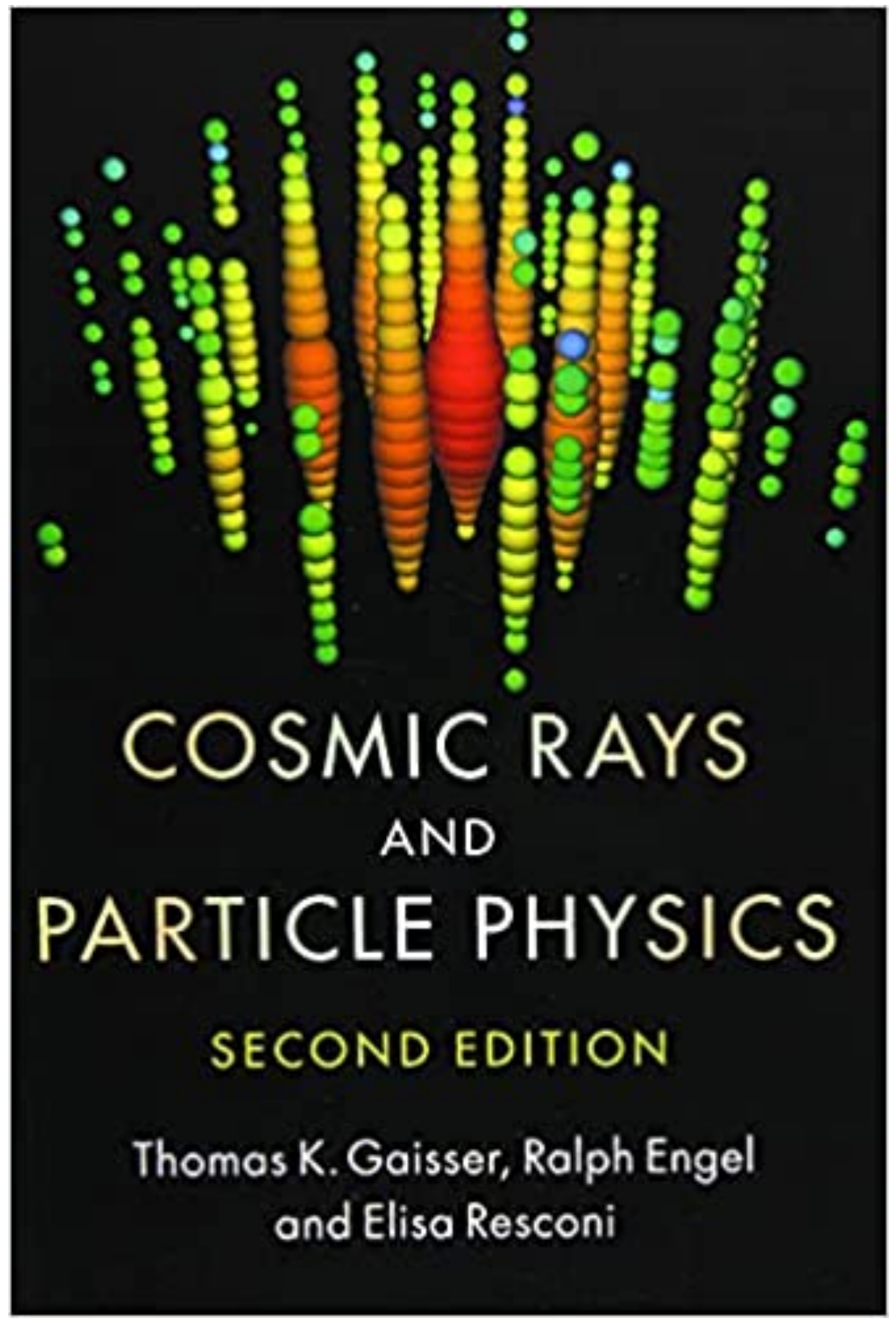
journal homepage: www.elsevier.com/locate/astropartphys

ELSEVIER

Review

Ultra high energy cosmic rays
The intersection of the Cosmic and Energy Frontiers

A. Coleman [1,a], J. Eser [2,a], E. Mayotte [3,a], F. Sarazin [3,a,*], F.G. Schröder [1,4,a,*], D. Soldin [1,5,a], T.M. Venters [6,a,*], R. Aloisio [7,b], J. Alvarez-Muñiz [8,b], R. Alves Batista [9,b], D. Bergman [10,b], M. Bertaina [11,b], L. Caccianiga [12,b], O. Deligny [13,b], H.P. Dembinski [14,b], P.B. Denton [15,b], A. di Matteo [16,b], N. Globus [17,18,b], J. Glombitza [19,b], G. Golup [20,b], A. Haungs [4,b], J.R. Hörandel [21,b], T.R. Jaffe [22,b], J.L. Kelley [23,b], J.F. Krizmanic [6,b], L. Lu [23,b], J.N. Matthews [10,b], I. Mariş [24,b], R. Mussa [16,b], F. Oikonomou [25,b], T. Pierog [4,b], E. Santos [26,b], P. Tinyakov [24,b], Y. Tsunesada [27,28,b], M. Unger [4,b], A. Yushkov [26,b], M.G. Albrow [29,c], L.A. Anchordoqui [30,c], K. Andeen [31,c], E. Arnone [11,16,c], D. Barghini [11,16,c], E. Bechtol [23,c], J.A. Bellido [32,c], M. Casolino [33,34,c], A. Castellina [16,35,c], L. Cazon [8,c], R. Conceição [36,c], R. Cremonini [37,c], H. Dujmovic [4,c], R. Engel [4,5,c], G. Farrar [38,c], F. Fenu [11,16,c], S. Ferrarese [11,c], T. Fujii [39,c], D. Gardiol [16,35,c], M. Gritsevich [40,41,c], P. Homola [42,c], T. Huege [4,43,c], K.-H. Kampert [44,c], D. Kang [4,c], E. Kido [45,c], P. Klimov [46,c], K. Kotera [43,47,c], B. Kozelov [48,c], A. Leszczyńska [1,5,c], J. Madsen [23,c], L. Marcelli [34,c], M. Marisaldi [49,c], O. Martineau-Huynh [50,c], S. Mayotte [3,c], K. Mulrey [21,c], K. Murase [51,52,c], M.S. Muzio [51,c], S. Ogio [28,c], A.V. Olinto [2,c], Y. Onel [53,c], T. Paul [30,c], L. Piotrowski [54,c], M. Plum [55,c], B. Pont [21,c], M. Reininghaus [4,c], B. Riedel [23,c], F. Riehn [36,c], M. Roth [4,c], T. Sako [56,c], F. Schlüter [4,57,c], D.H. Shoemaker [58,c], J. Sidhu [59,c], I. Sidelnik [20,c], C. Timmermans [21,60,c], O. Tkachenko [4,c], D. Veberic [4,c], S. Verpoest [61,c], V. Verzi [34,c], J. Vícha [26,c], D. Winn [53,c], E. Zas [8,c], M. Zotov [46,c]



[A. Coleman et al., Astropart. Phys. 147 (2023)]

Outline

Lecture 1 (Monday, 9 am):

- Introduction to Astroparticle Physics
- Cosmic Rays
- Air shower Physics

→ Theory / Phenomenology

Lecture 2 (Monday, 2 pm):

- Indirect Detection of Cosmic Rays
- Recent Results and Open Questions

→ Experiment

Lecture 3 (Tuesday, 9 am):

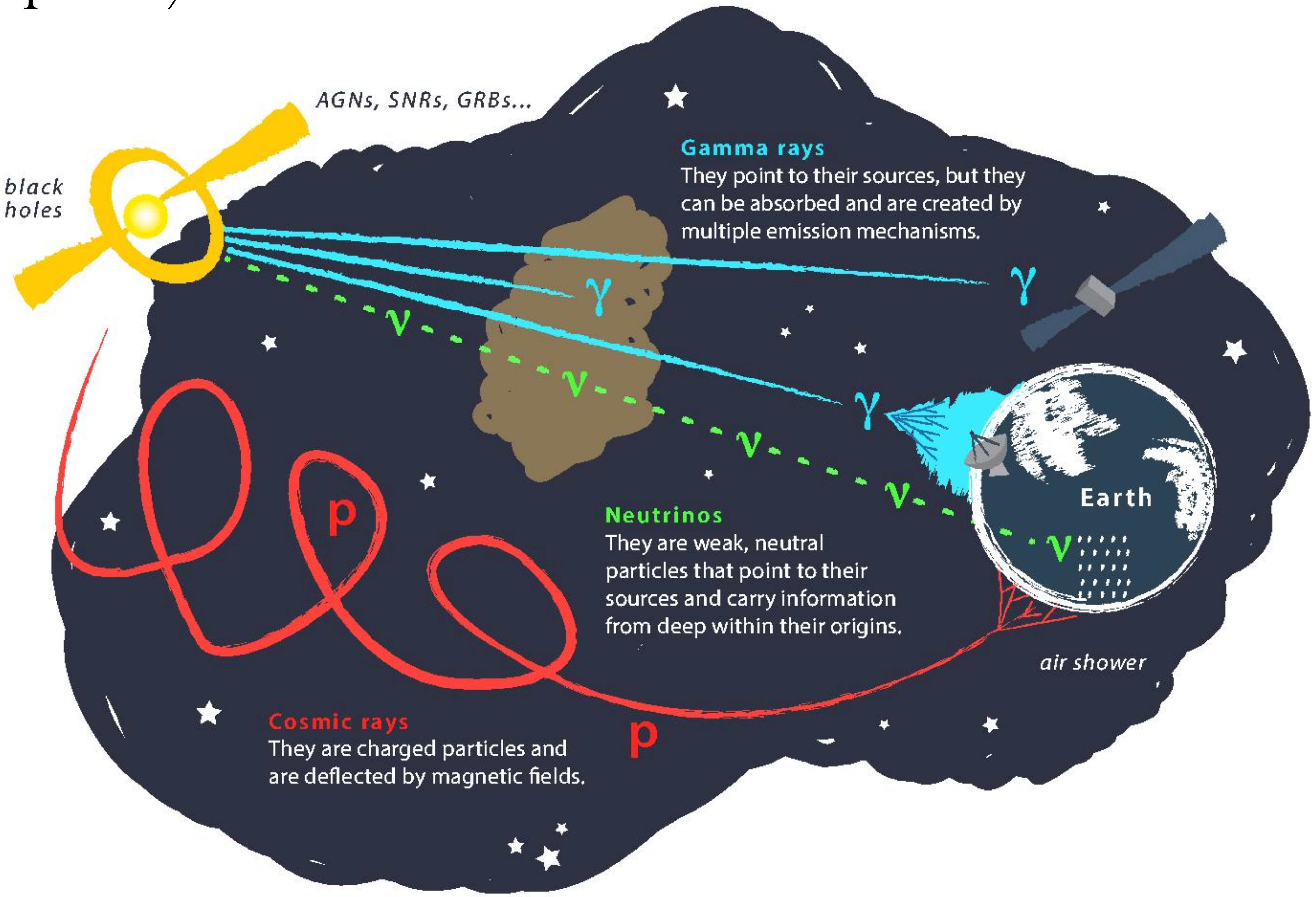
- (Possible continuation)
- Exercise & Discussion!

→ Discussion

Astroparticle Physics

<u>What is astroparticle physics?</u>

- Studies of elementary particles of astrophysical origin and their relation to astrophysics and cosmology (Wikipedia)
- <u>Astroparticles:</u>
  - Cosmic Rays
  - Neutrinos
  - Gamma Rays
  - Dark Matter?
- Let's start with <u>cosmic rays</u>!



Cosmic Rays

Cosmic Rays

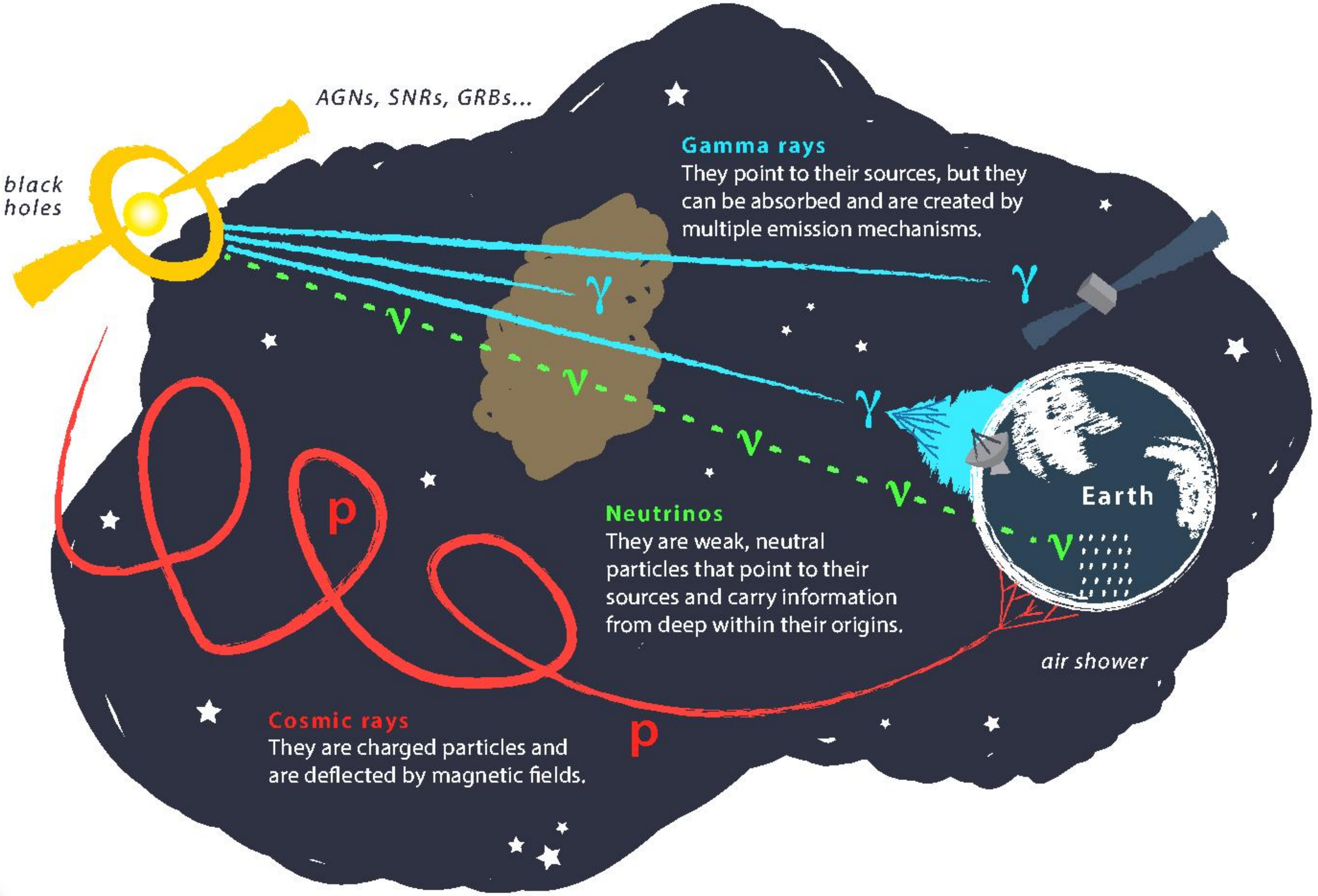
- D. Pacini (1910):
  - Ionization in the atmosphere is due to extra-terrestrial radiation
- V. Hess (1911/12, Nobel prize 1936):
  - First prove that radiation is of extra-terrestrial origin
- Many experiments followed over the last 100 years…
  - Comic rays (CRs) are charged particles, mostly protons, which reach Earth from Space
  - CRs can have extremely high energies…
- However, many open questions remain after more than 100 years of research!

[picture credit: www.wikipedia.org]

Open Questions

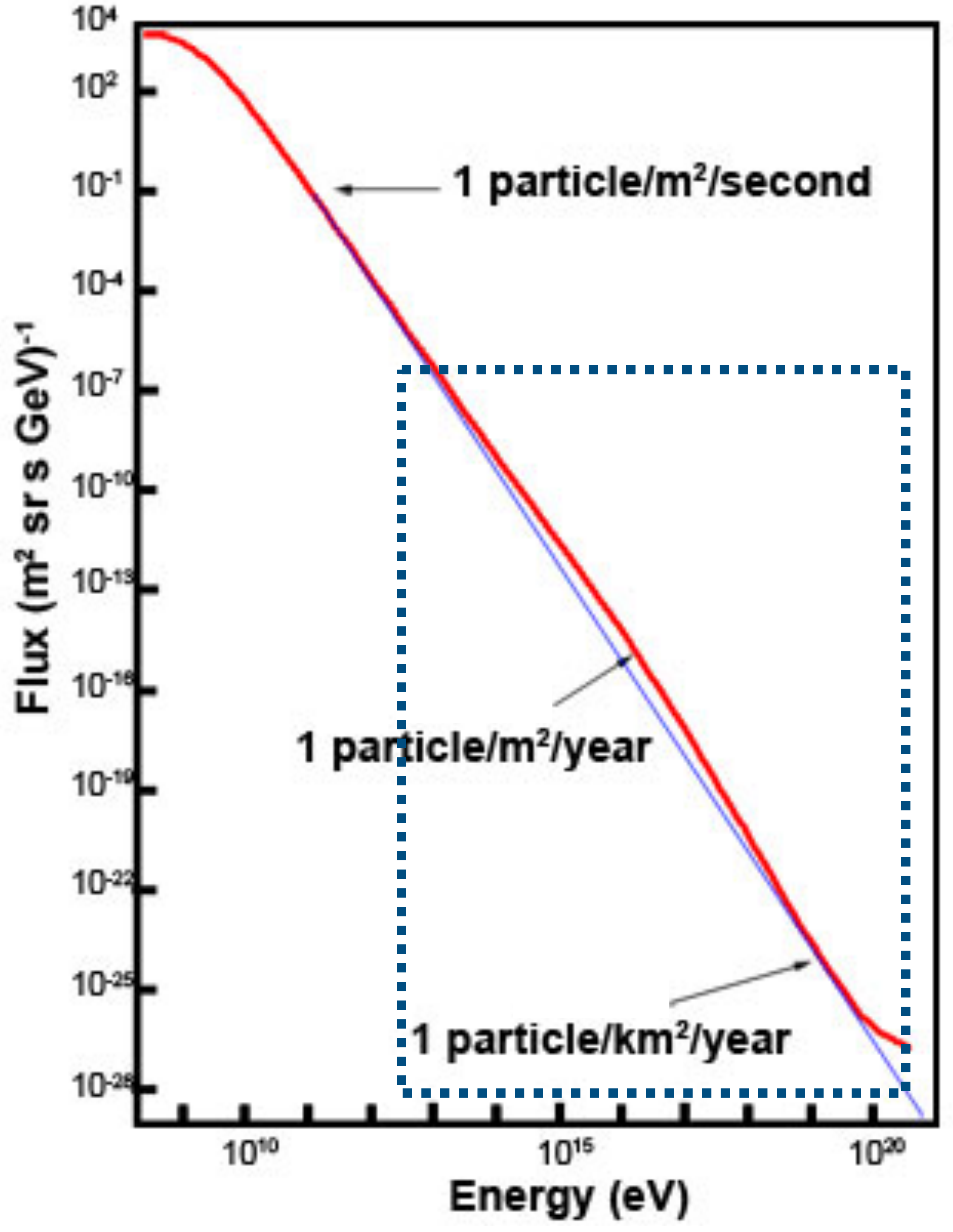
- What are the sources of high-energy CRs?
- What are the acceleration mechanisms of CRs?
- What is their mass composition? (later more…)
- What is the origin of features observed in the CR spectrum? (later more…)
- …

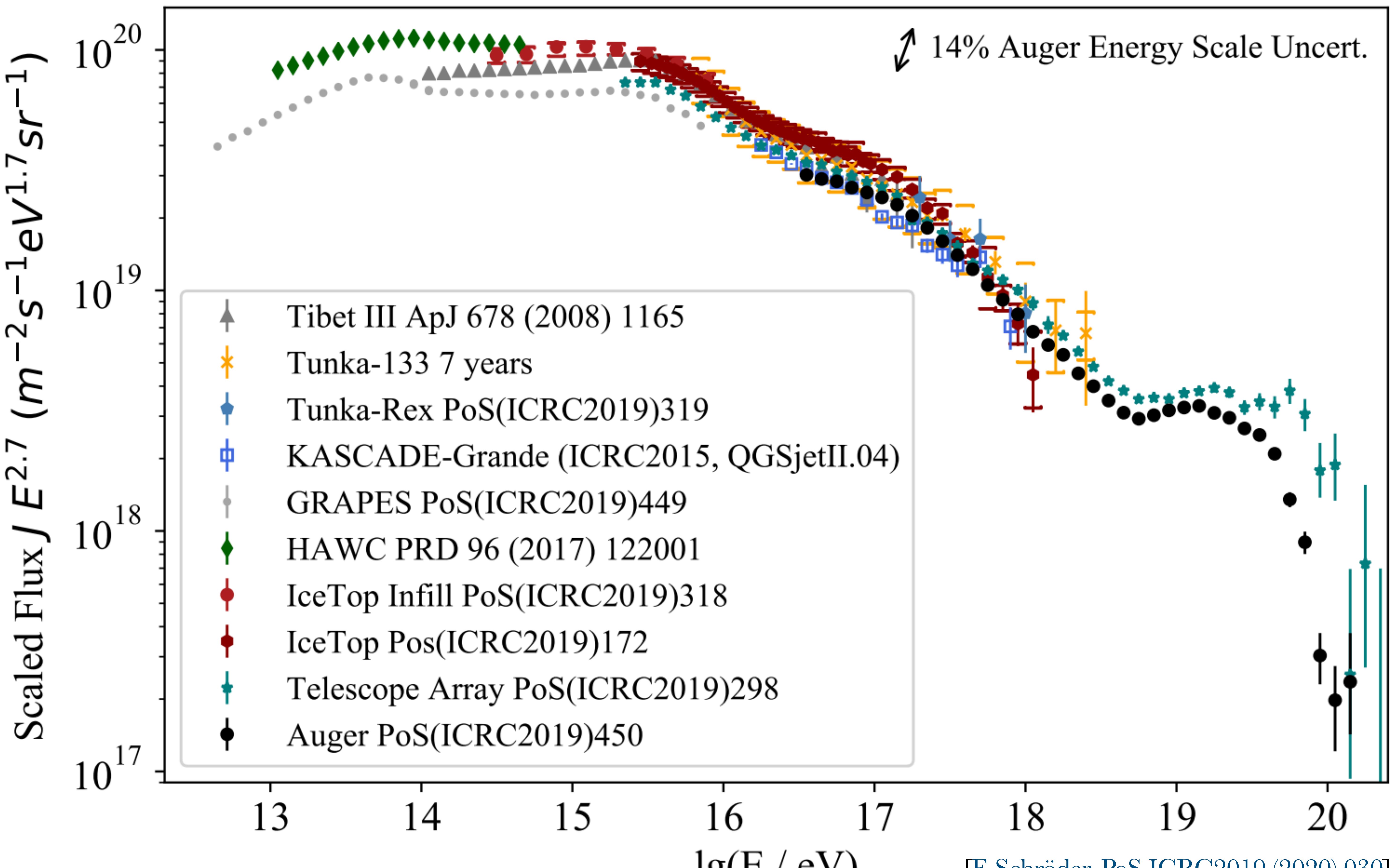
Can only be answered with multimessenger observations!



Cosmic Rays

- Comic rays (CRs) are dominated by atomic nuclei
- All-particle flux known over many orders of magnitude in E_0
- Various prominent features observed in spectrum

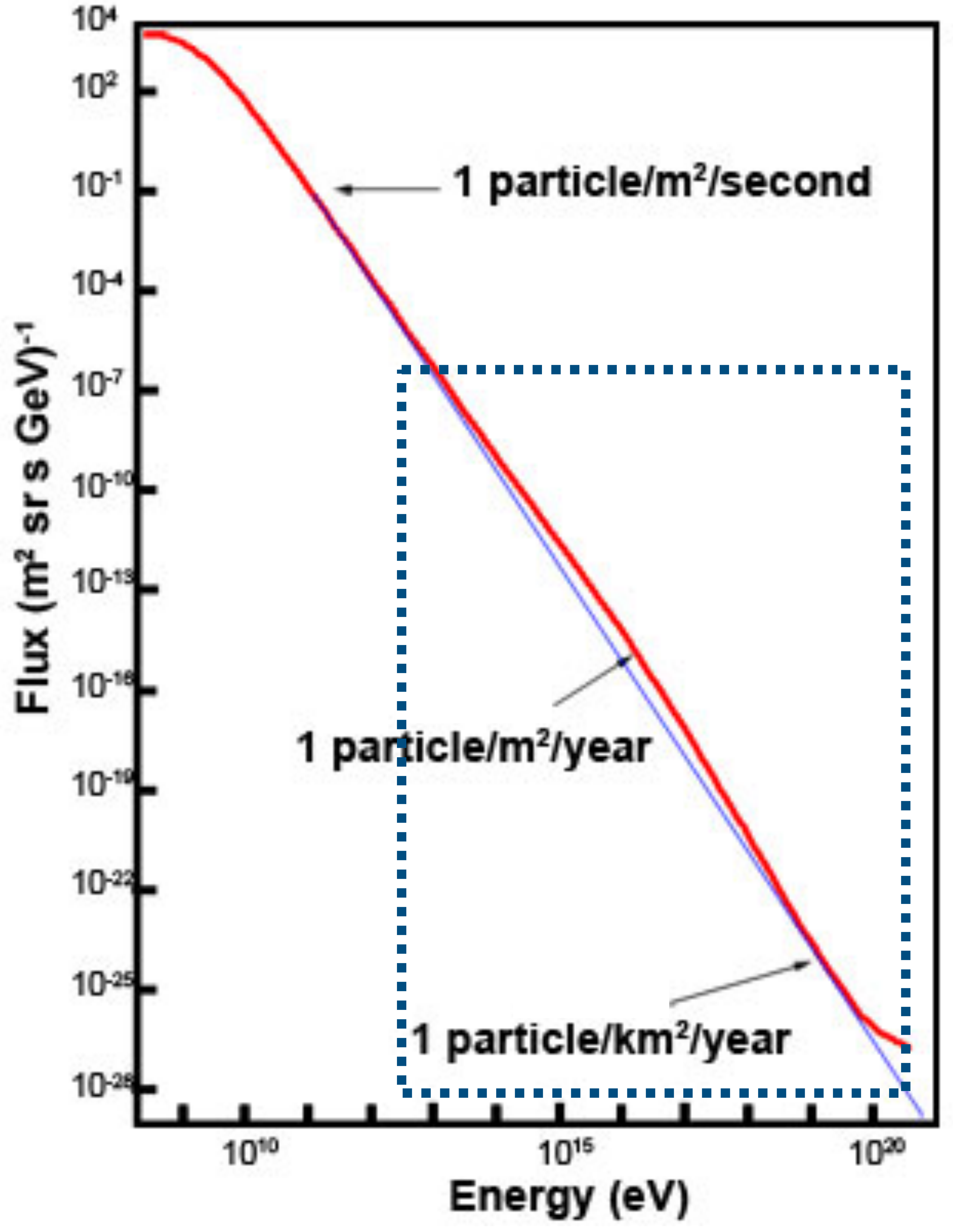


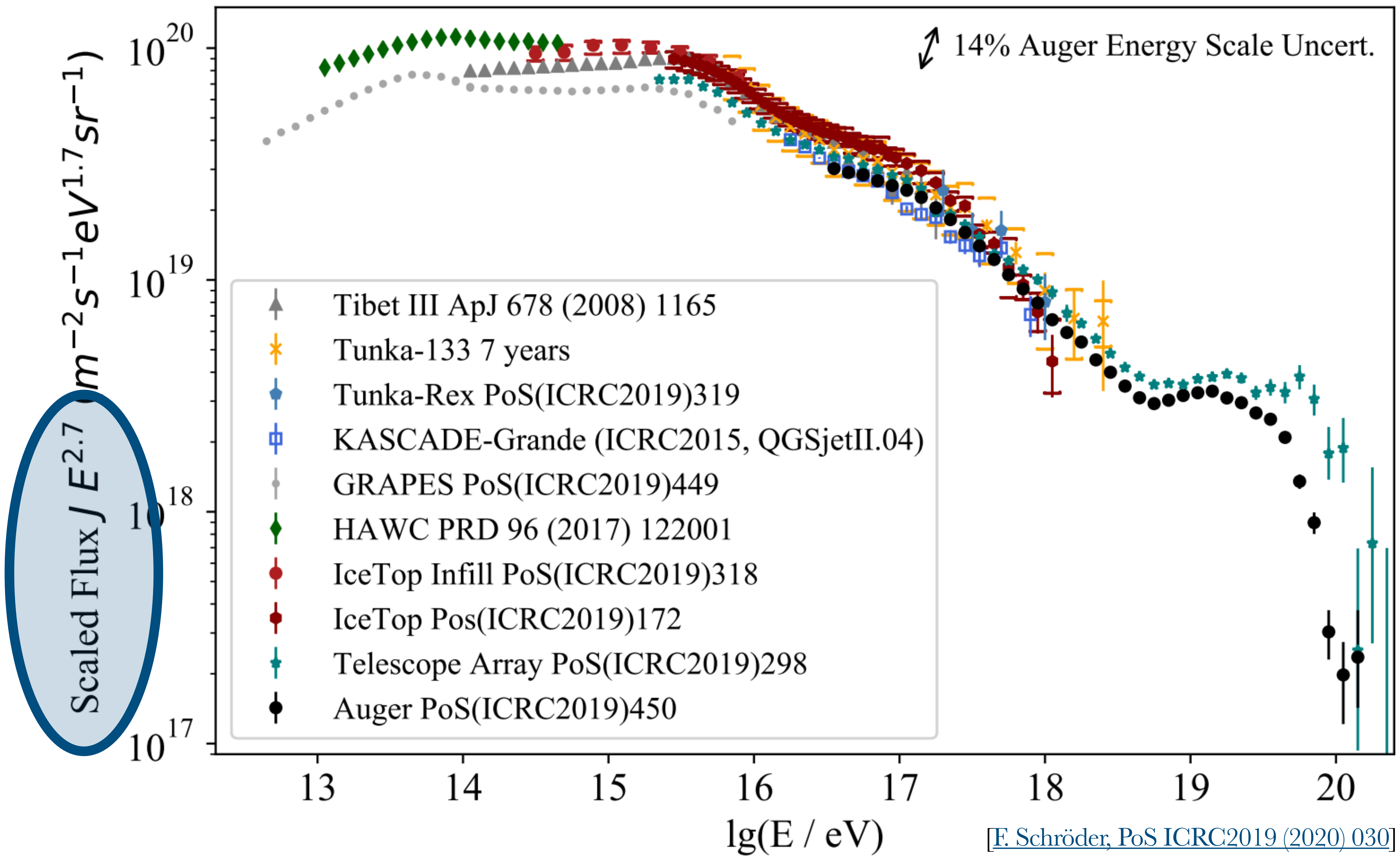


[F. Schröder, PoS ICRC2019 (2020) 030]

Cosmic Rays

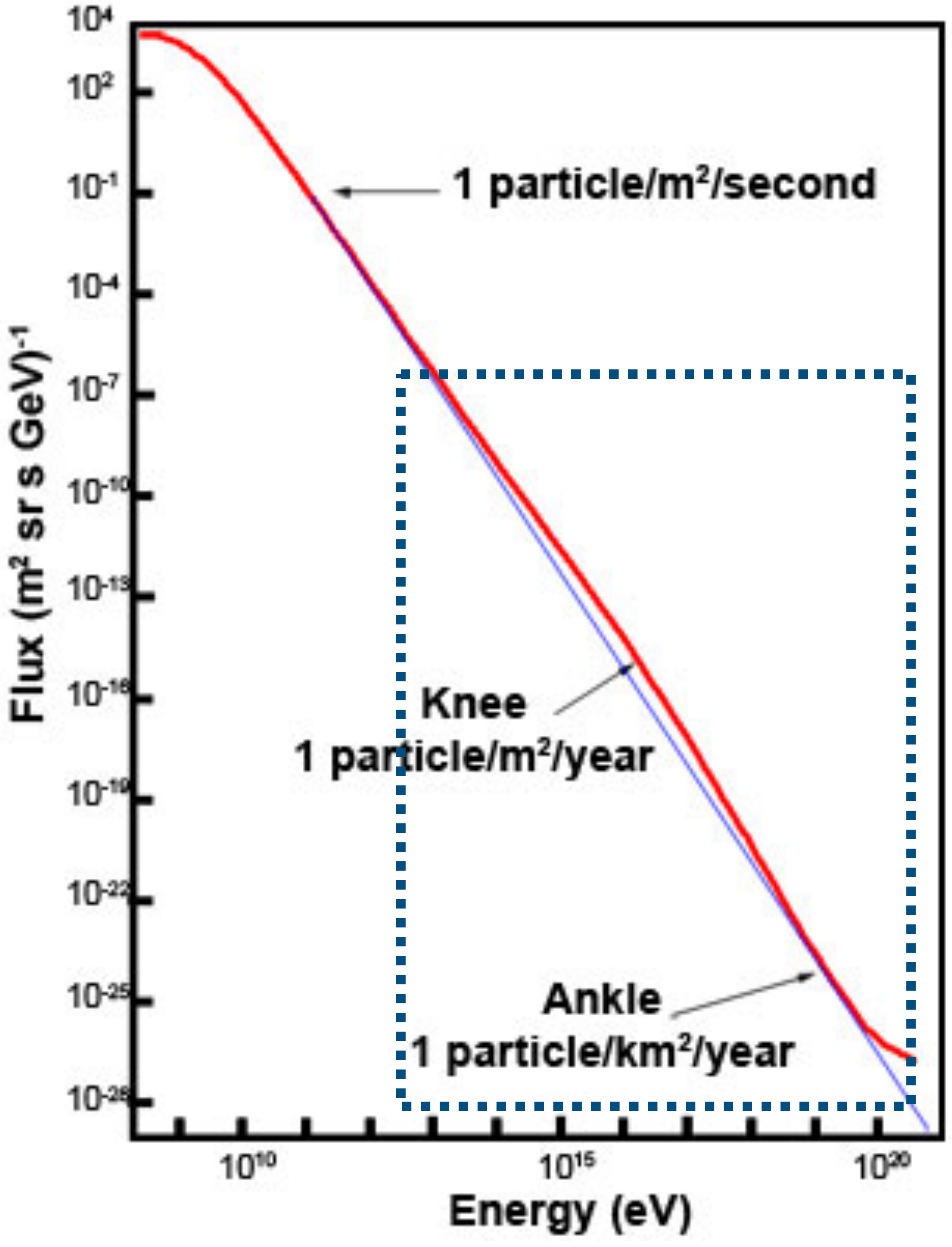
- Comic rays (CRs) are dominated by atomic nuclei
- All-particle flux known over many orders of magnitude in E_0
- Various prominent features observed in spectrum

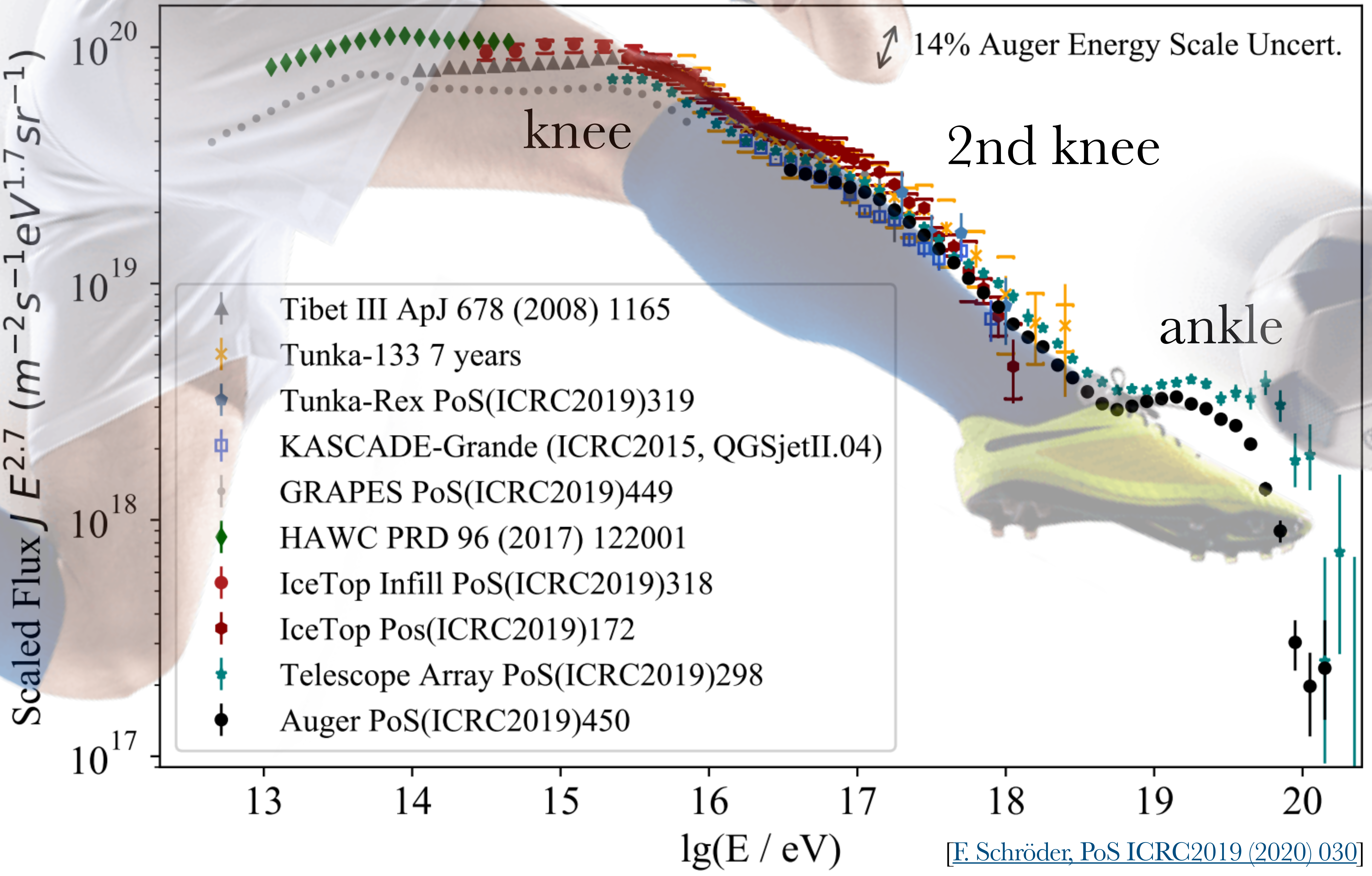




[F. Schröder, PoS ICRC2019 (2020) 030]

Cosmic Rays





[F. Schröder, PoS ICRC2019 (2020) 030]

Cosmic Rays

First order approximation:

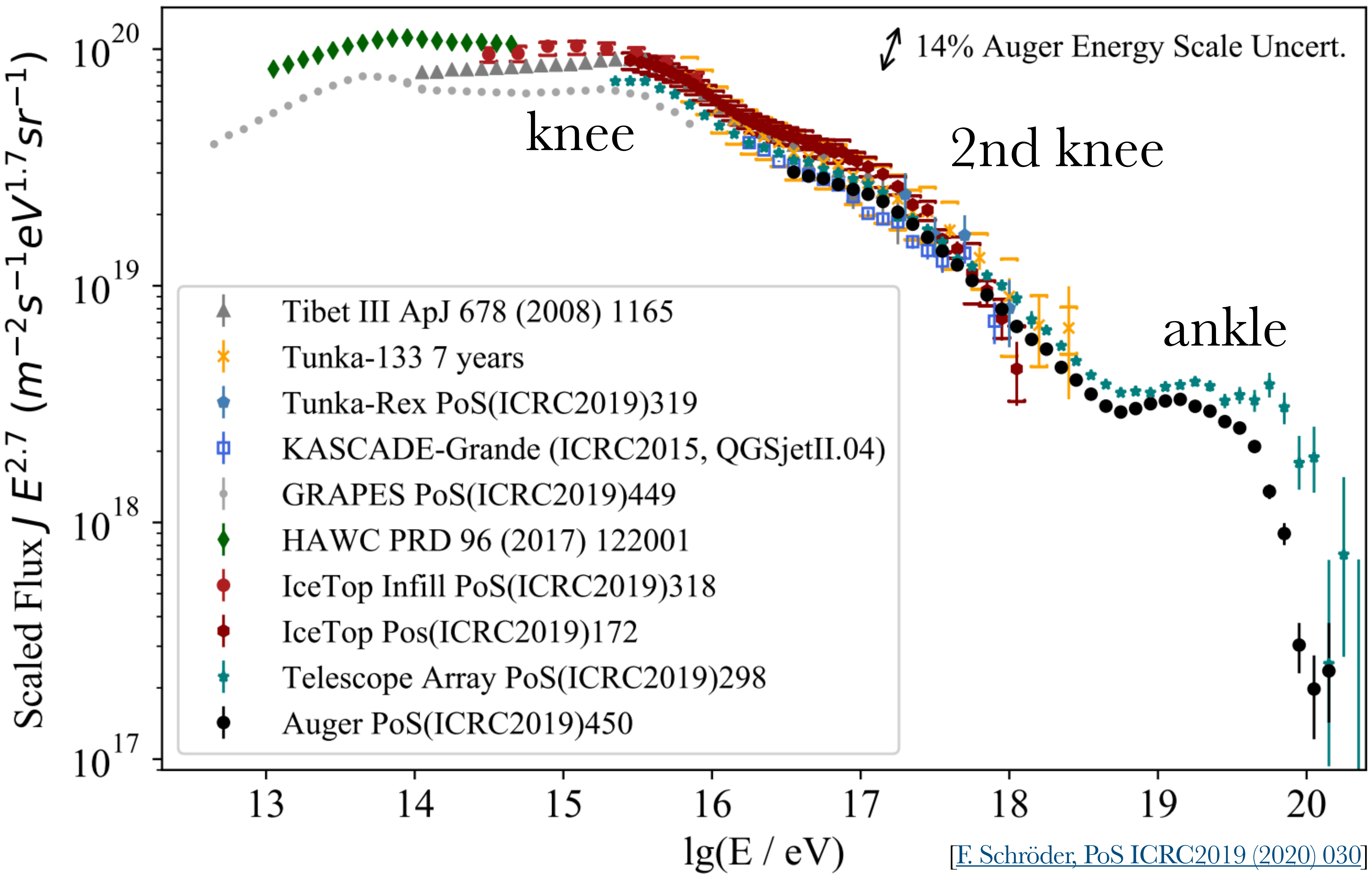
- Simple power law

$$\frac{d\Phi}{dE_0} = \frac{dN}{dtdAd\Omega dE_0} \propto E_0^{-\gamma}$$

- Spectral index γ
- Simple approximation

$$\frac{d\Phi}{dE_0} \simeq 1.8 \cdot E_0^{-\gamma} \frac{\text{nucleons}}{\text{cm}^2\,\text{s}\,\text{sr}\,\text{GeV/A}}$$

- More in the exercise!



[F. Schröder, PoS ICRC2019 (2020) 030]

Cosmic Rays

First order approximation:

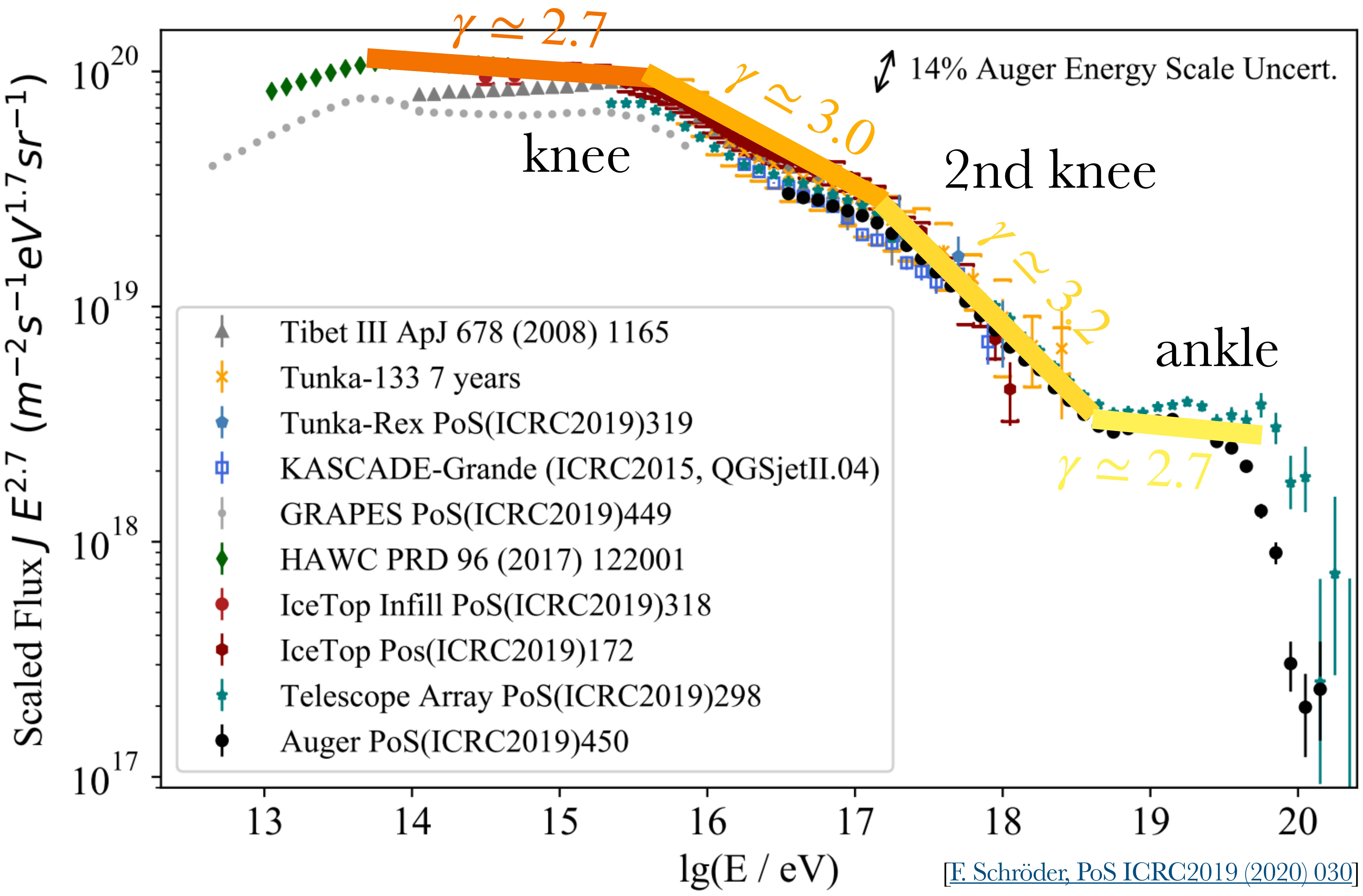
- Simple power law

$$\frac{d\Phi}{dE_0} = \frac{dN}{dt dA d\Omega dE_0} \propto E_0^{-\gamma}$$

- Spectral index γ
- Simple approximation

$$\frac{d\Phi}{dE_0} \simeq 1.8 \cdot E_0^{-\gamma} \frac{\text{nucleons}}{\text{cm}^2\,\text{s}\,\text{sr}\,\text{GeV/A}}$$

  - $\gamma \simeq 2.7$ up to $E_{\text{knee}} \simeq 4\,\text{PeV}$
  - $\gamma \simeq 3.0$ up to $E_{\text{2nd knee}} \simeq 0.6\,\text{EeV}$
  - $\gamma \simeq 3.2$ up to $E_{\text{2nd knee}} \simeq 0.6\,\text{EeV}$
  - $\gamma \simeq 2.7$ above $E_{\text{ankle}} \simeq 4\,\text{EeV}$
- Mass number A



[F. Schröder, PoS ICRC2019 (2020) 030]

Cosmic Rays

First order approximation:

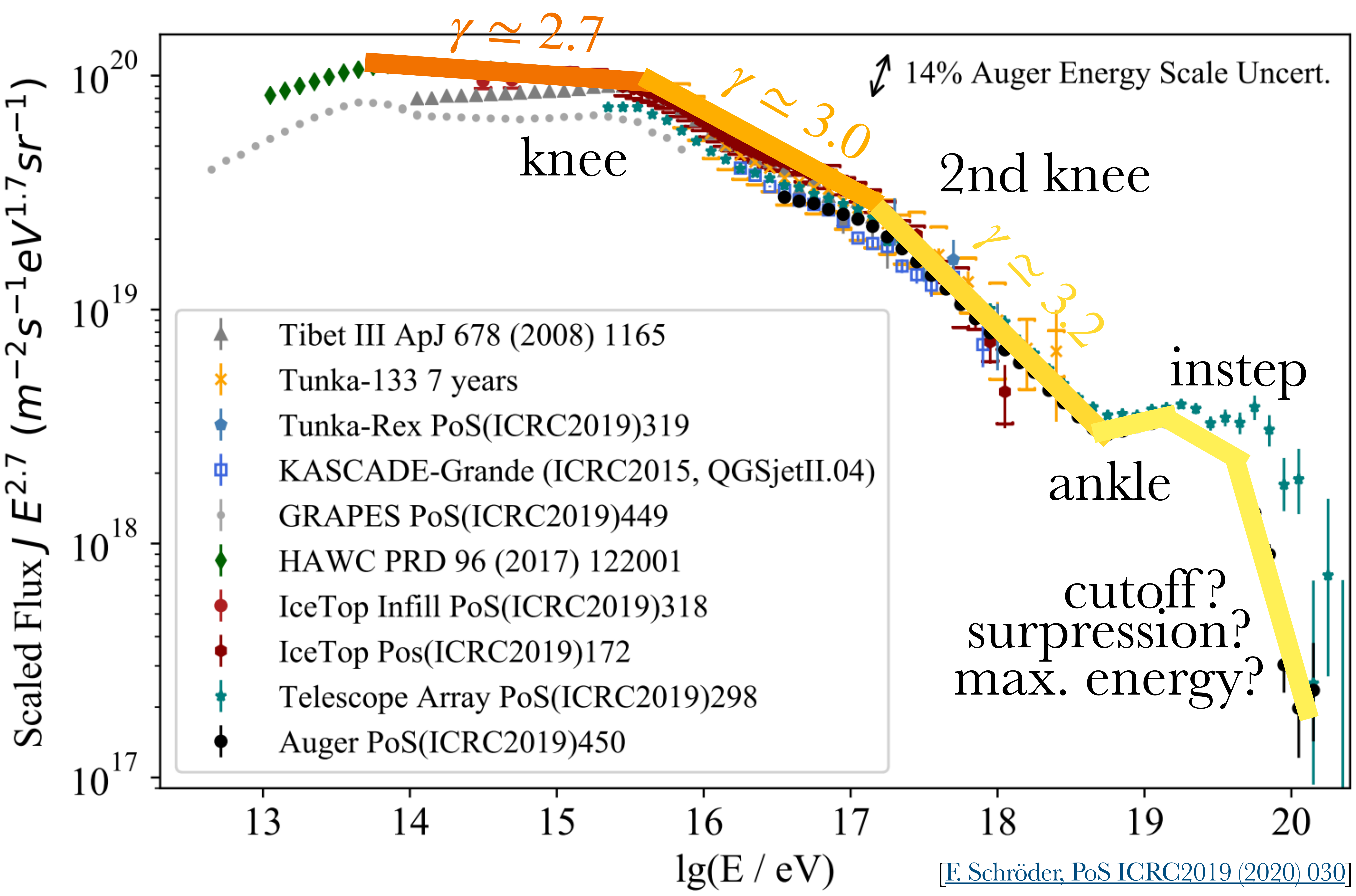
- Simple power law

$$\frac{d\Phi}{dE_0} = \frac{dN}{dt dA d\Omega dE_0} \propto E_0^{-\gamma}$$

- Spectral index γ
- Simple approximation

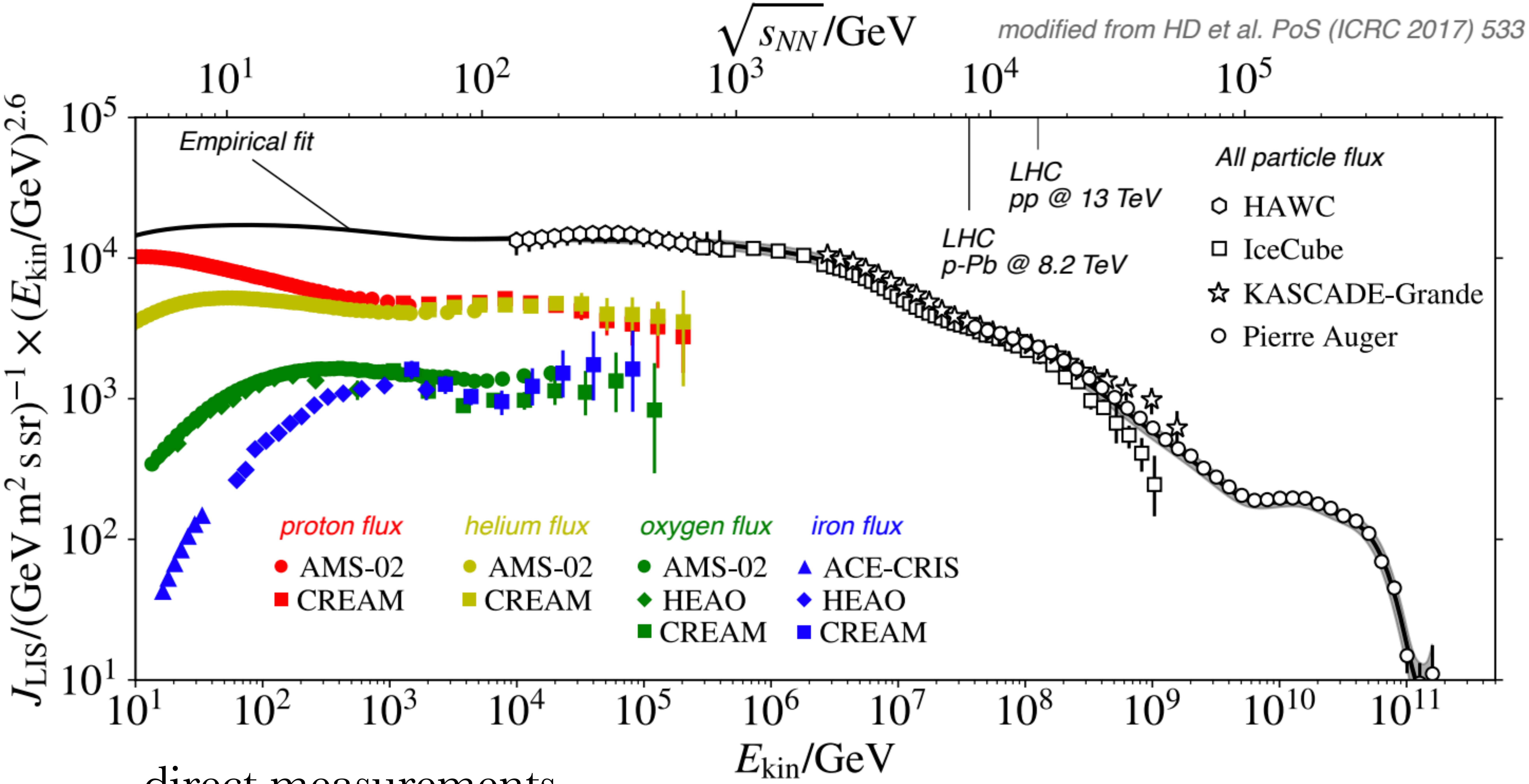
$$\frac{d\Phi}{dE_0} \simeq 1.8 \cdot E_0^{-\gamma} \frac{\text{nucleons}}{\text{cm}^2\,\text{s}\,\text{sr}\,\text{GeV/A}}$$

  - $\gamma \simeq 2.7$ up to $E_{\text{knee}} \simeq 4\,\text{PeV}$
  - $\gamma \simeq 3.0$ up to $E_{\text{2nd knee}} \simeq 0.6\,\text{EeV}$
  - $\gamma \simeq 3.2$ up to $E_{\text{2nd knee}} \simeq 0.6\,\text{EeV}$
  - $\gamma \simeq 2.7$ above $E_{\text{ankle}} \simeq 4\,\text{EeV}$
- Mass number A



[F. Schröder, PoS ICRC2019 (2020) 030]

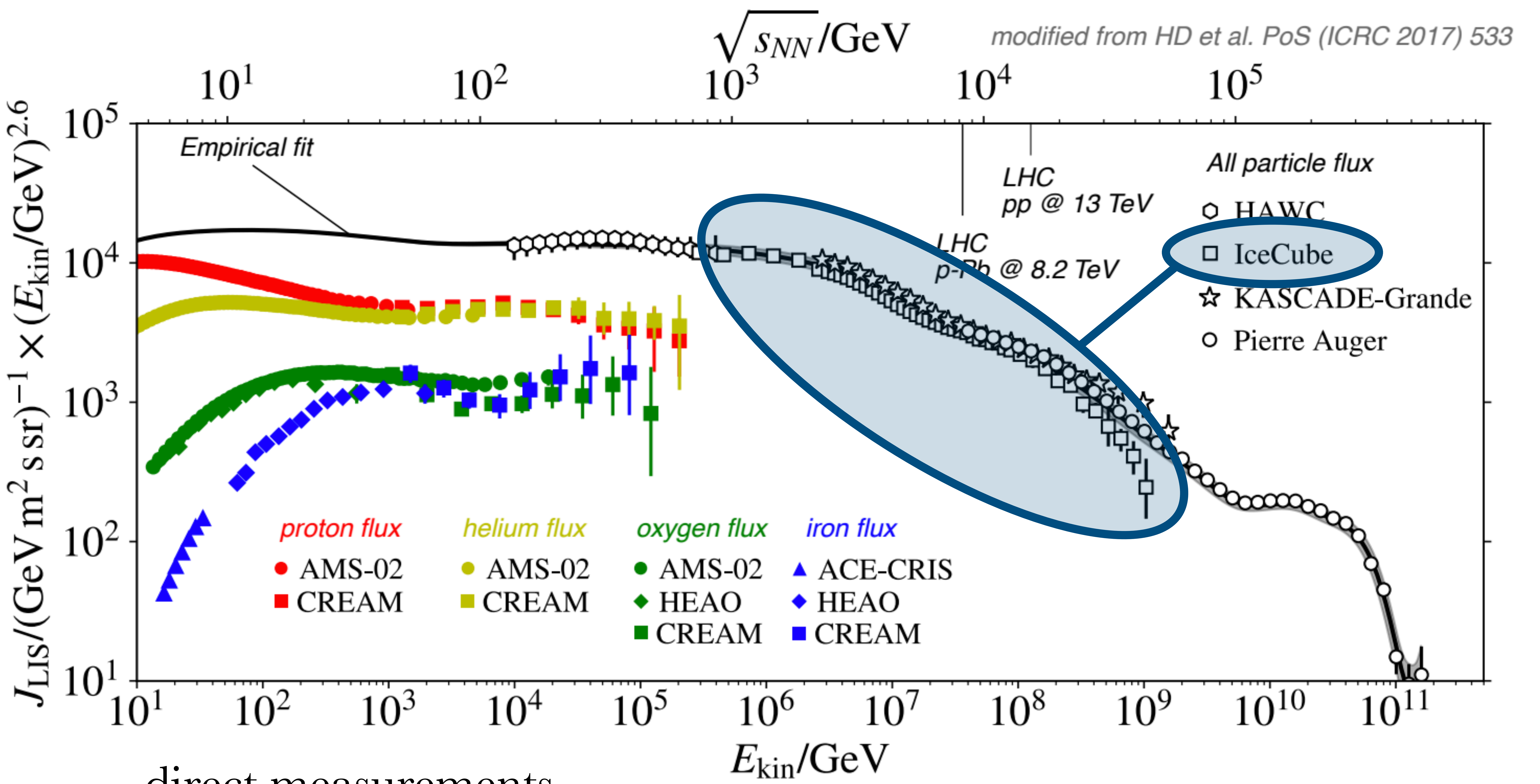
CR All-Particle Spectrum



direct measurements
(satellite or balloon experiments)

indirect measurements
(large ground-based experiments)

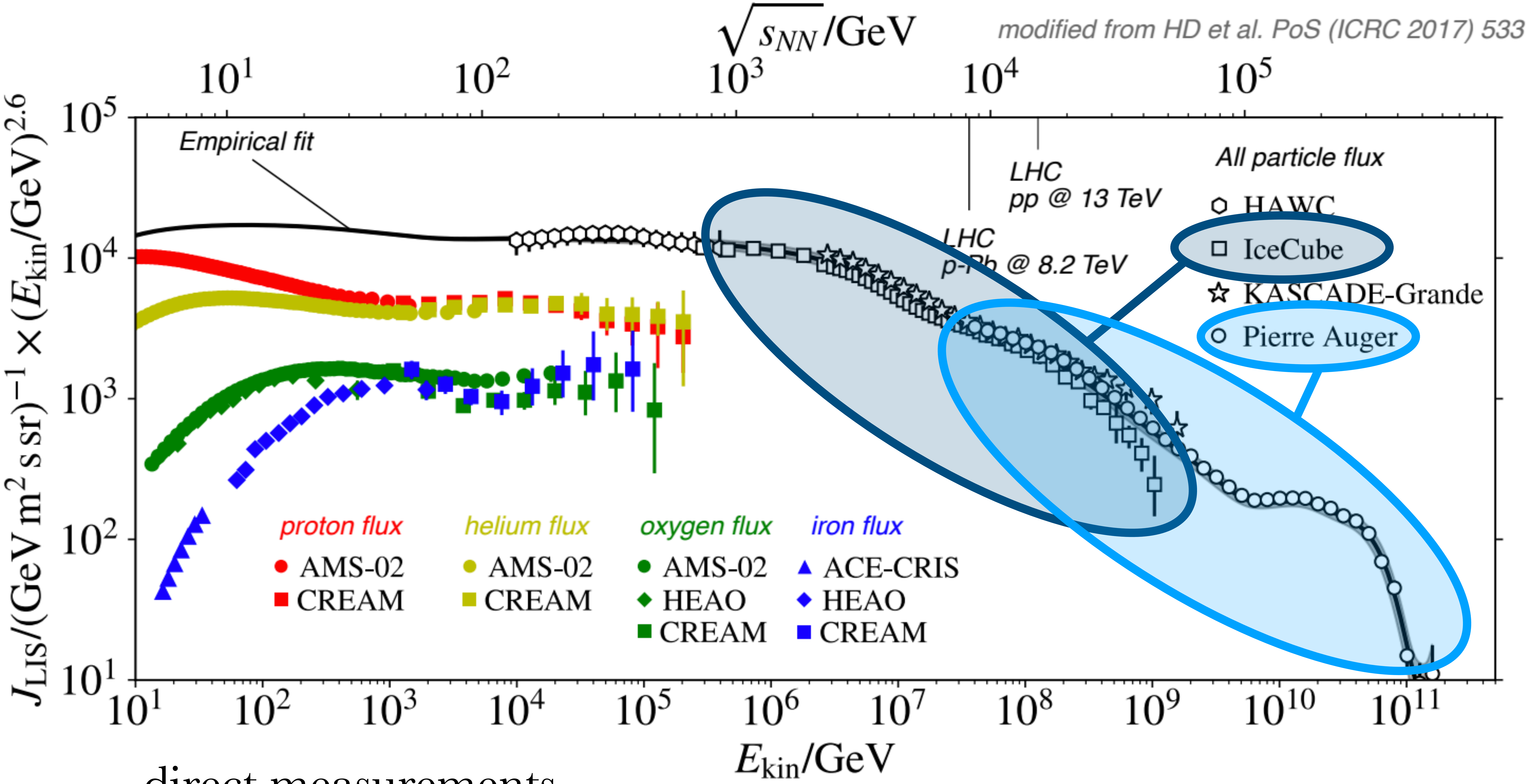
CR All-Particle Spectrum



direct measurements
(satellite or balloon experiments)

indirect measurements
(large ground-based experiments)

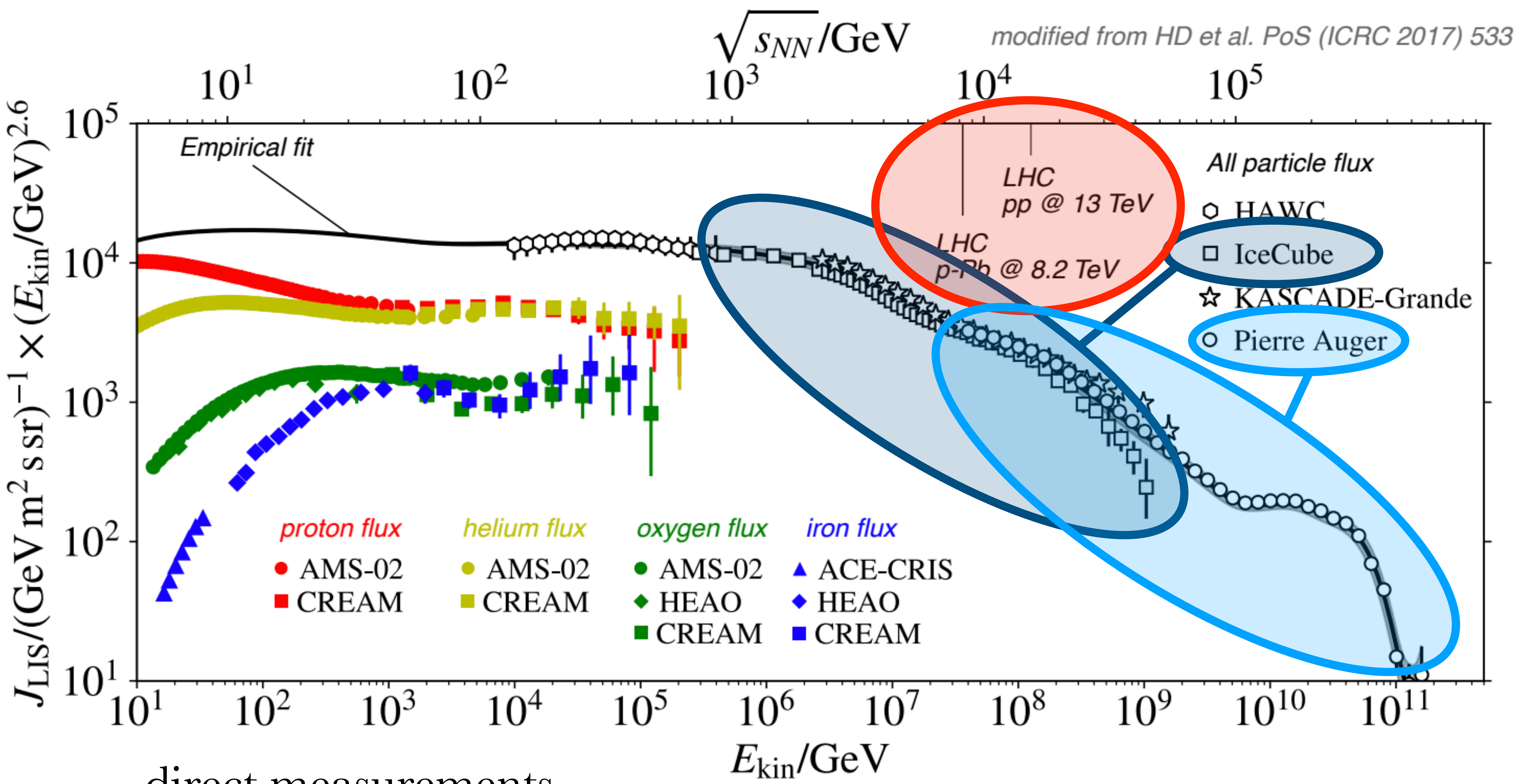
CR All-Particle Spectrum



direct measurements
(satellite or balloon experiments)

indirect measurements
(large ground-based experiments)

CR All-Particle Spectrum



direct measurements
(satellite or balloon experiments)

indirect measurements
(large ground-based experiments)

Interlude: CM vs. Lab Frame

- Four-momentum:

$$\vec{P} = (E, p_x, p_y, p_z) \quad \text{(natural units)}$$

$$\Rightarrow \vec{P}\vec{P} = E^2 + \vec{p}\,\vec{p} = m^2$$

$$\vec{P}\vec{P} \text{ is conserved!}$$

- Invariant mass:

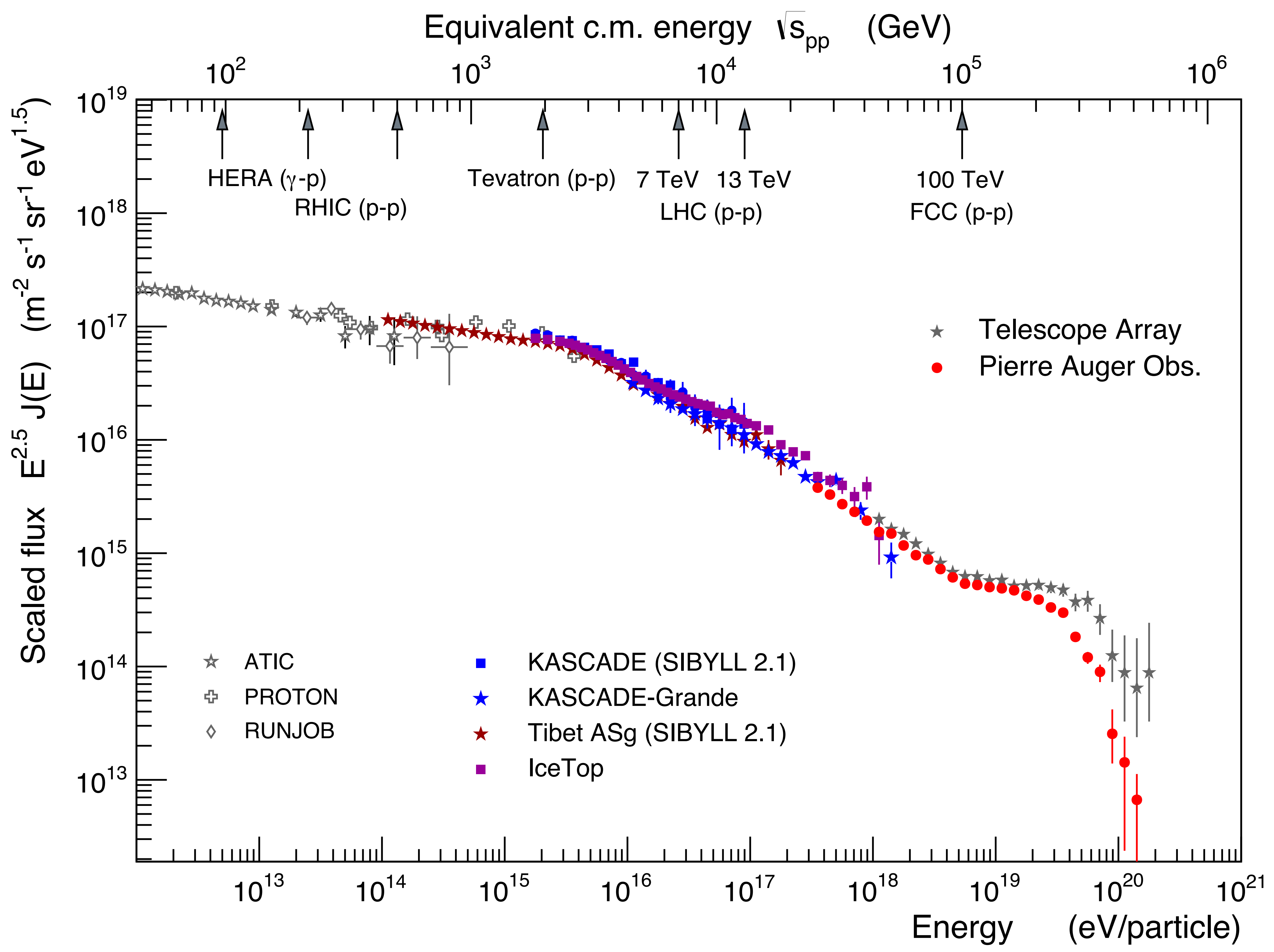
$$s = (\vec{P}_1 + \vec{P}_2)^2$$

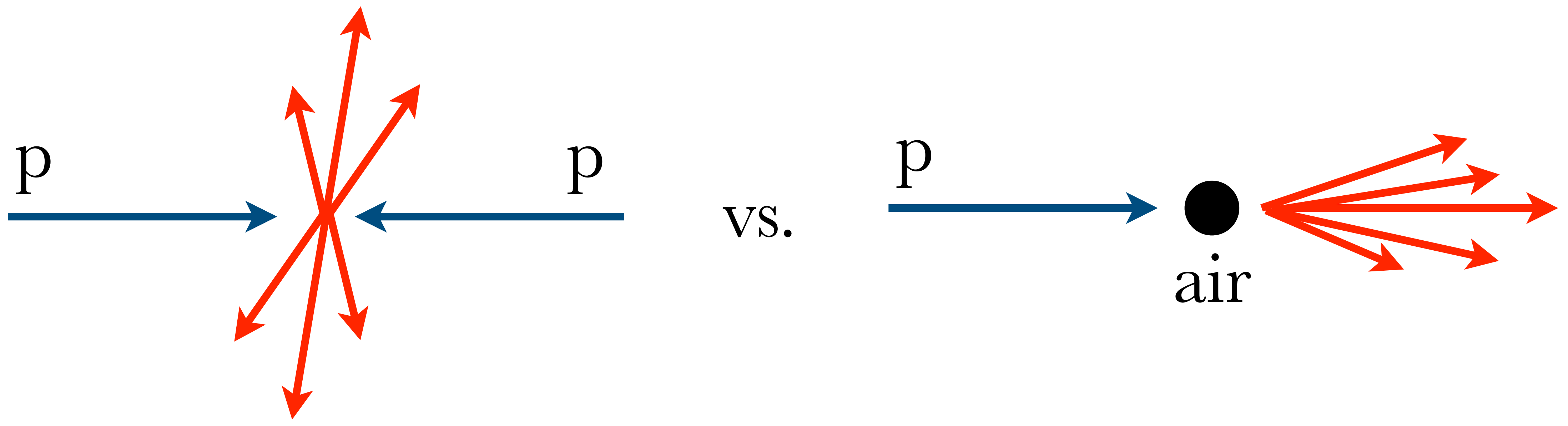
- Center-of-mass energy:

$$\vec{P}_1 = \vec{P}_2 = (E, \vec{p}) \quad \Rightarrow s = ?$$

- Laboratory energy:

$$\vec{P}_1 = (E, \vec{p}_1) \text{ and } \vec{P}_2 = (m, 0) \quad \Rightarrow s = ?$$





Interlude: CM vs. Lab Frame

- Four-momentum:

$$\vec{P} = (E, p_x, p_y, p_z) \quad \text{(natural units)}$$

$$\Rightarrow \vec{P}\vec{P} = E^2 + \vec{p}\vec{p} = m^2$$

$\vec{P}\vec{P}$ is conserved!

- Invariant mass:

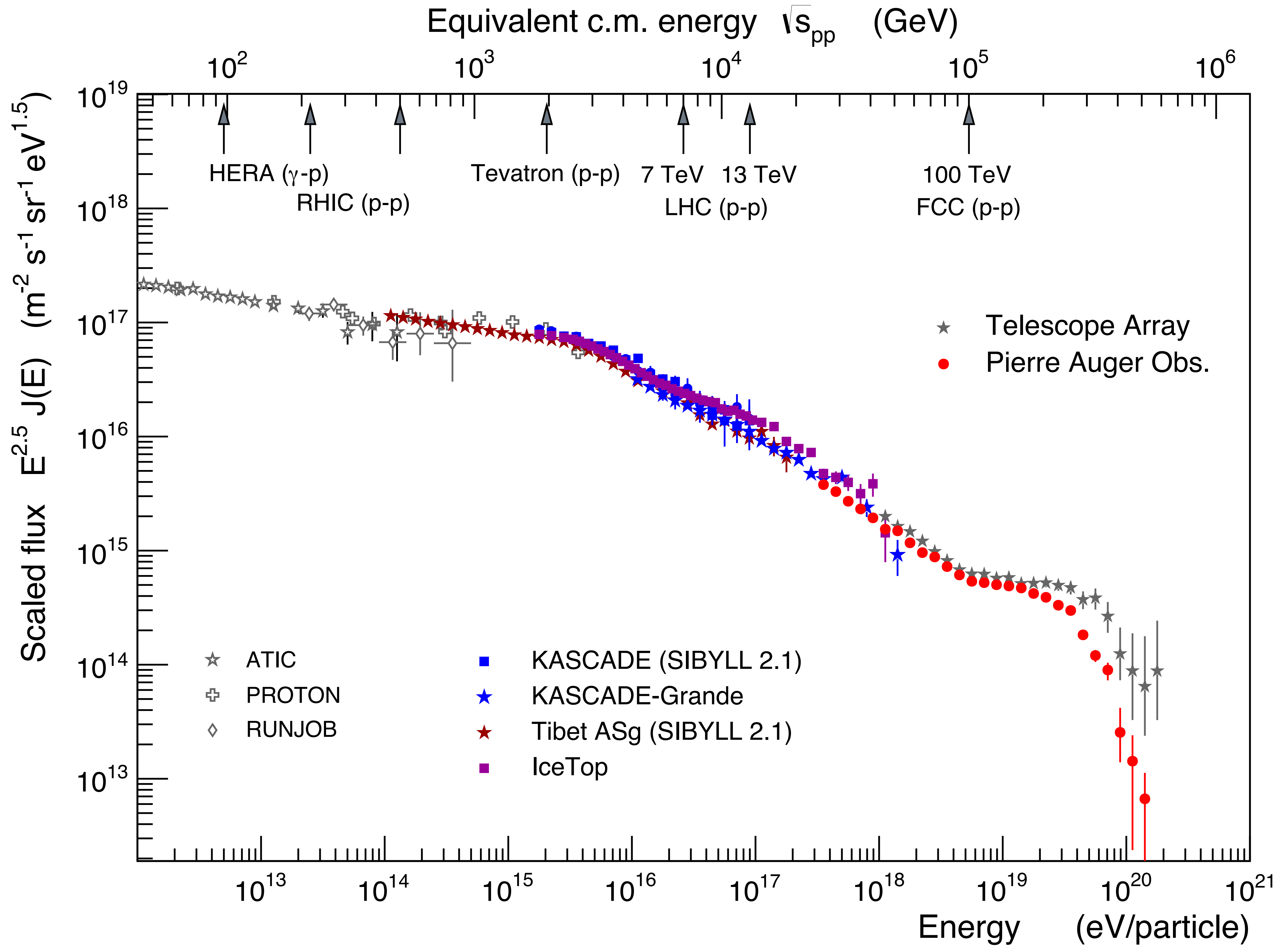
$$s = (\vec{P}_1 + \vec{P}_2)^2$$

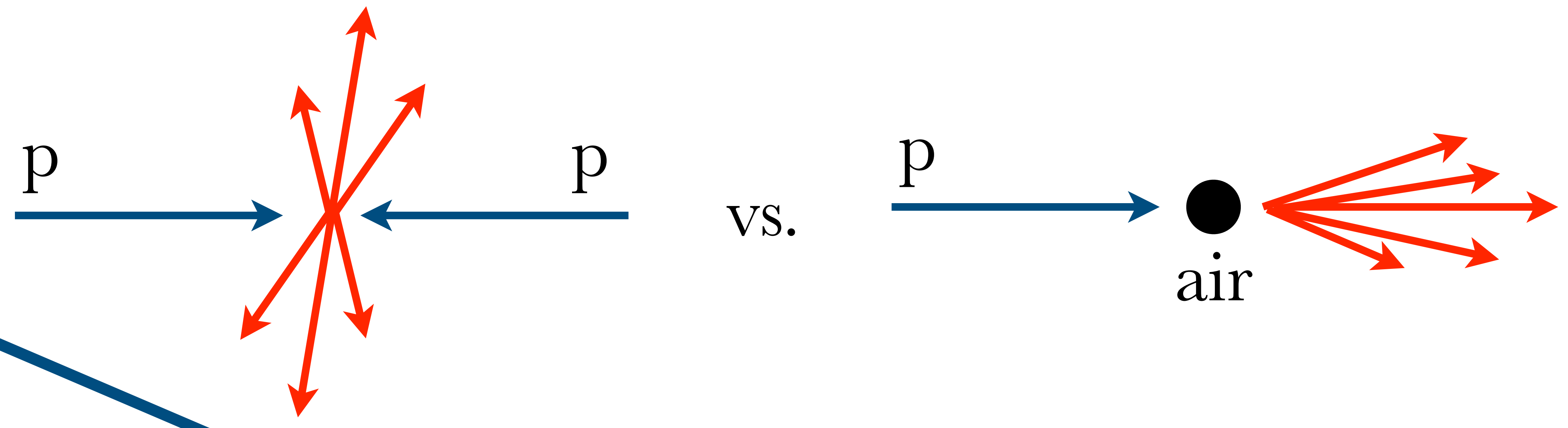
- Center-of-mass energy:

$$\vec{P}_1 = \vec{P}_2 = (E, \vec{p}) \quad \Rightarrow s = ?$$

- Laboratory energy:

$$\vec{P}_1 = (E, \vec{p}_1) \text{ and } \vec{P}_2 = (m, \vec{p}_2) \quad \Rightarrow s = ?$$

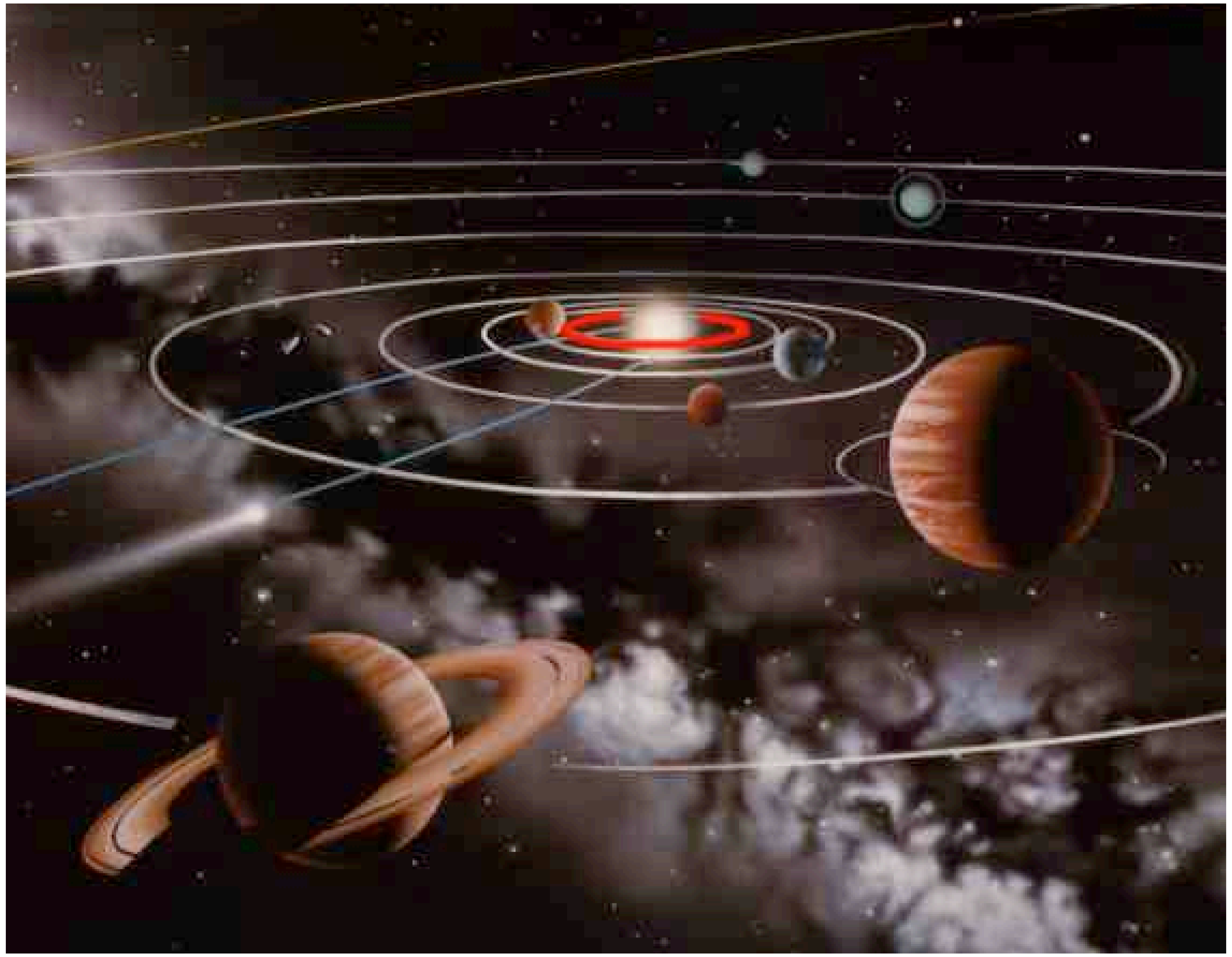




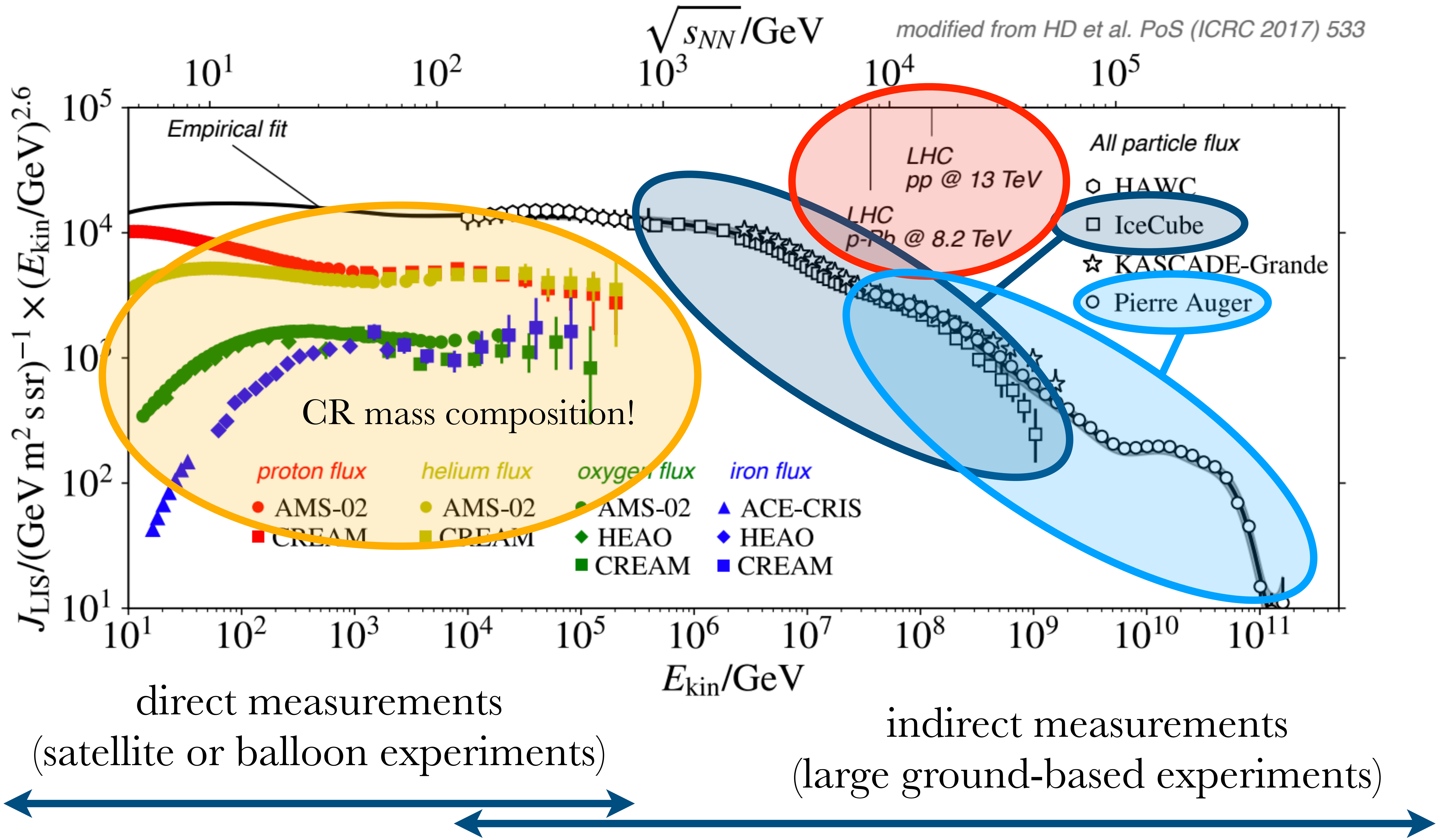
Exercise!

Interlude: Ultra-High Energy

- Large Hadron Collider (LHC), 27 km circumference, superconducting magnets
- Need accelerator of size of Mercury's orbit to reach 10^{20} eV with current technology!



CR All-Particle Spectrum



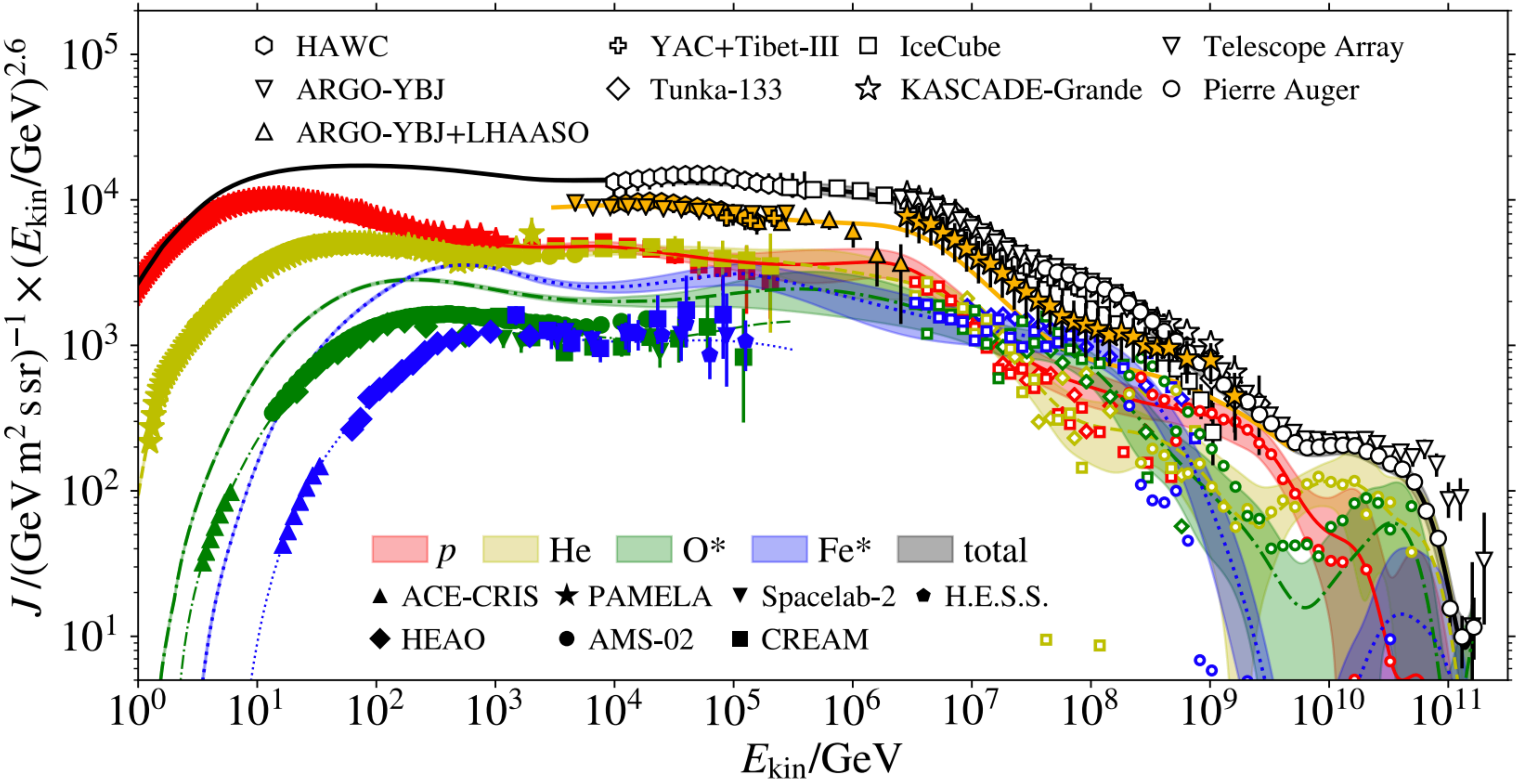
direct measurements
(satellite or balloon experiments)

indirect measurements
(large ground-based experiments)

CR Mass Composition

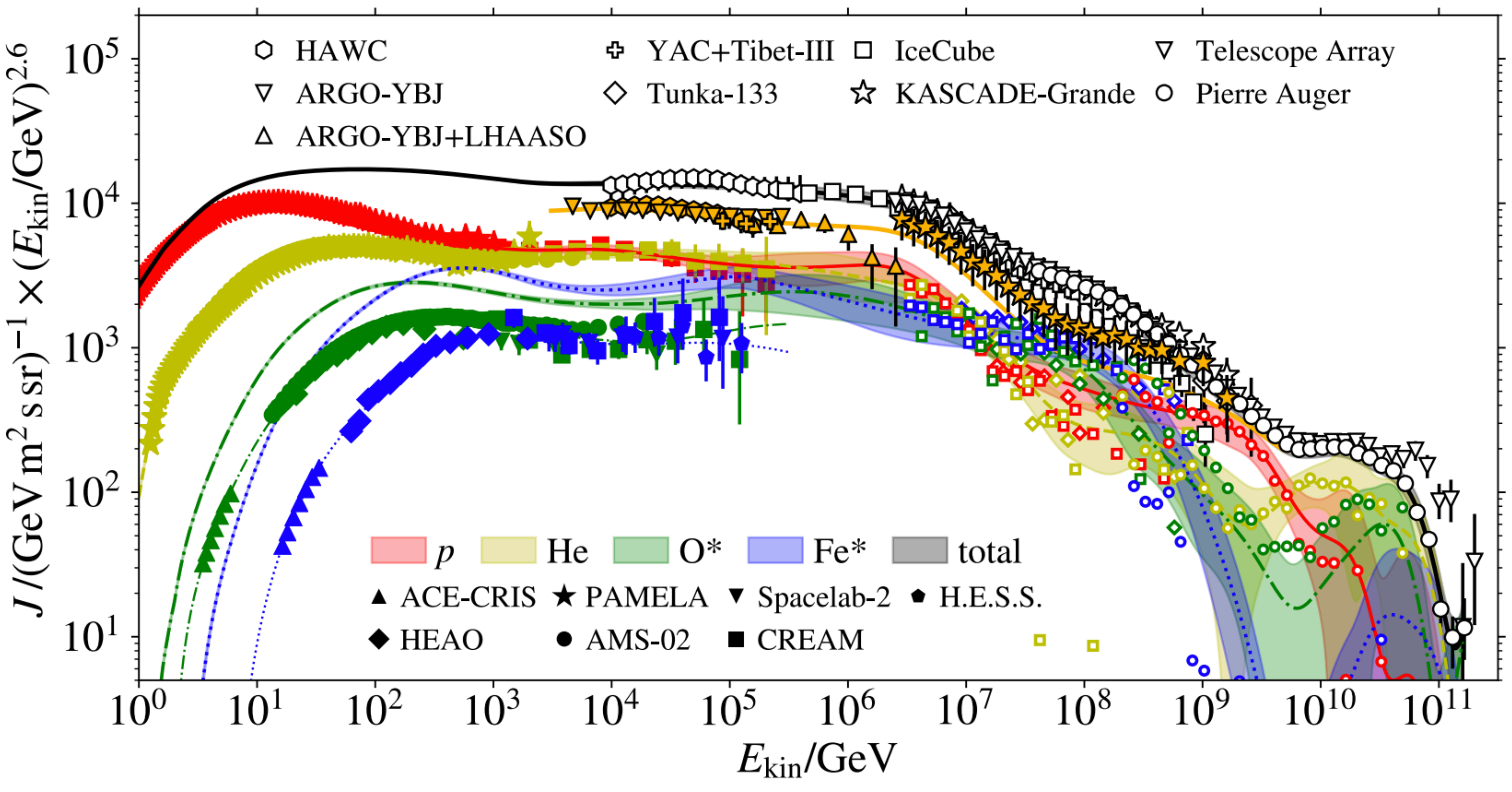
- **G**lobal **S**pline **F**it (GSF) flux model

[H.P. Dembinski, R. Engel, A. Fedynitch, T. K. Gaisser, F. Riehn, T. Stanev, PoS ICRC2017 (2017) 533]

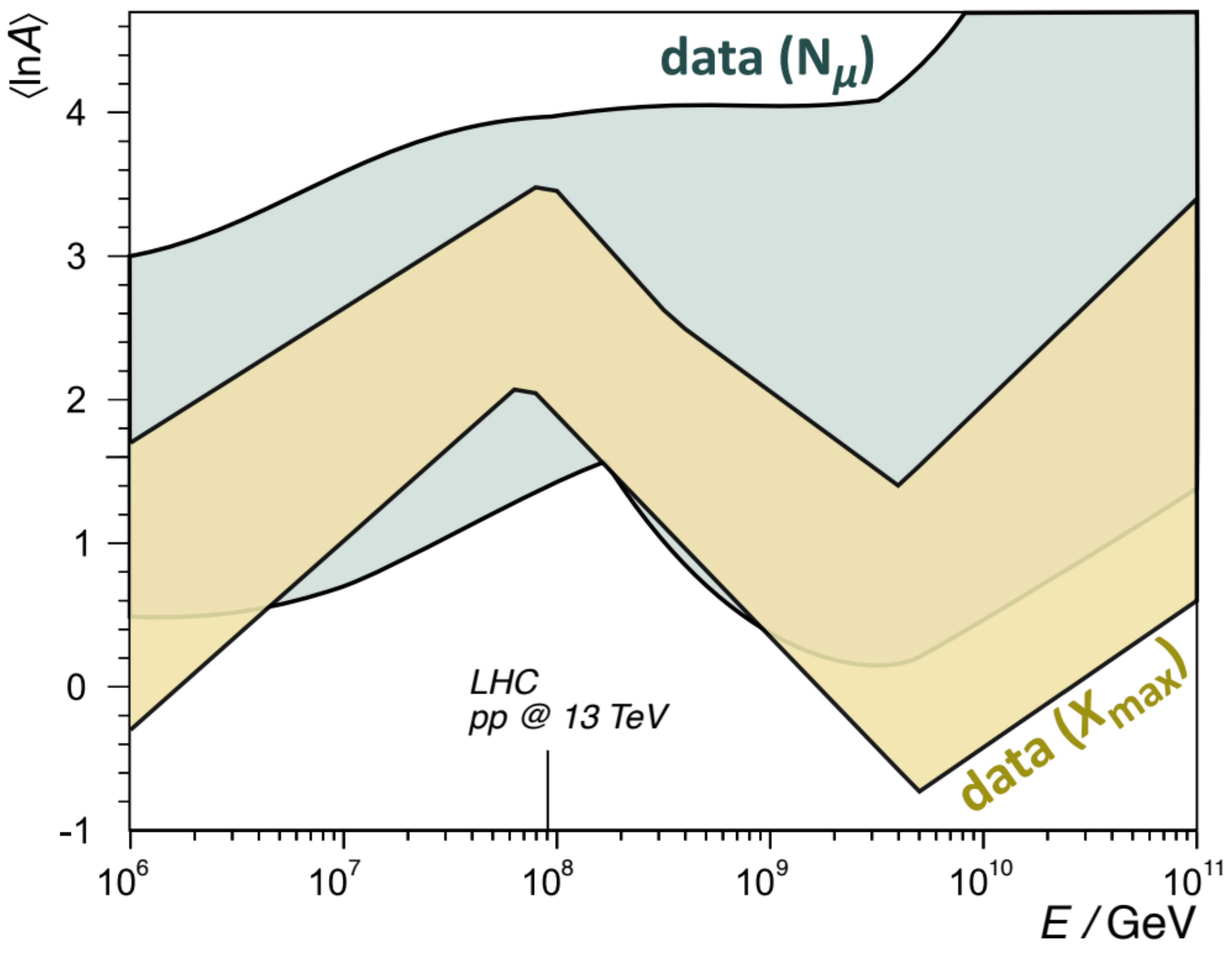


CR Mass Composition

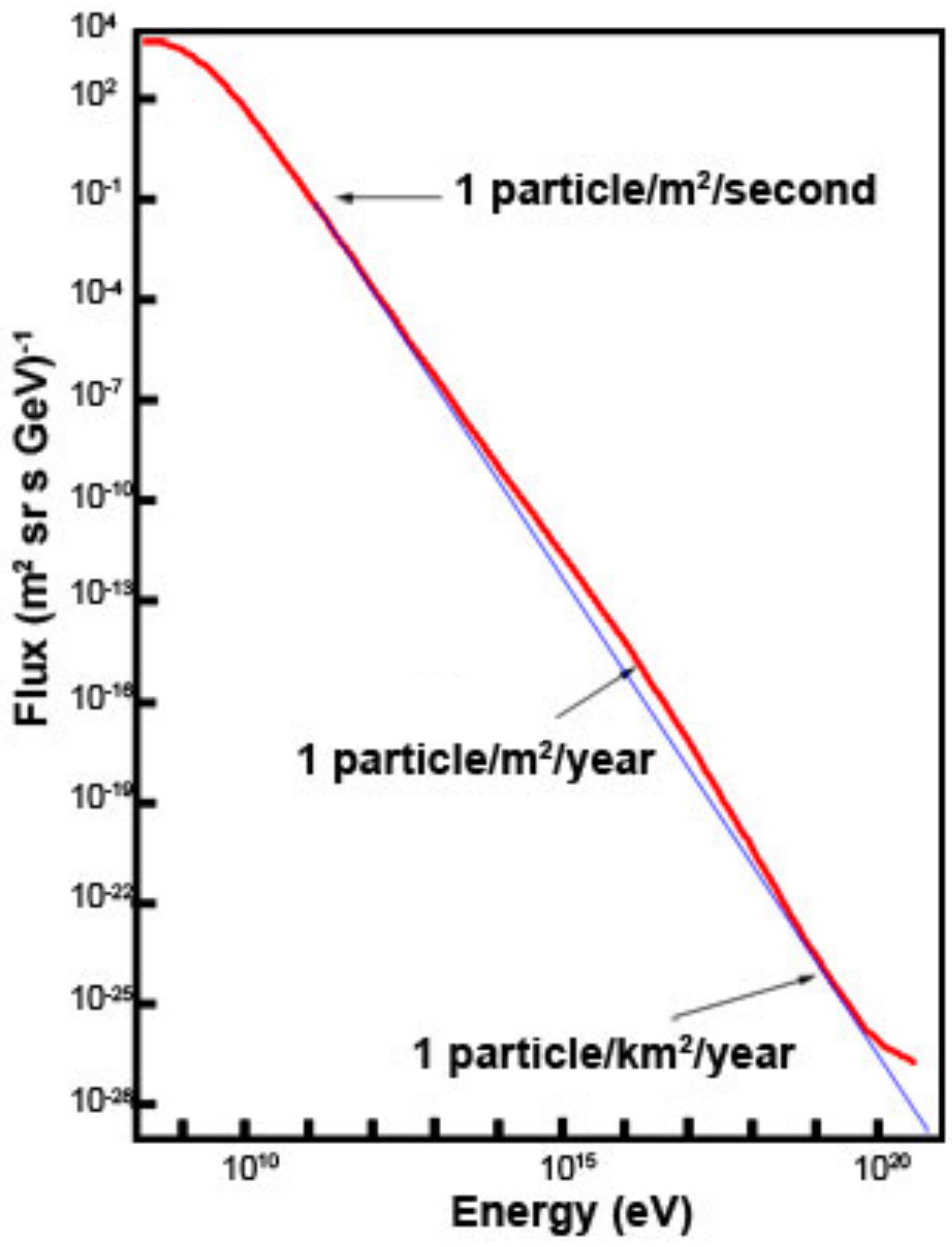
- CR mass composition measured over many order of magnitude in E_0
- Often given in terms of mean logarithmic mass, $\langle \ln A \rangle$
- Very large uncertainties, in particular towards high energies!
- Because CR properties are inferred indirectly from "air shower" measurements



[H.P. Dembinski, R. Engel, A. Fedynitch, T. K. Gaisser, F. Riehn, T. Stanev, PoS ICRC2017 (2017) 533]

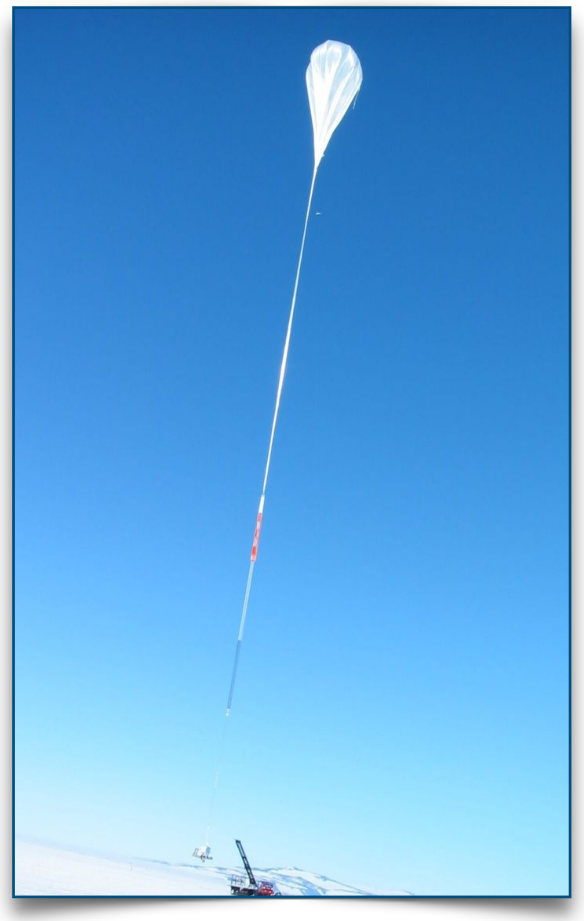


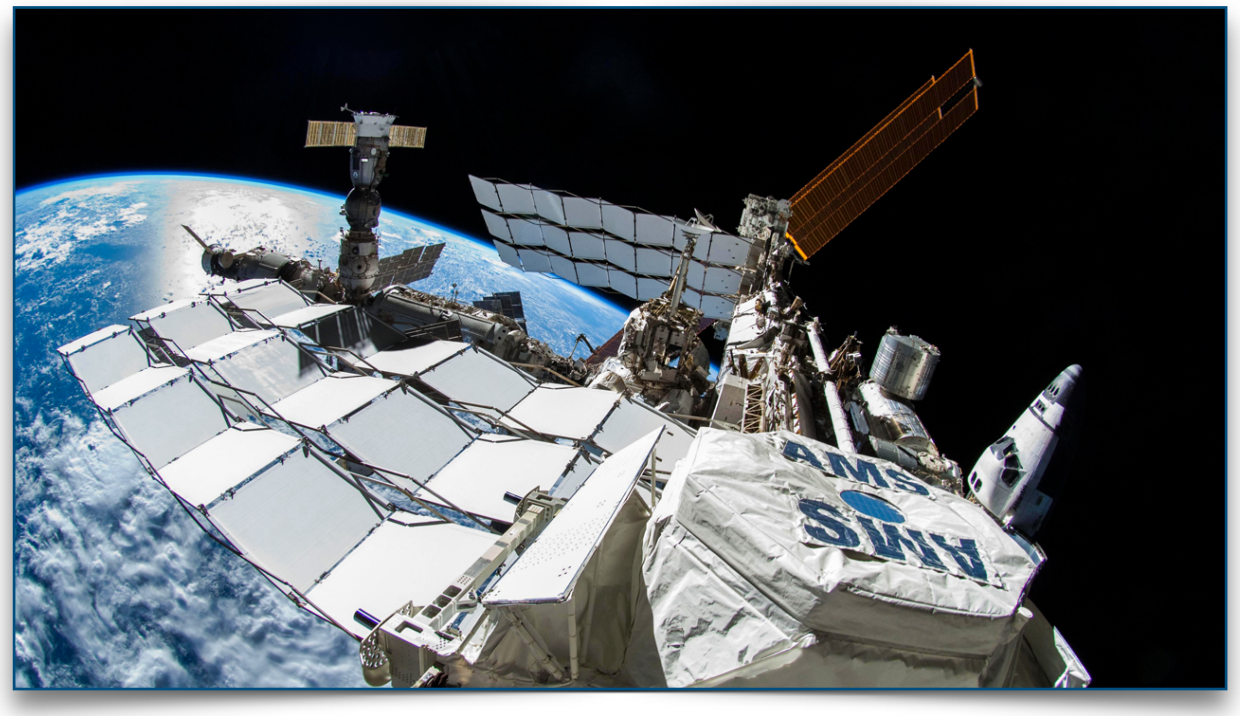
How to Detect Cosmic Rays?

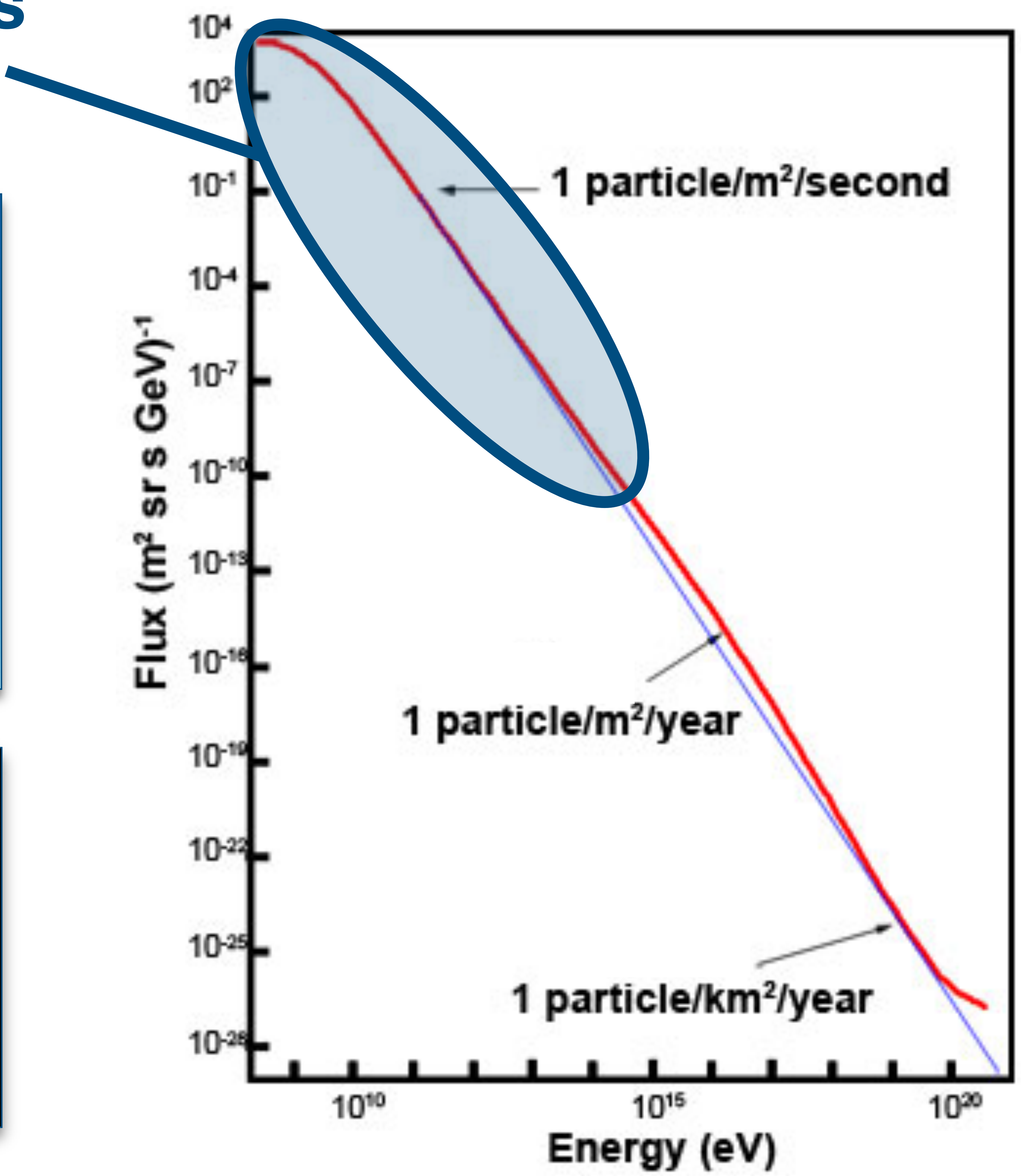


How to Detect Cosmic Rays?

Direct measurements (balloon / space)

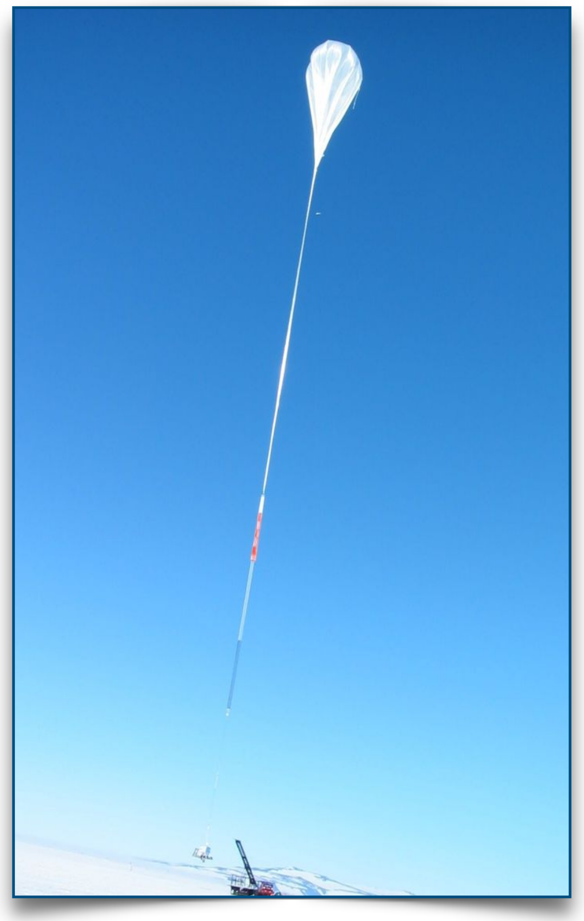


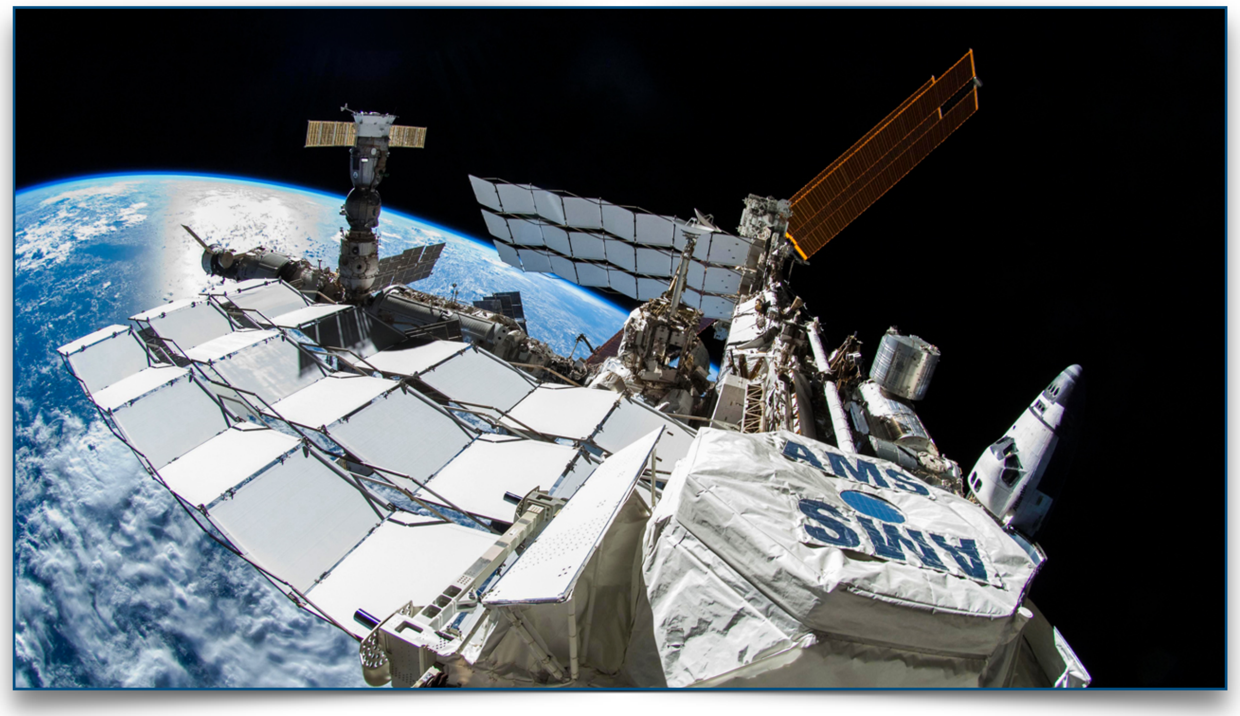


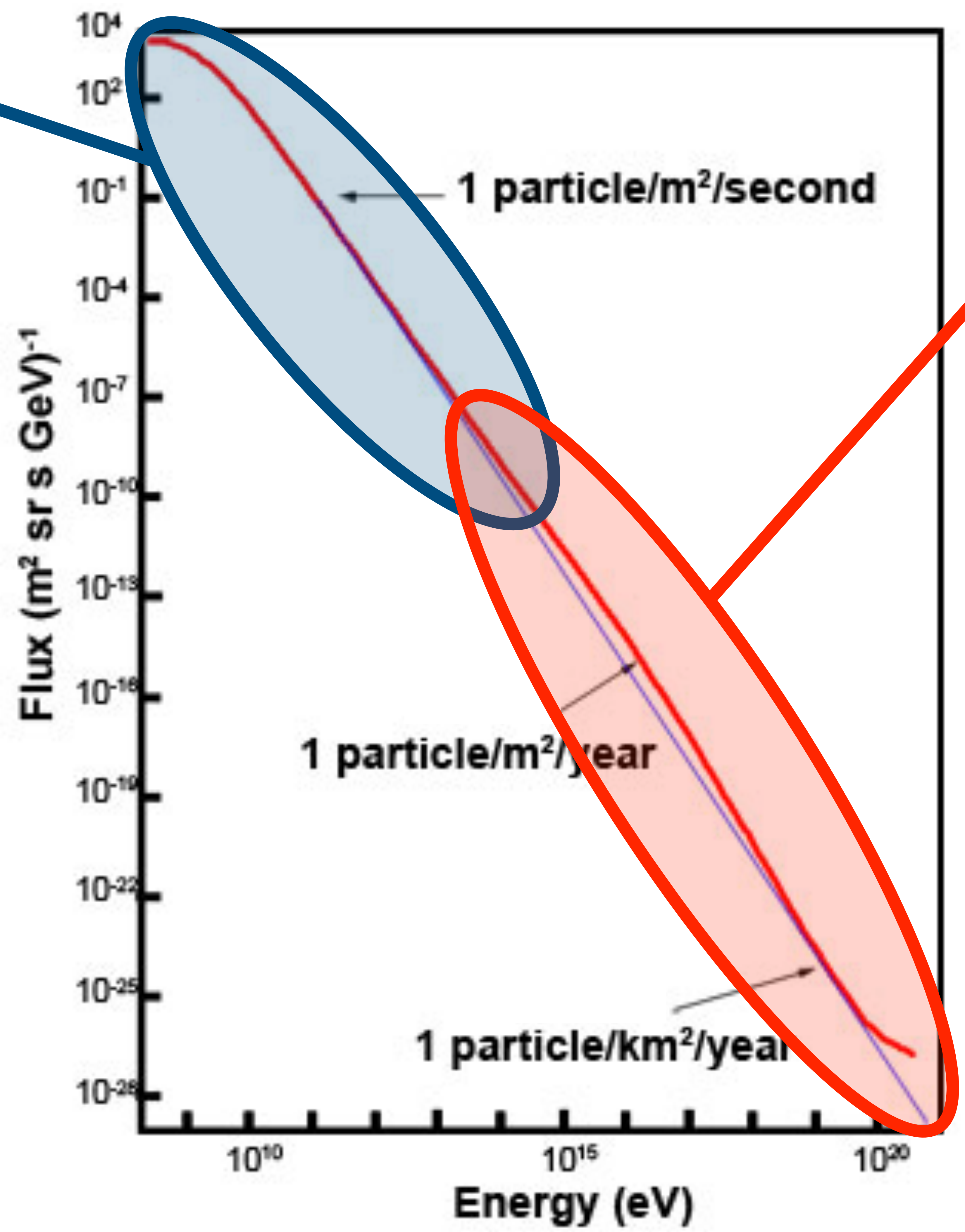


How to Detect Cosmic Rays?

Direct measurements (balloon / space)





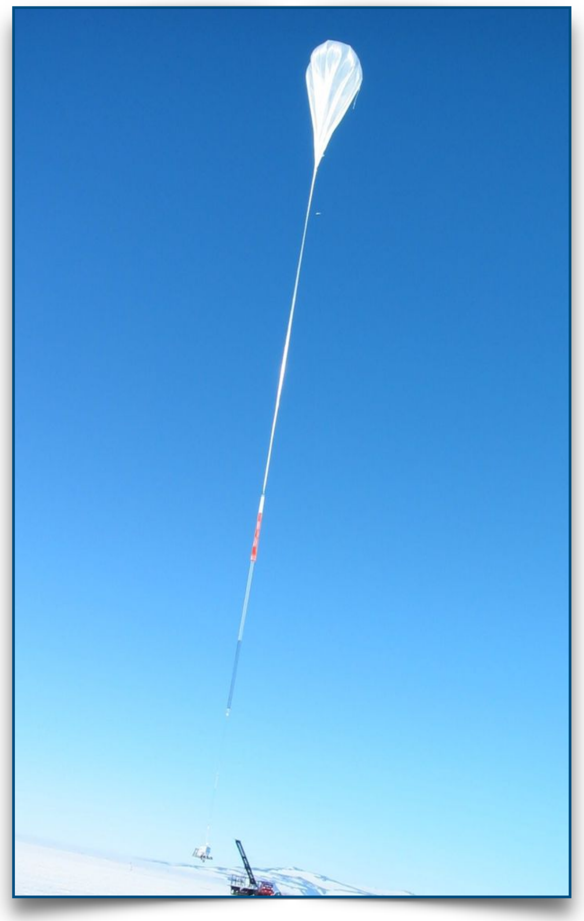


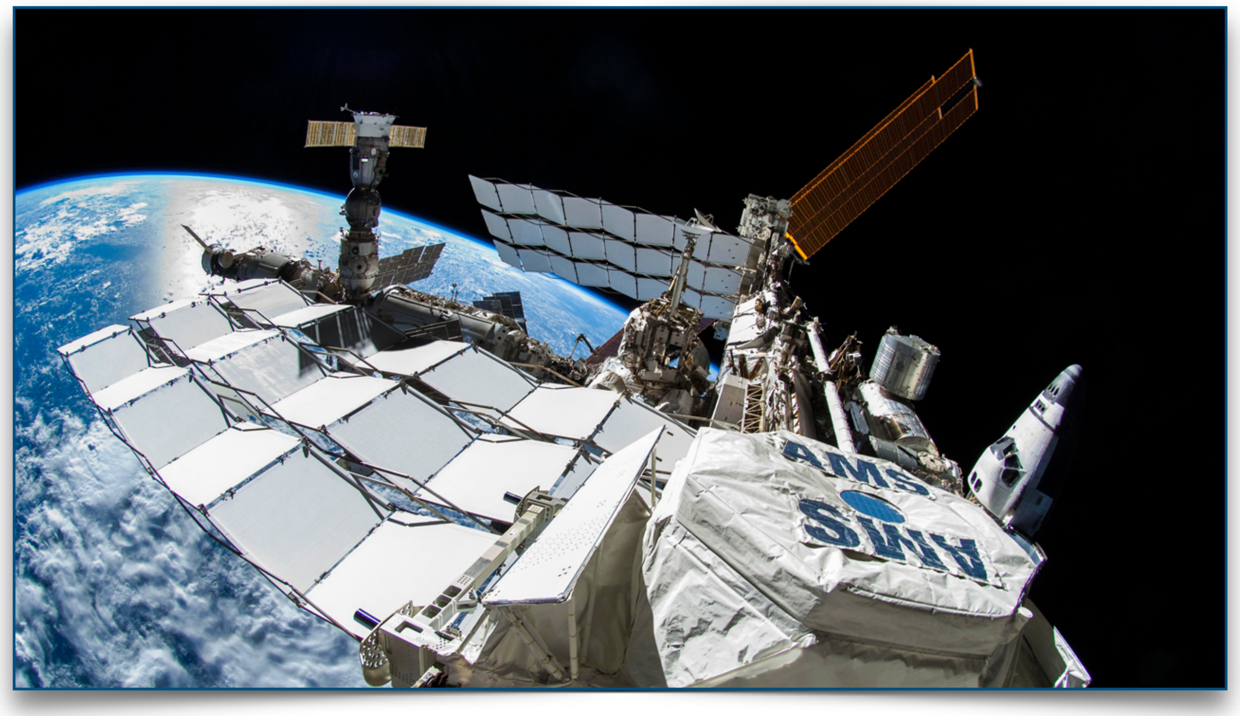
Indirect measurements* (ground-based)

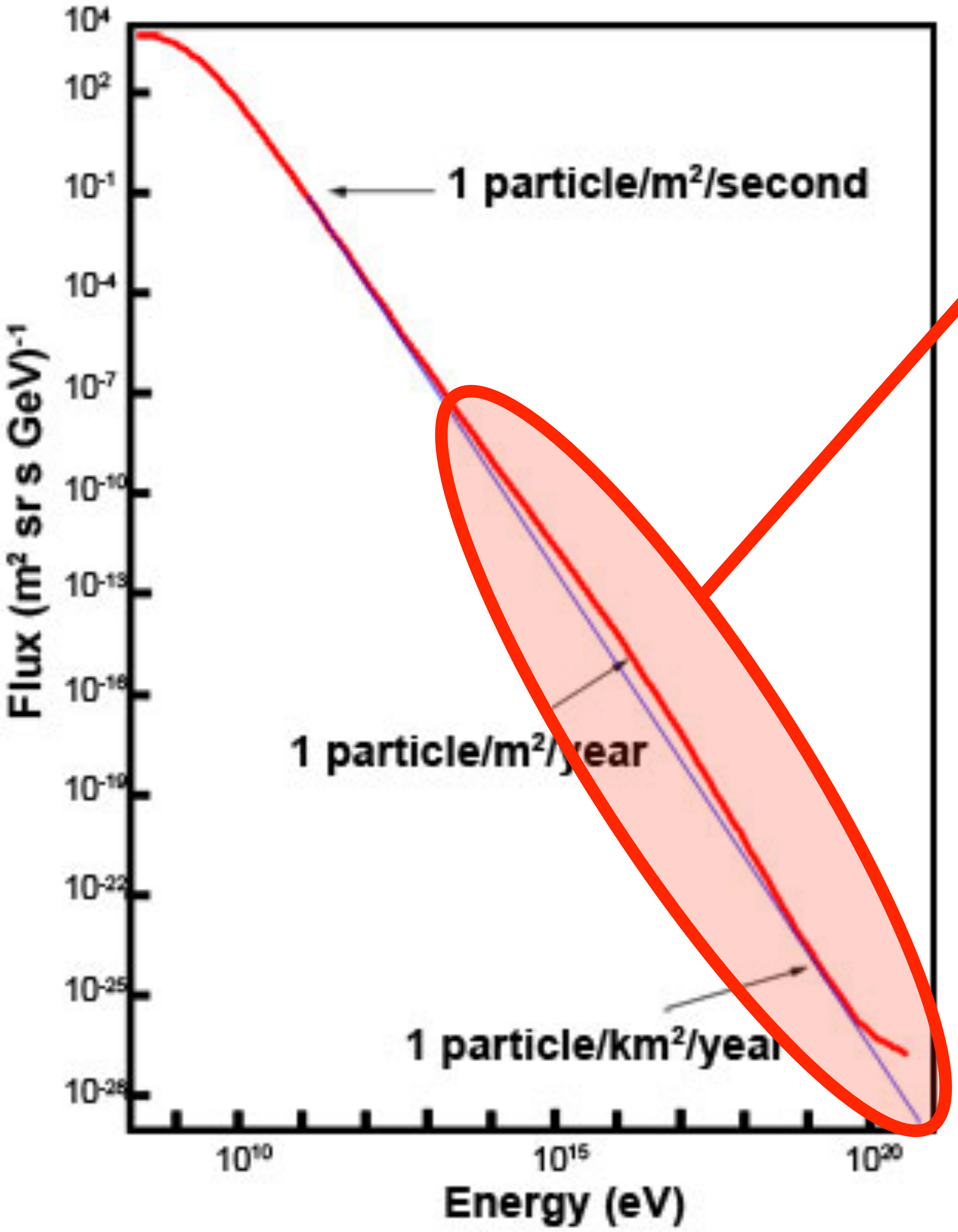
* More about the indirect detection of cosmic rays in Lecture 2!

How to Detect Cosmic Rays?

Direct measurements (balloon / space)





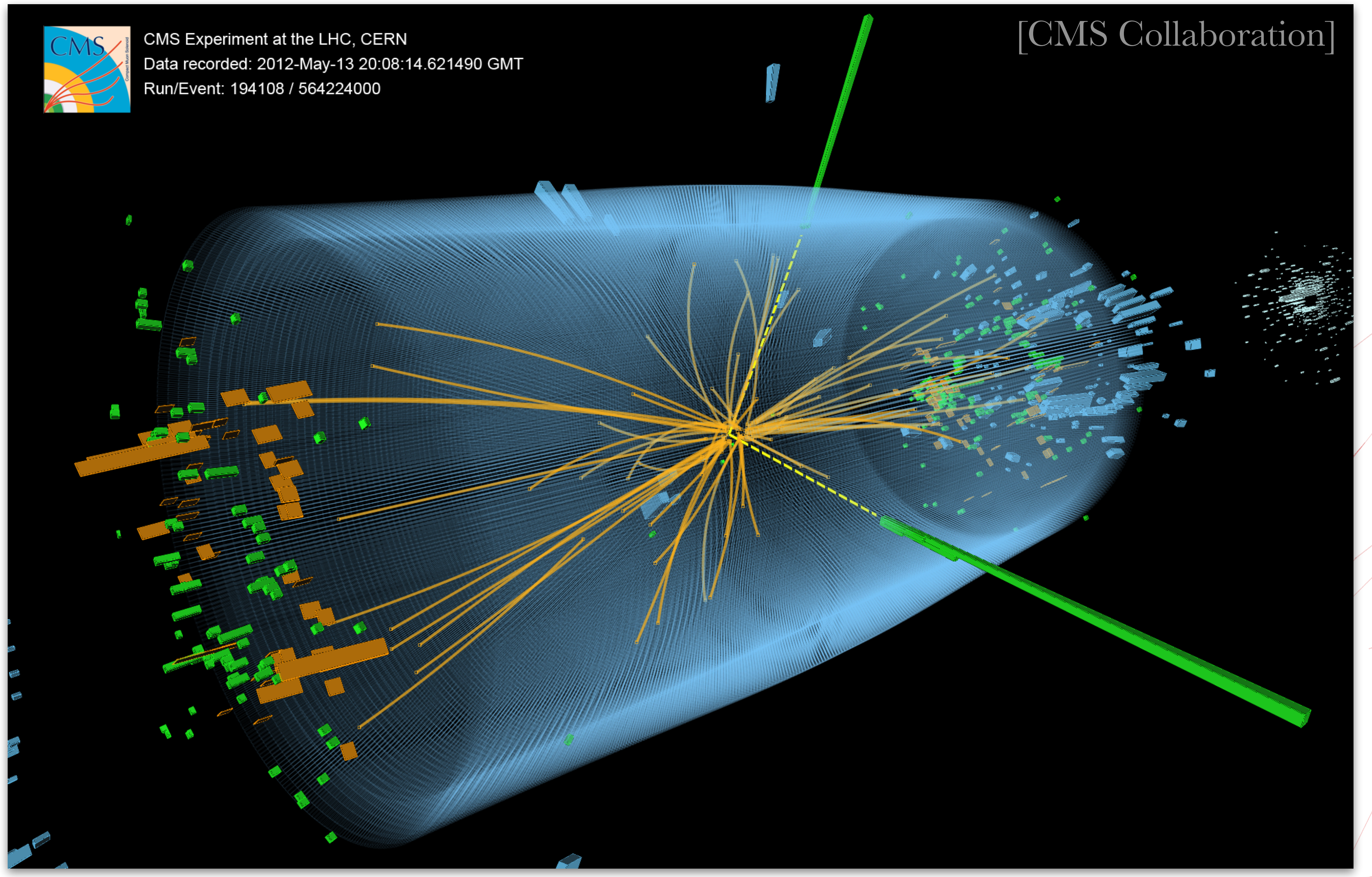


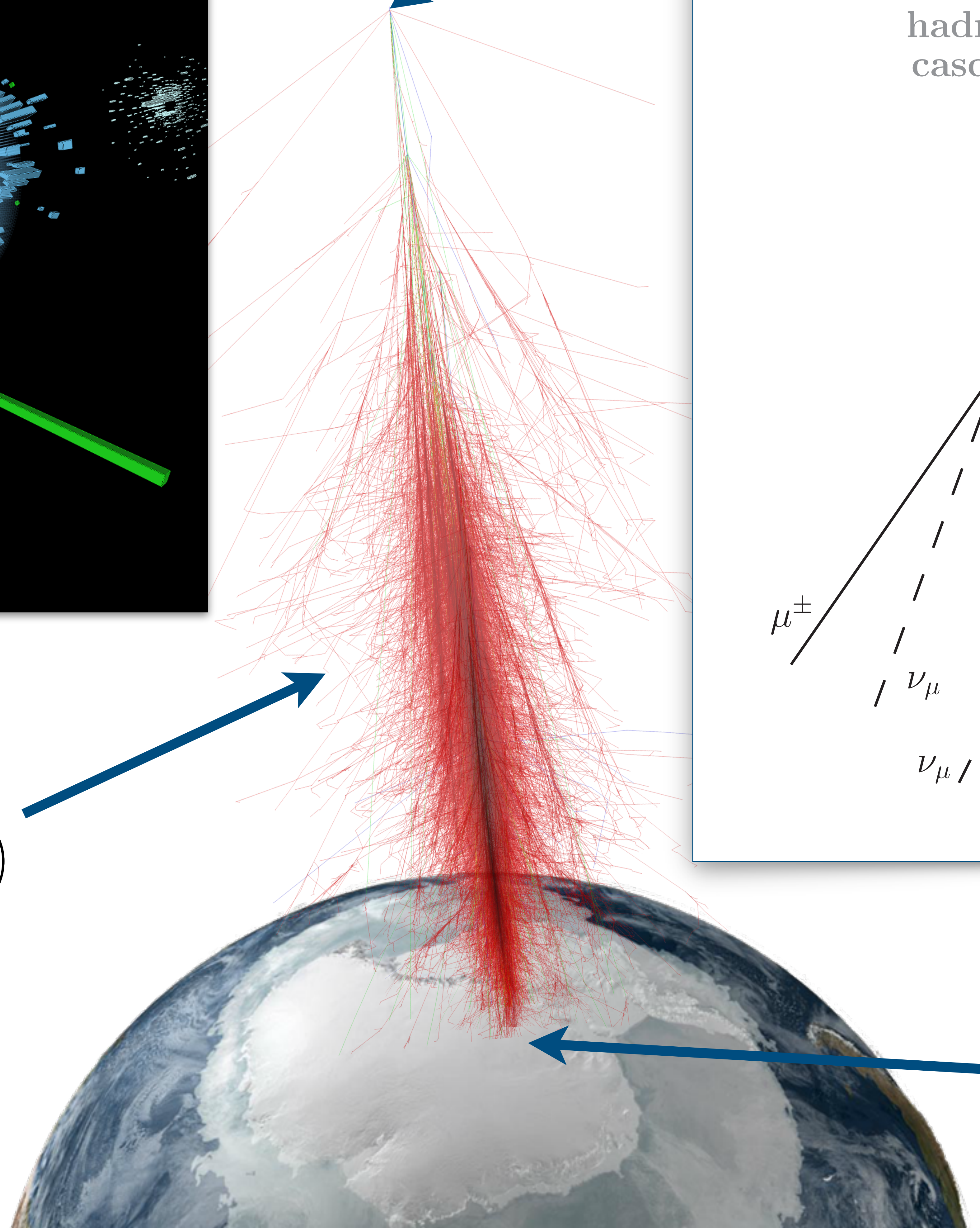
This Lecture!

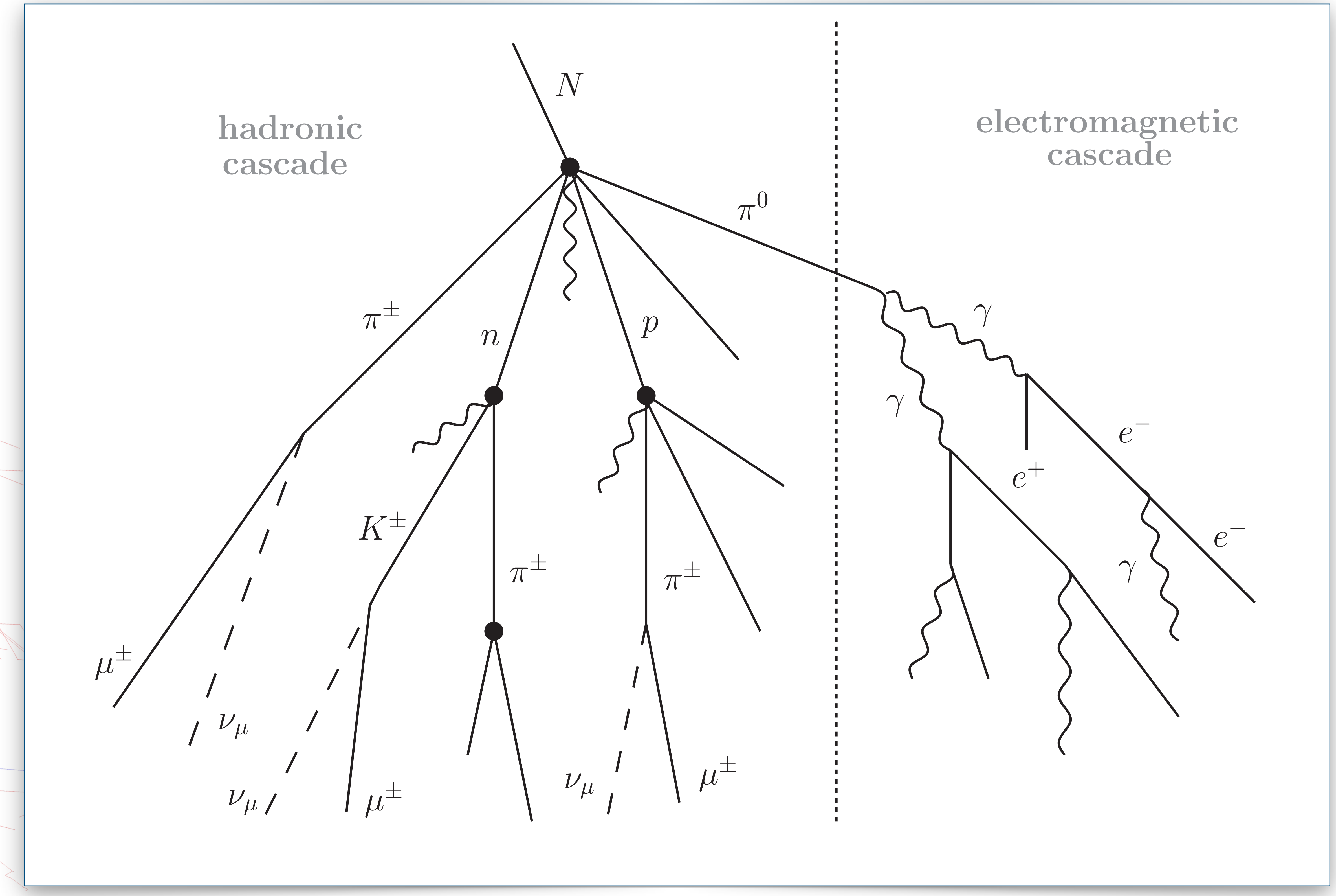
* More about the indirect detection of cosmic rays in Lecture 2!

Extensive Air Showers

Cosmic Ray Interaction





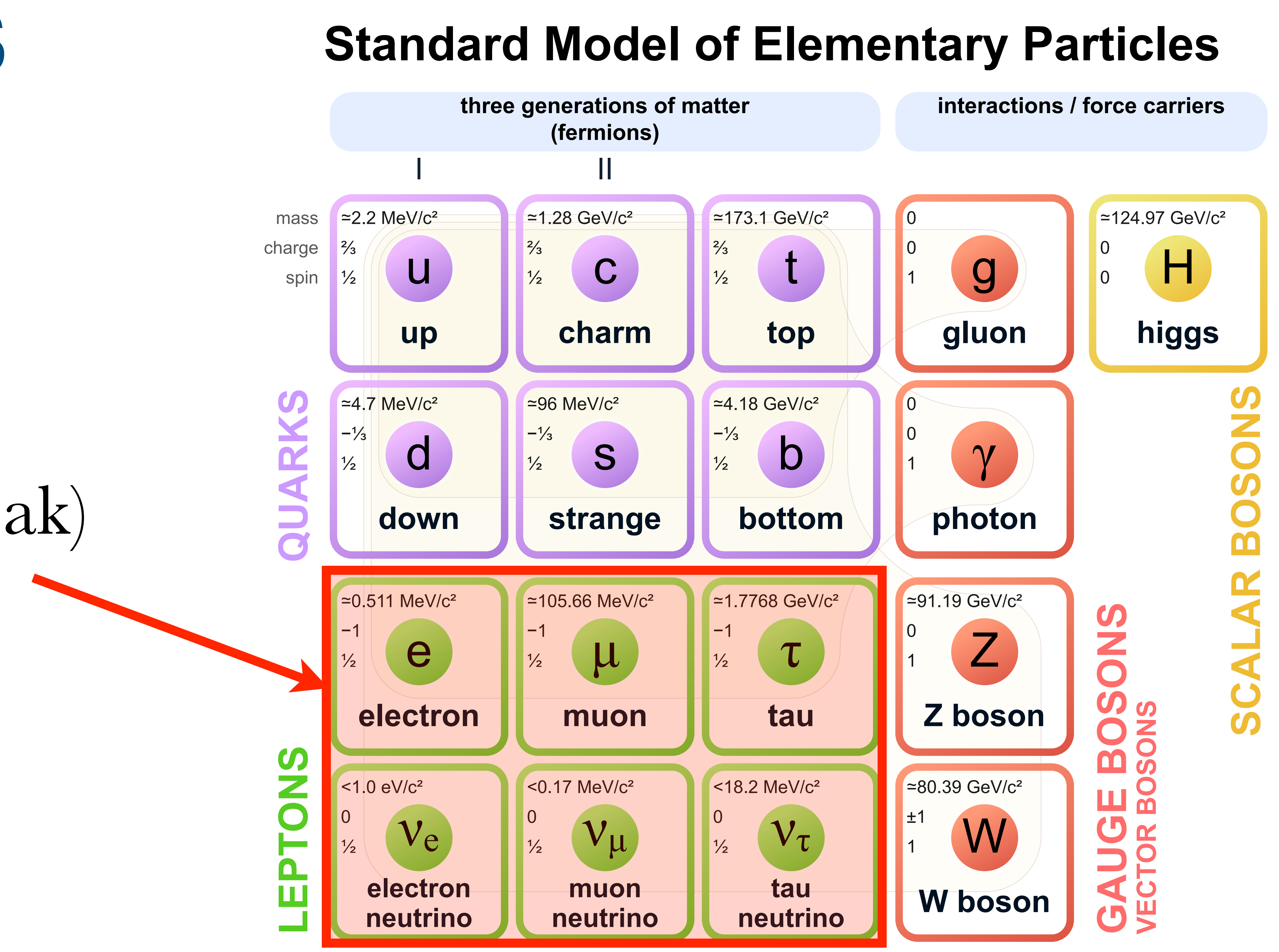


Extensive Air Shower (EAS)

Ground-Based Particle Detector

Basics: Particle Physics

- **S**tandard **M**odel (SM) of Particle Physics
  - Leptons:
    - elementary particles
    - No strong interactions (only em and weak)

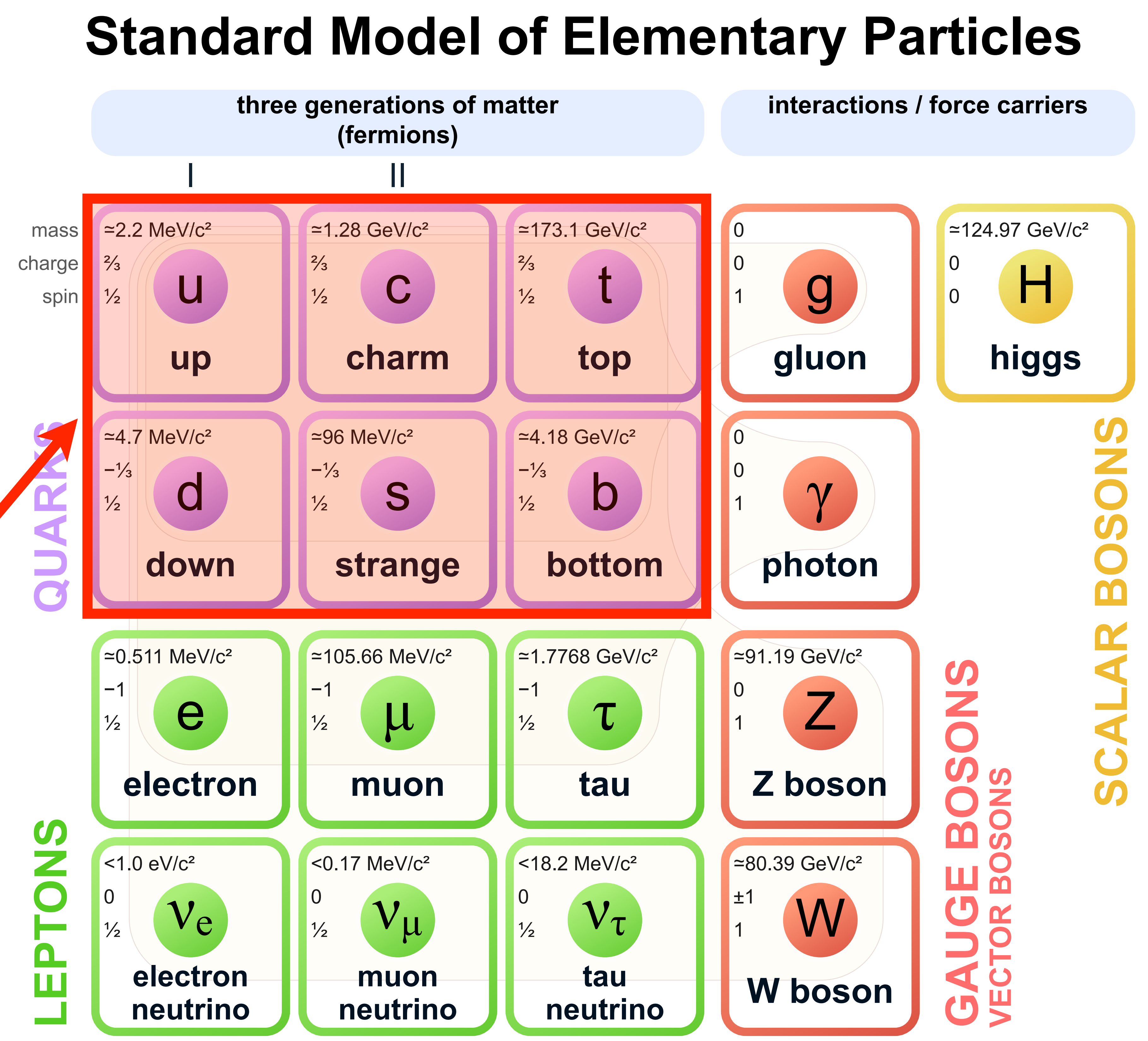


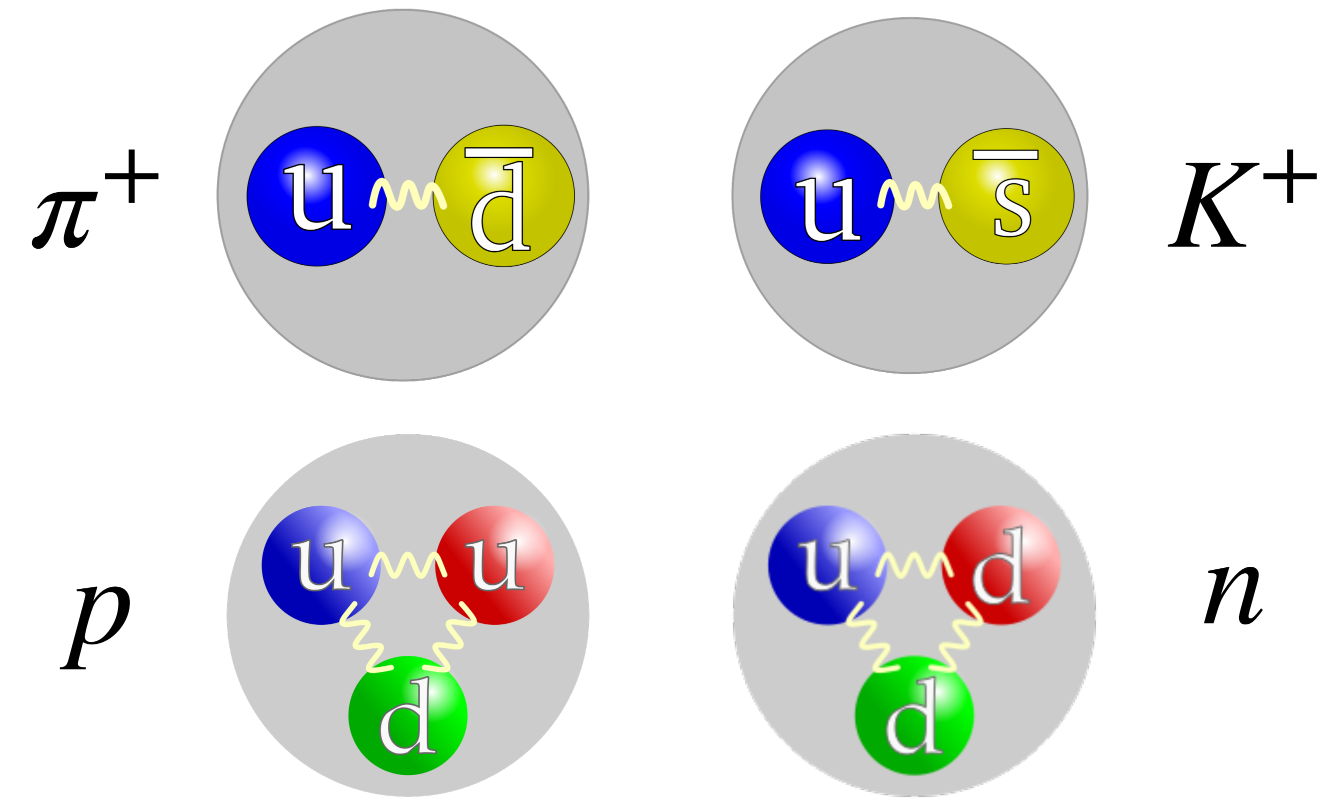
Basics: Particle Physics

- **S**tandard **M**odel (SM) of Particle Physics
  - <u>Leptons:</u>
    - elementary particles
    - <u>No strong interactions</u> (only em and weak)
  - <u>Hadrons:</u>
    - Composite particles
    - 2+ quarks held together by <u>strong force</u>
    - <u>Mesons:</u> even number of quarks (2+), e.g.

      $\pi^+\ (u\bar{d})$, $\pi^-\ (d\bar{u})$, $\pi^0\ (u\bar{u}$ or $d\bar{d})$, $K^+\ (u\bar{s})$,
      $K^-\ (s\bar{u})$, $K^0\ (d\bar{s}$ or $s\bar{d})$, $D^+\ (c\bar{d})$, $D^0\ (c\bar{u})$, …
    - <u>Baryons:</u> odd number of quarks (3+), e.g.

      $p\ (uud)$, $n\ (udd)$, …



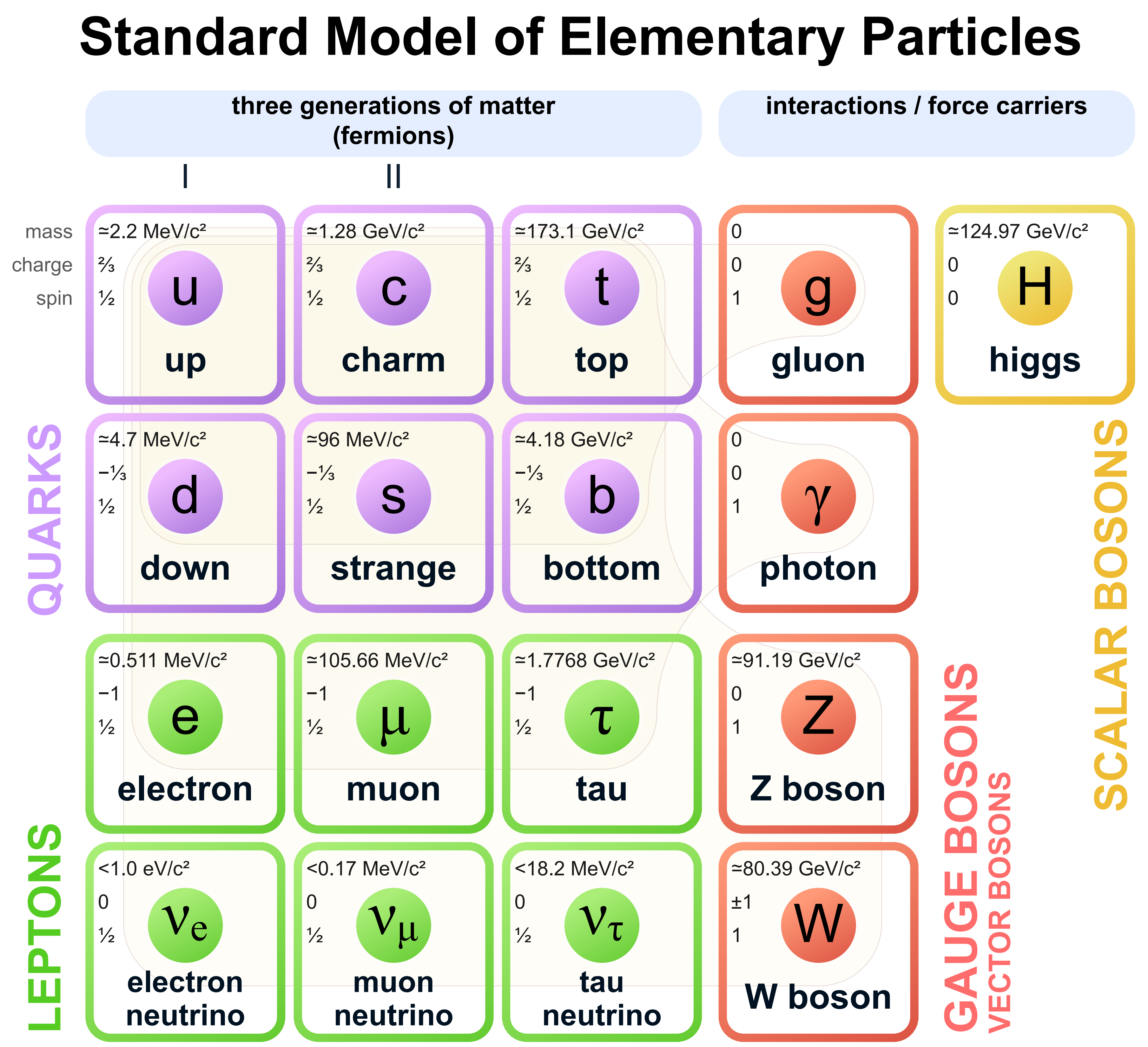


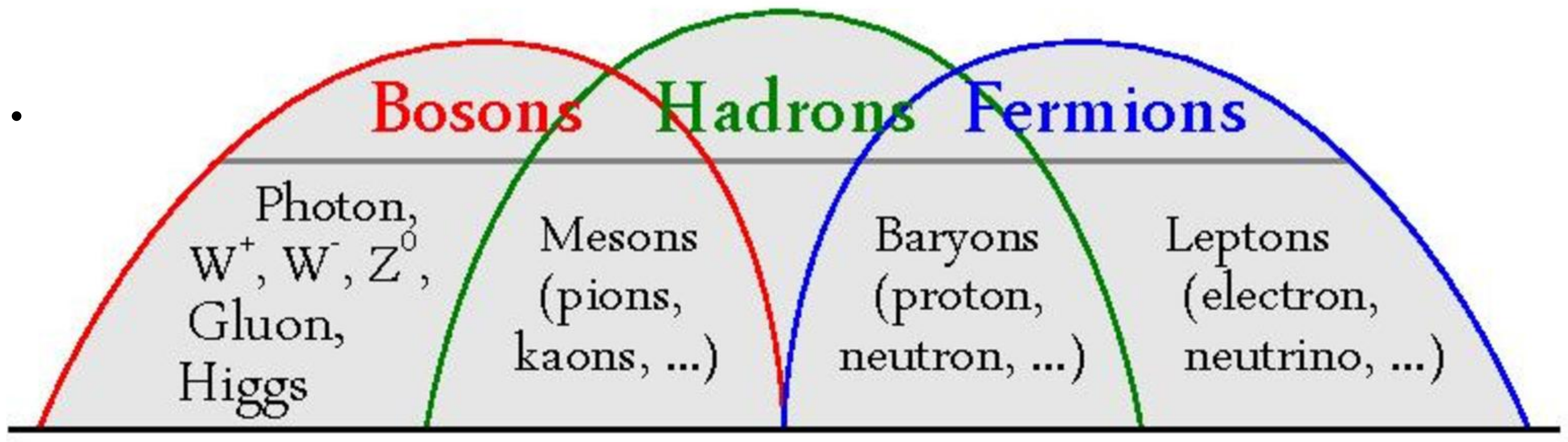
Basics: Particle Physics

- **S**tandard **M**odel (SM) of Particle Physics
  - Leptons:
    - elementary particles
    - No strong interactions (only em and weak)
  - Hadrons:
    - Composite particles
    - 2+ quarks held together by strong force
    - Mesons: even number of quarks (2+), e.g.

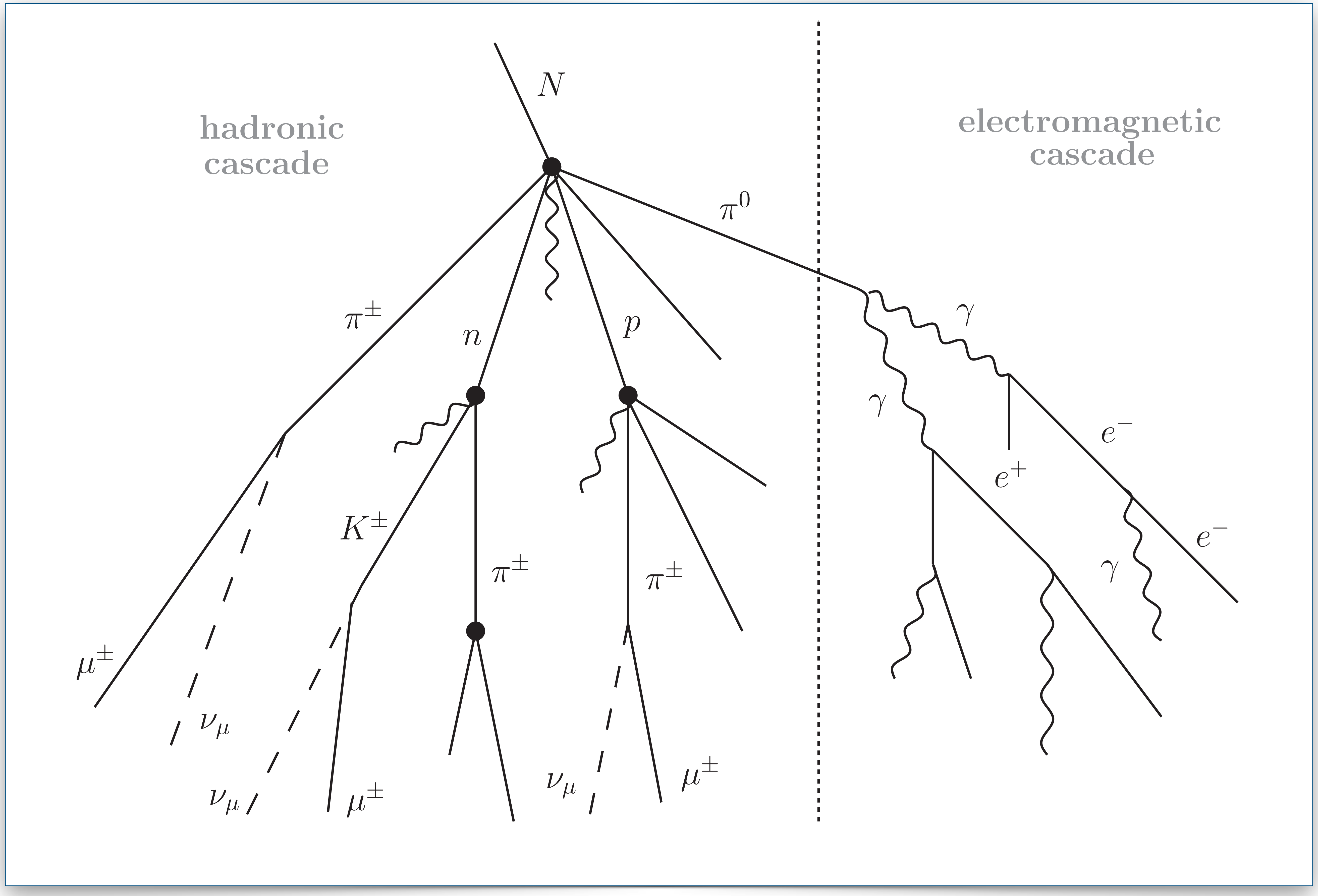
      $\pi^+ (u\bar{d})$, $\pi^- (d\bar{u})$, $\pi^0 (u\bar{u}$ or $d\bar{d})$, $K^+ (u\bar{s})$, $K^- (s\bar{u})$, $K^0 (d\bar{s}$ or $s\bar{d})$, $D^+ (c\bar{d})$, $D^0 (c\bar{u})$, …
    - Baryons: odd number of quarks (3+), e.g.

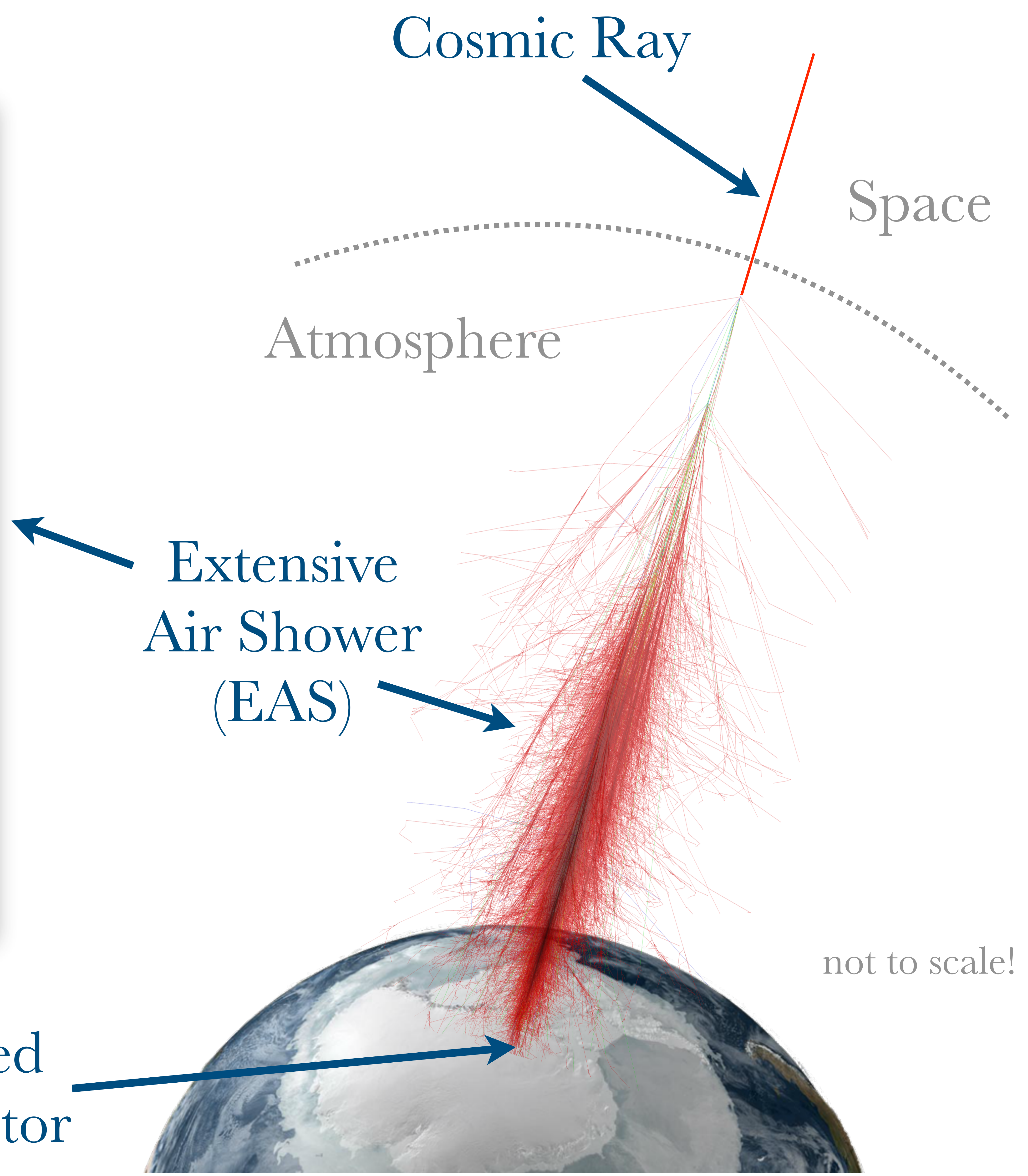
      $p\ (uud)$, $n\ (udd)$, …





Extensive Air Showers



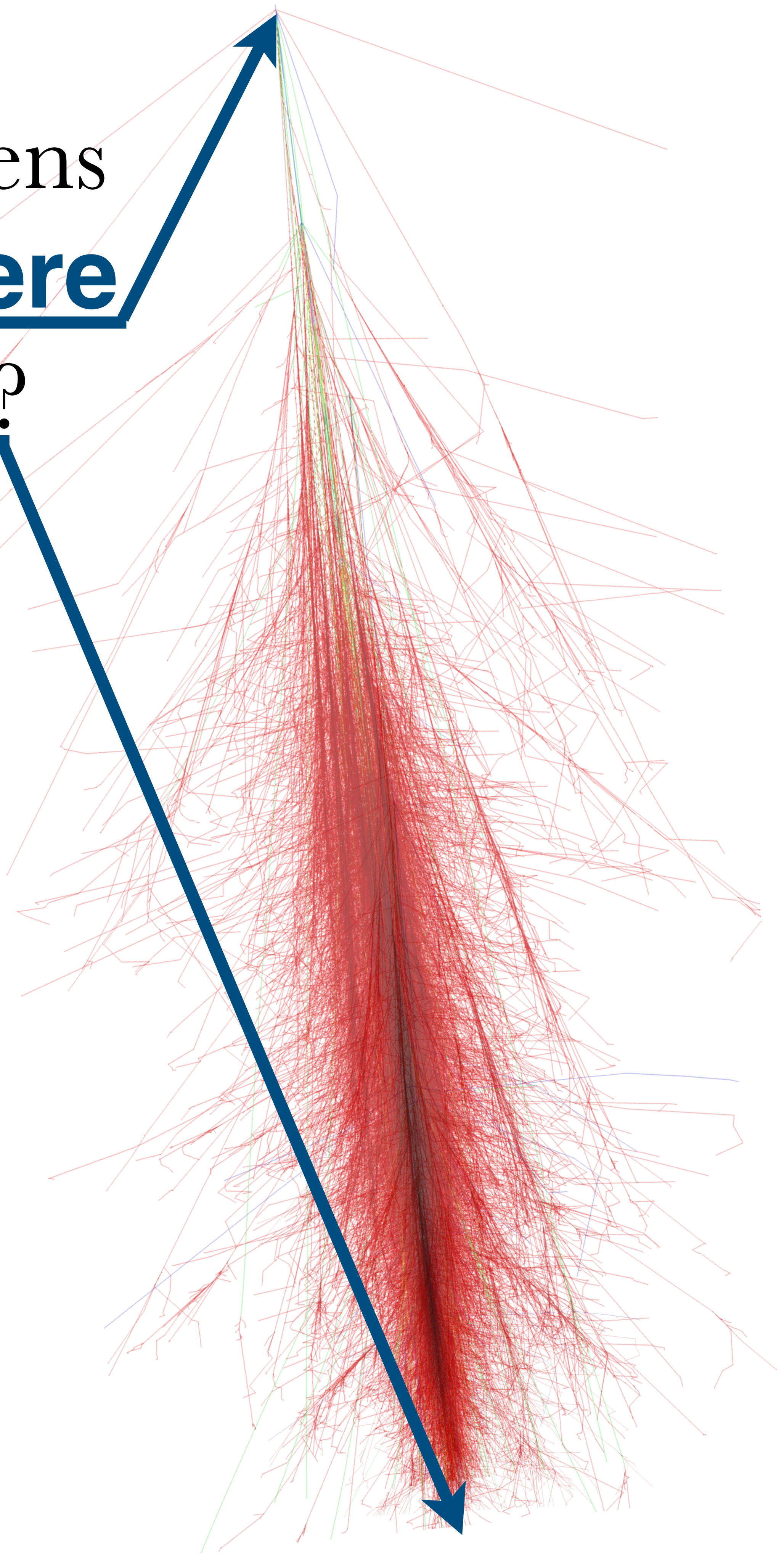


- EAS are the connection between cosmic ray and particle physics!

Extensive Air Showers (EAS)

- CR properties are inferred from the (secondary) particles measured at the ground

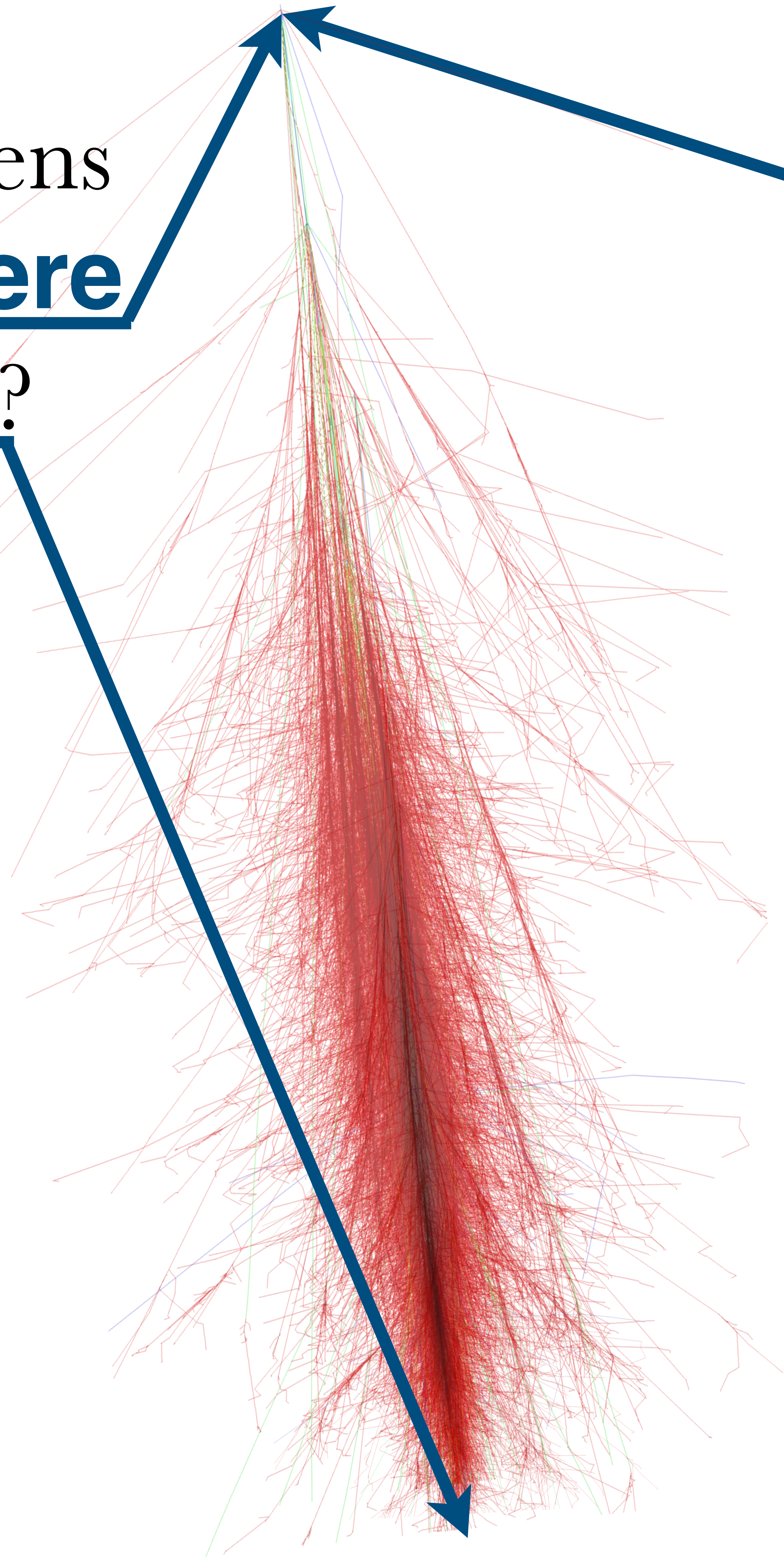
What happens between **here** and **here**?

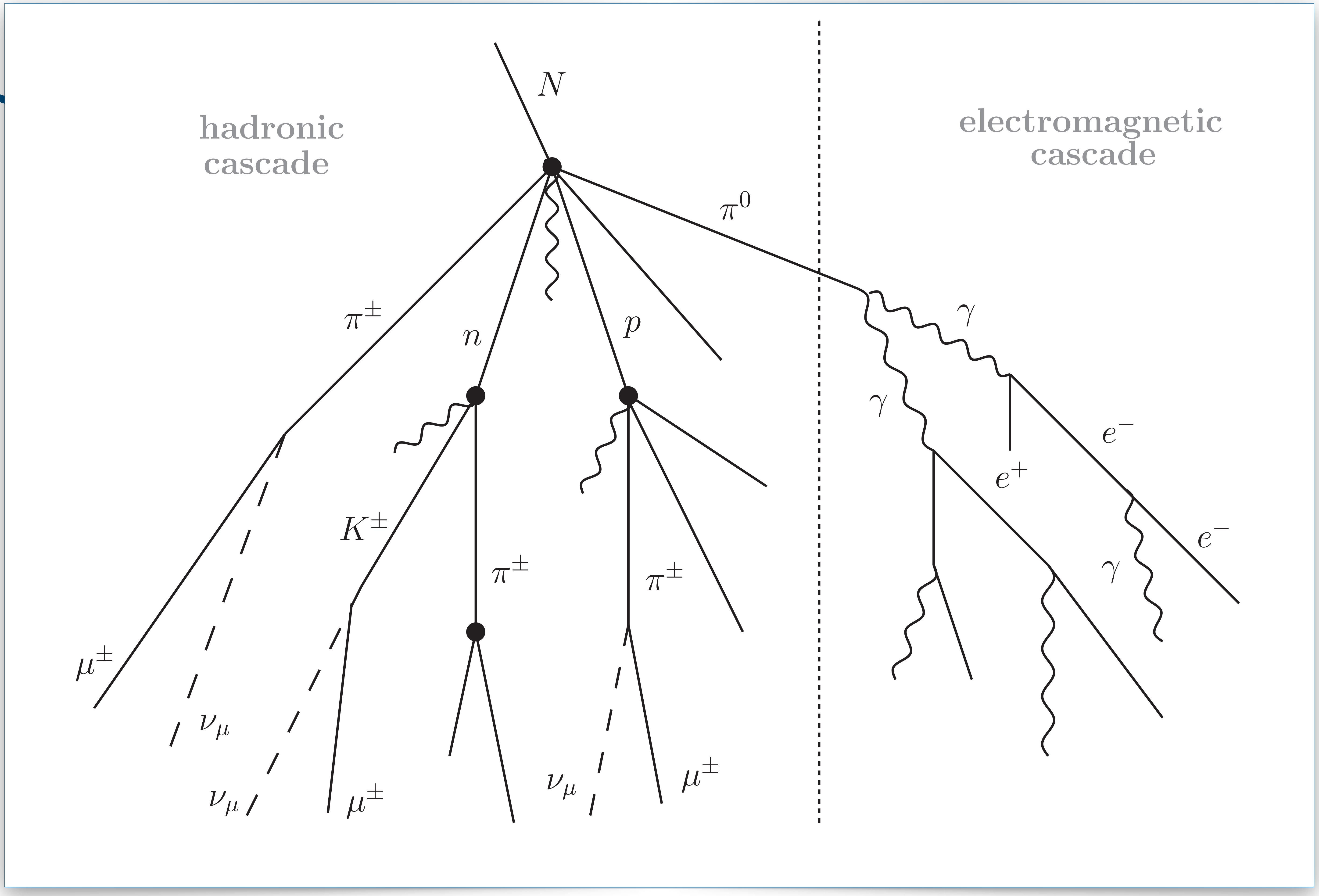


Extensive Air Showers (EAS)

- CR properties are inferred from the (secondary) particles measured at the ground

What happens between **here** and **here**?

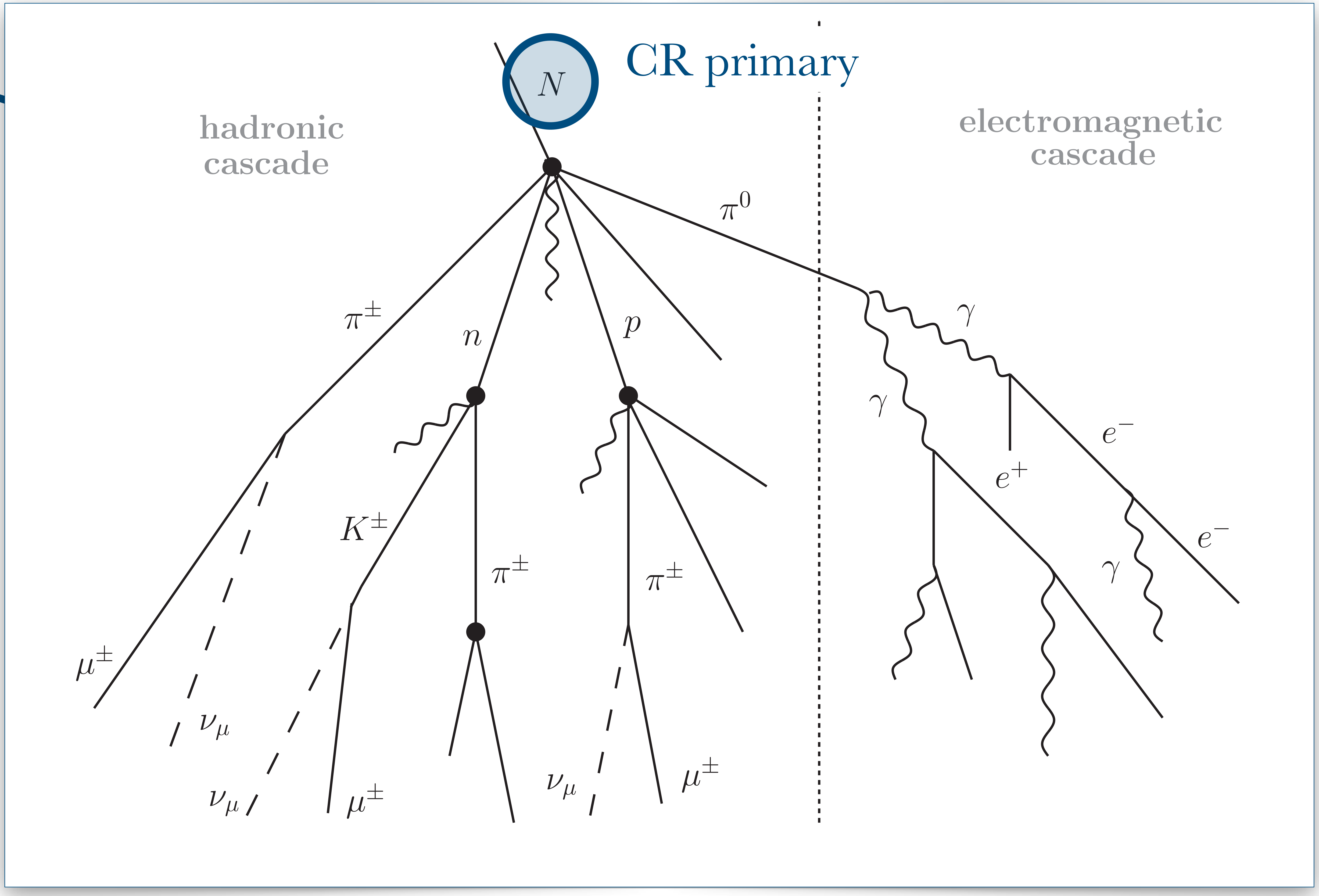




Extensive Air Showers (EAS)

- CR properties are inferred from the (secondary) particles measured at the ground

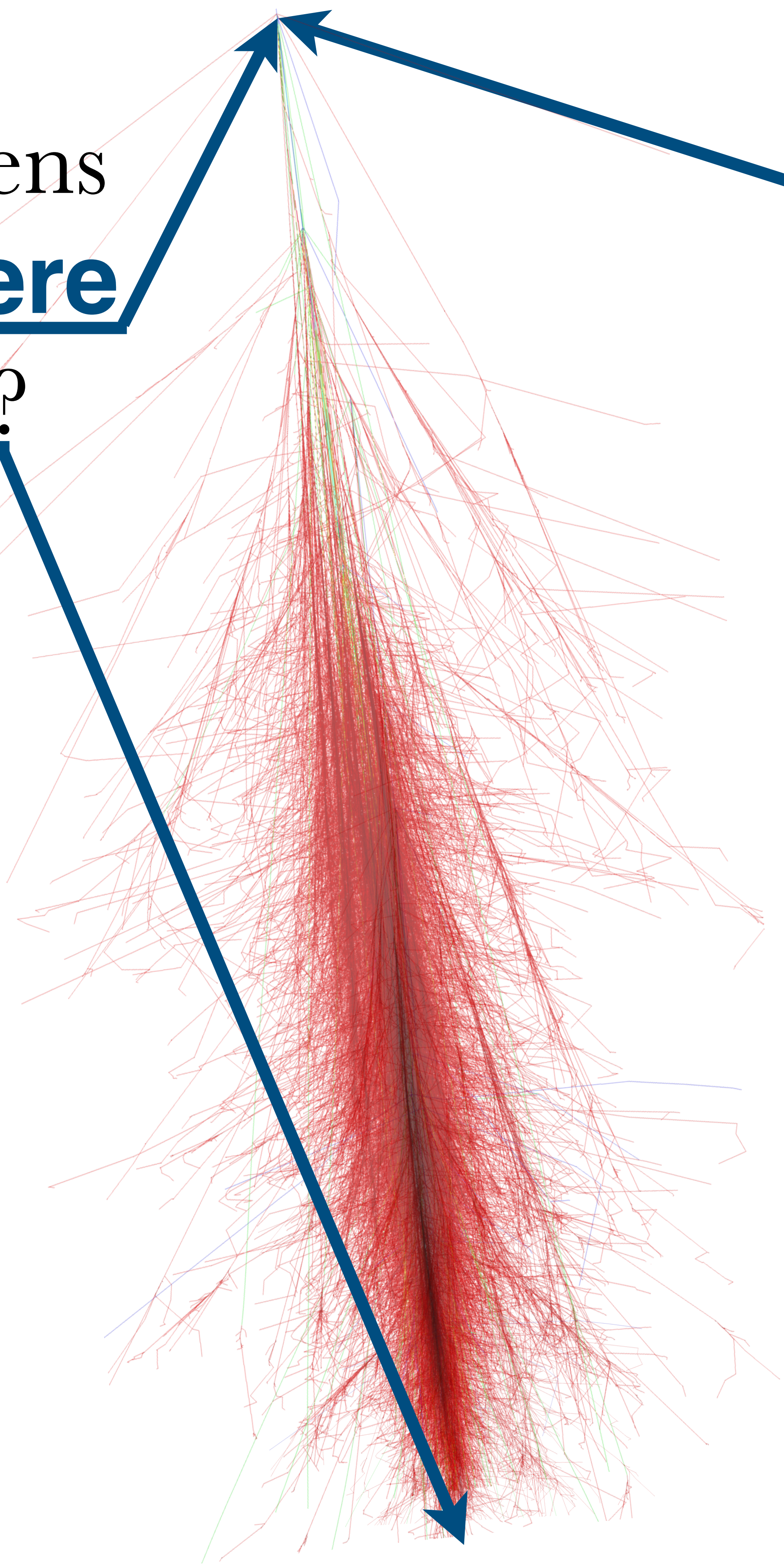
What happens between **here** and **here**?

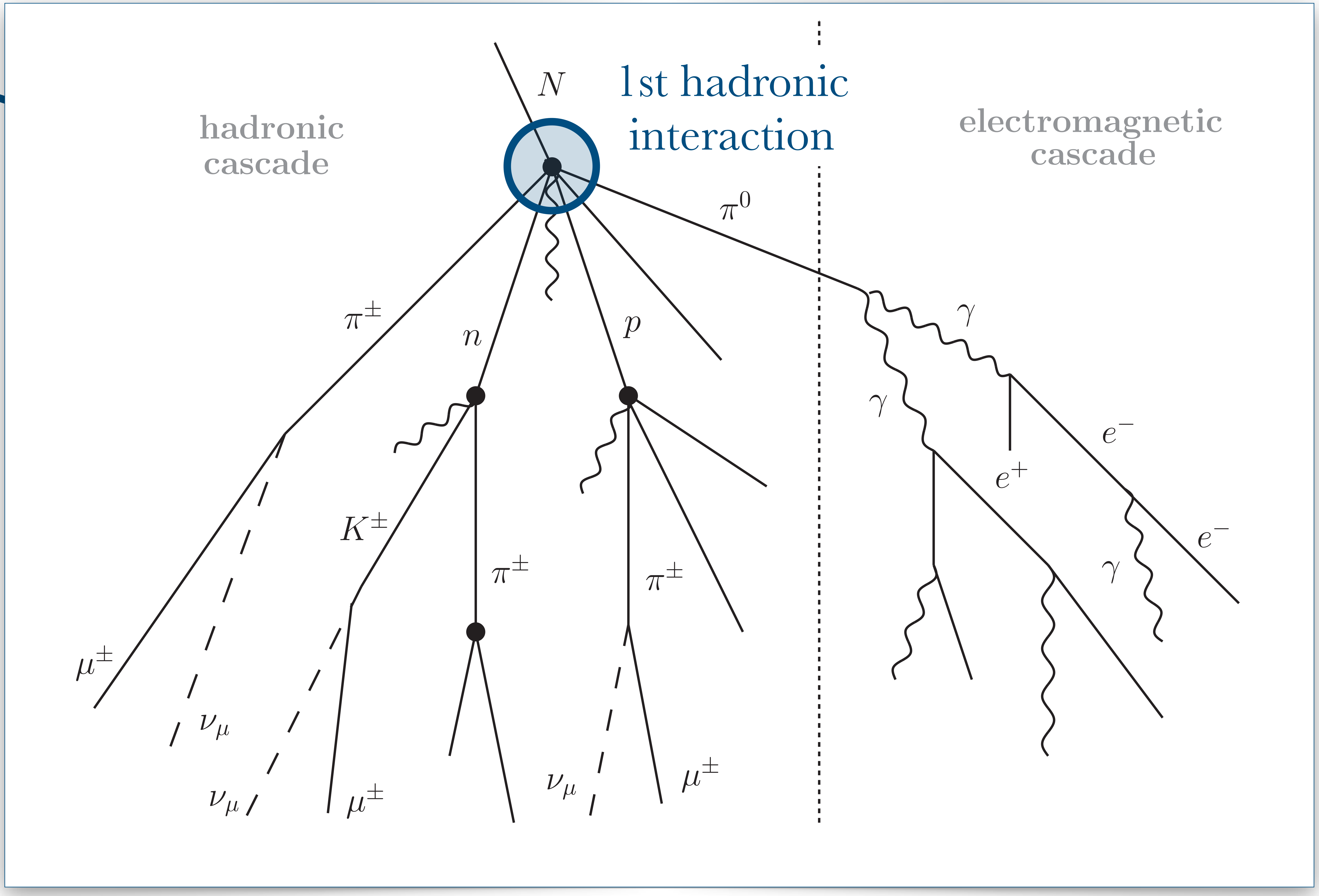


Extensive Air Showers (EAS)

- CR properties are inferred from the (secondary) particles measured at the ground

What happens between **here** and **here**?

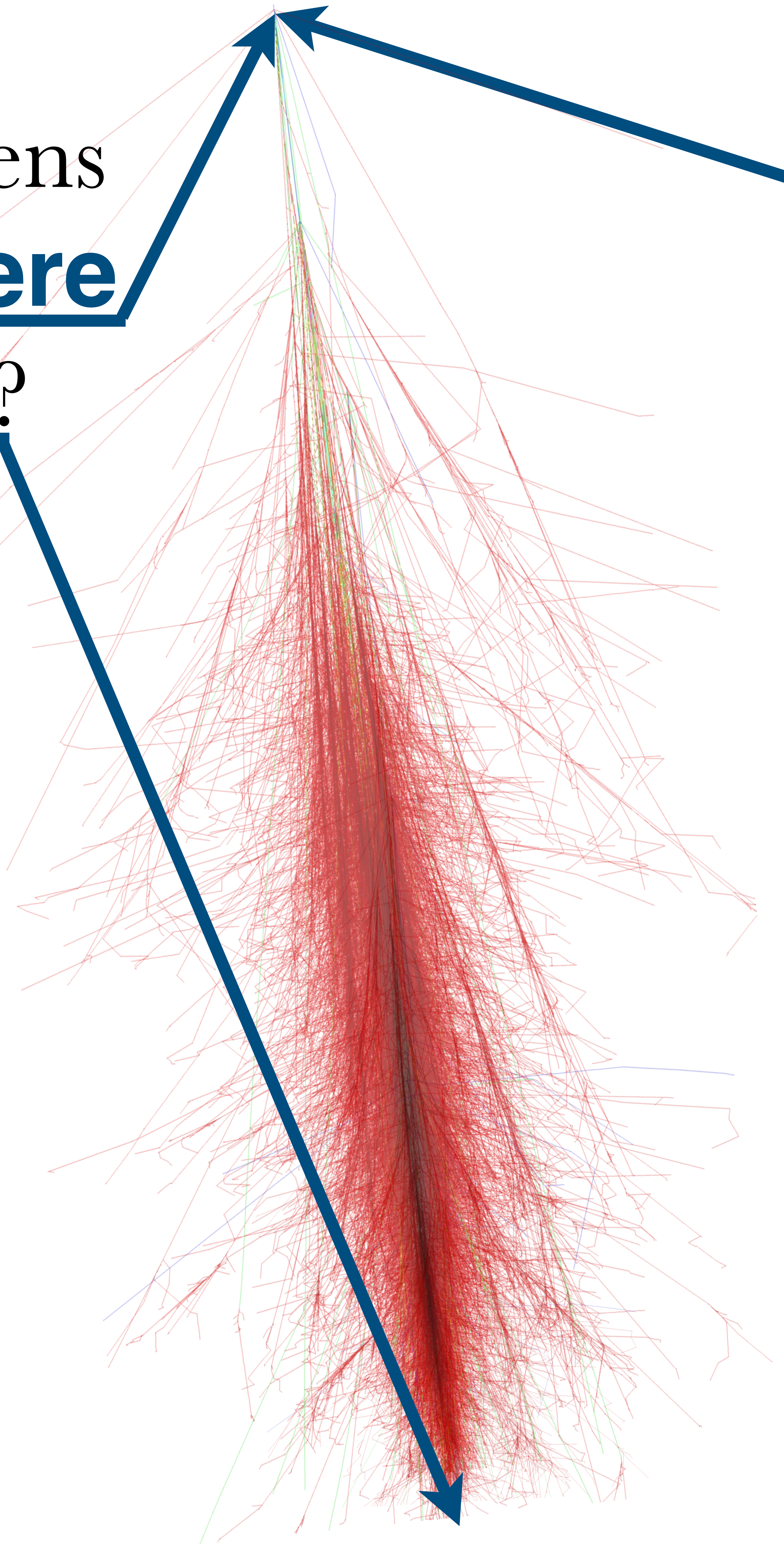


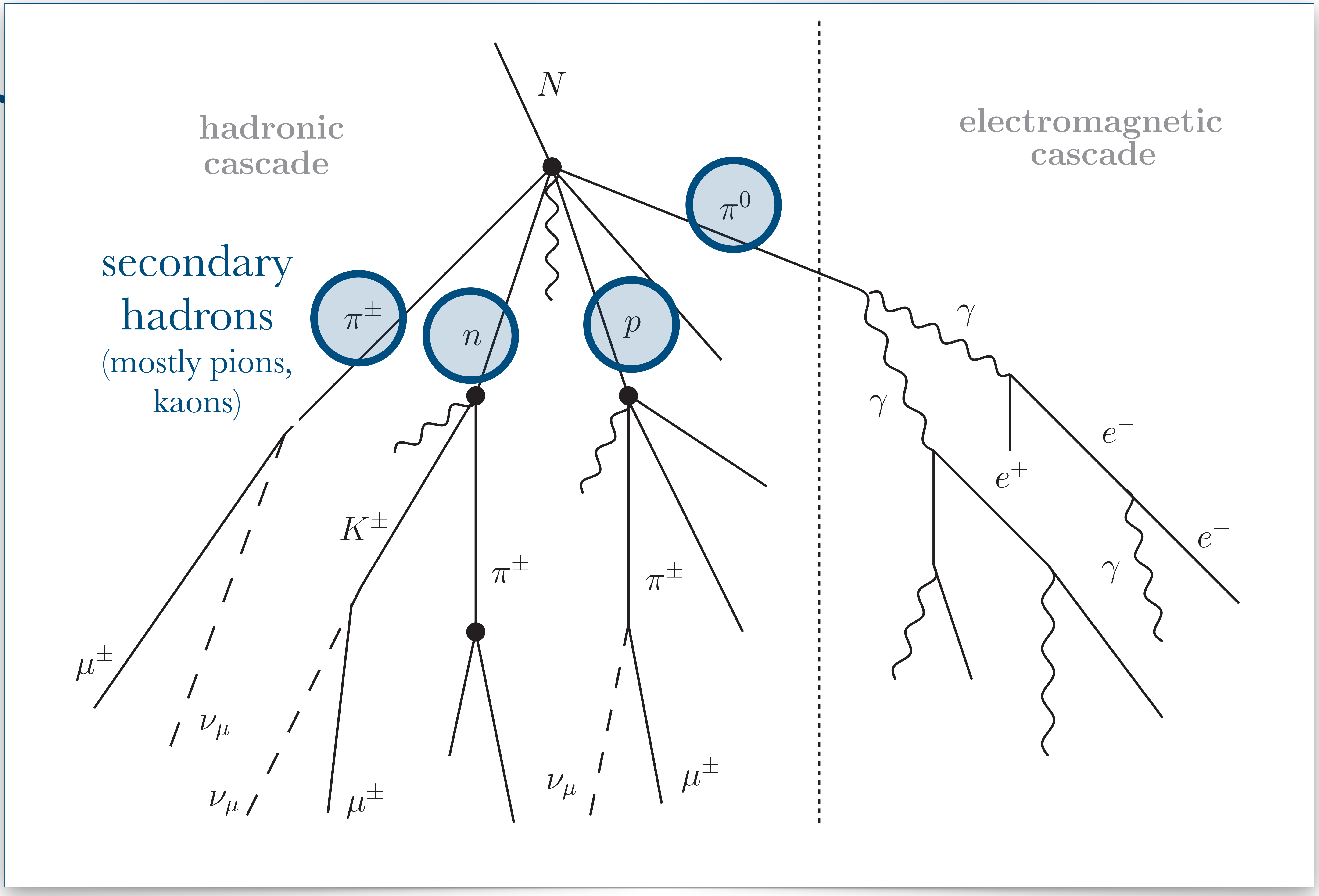


Extensive Air Showers (EAS)

- CR properties are inferred from the (secondary) particles measured at the ground

What happens between **here** and **here**?

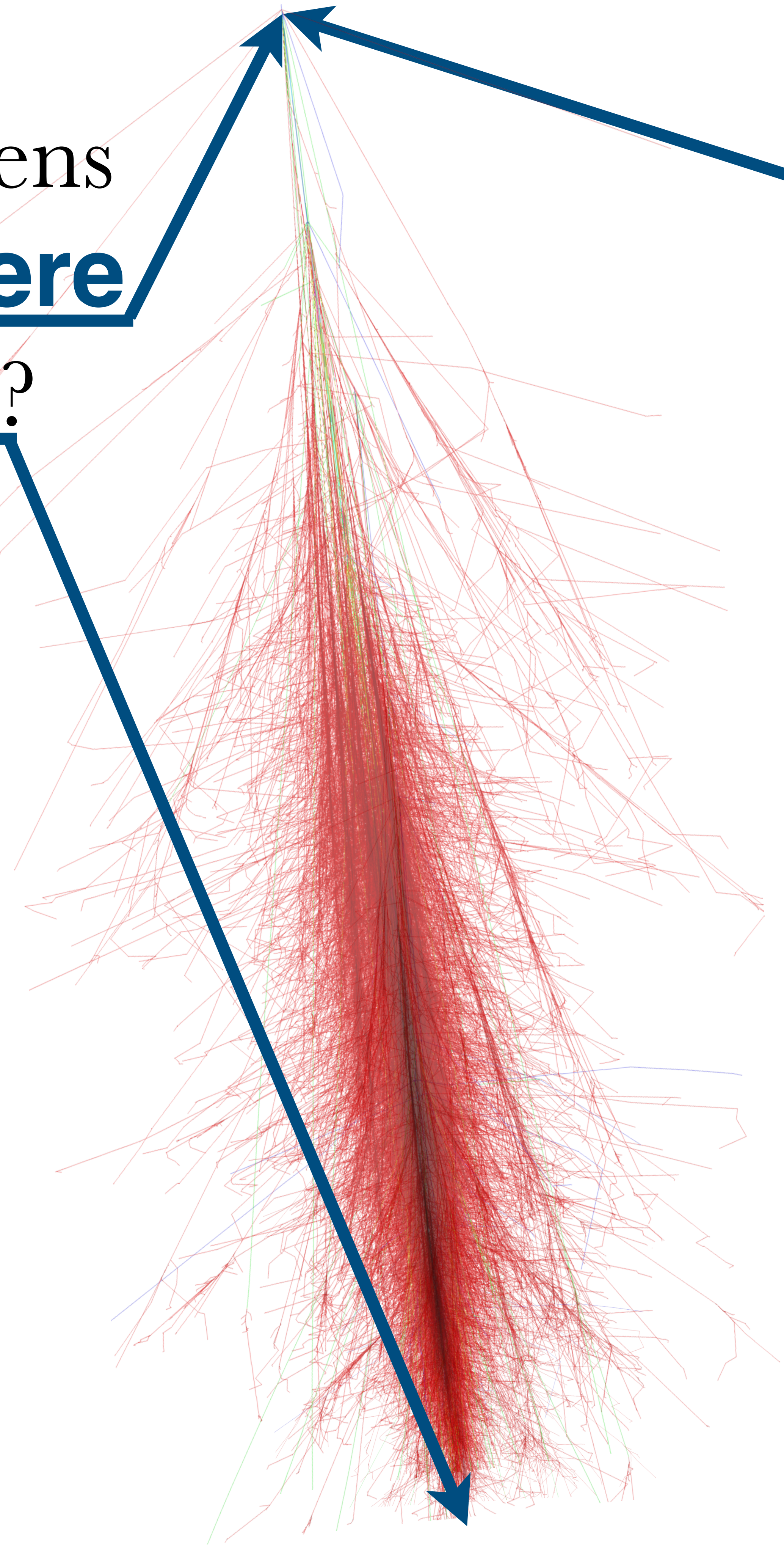


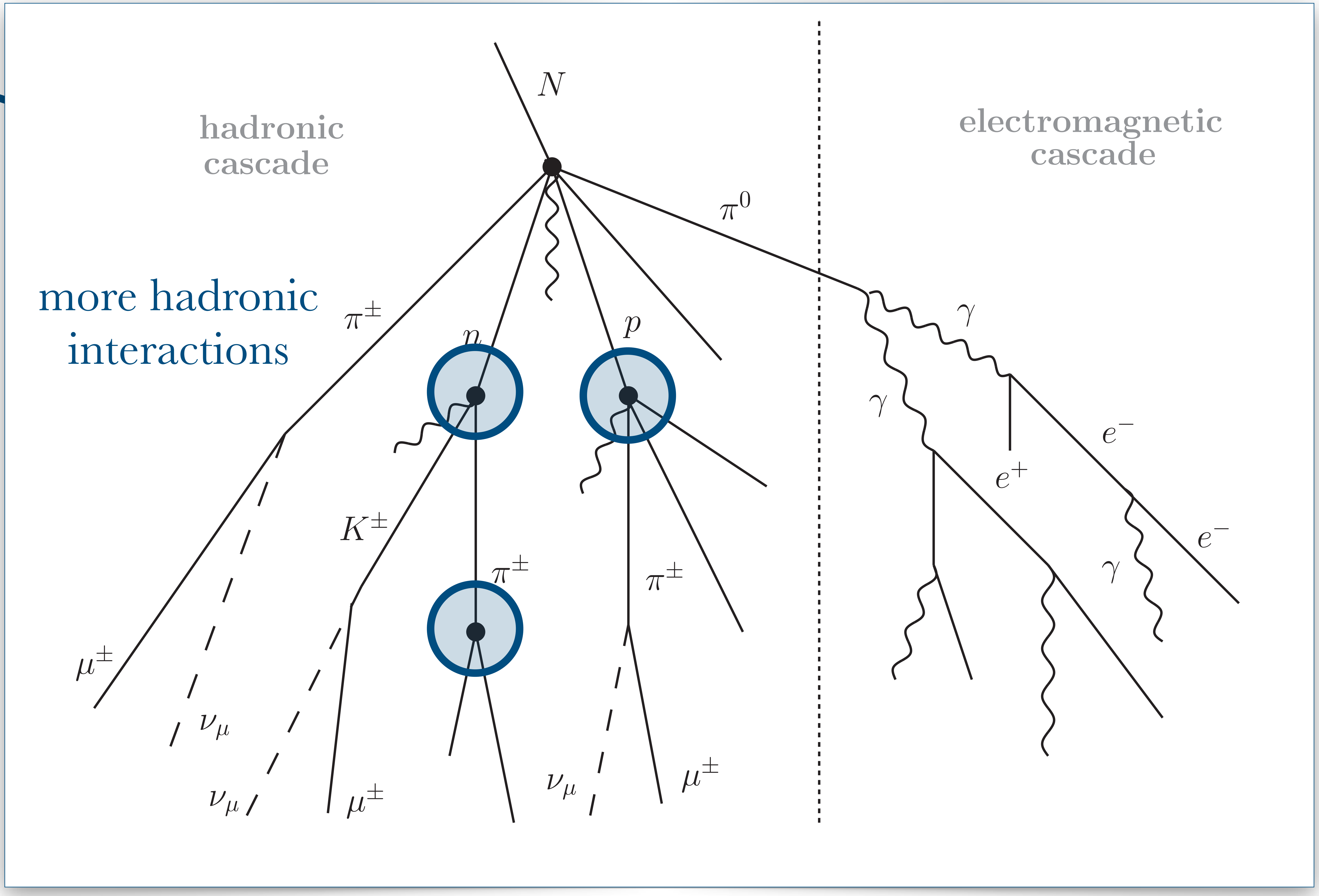


Extensive Air Showers (EAS)

- CR properties are inferred from the (secondary) particles measured at the ground

What happens between **here** and **here**?

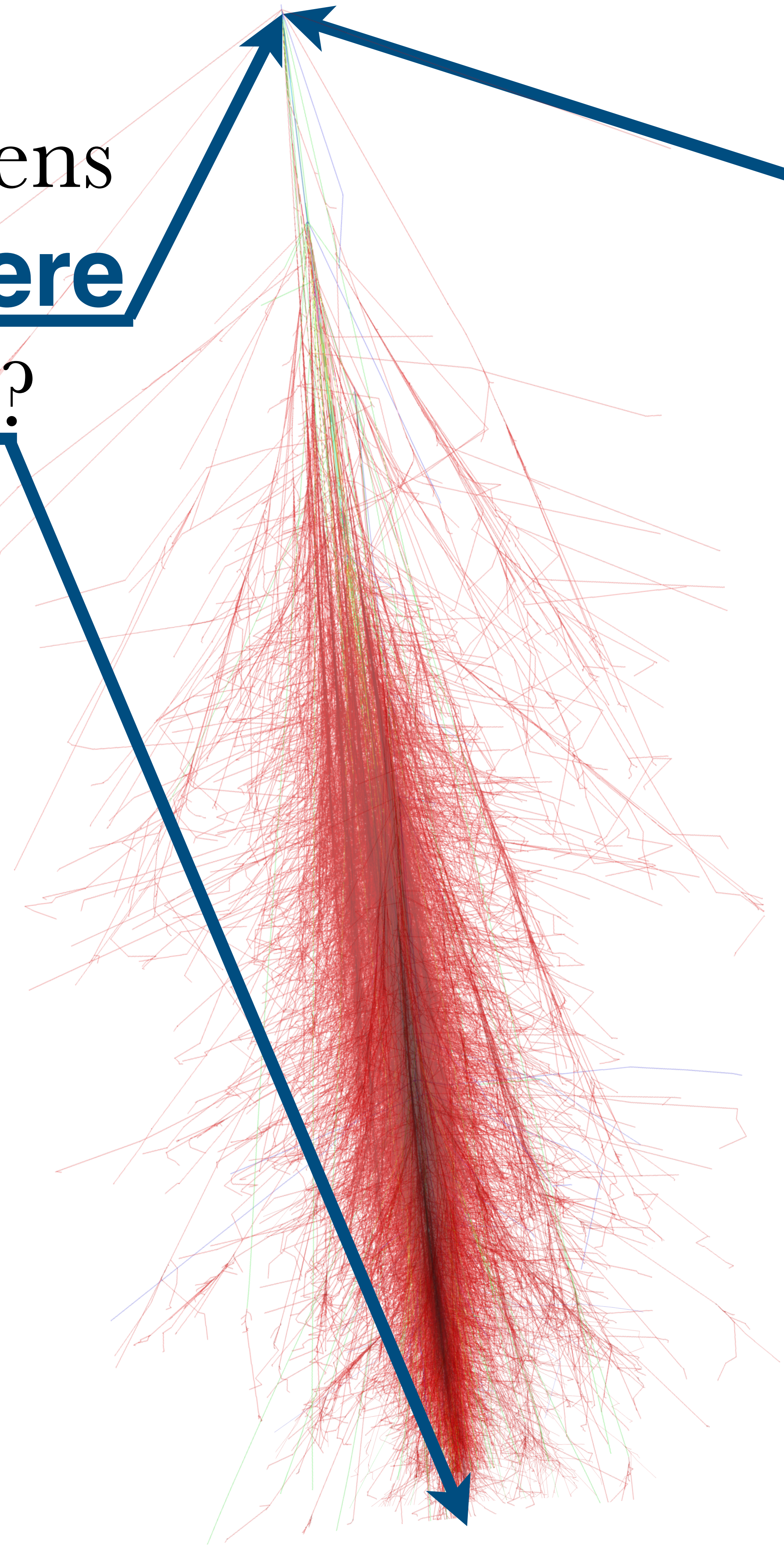


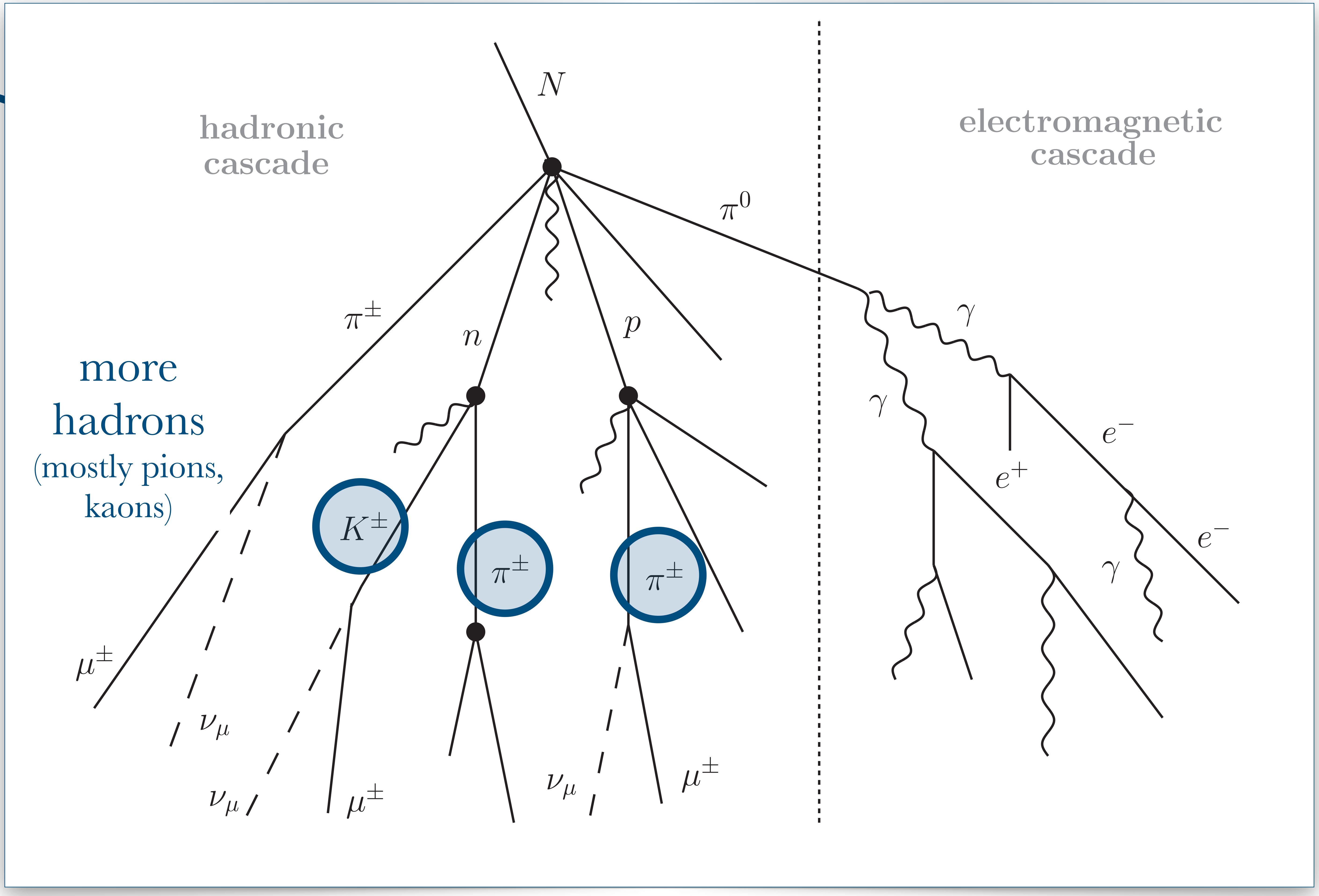


Extensive Air Showers (EAS)

- CR properties are inferred from the (secondary) particles measured at the ground

What happens between **here** and **here**?

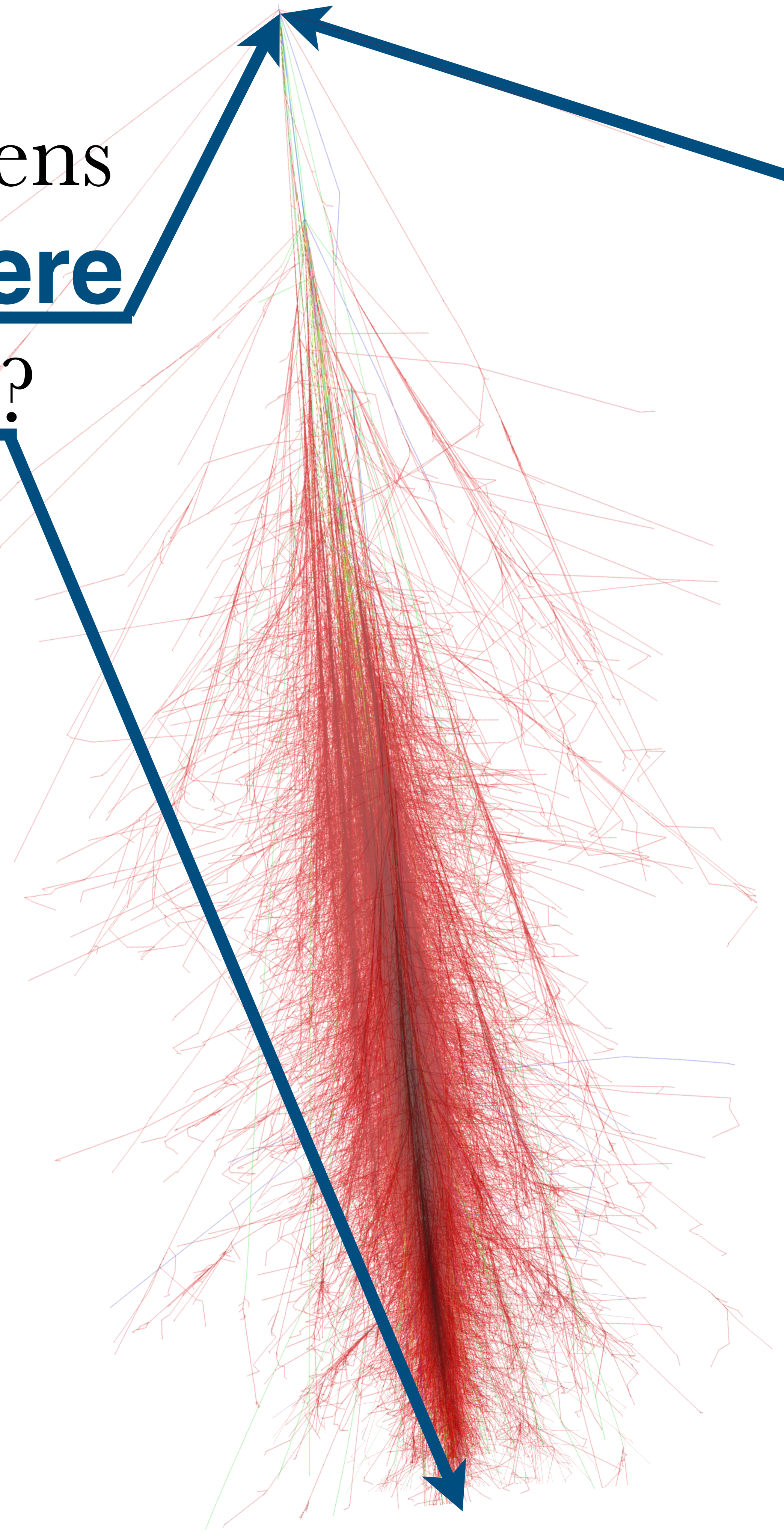


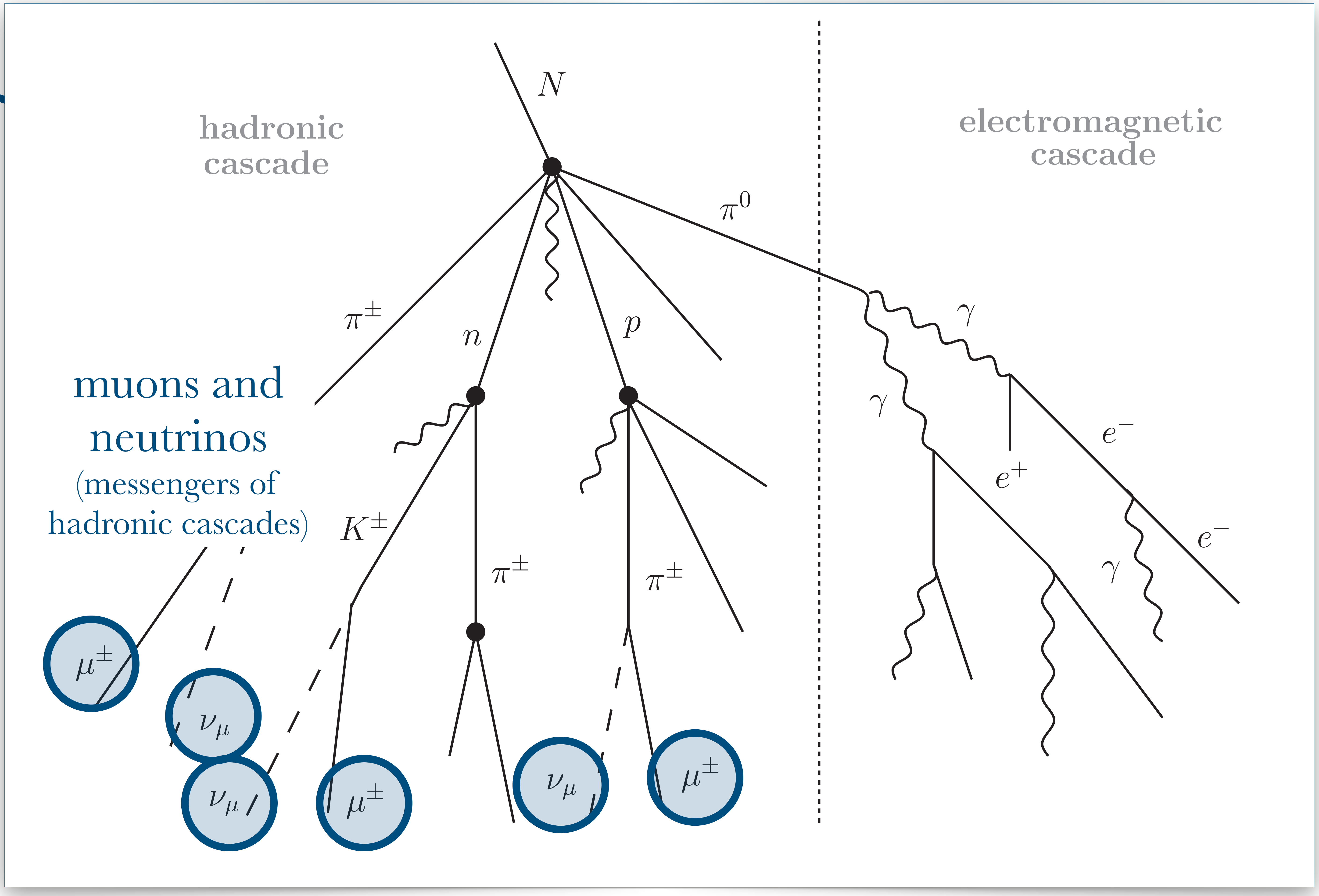


Extensive Air Showers (EAS)

- CR properties are inferred from the (secondary) particles measured at the ground

What happens between **here** and **here**?

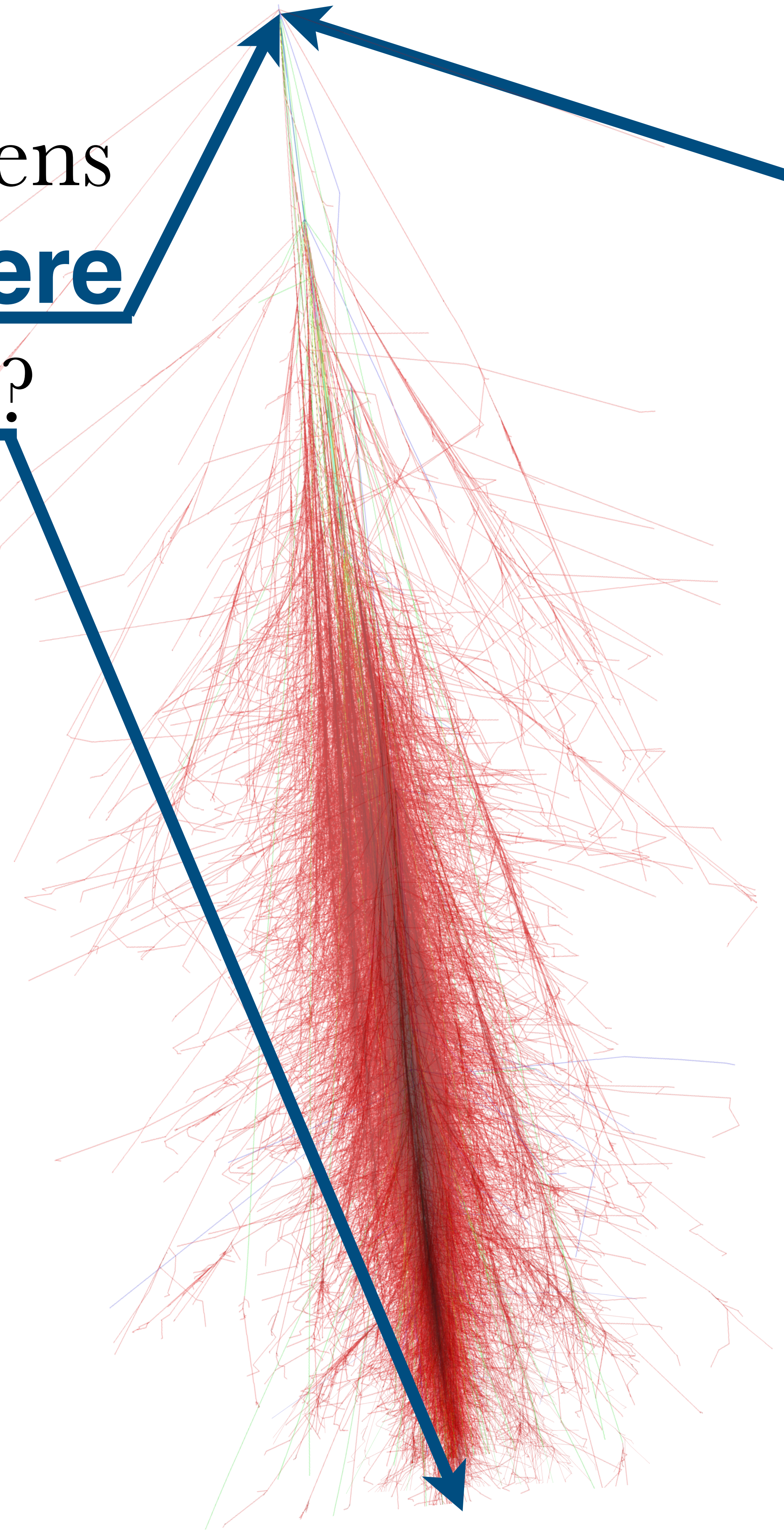


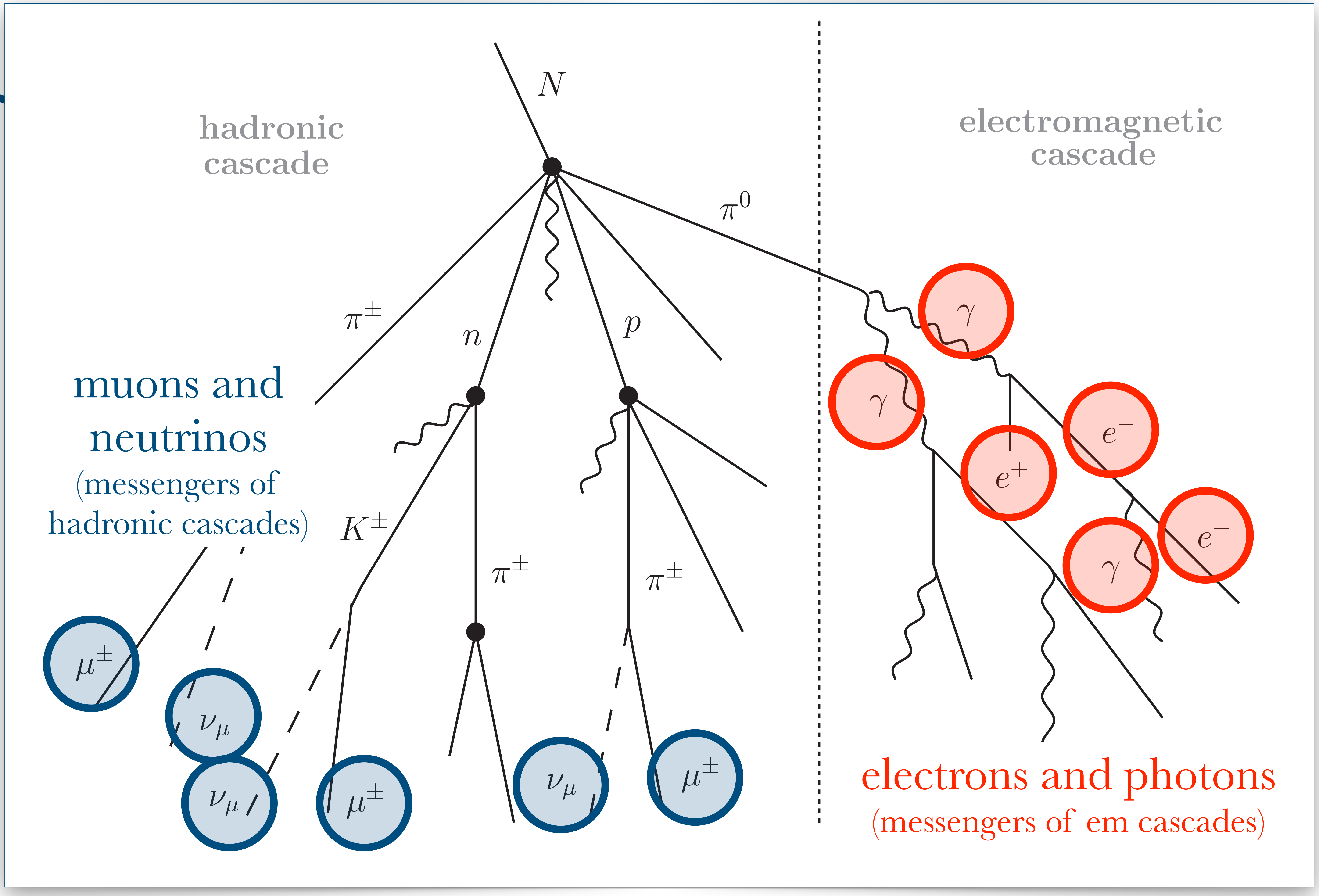


Extensive Air Showers (EAS)

- CR properties are inferred from the (secondary) particles measured at the ground

What happens between **here** and **here**?

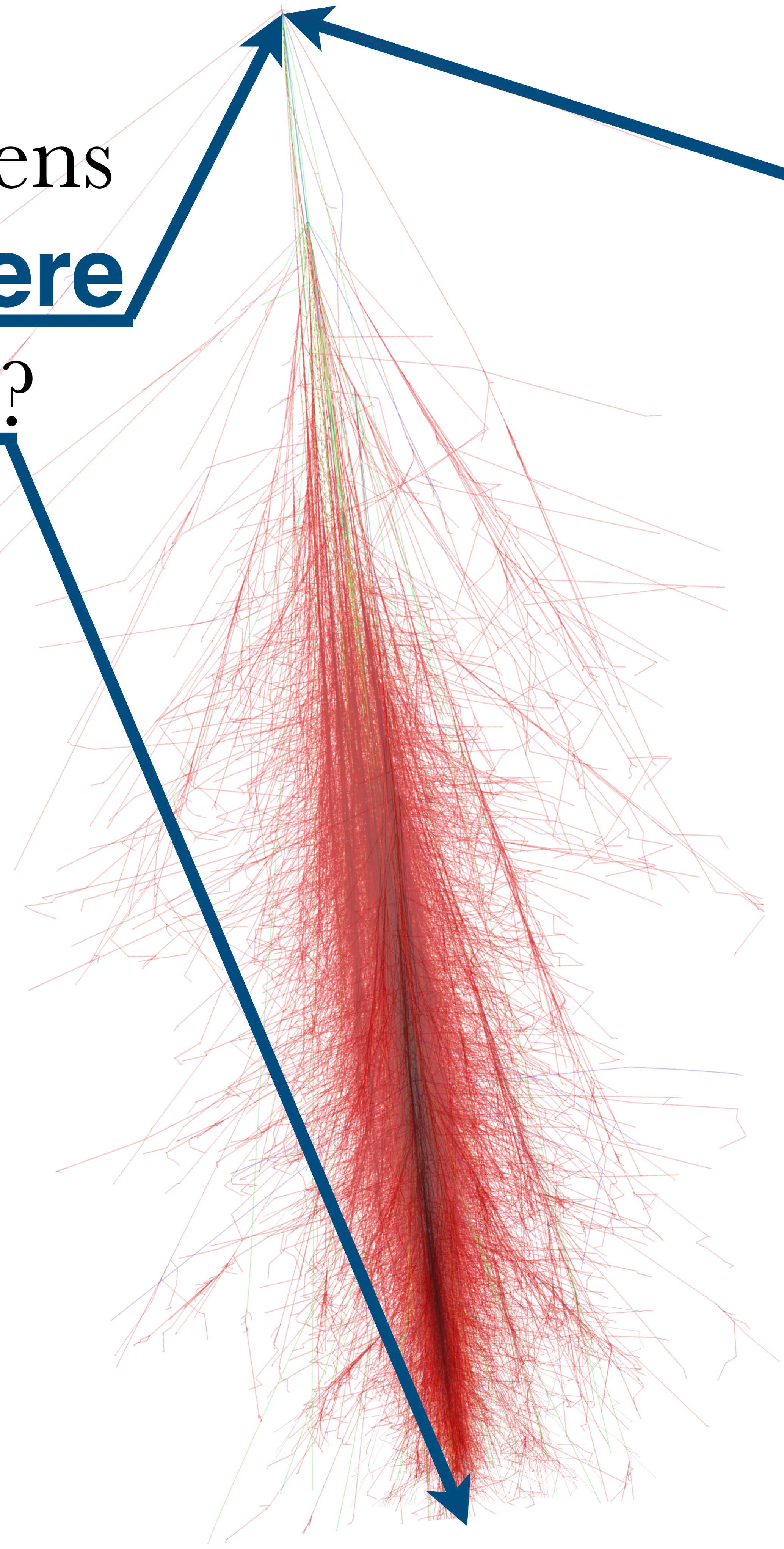


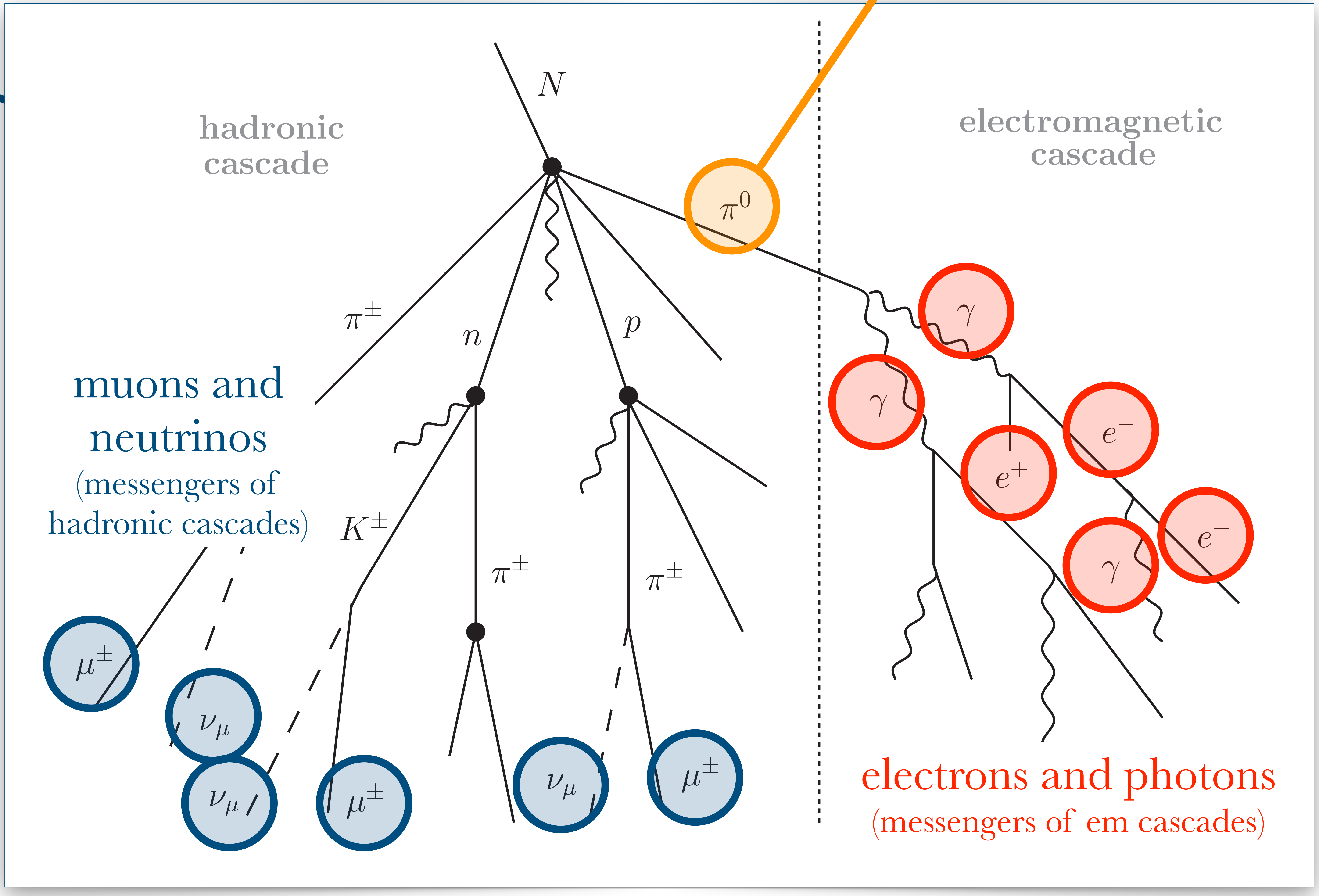


Extensive Air Showers (EAS)

- CR properties are inferred from the (secondary) particles measured at the ground

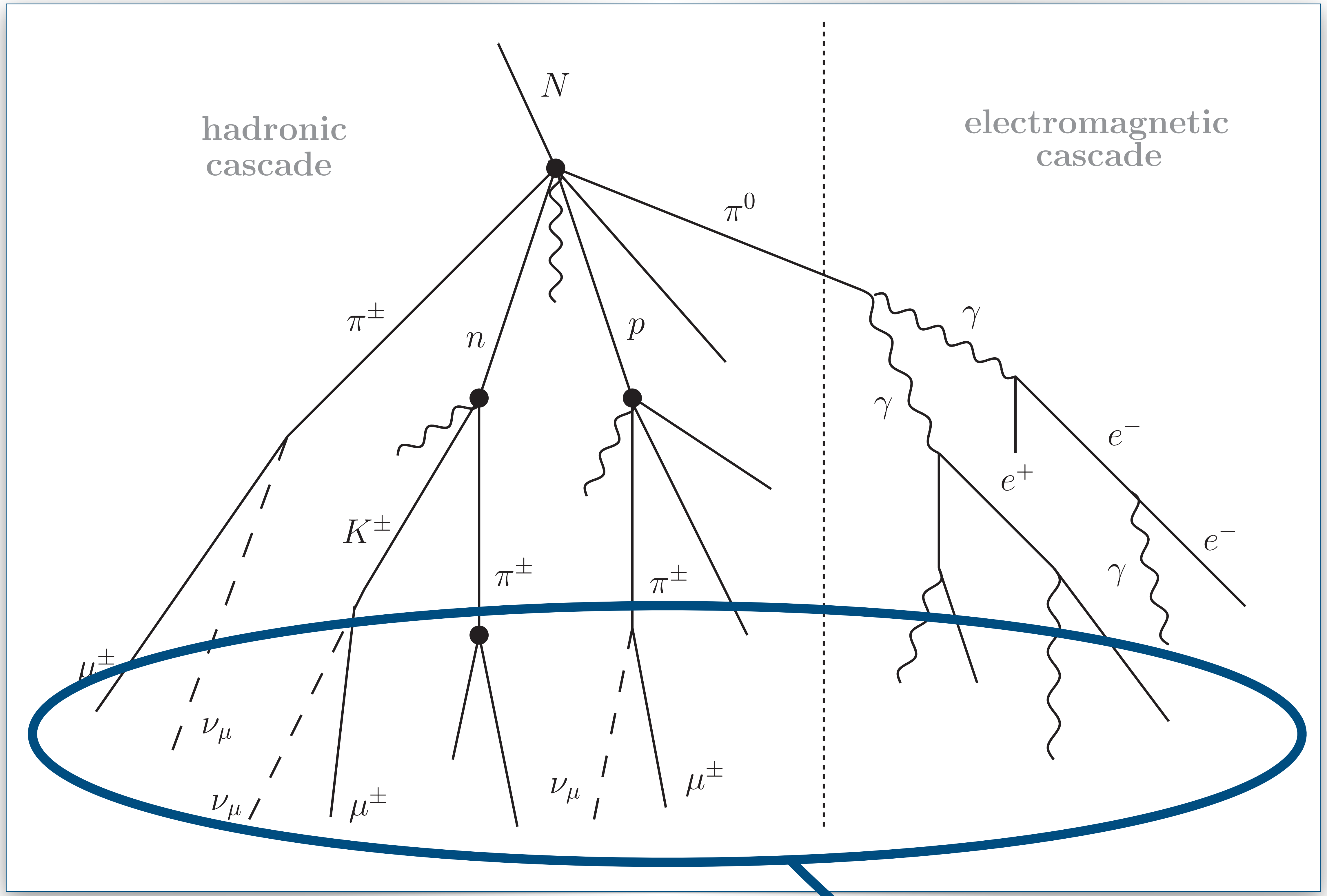
What happens between **here** and **here**?





Plays an important role, transferring energy from the hadronic to the electromagnetic cascade!

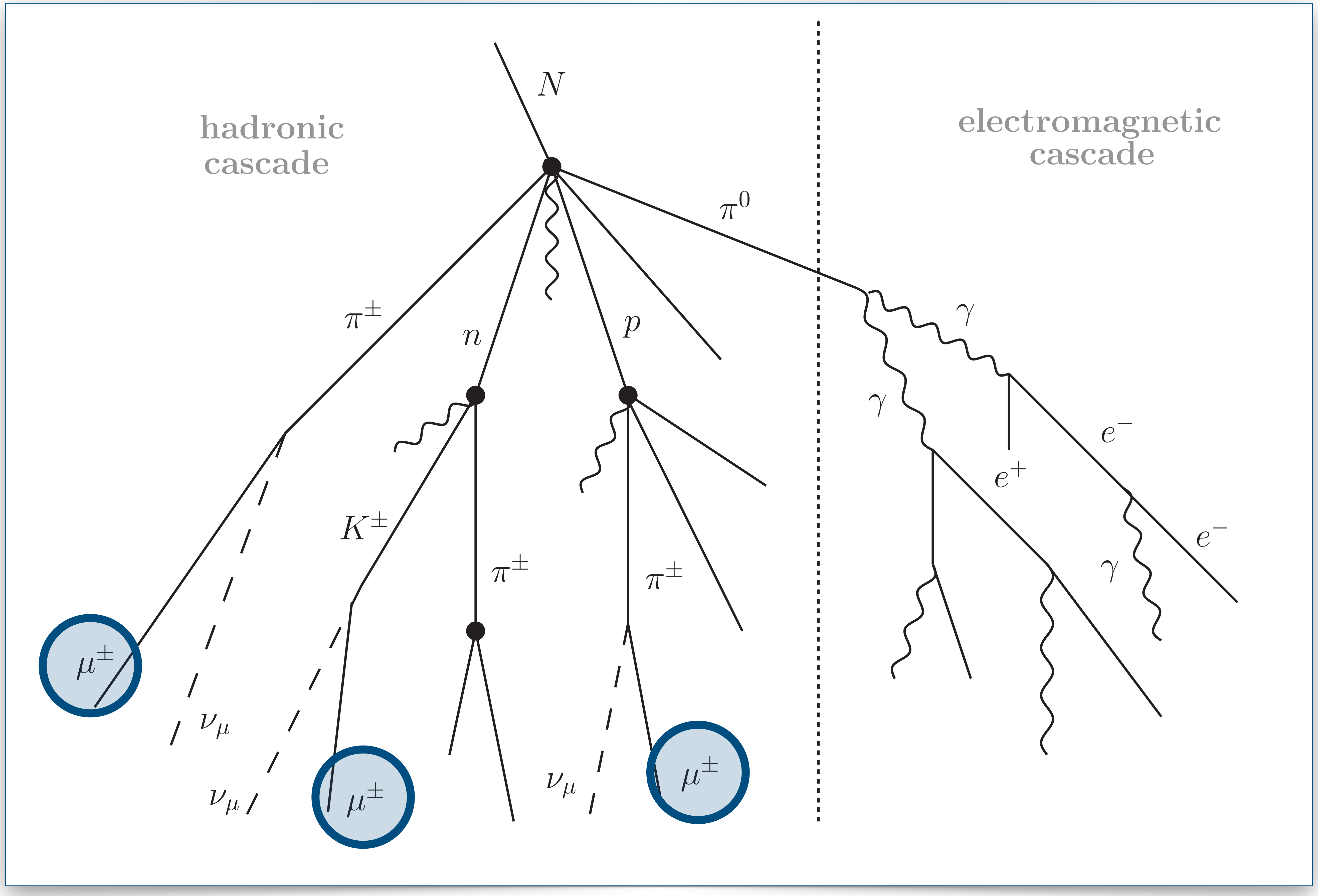
Extensive Air Showers (EAS)



- Observation: We see the complex "mess" after multiple collisions
- Goal: Find out what initiated the collision
- Not trivial…

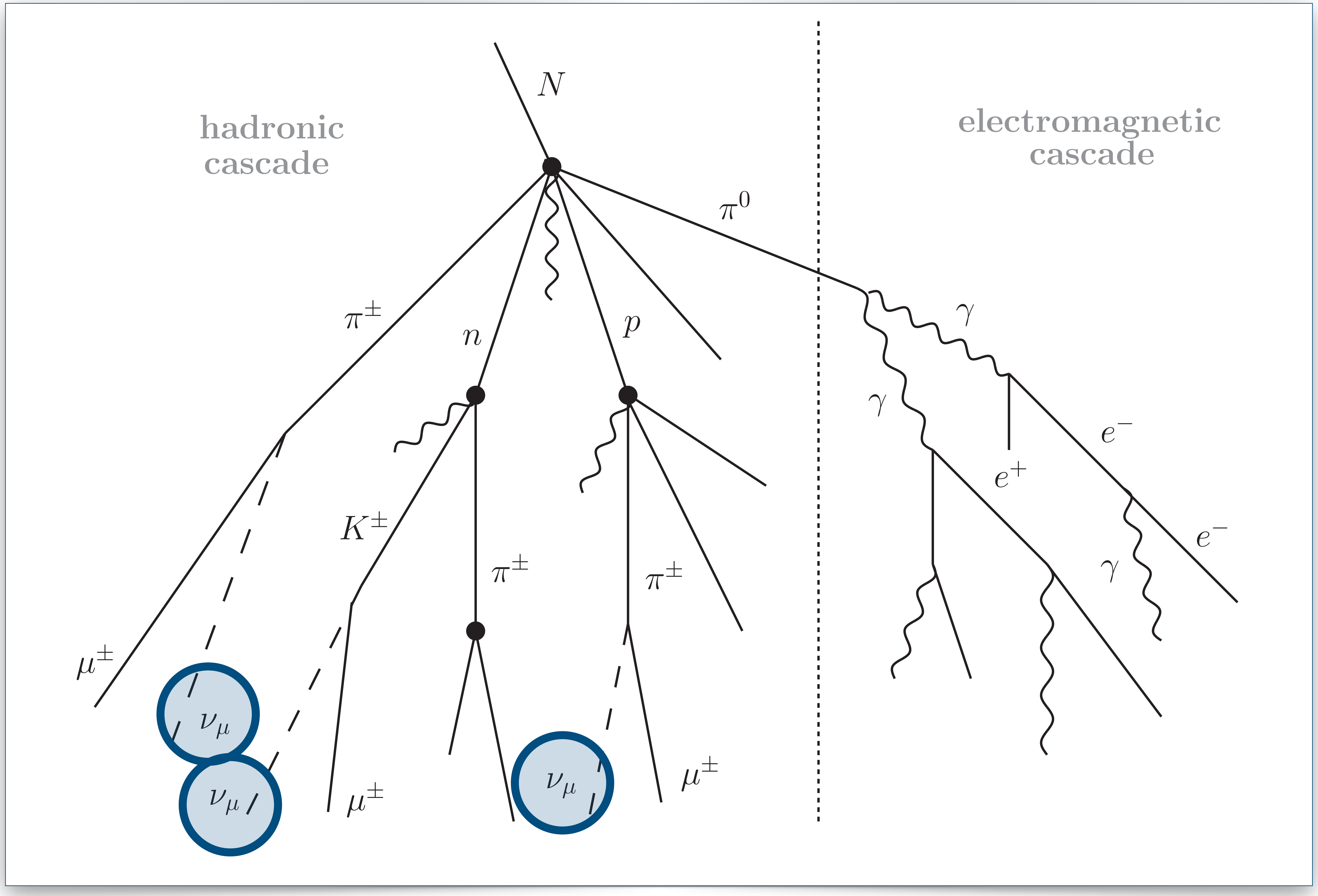
Basics: Muons

- Muons are the "heavy siblings" of the electron (about 200x heavier)
- First discovered 1936 in measurements of EAS
- Mainly produced through pion (kaon) decays in EAS
- About 100 muons per square meter per second at ground level
- Highly penetrating particles, can traverse several kilometer of rock
- Life-time: 2.2 μs , Mass: 105.66 MeV
- Distance traveled in an EAS: $l = \gamma c t$, where $\gamma = E_\mu / m_\mu$ (more in the exercise)



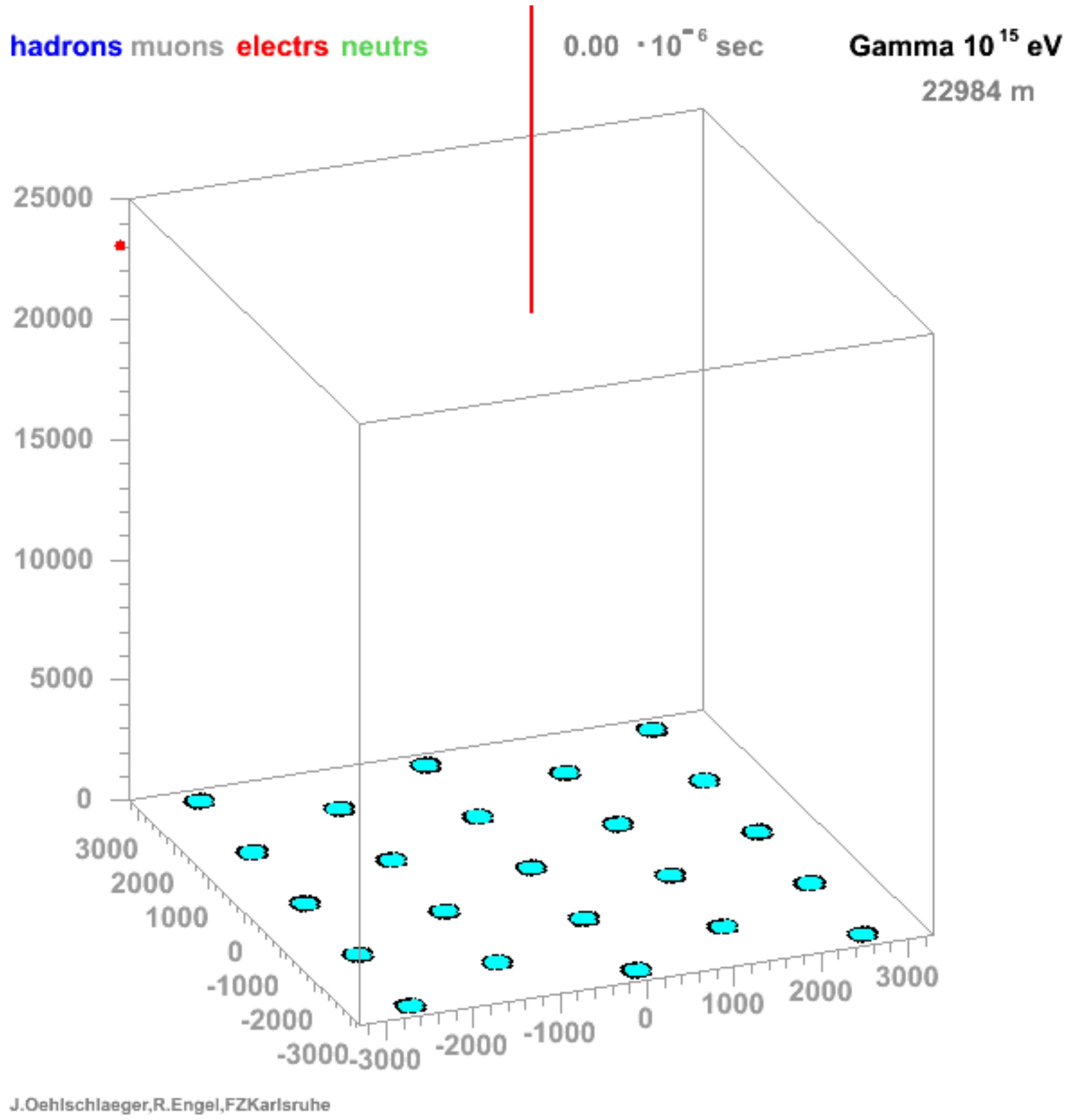
Basics: Neutrinos

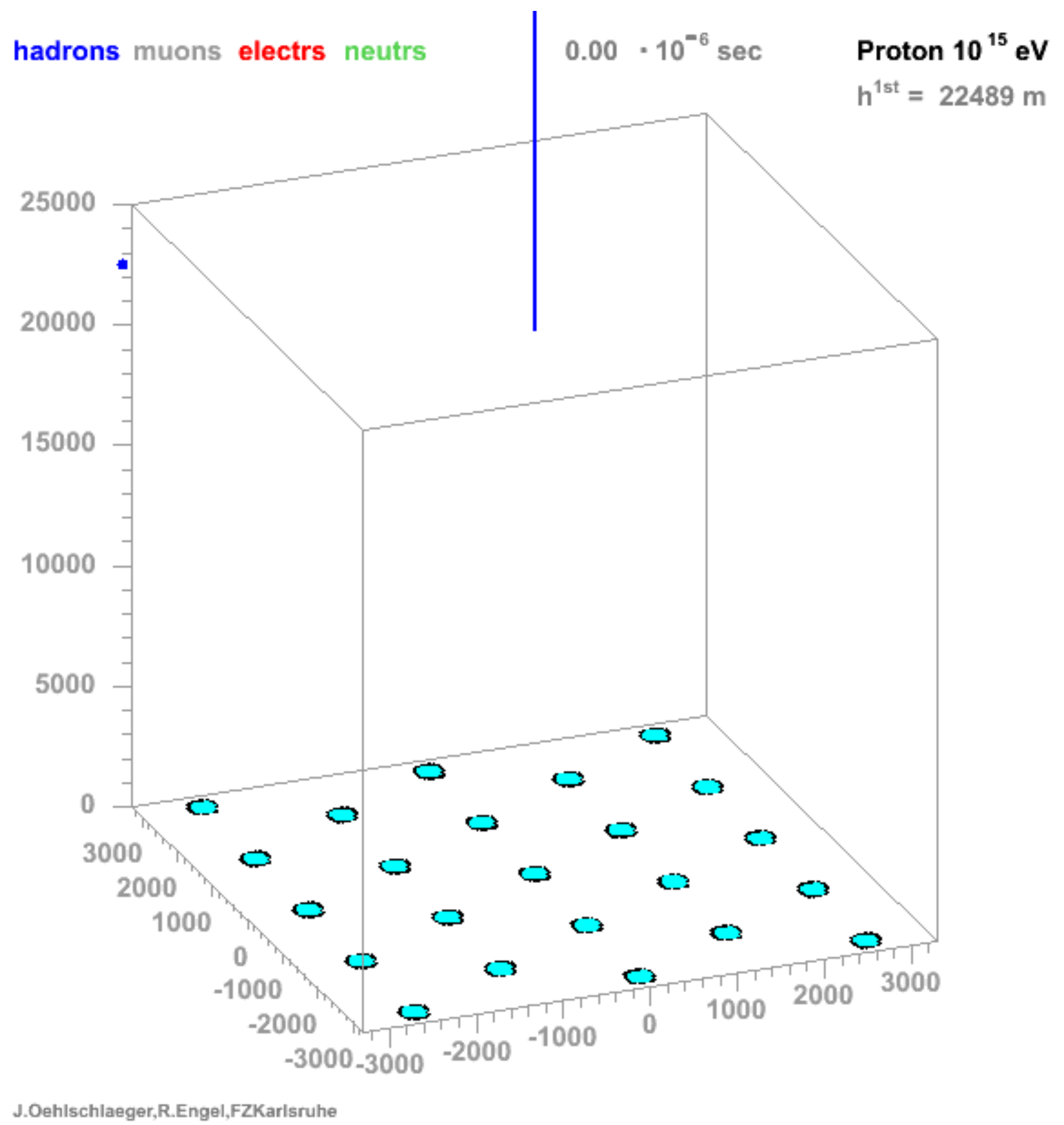
- Neutrinos interact only via the weak interaction!
- Mostly pass through normal matter unimpeded and undetected
- Three flavors: ν_e, ν_μ, ν_τ
- Can oscillate between flavors!
- Mass: $< 0.120\,\mathrm{eV}$
- Mainly produced through nuclear interactions in the sun
- About 65 billion neutrinos per square centimeter per second at ground level!

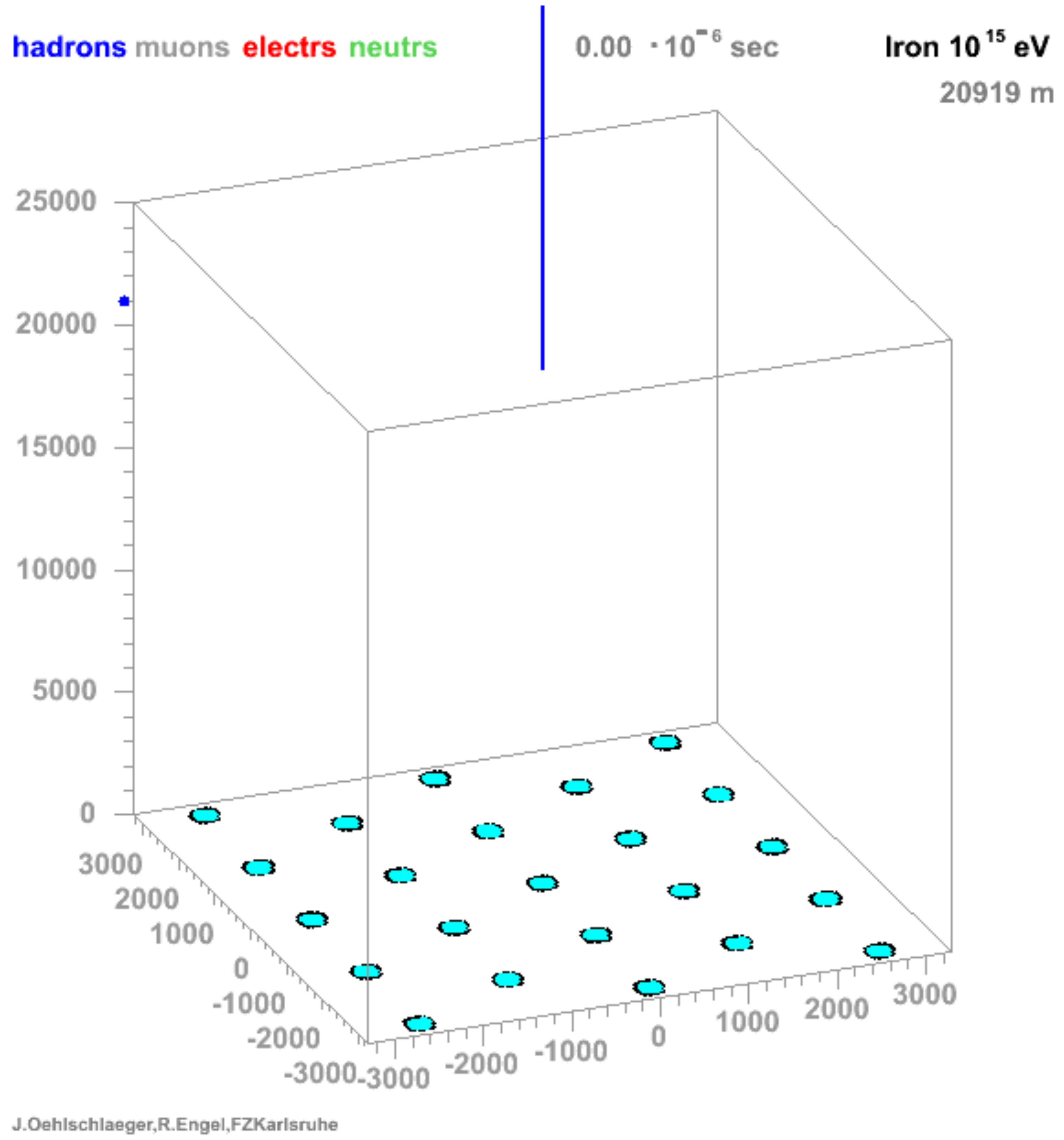


Some EAS Simulations…
(later more details)

Extensive Air Showers







- Simulated gamma, proton, and iron showers at $E_0 = 10^{15}\,\mathrm{eV}$

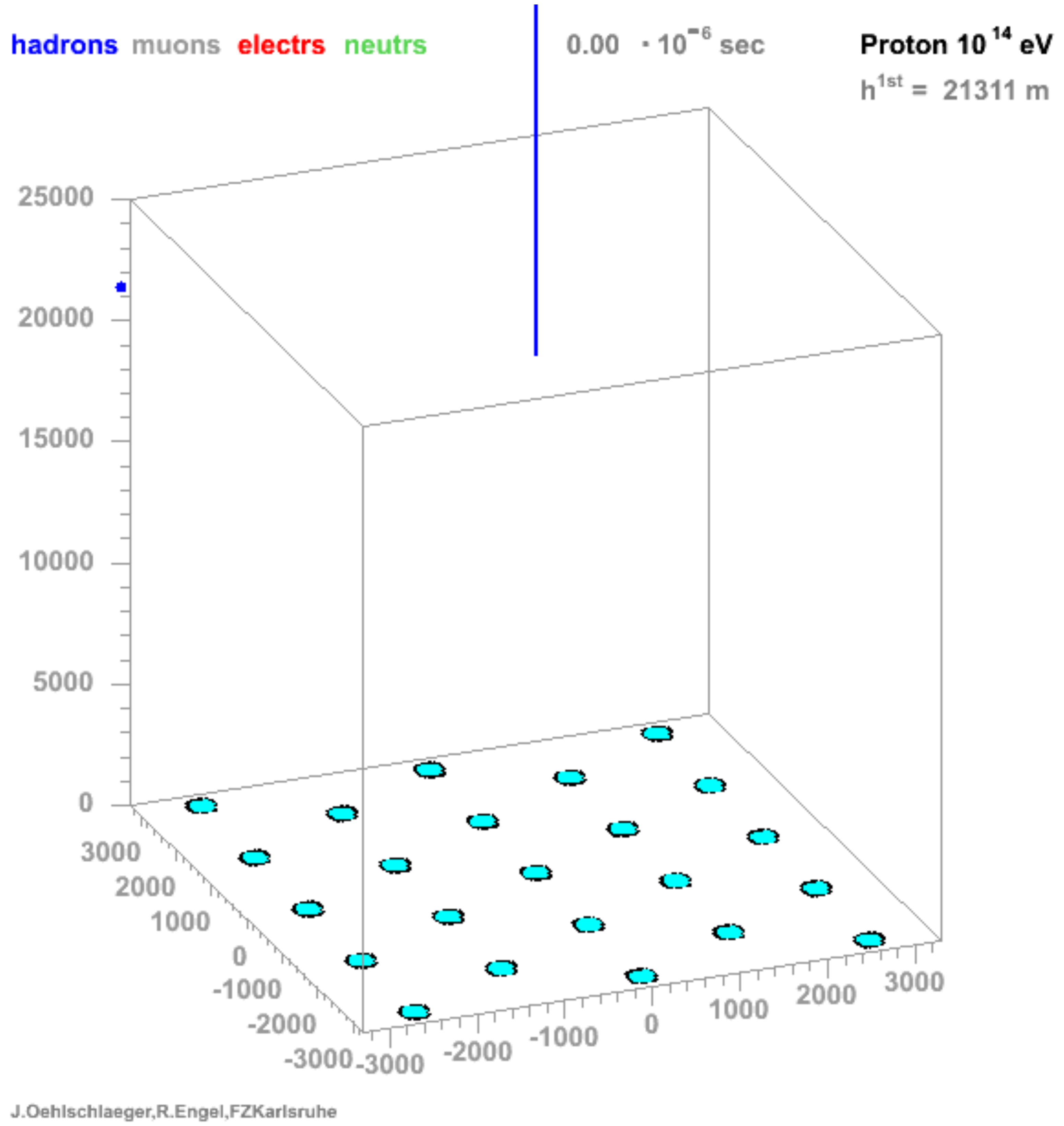
[https://www.iap.kit.edu/corsika/]

- Later more about EAS simulations…

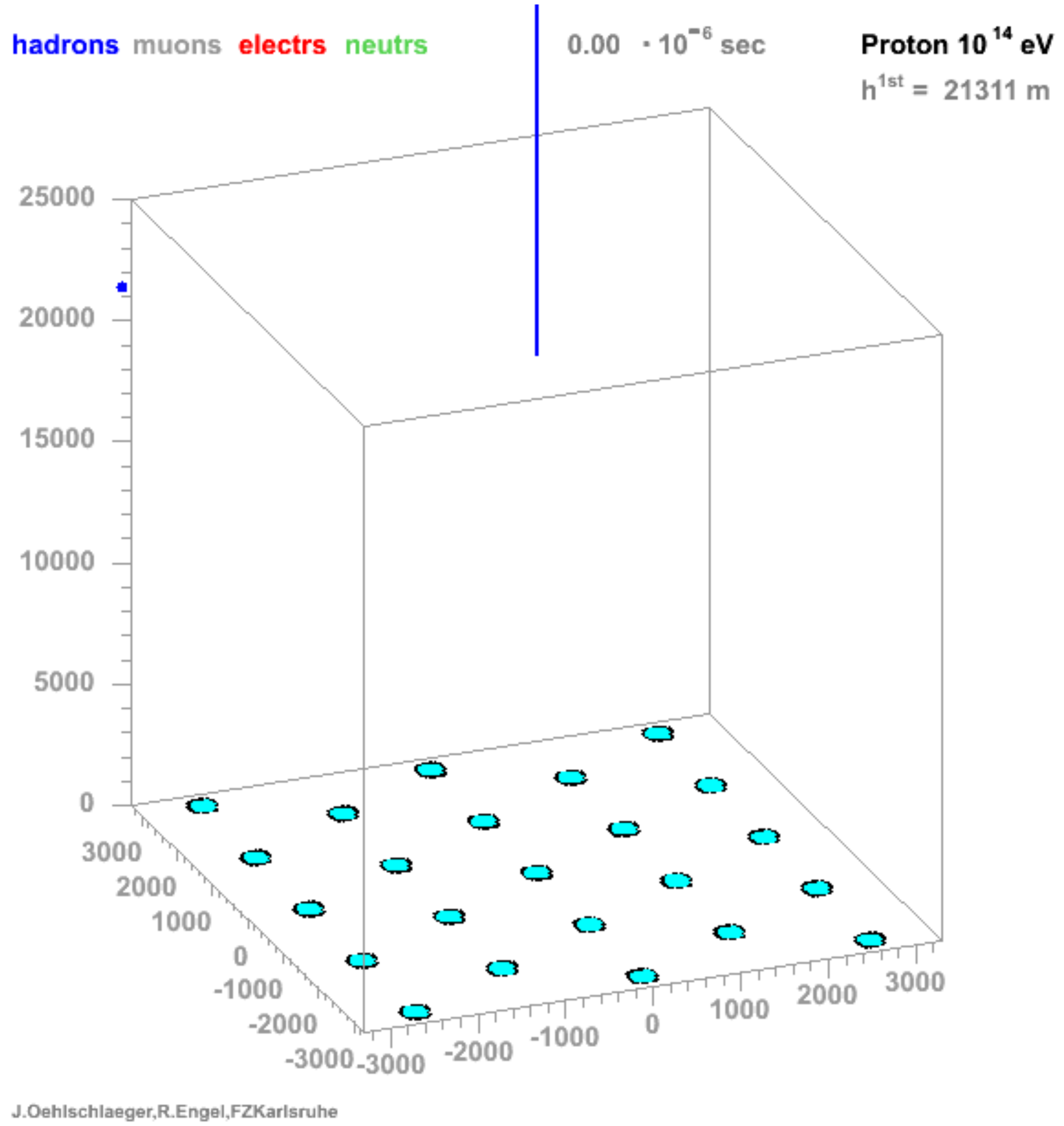
Extensive Air Showers

- Simulated proton shower at 10^{14} eV

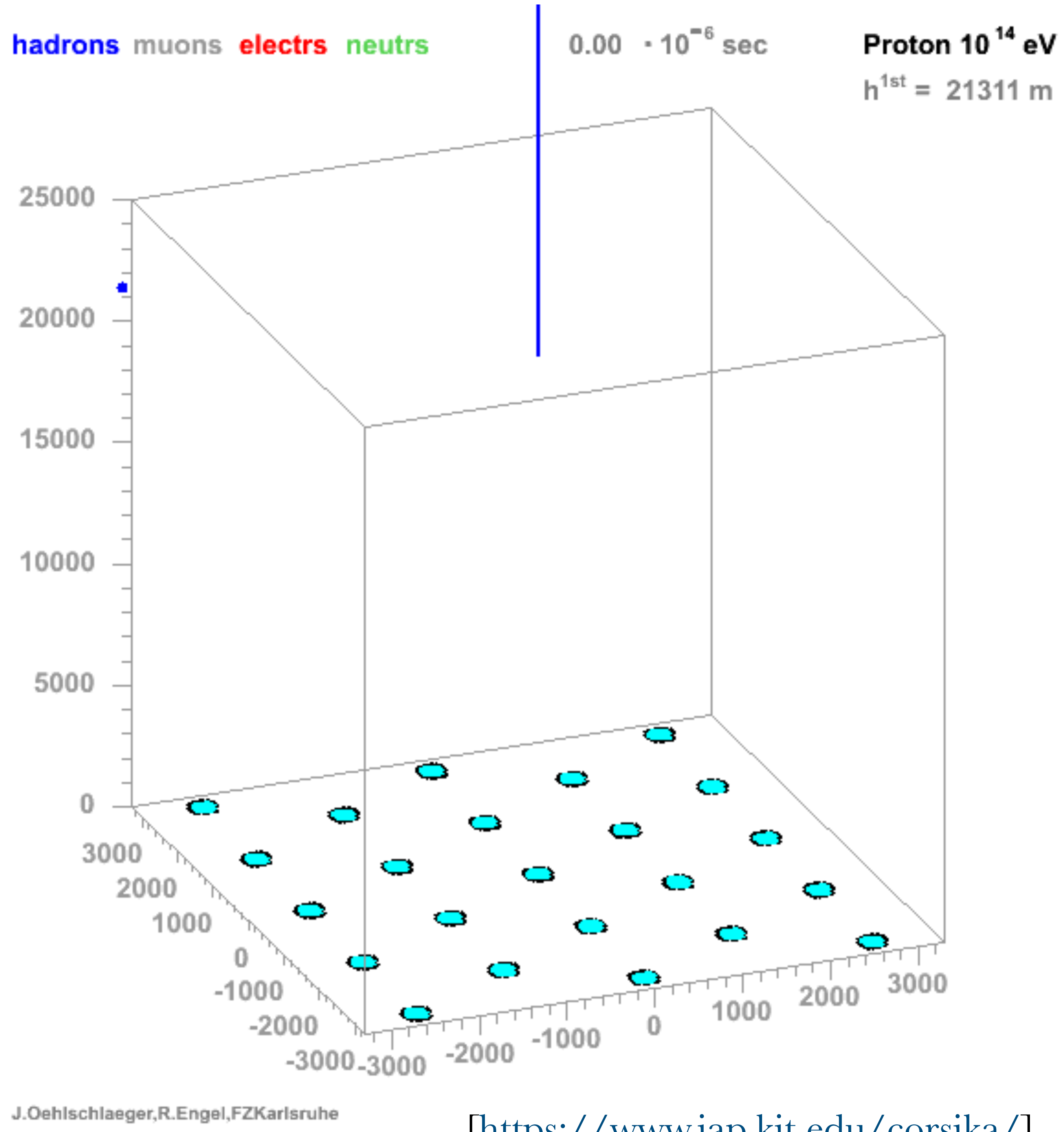
All particles



No electrons



No electrons, no muons

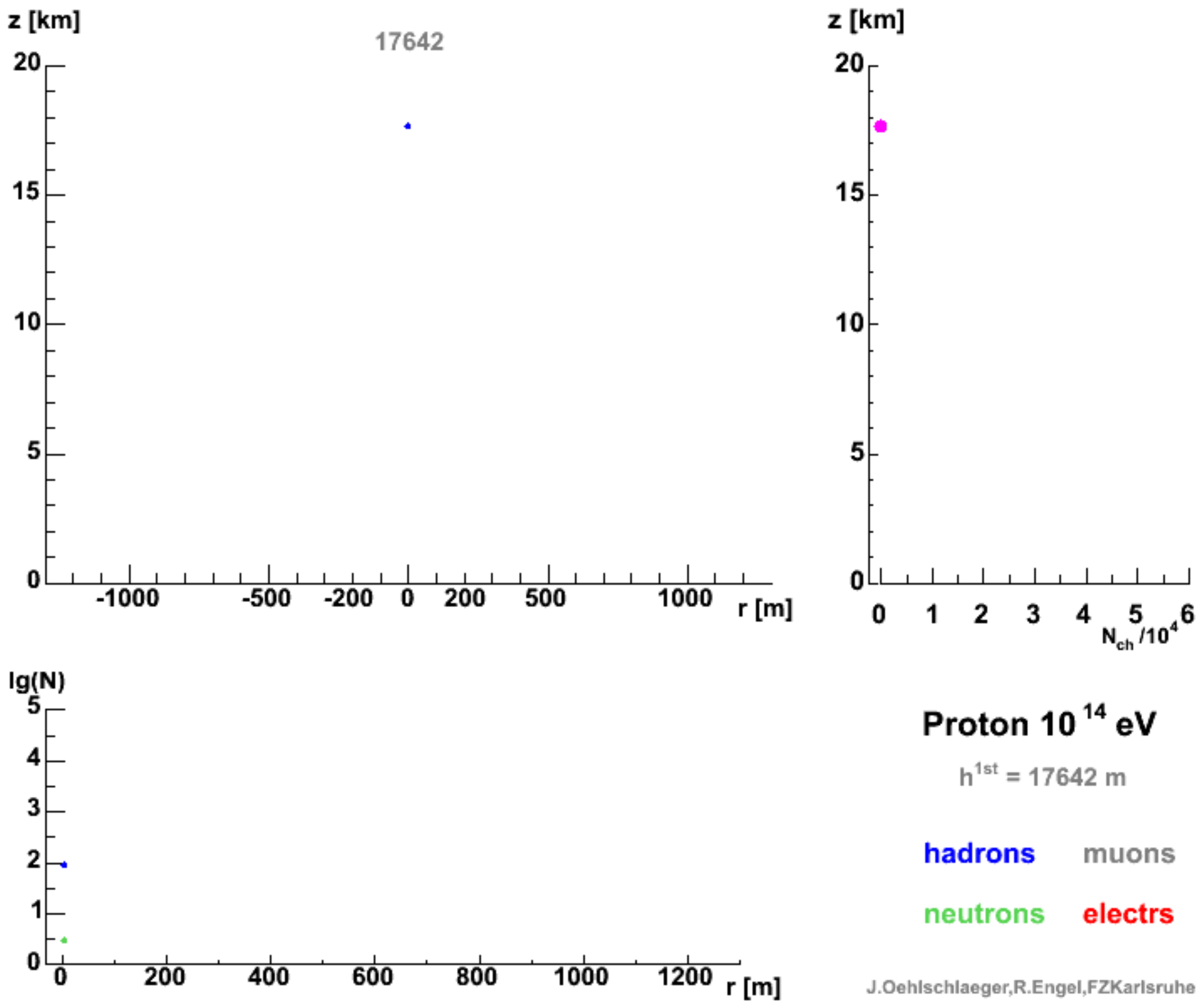


[https://www.iap.kit.edu/corsika/]

Extensive Air Showers

[https://www.iap.kit.edu/corsika/]

- EAS simulation (proton, 10^{15} eV)
  - Shower front
  - Longitudinal profile (X_{max})
  - Lateral profile

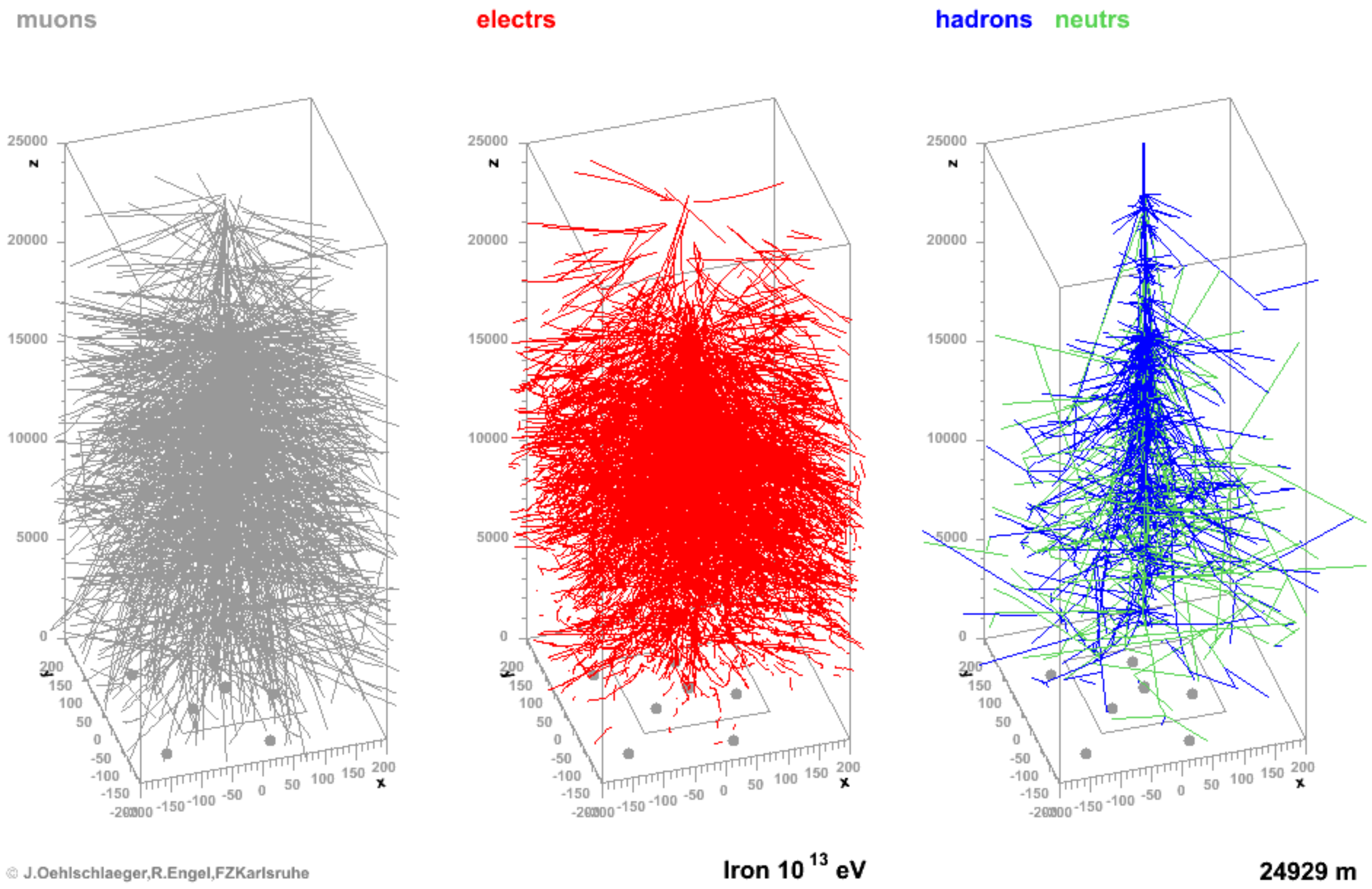


Extensive Air Showers

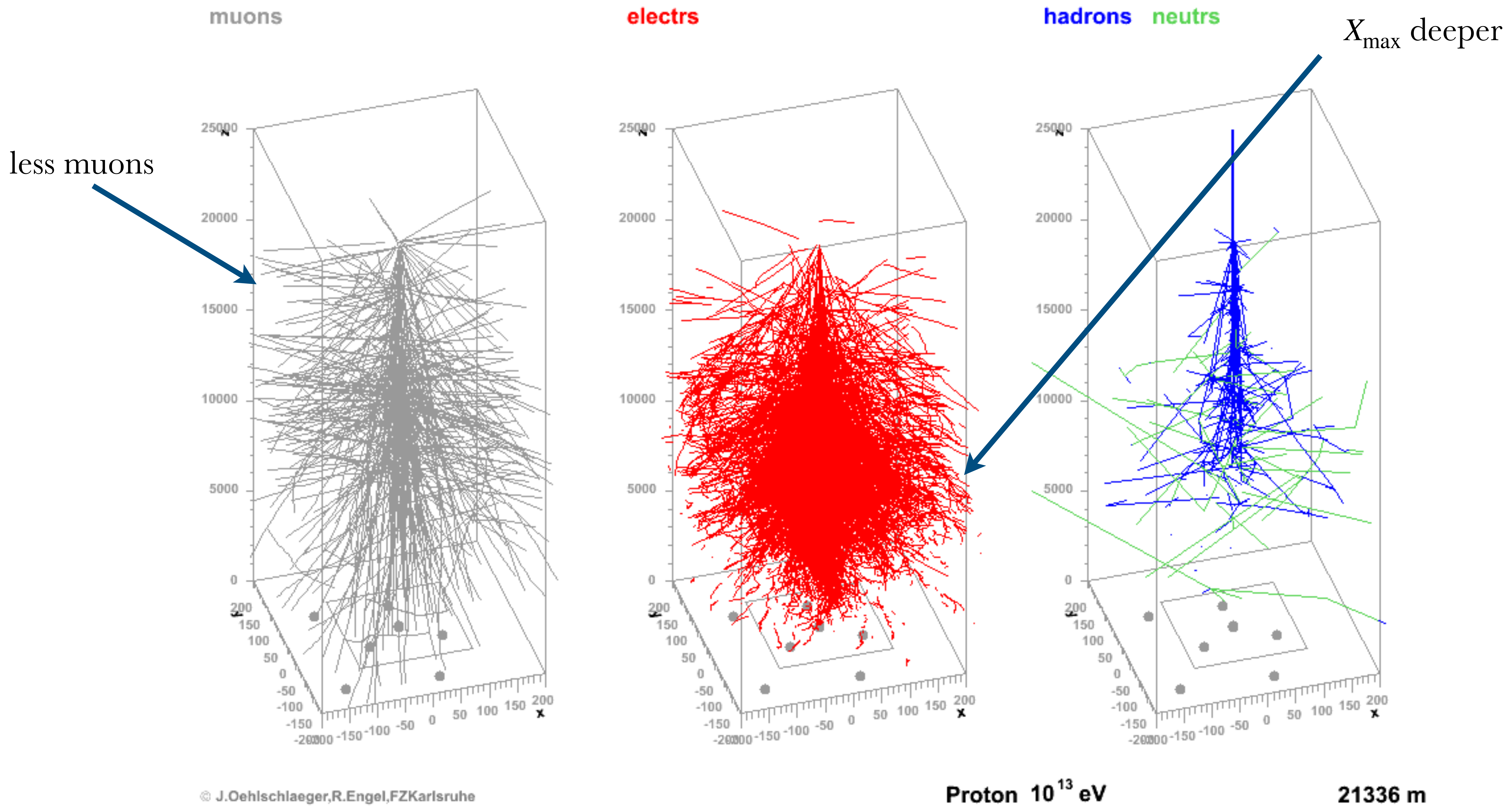
- EAS simulation (proton, 10^{15} eV)
  - Shower front
  - Secondary particles measured with detectors at the ground
- Detector simulation
  - Typically based on GEANT4
  - Detailed model for detector response (PMT, electronics, …)
  - Not in this lecture…
- This lecture:
  - What happens during EAS development in detail?

[https://www.iap.kit.edu/corsika/]

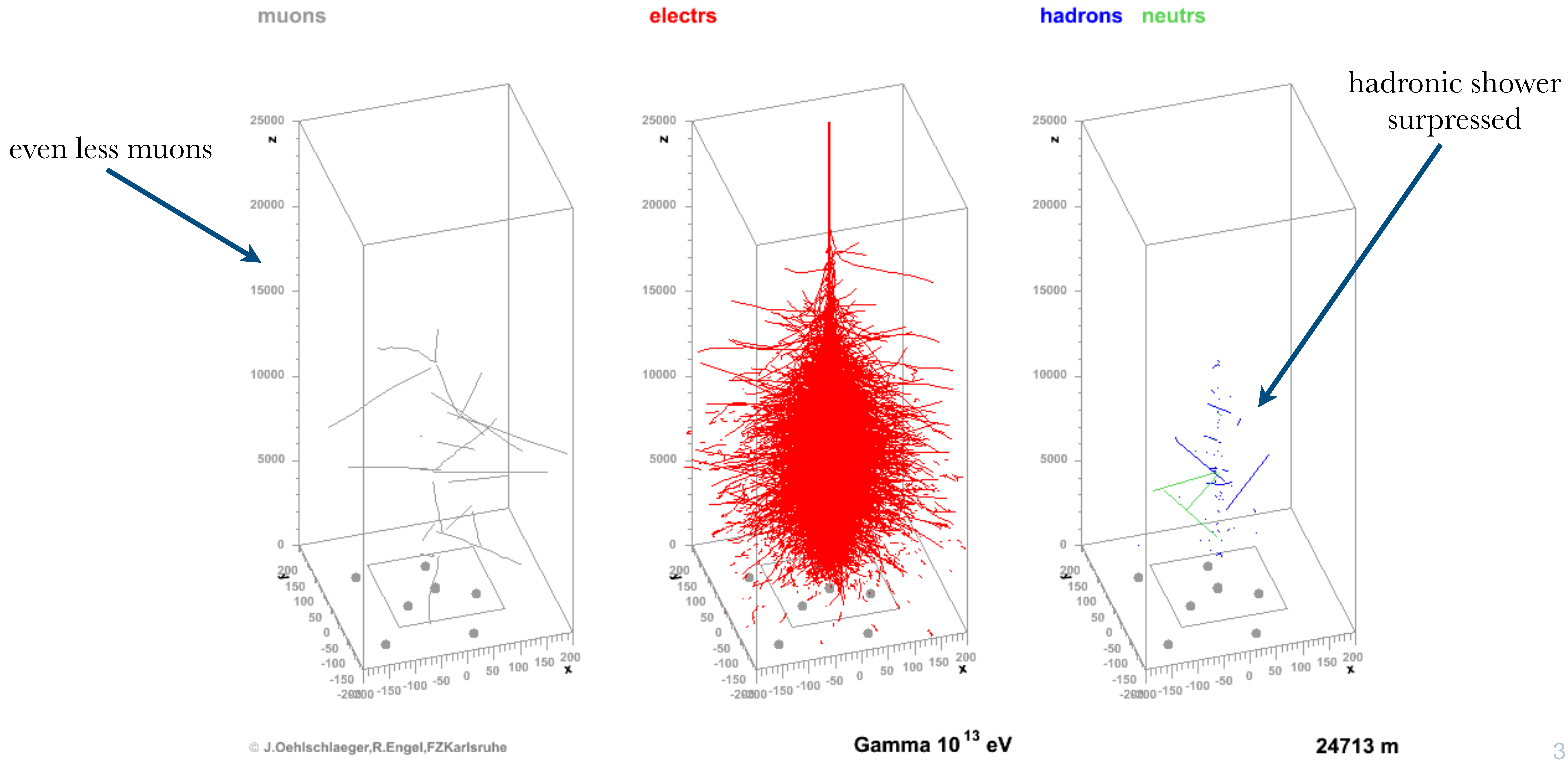
EAS Particles (Iron Shower)



EAS Particles (Proton Shower)



EAS Particles (Gamma Shower)

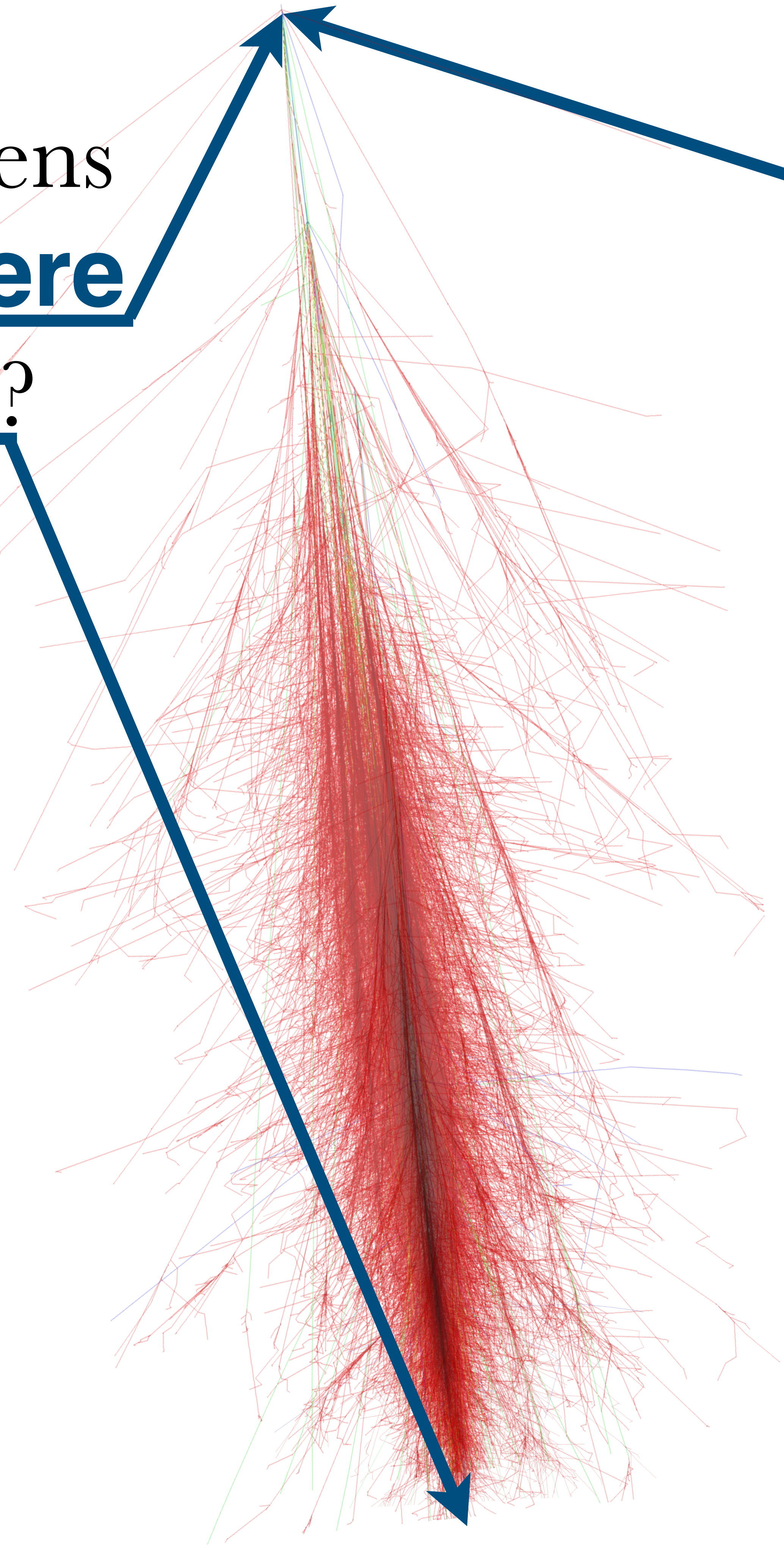


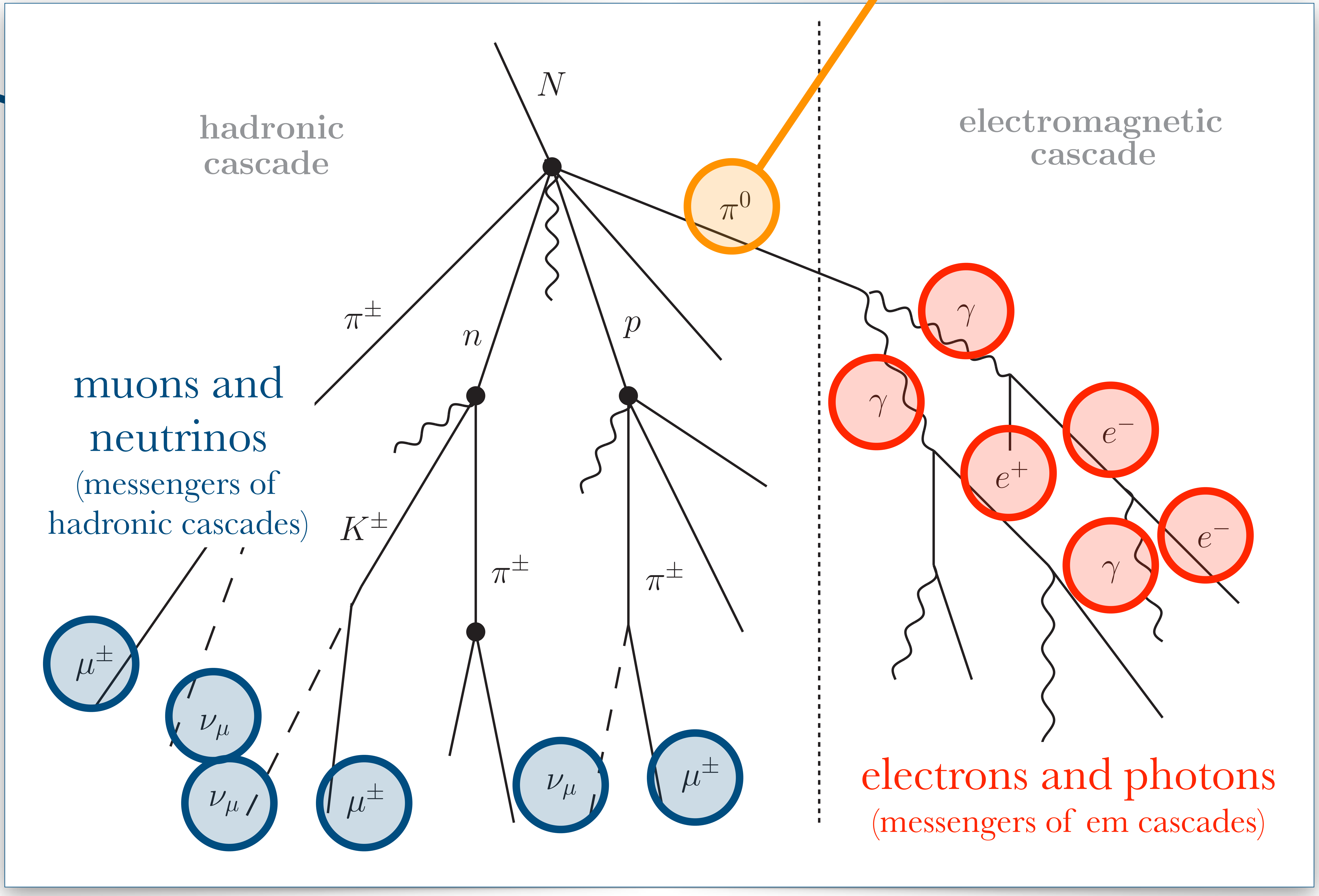
Extensive Air Showers (EAS)

Plays an important role, transferring energy from the hadronic to the electromagnetic cascade!

- CR properties are inferred from the (secondary) particles measured at the ground

What happens between **here** and **here**?





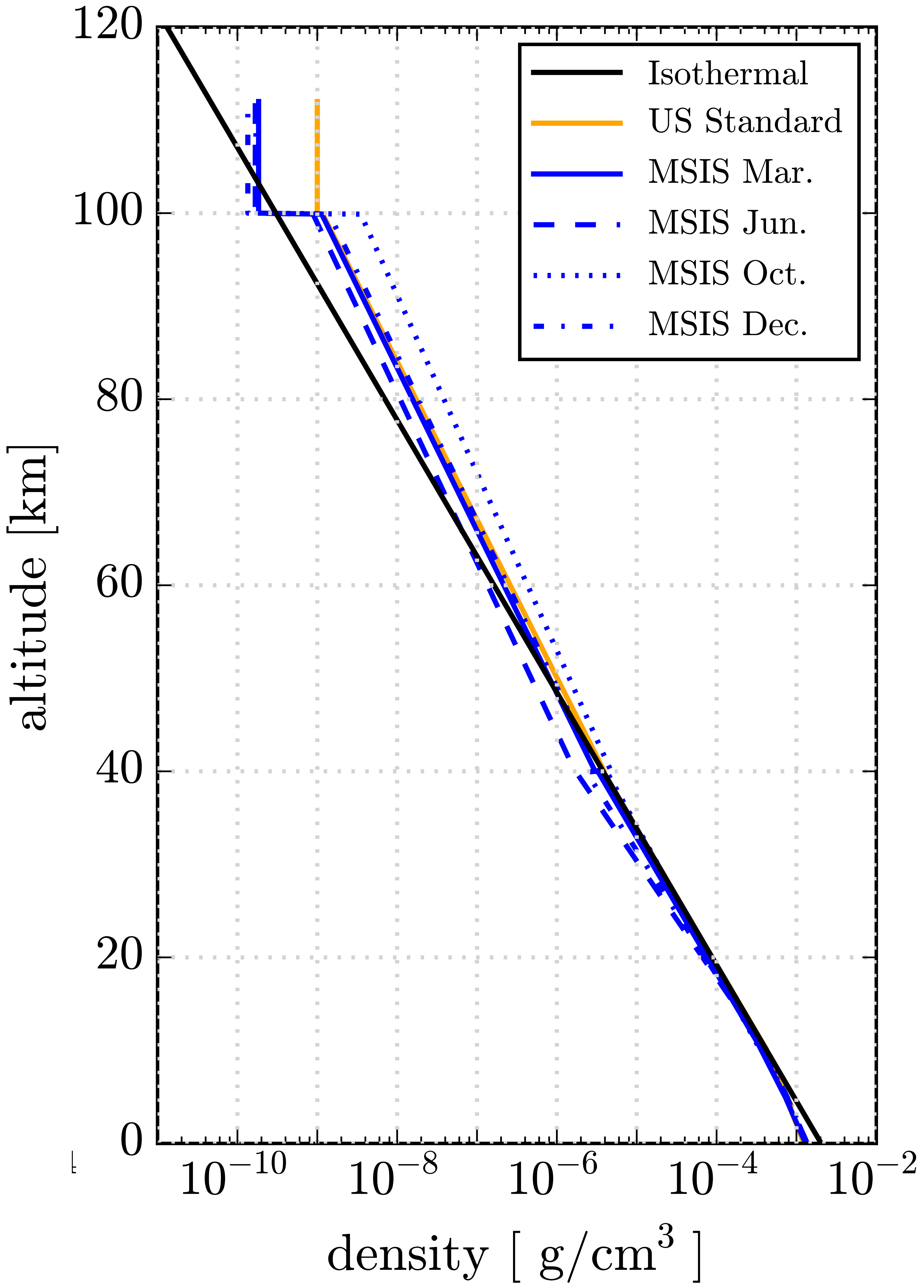
Theoretical Description of EAS

Atmosphere

- Density of the air, ρ_{air}, plays an important role for the EAS development!
- Atmospheric slant depth:

$$X = \int \rho_{\text{air}} \, dl$$

(integral taken along shower axis)



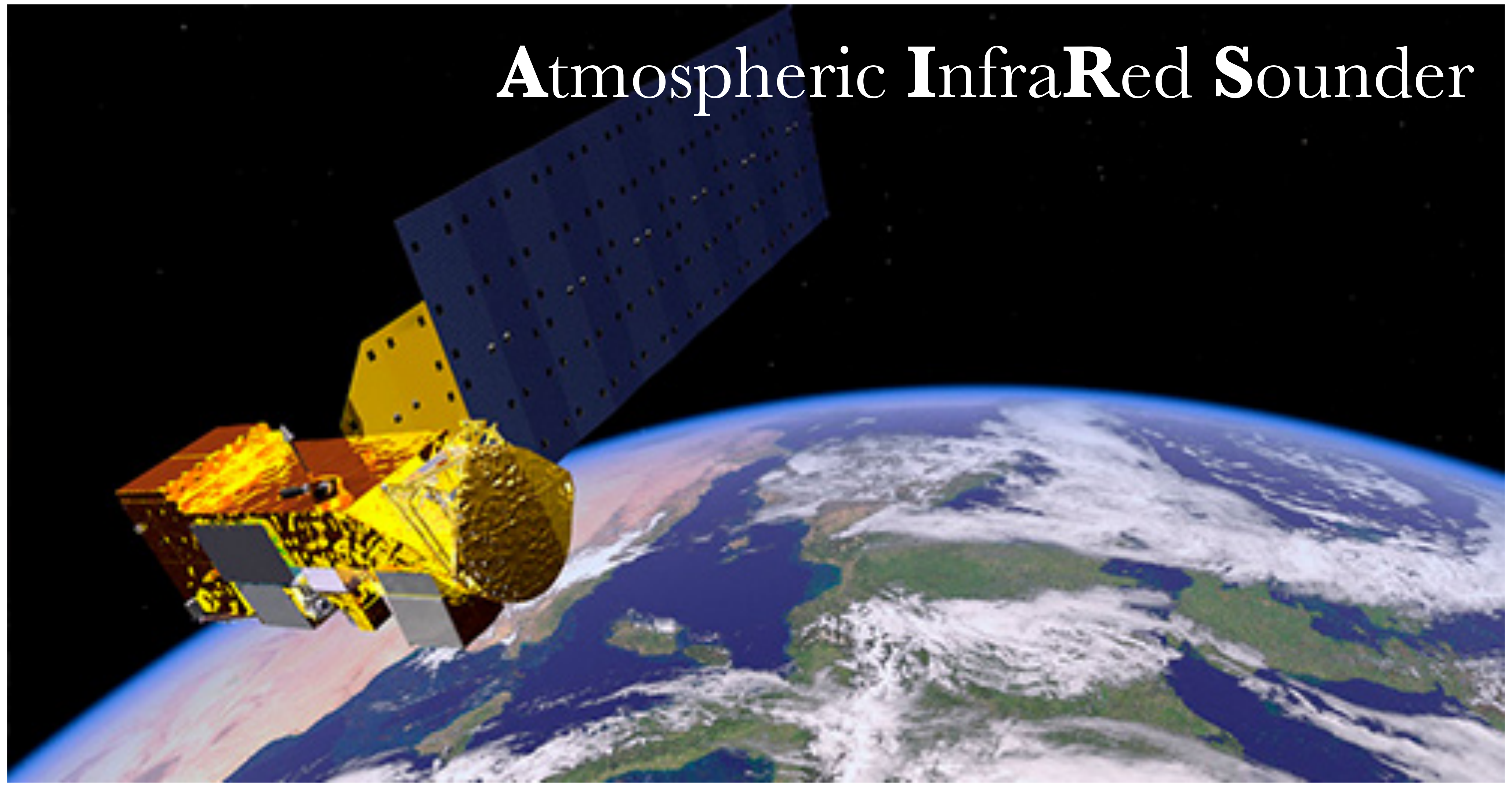
Atmosphere (South Pole)

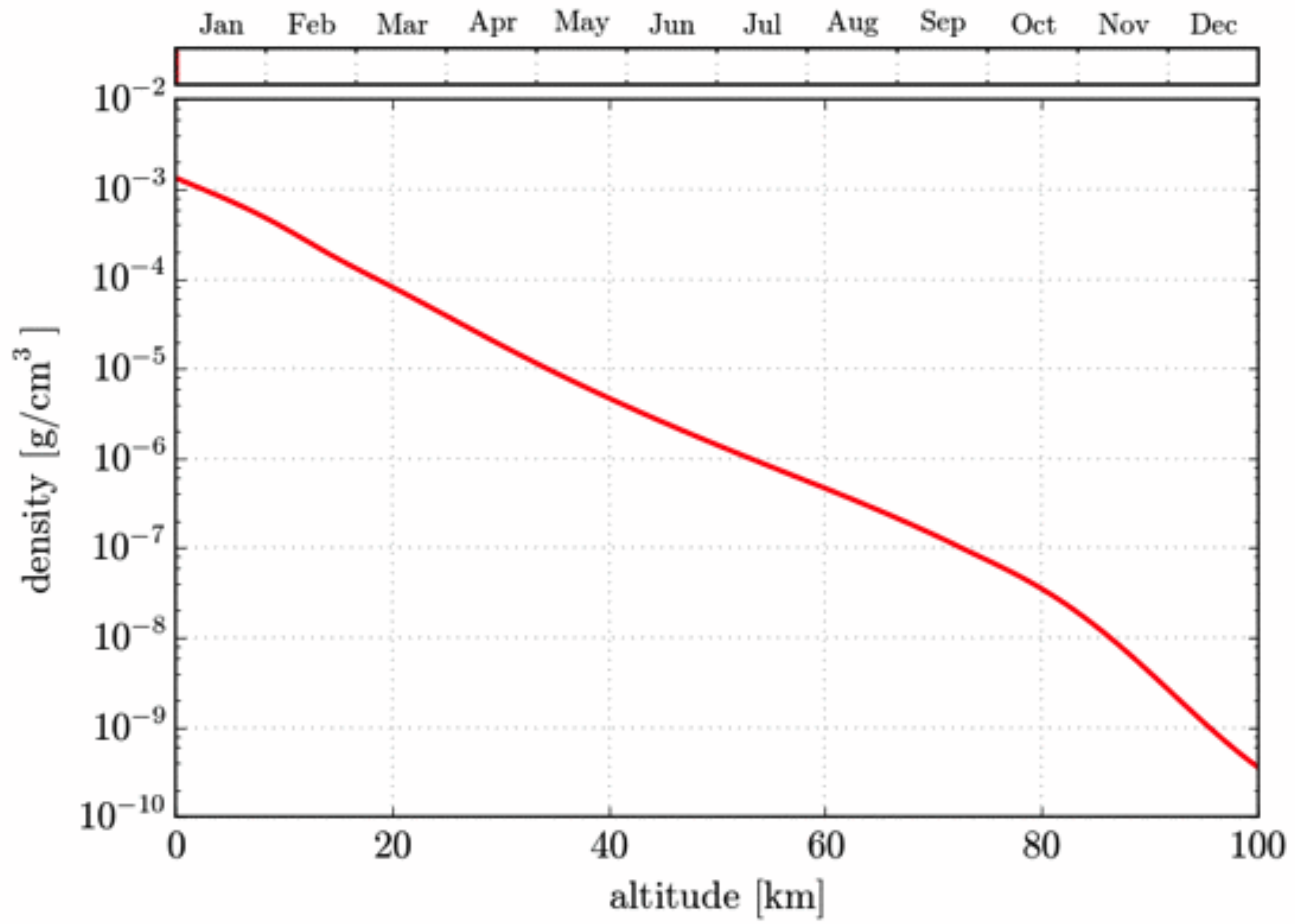
- AIRS (**A**tmospheric **I**nfra**R**ed **S**ounder) data
- Typical model: 5-layer parametrization (MSIS) in terms of mass overburden

$$T(H) = a_i + b_i^{H/c_i}\ ,\ \ H_{\text{layer}} = 4{,}10{,}40{,}100\,\text{km}$$

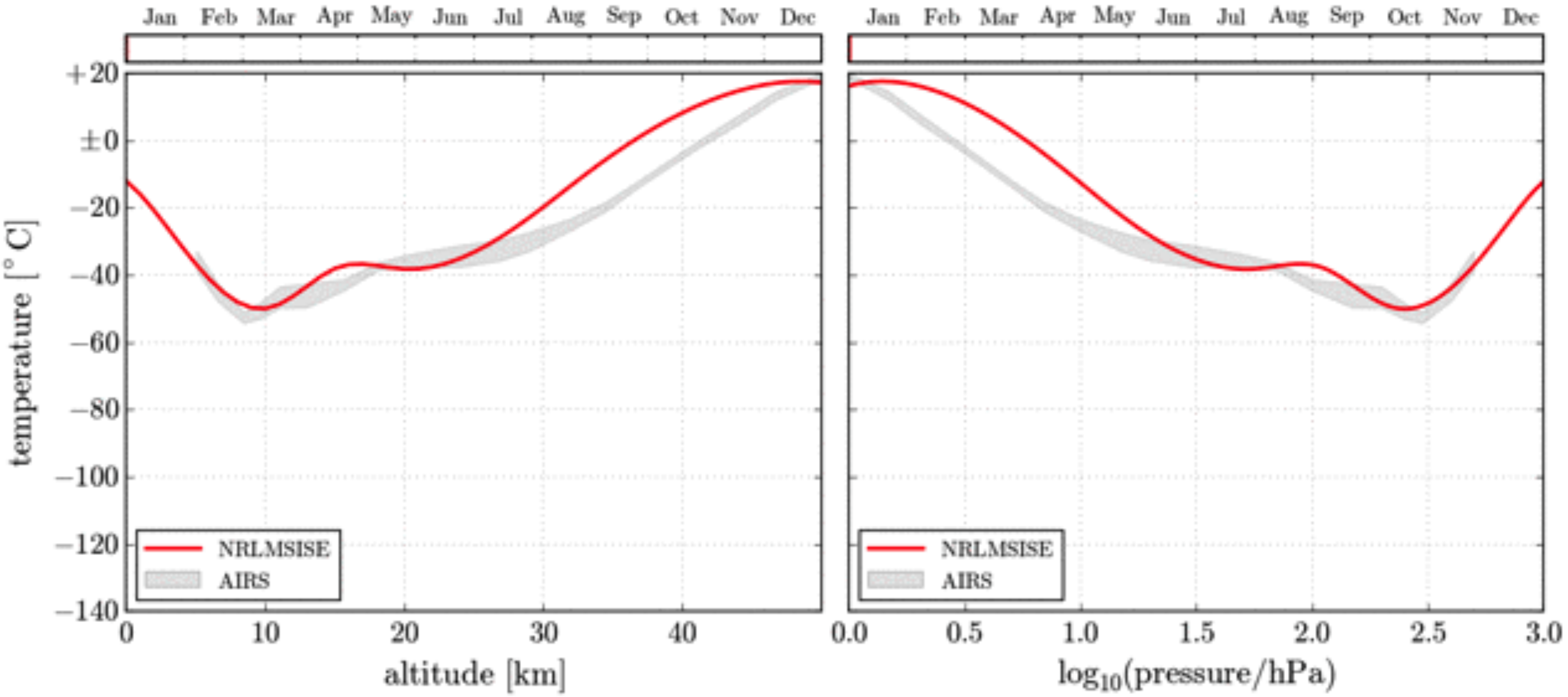
$$T(H) = a_5 + b_5 \cdot H/c_i\ ,\ \ H_{\max} = 112.8\,\text{km}$$

- Typical height of 1st interaction: $H \simeq 20\,\text{km}$





Atmosphere (South Pole)



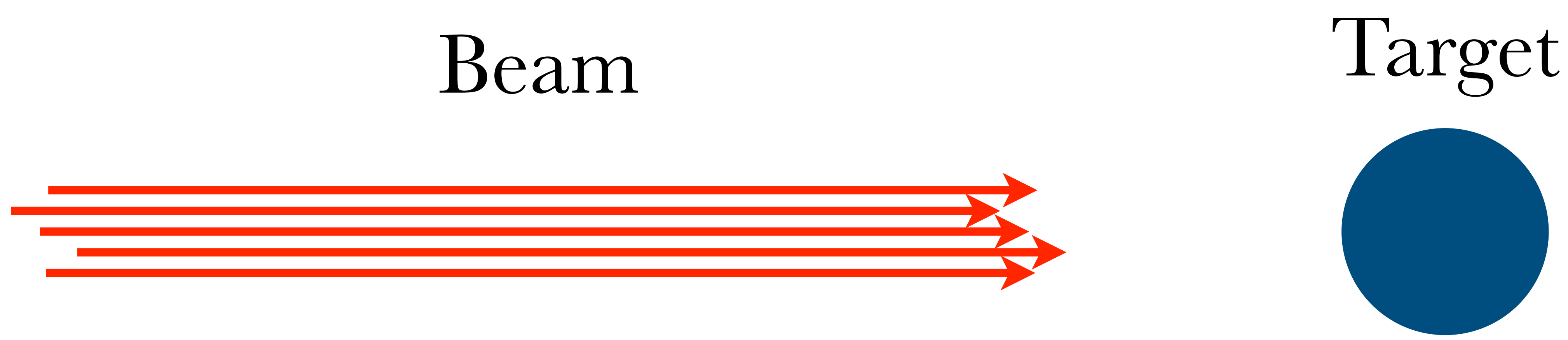
Cross Section

- Particle flux:

$$\Phi = \frac{dN_{\text{beam}}}{dAdt}$$

- Cross section:

$$\sigma = \frac{1}{\Phi}\frac{dN_{\text{int}}}{dt}$$

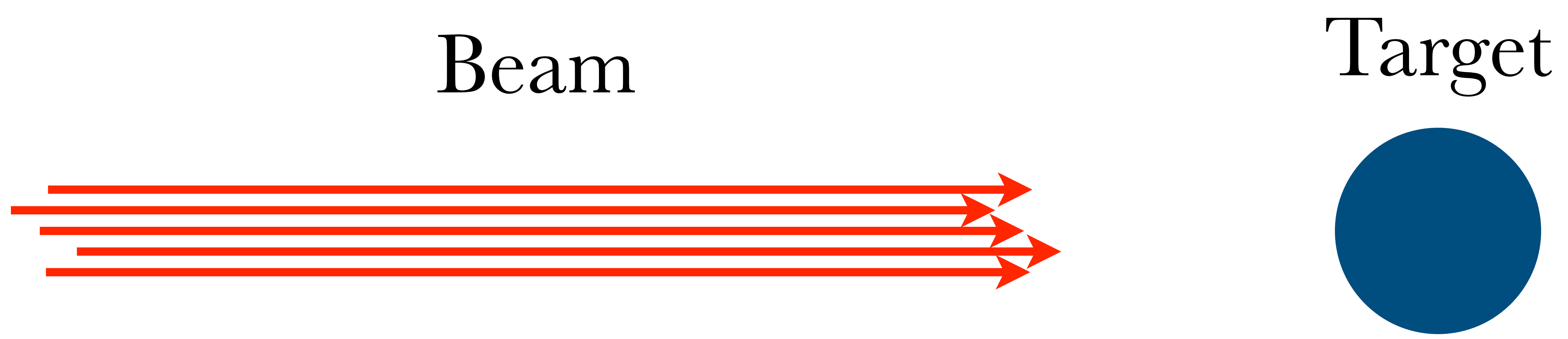


Units of an area, typically "barn": 1 barn = 10^{-28} cm^2

Cross Section

- Particle flux:

$$\Phi = \frac{dN_{\text{beam}}}{dA\,dt}$$



- Cross section:

$$\sigma = \frac{1}{\Phi}\frac{dN_{\text{int}}}{dt}$$

flux of particles on single target

interaction rate

Units of an area, typically "barn": 1 barn $= 10^{-28}\,\text{cm}^2$

Interaction Length

- Cross section:

$$\sigma = \frac{1}{\Phi}\frac{dN_{\text{int}}}{dt}$$

- Now, we can write

$$\frac{dN_{\text{int}}}{dtdV} = \frac{\rho_{\text{target}}}{\langle m_{\text{target}}\rangle}\sigma\Phi \qquad \text{with} \quad dX = \rho_{\text{target}}\, dl$$

$$\Rightarrow \frac{d\Phi}{dX} = -\frac{\sigma}{m_{\text{target}}}\Phi \equiv -\frac{1}{\lambda_{\text{int}}}\Phi$$

- Interaction length (units of g/cm^2):

$$\lambda_{\text{int}} = \frac{\langle m_{\text{target}}\rangle}{\sigma_{\text{int}}}$$

- Interaction length in air:

$$\lambda_{\text{int}} = \frac{\langle m_{\text{air}}\rangle}{\sigma_{\text{int}}} = \frac{24160\,\text{mb g/cm}^2}{\sigma_{\text{int}}}$$

- Typical values:

$$\lambda_{\gamma\to e^+e^-} \approx 46\,\text{g/cm}^2$$

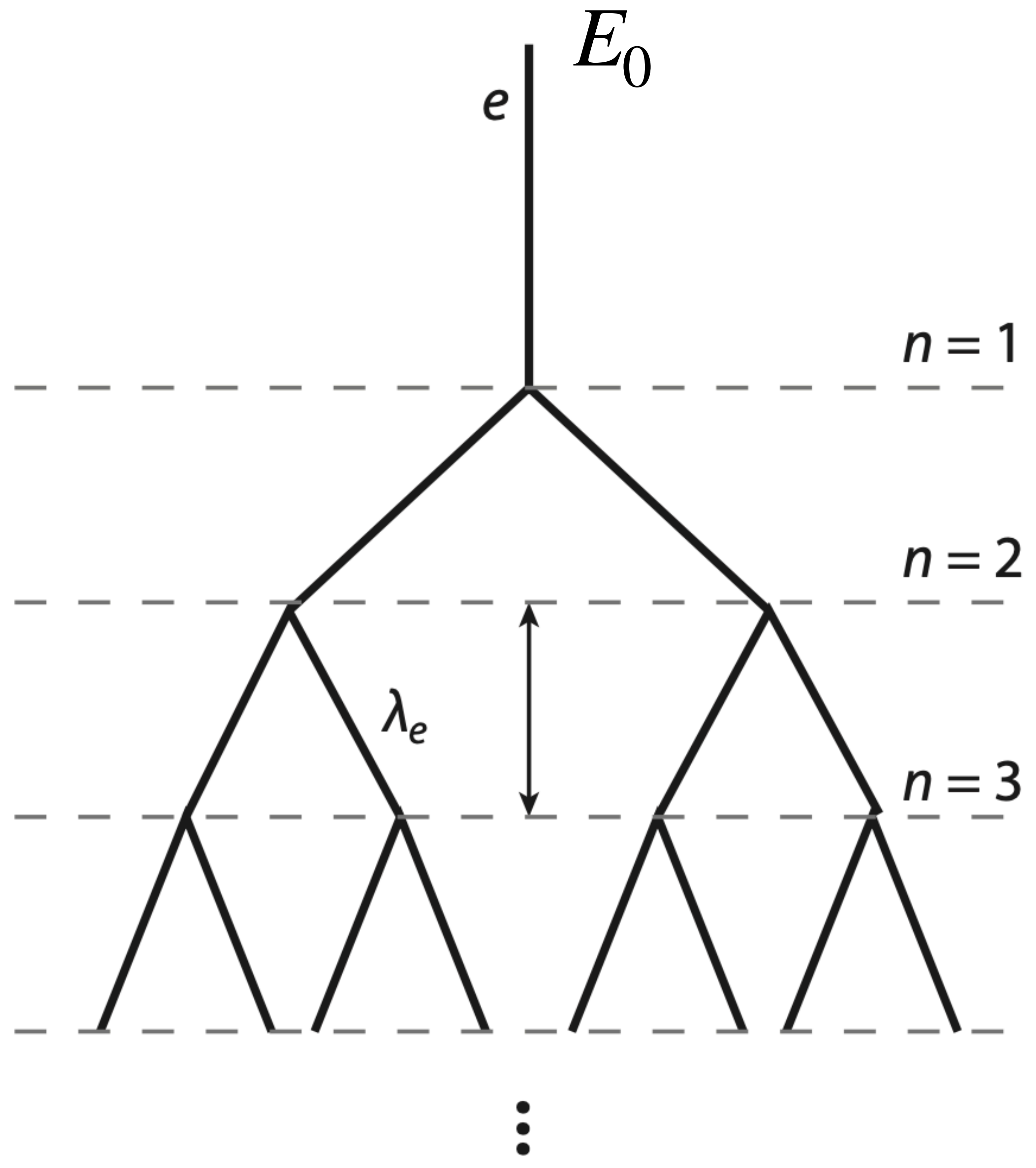
$$\lambda_{\pi} \approx \lambda_K \approx 120\,\text{g/cm}^2$$

$$\lambda_{\text{p}} \approx 80\,\text{g/cm}^2$$

$$\lambda_{\text{Fe}} \approx 10\,\text{g/cm}^2$$

Electromagnetic Cascade

- Qualitative description: Heitler model
  - Primary electron (gamma) with energy E_0
  - Particle number doubles with each generation n
  - Energy equally distributed
  - Cascade stops when particle energy drops below a critical energy ξ_C
  - Energy at shower maximum ($X_{\max}$): $E = \xi_C$
  - What is ξ_C?



[Heitler in The Quantum Theory of Radiation, (1954)]

Energy Loss of Charged Particles

- Ionization energy loss (Bethe-Bloch formula):

$$\frac{dE_{\text{ion}}}{dX} = -\,\alpha(E) \quad \text{with} \quad \alpha \approx 2.4\,\text{MeV}/(\text{g/cm}^2)$$

- Radiative energy loss (Bremsstrahlung):

$$\frac{dE_{\text{Brems}}}{dX} = -\,\frac{E}{X_0} \quad \text{with} \quad X_0 \approx 36\,\text{g/cm}^2$$

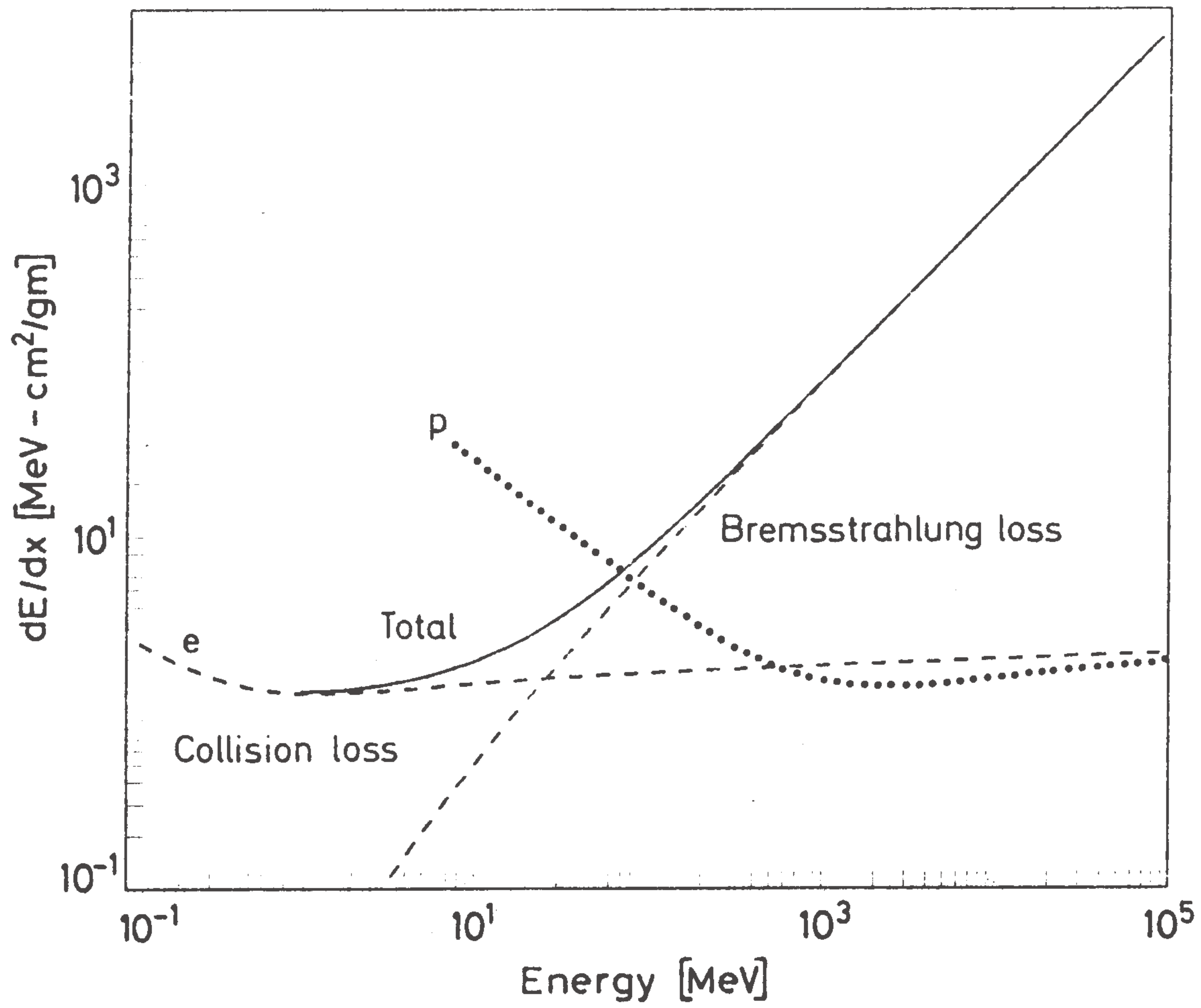
(radiation length)

- Stopping power:

$$\frac{dE}{dX} = -\,\alpha(E) - \frac{E}{X_0}$$

- Critical energy at which both are equal:

$$\xi_C = \alpha X_0 \approx 85\,\text{MeV}$$



Electromagnetic Cascade

- Heitler model: [Heitler in The Quantum Theory of Radiation, (1954)]
  - Primary energy E_0
  - After $n = X/\lambda_{em}$ branchings:

    $$N(X) = 2^{X/\lambda_{em}}$$

  - Energy per particle:

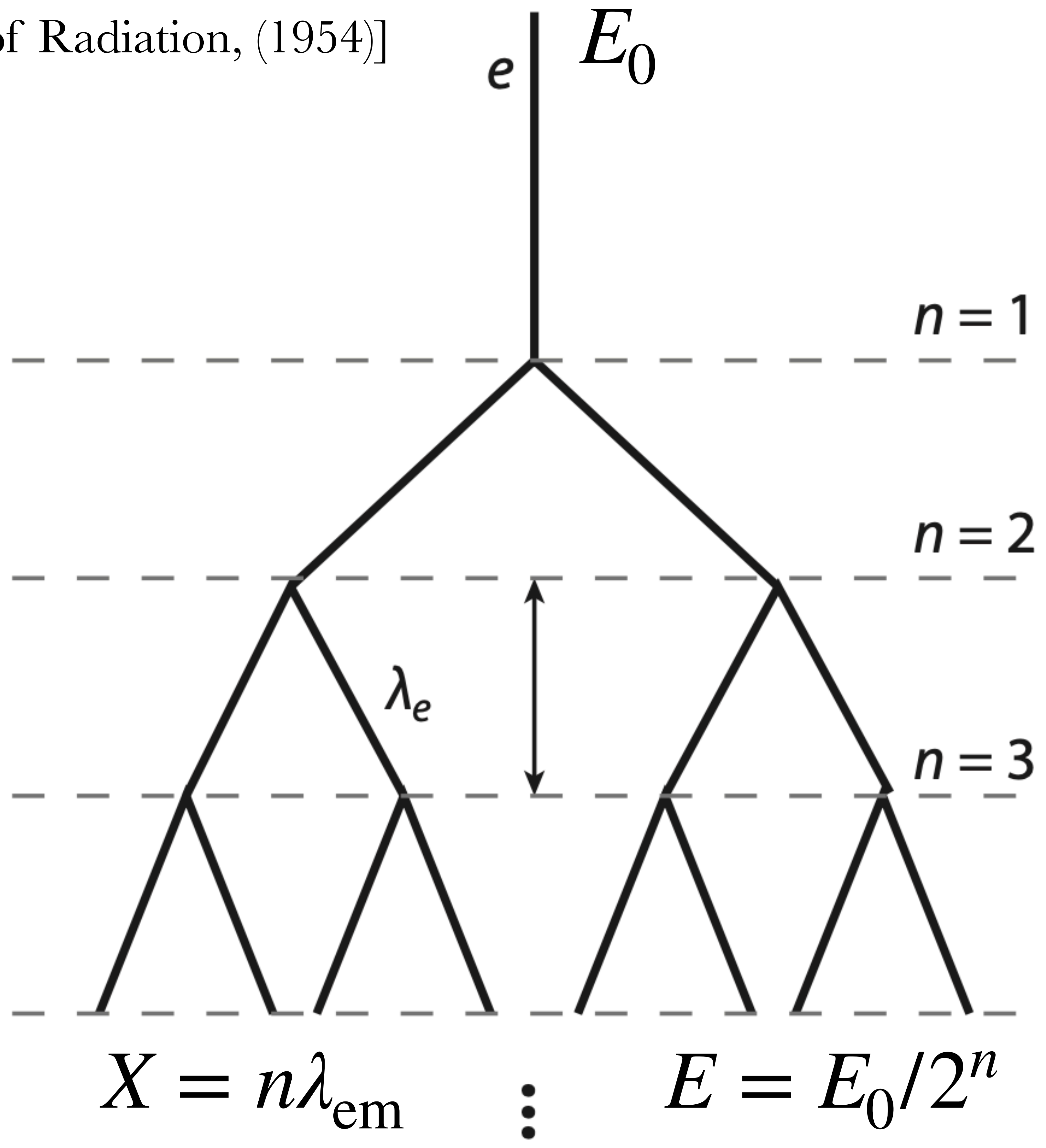
    $$E(X) = E_0/N(X)$$

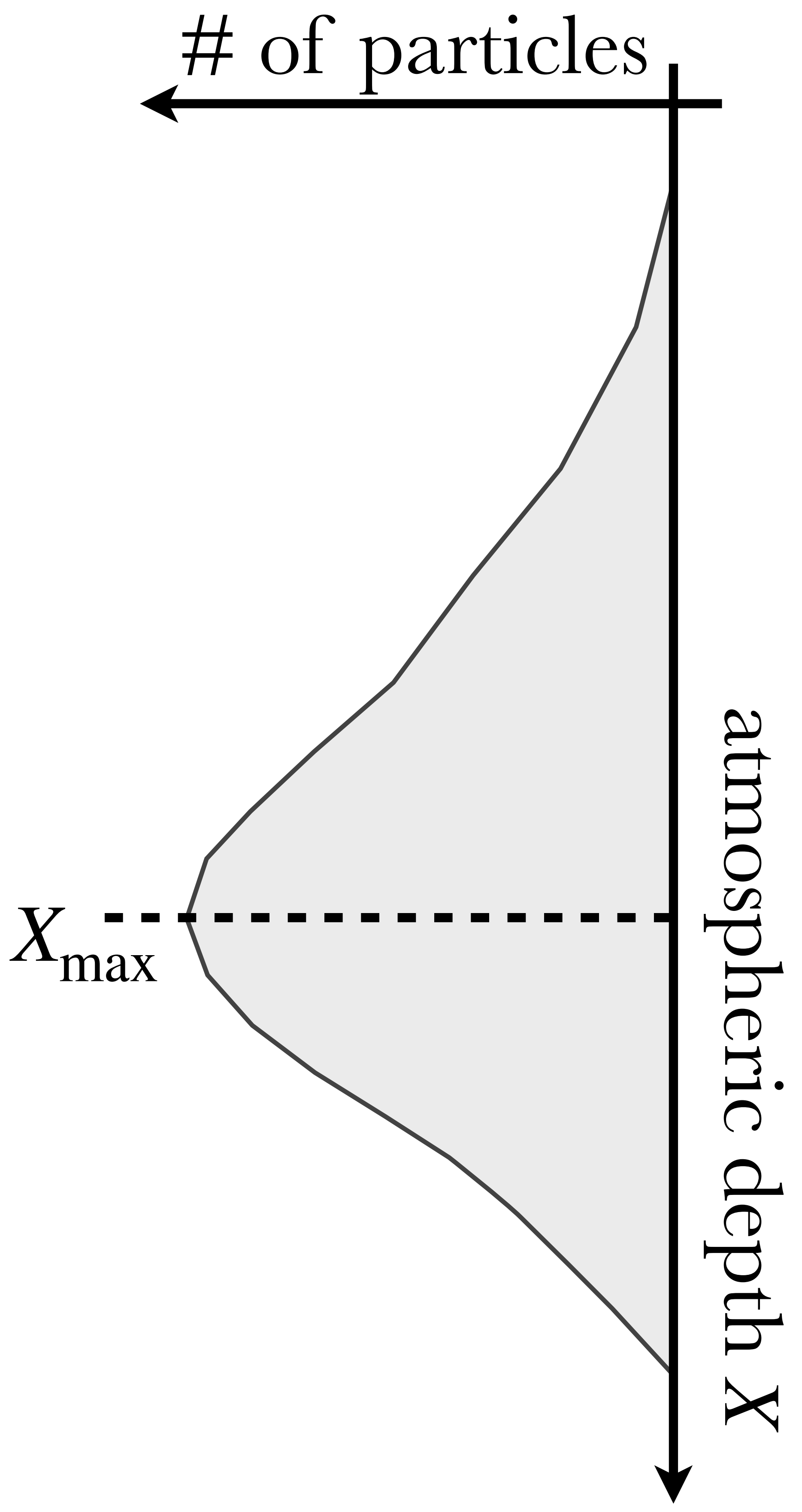
    $$\Rightarrow E(X_{max}) = E_0/\xi_C$$

    $$\Rightarrow X_{max} = \lambda_{em} \cdot \frac{\ln(E_0/\xi_C)}{\ln(2)}$$

  or with cascade equations (later)

$$X_{max} \approx X_0 \ln(E_0/\xi_C) \quad \text{and} \quad N_{max} \approx \frac{0.31}{\sqrt{\ln(E_0/\xi_C) - 0.33}} E_0/\xi_C$$





Hadronic Cascade

- Heitler-Matthews model of the air shower development

[Matthews, Astropart.Phys. 22 (2005)]

hadronic energy

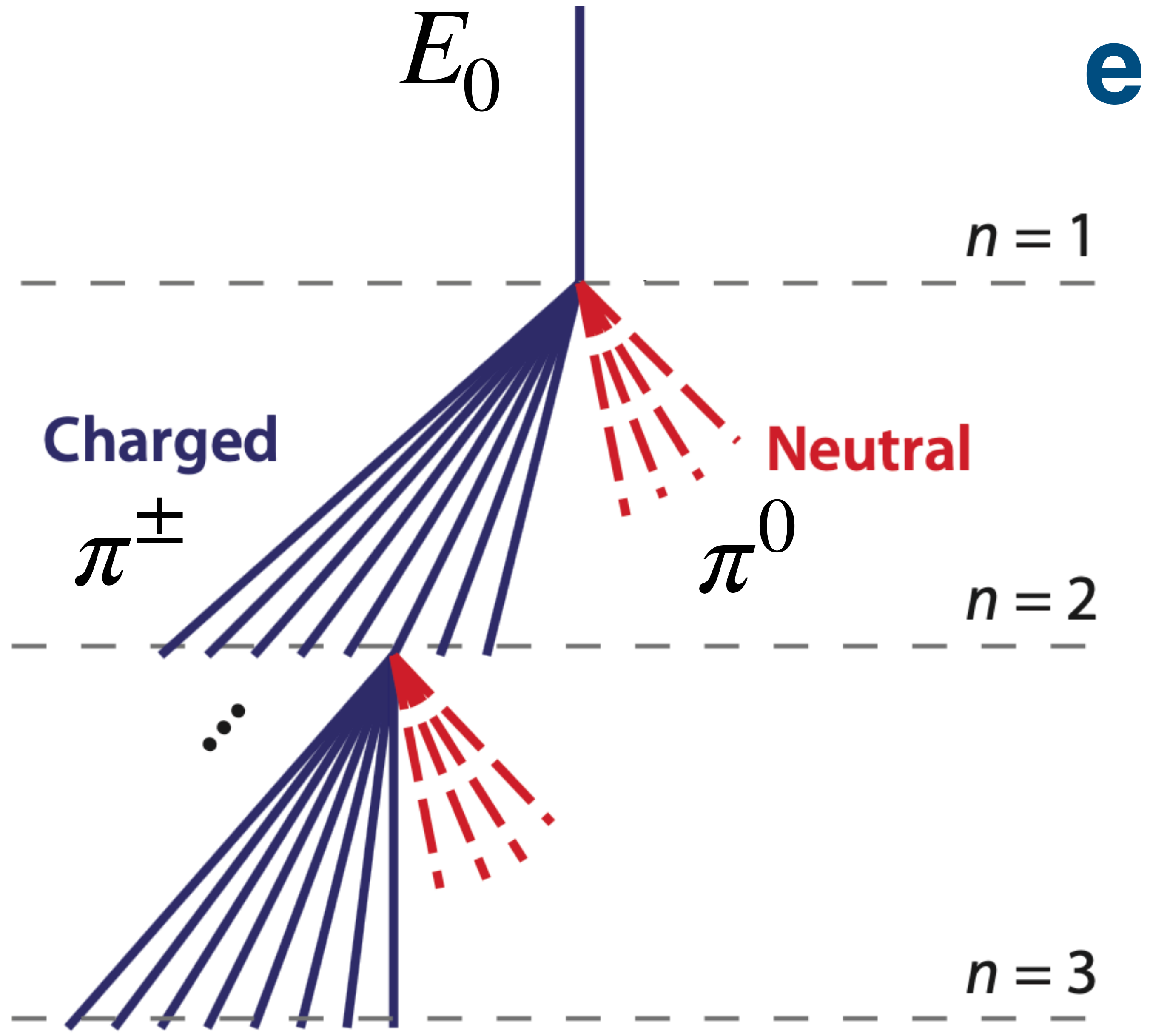
$$\frac{2}{3}\cdot E_0$$

$$\frac{2}{3}\cdot\left(\frac{2}{3}\cdot E_0\right)$$

$$\vdots$$

$$E_{\text{had}} = \left(\frac{2}{3}\right)^n \cdot E_0$$

E_0



after n generations

electromagnetic energy

$$\frac{1}{3}\cdot E_0$$

$$\frac{1}{3}\cdot E_0 + \frac{1}{3}\cdot\left(\frac{2}{3}\cdot E_0\right)$$

$$\vdots$$

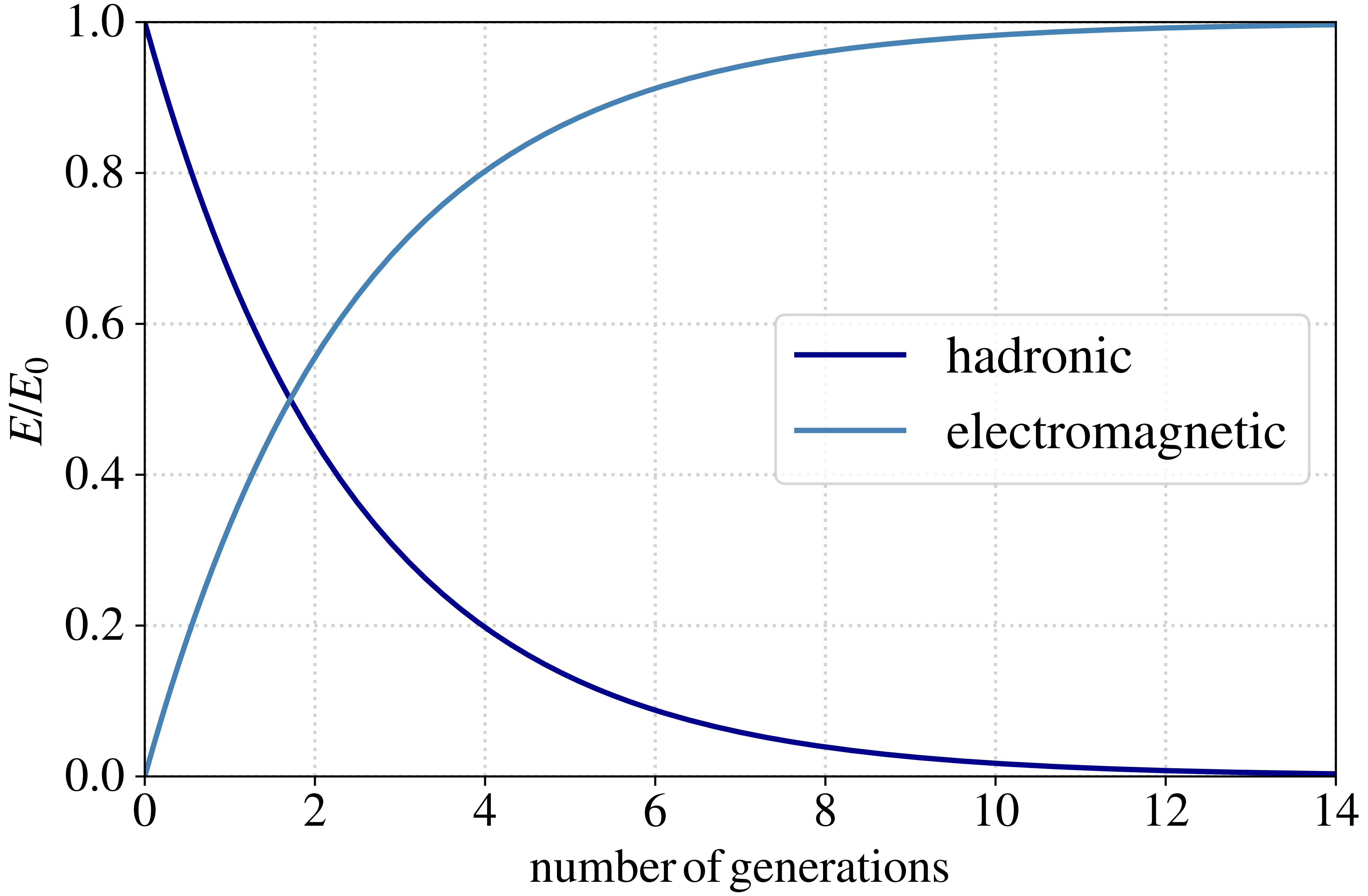
$$E_{\text{em}} = \left[1-\left(\frac{2}{3}\right)^n\right]\cdot E_0$$

- Cascade stops when energy drops below a critical energy, ξ_C, and:

$$\pi^\pm \rightarrow \mu^\pm + \nu_\mu(\bar{\nu}_\mu) \qquad\qquad \pi^0 \rightarrow \gamma\gamma$$

Heitler-Matthews Model of EAS

- Simplified model of the air shower development (only charged and neutral pions)



- After 5 (6) generations: $E_{\text{had}} \sim 12\,\%\,(8\,\%)$
- Example IceTop:
  - Atmospheric depth: $X \sim 690\,\text{g/cm}^2$
  - Pion interaction length: $\lambda_\pi \sim 120\,\text{g/cm}^2$

  $\Rightarrow \; n = X/\lambda_\pi = 690/120 = 5.75 \simeq 6$
  - Pions decay below critical energy

    $\xi_\pi = 115\,\text{GeV} \quad (\xi_K = 850\,\text{GeV})$

$$E_{\text{had}} = \left(\frac{2}{3}\right)^n \cdot E_0 \qquad \text{after } n \text{ generations} \qquad E_{\text{em}} = \left[1 - \left(\frac{2}{3}\right)^n\right] \cdot E_0$$

Superposition Model

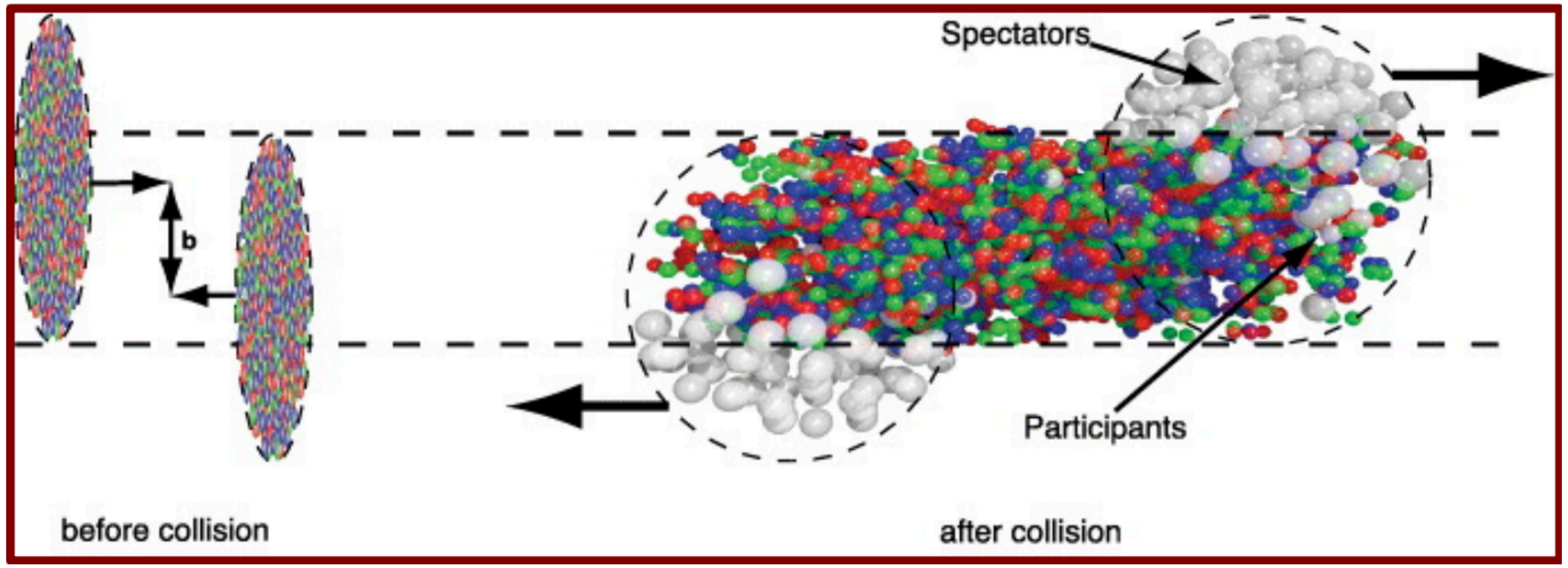
- Assumption: nucleus of mass A and energy E_0 correspond to A nucleons of energy

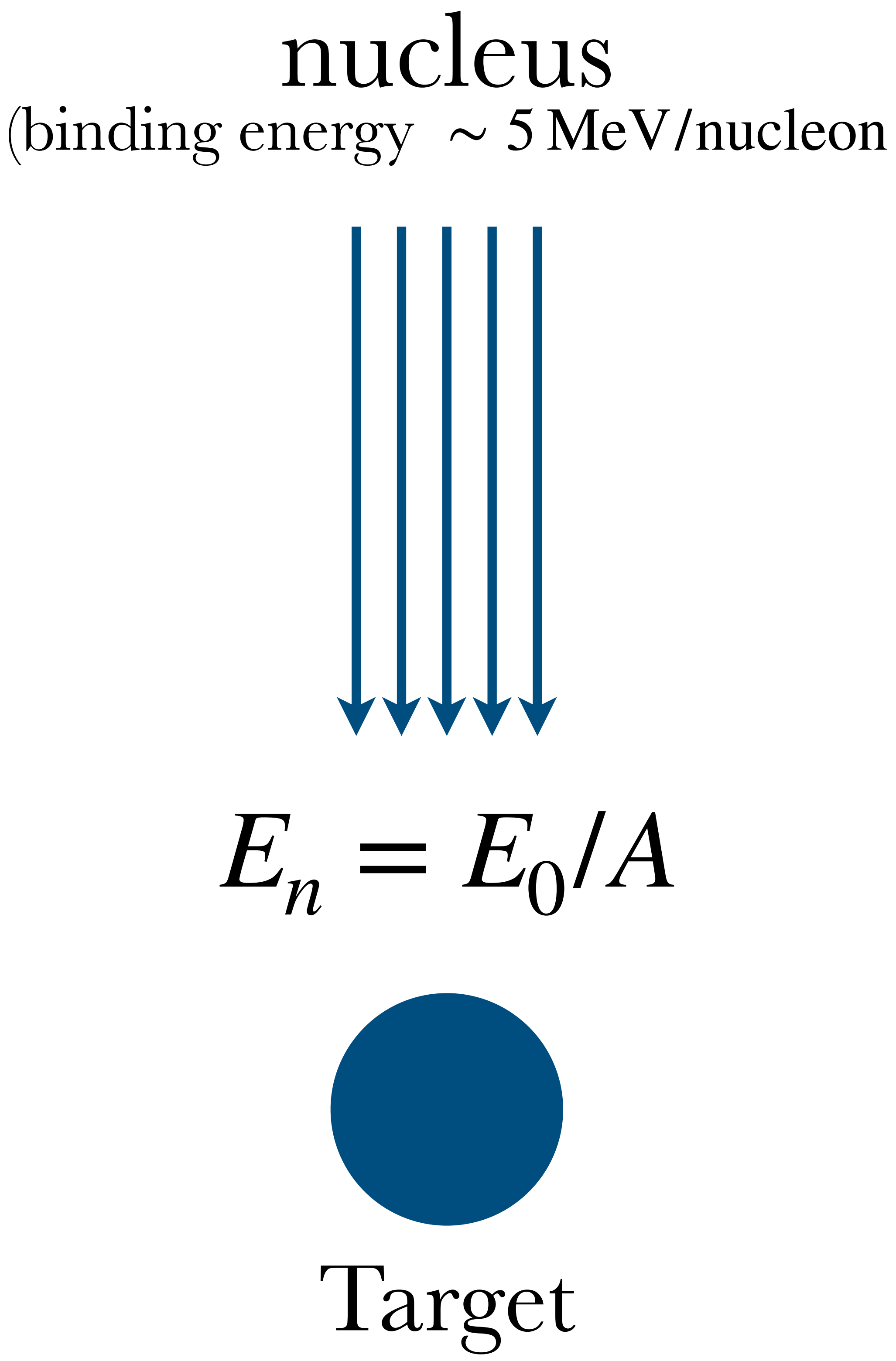
$$E_n = E_0/A$$

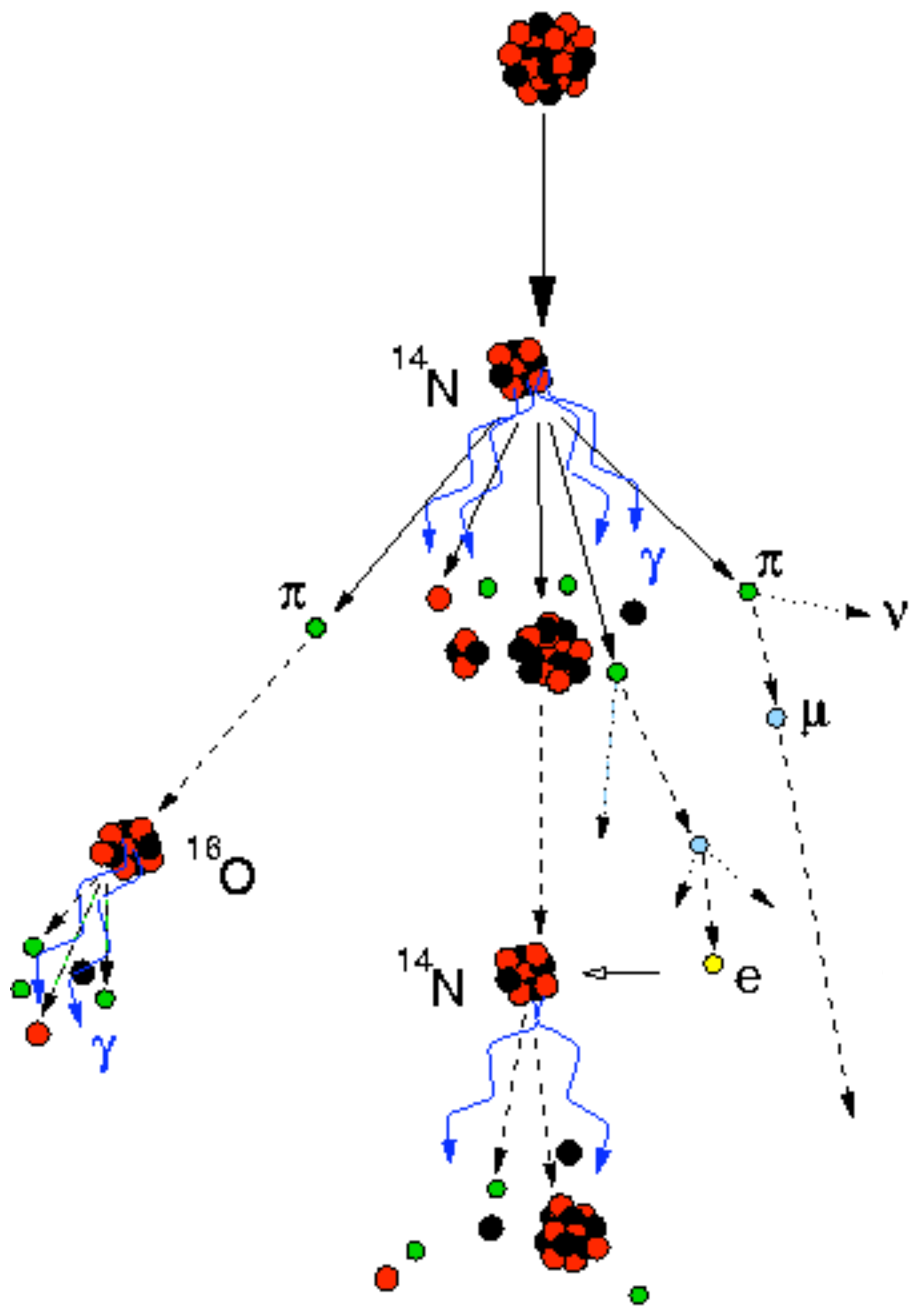
- Glauber approximation:

$$\sigma_{\text{Fe-air}} = \frac{A}{n_{\text{part}}} \cdot \sigma_{\text{p-air}} \quad ,$$

where n_{part} is the number of participants







Heitler-Matthews Model of EAS

- Number of muons, N_μ, follows charged hadrons, N_{ch}, as

  $$N_\mu = N_{ch}^n \quad \text{where} \quad E = E_0 / N_{tot}^n \sim \xi_C$$

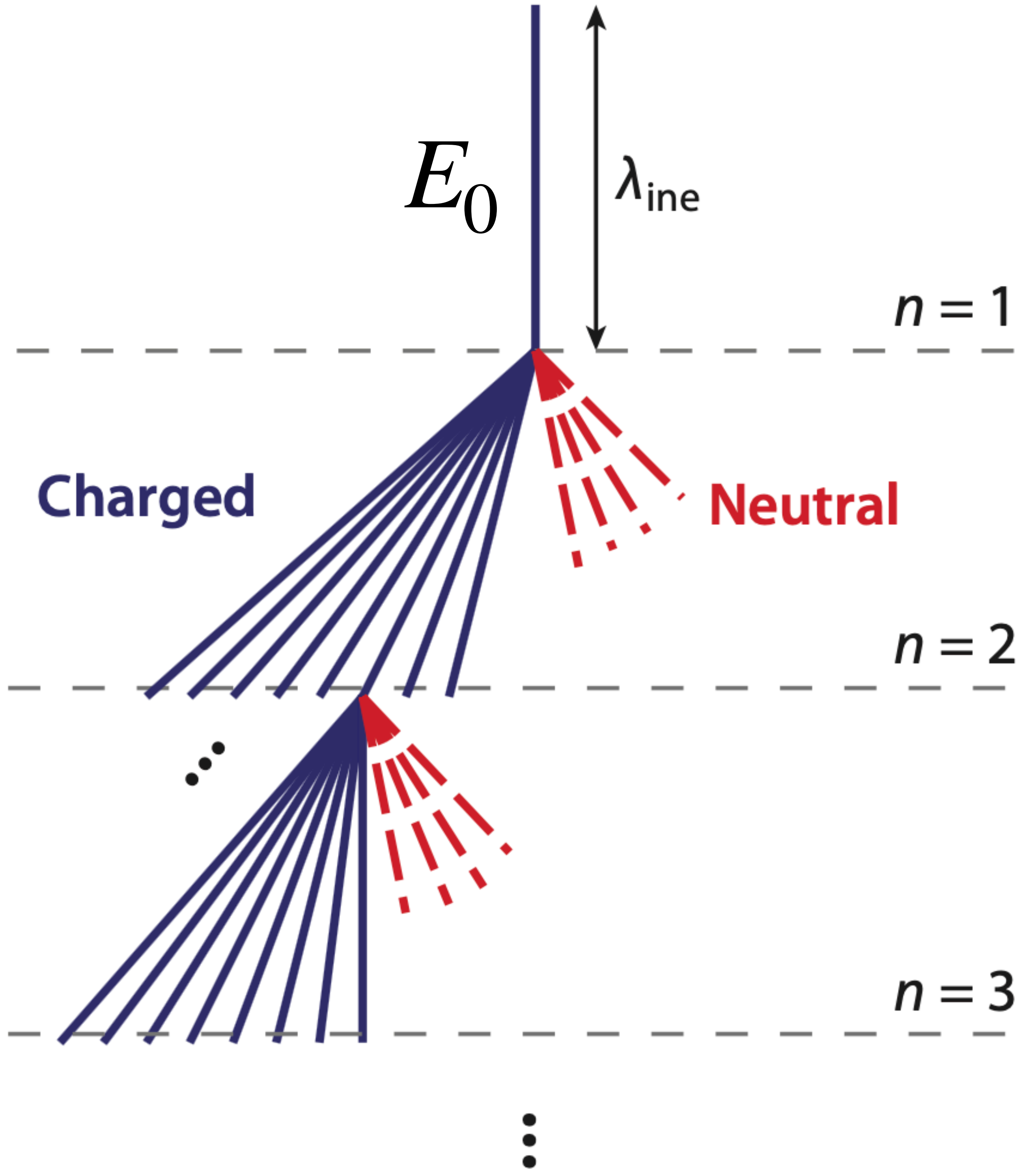
  with total number of particles, N_{tot}, from each interaction
- The (average) number of muons is then given by

  $$N_\mu = A \cdot \left(\frac{E_0}{A \cdot \xi_C} \right)^\beta, \quad \beta = \frac{\ln N_{ch}}{\ln N_{tot}} \simeq 0.82 \ldots 0.94$$

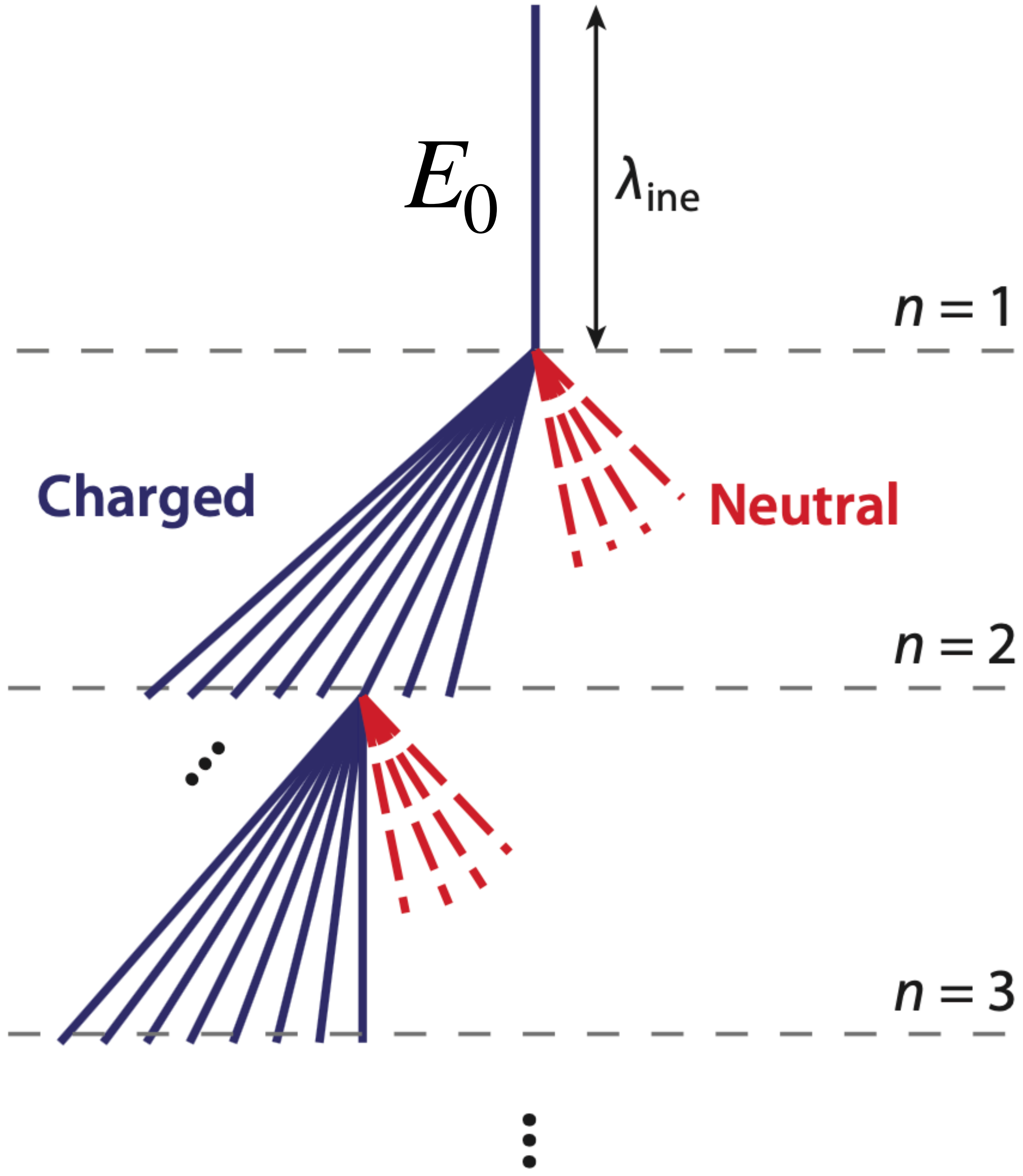
- β needs to be obtained from simulation as there are not only pions
- Processes that transfer energy between EM and hadron components crucial!

$$\pi^\pm + p \rightarrow \pi^0 + X$$
$$\pi^0 \rightarrow \gamma\gamma \qquad \text{contribution to EM component}$$

$$\pi^\pm + p \rightarrow \rho^0 + X \qquad \text{contribution to hadronic component}$$
$$\rho^0 \rightarrow \pi^+\pi^-$$



Heitler-Matthews Model of EAS



- Number of muons, N_μ, follows charged hadrons, N_{ch}, as

  $$N_\mu = N_{\text{ch}}^n \quad \text{where} \quad E = E_0/N_{\text{tot}}^n \sim \xi_C$$

  with total number of … n each interaction

  CR energy

- The (average) number of muons is then given by

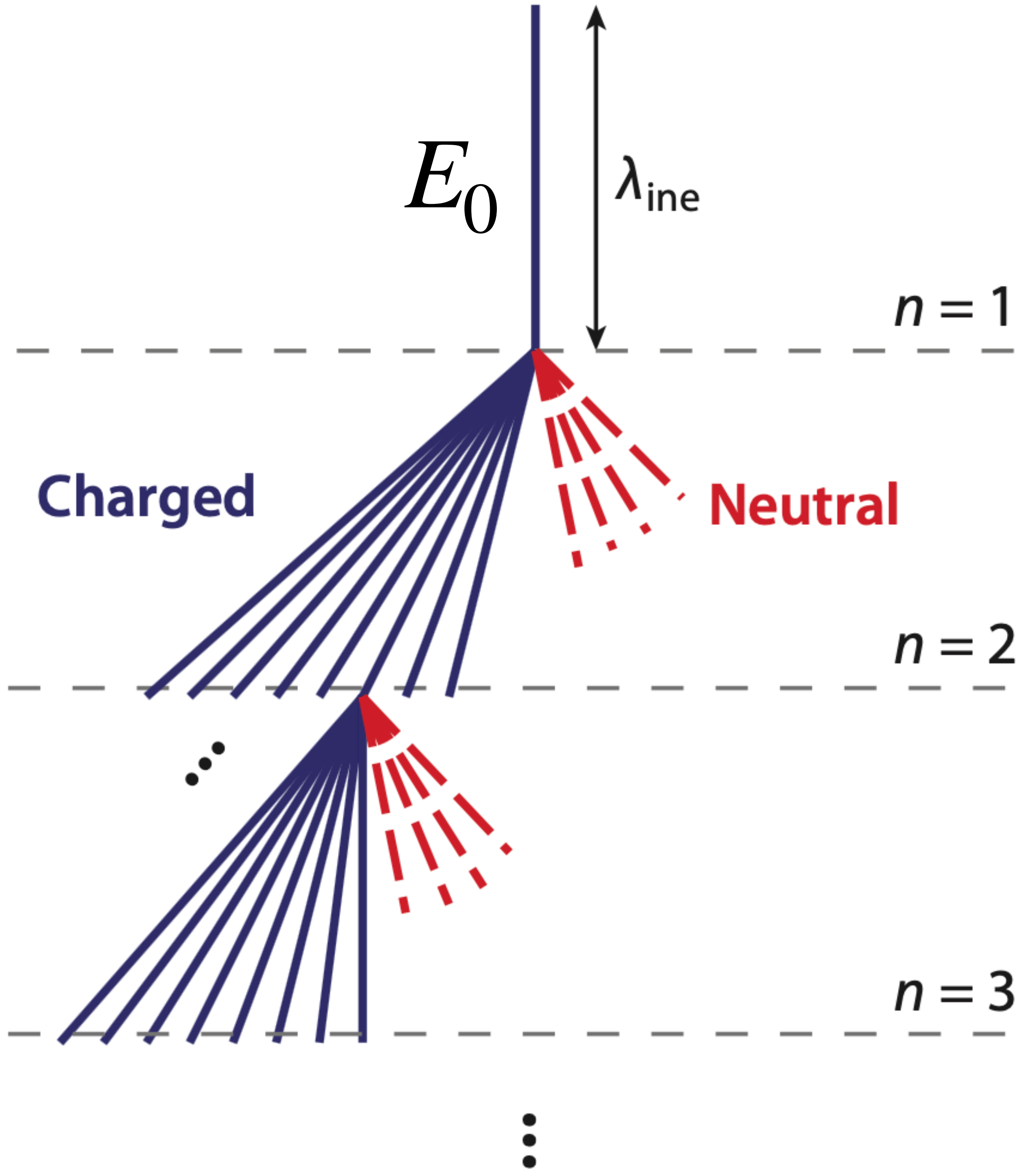
  $$N_\mu = A \cdot \left(\frac{E_0}{A \cdot \xi_C}\right)^\beta, \quad \beta = \frac{\ln N_{\text{ch}}}{\ln N_{\text{tot}}} \simeq 0.82 \ldots 0.94$$

- β needs to be obtained from simulation as there are not only pions
- Processes that transfer energy between EM and hadron components crucial!

  $$\pi^\pm + p \to \pi^0 + X$$
  $$\pi^0 \to \gamma\gamma$$
  contribution to EM component

  $$\pi^\pm + p \to \rho^0 + X$$
  $$\rho^0 \to \pi^+\pi^-$$
  contribution to hadronic component

Heitler-Matthews Model of EAS



- Number of muons, N_μ, follows charged hadrons, N_{ch}, as

$$N_\mu = N_{\text{ch}}^n \quad \text{where} \quad E = E_0 / N_{\text{tot}}^n \sim \xi_C$$

CR mass

CR energy

ber of … n each interaction

- The (average) number of muons is then given by

$$N_\mu = A \cdot \left(\frac{E_0}{A \cdot \xi_C} \right)^\beta, \quad \beta = \frac{\ln N_{\text{ch}}}{\ln N_{\text{tot}}} \simeq 0.82 \ldots 0.94$$

- β needs to be obtained from simulation as there are not only pions
- Processes that transfer energy between EM and hadron components crucial!

$$\pi^\pm + p \to \pi^0 + X$$
$$\pi^0 \to \gamma\gamma$$

contribution to EM component

$$\pi^\pm + p \to \rho^0 + X$$
$$\rho^0 \to \pi^+ \pi^-$$

contribution to hadronic component

Basics: Muon Production

- Main decay channels for muon production:
  - Pions:

    $\pi^{\pm} \rightarrow \mu^{\pm} + \nu_{\mu}(\bar{\nu}_{\mu})$ $(\sim 100\,\%)$

  - Kaons:

    $K^{\pm} \rightarrow \mu^{\pm} + \nu_{\mu}(\bar{\nu}_{\mu})$ $(\sim 63.5\,\%)$

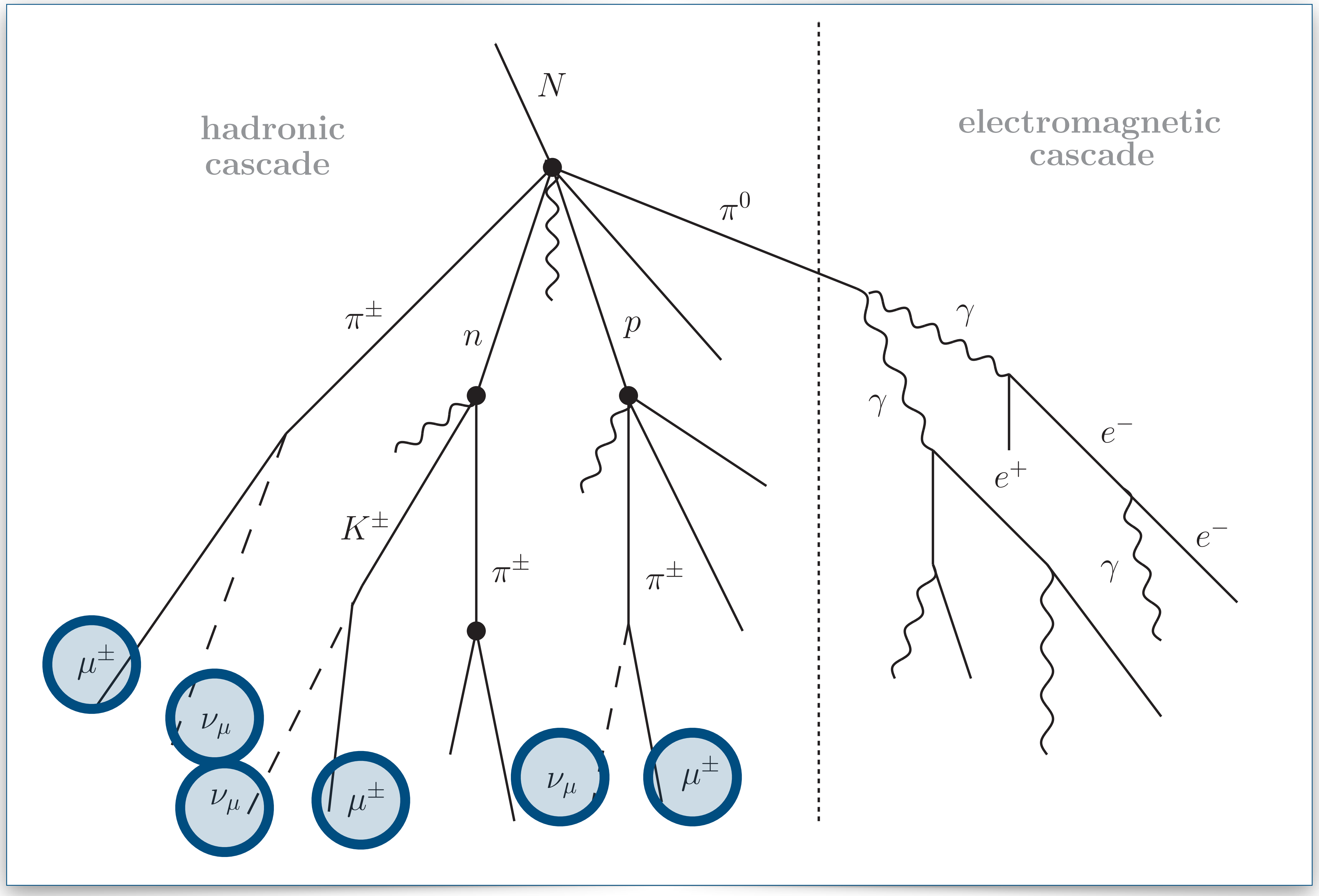
    $K_L^0 \rightarrow \pi^{\pm} + \mu^{\mp} + \nu_{\mu}(\bar{\nu}_{\mu})$ $(\sim 27.0\,\%)$

    $\pi^{\pm} \rightarrow \mu^{\pm} + \nu_{\mu}(\bar{\nu}_{\mu})$ $(\sim 100\,\%)$

    $K_L^0 \rightarrow \pi^{\pm} + e^{\mp} + \nu_{e}(\bar{\nu}_{e})$ $(\sim 38.7\,\%)$

    $\pi^{\pm} \rightarrow \mu^{\pm} + \nu_{\mu}(\bar{\nu}_{\mu})$ $(\sim 100\,\%)$

Muons are the tracers of hadronic interactions!



- Neutral pions transfer hadronic energy to electromagnetic cascade via

  $\pi^0 \rightarrow \gamma\gamma$ $(\sim 100\,\%)$

Basics: Muon Production

- Main decay channels for muon production:

  - Pions: "Conventional"

    $\pi^\pm \rightarrow \mu^\pm + \nu_\mu(\bar{\nu}_\mu)$ $(\sim 100\,\%)$

  - Kaons:

    $K^\pm \rightarrow \mu^\pm + \nu_\mu(\bar{\nu}_\mu)$ $(\sim 63.5\,\%)$

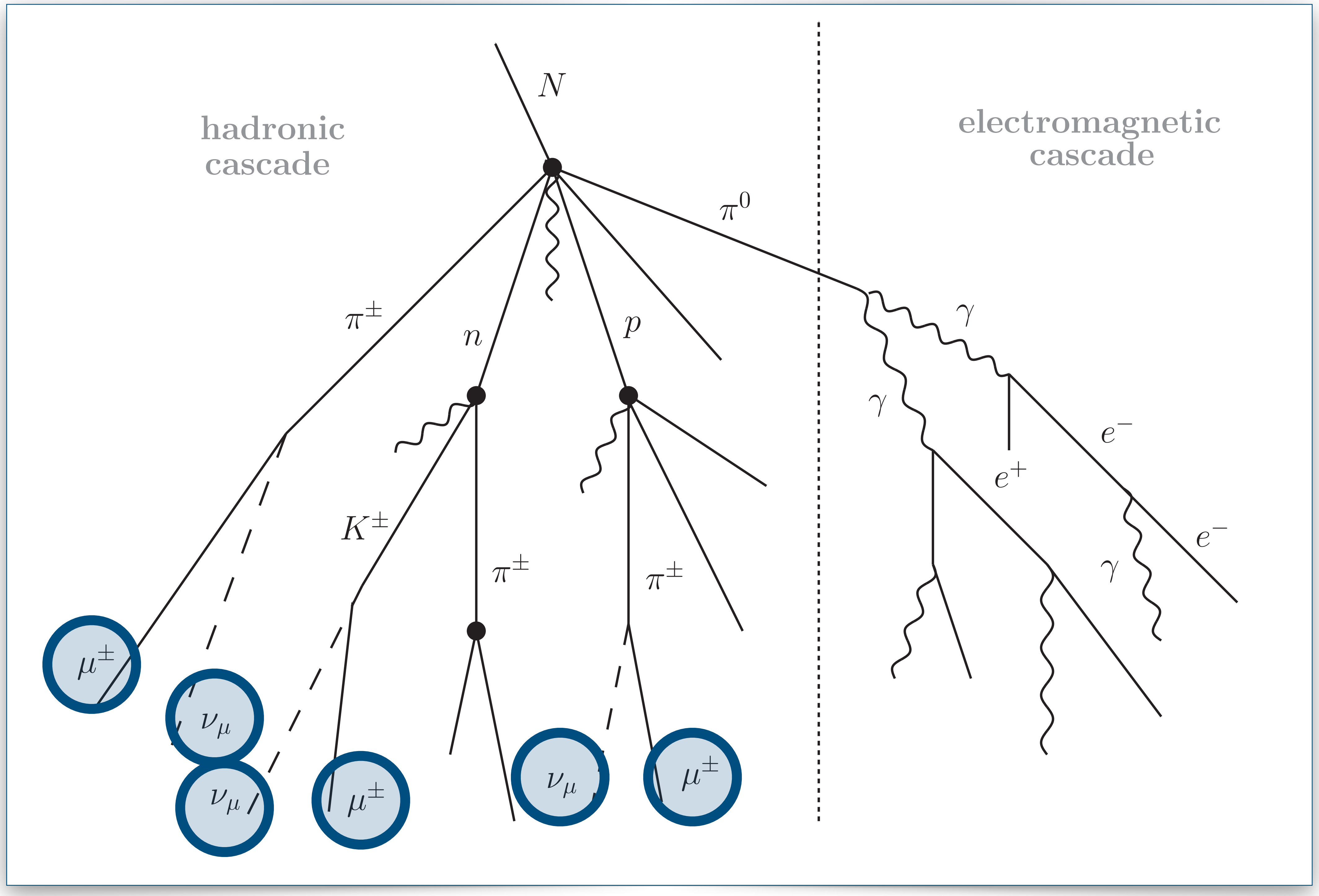
    $K^0_L \rightarrow \pi^\pm + \mu^\mp + \nu_\mu(\bar{\nu}_\mu)$ $(\sim 27.0\,\%)$

    $\pi^\pm \rightarrow \mu^\pm + \nu_\mu(\bar{\nu}_\mu)$ $(\sim 100\,\%)$

    $K^0_L \rightarrow \pi^\pm + e^\mp + \nu_e(\bar{\nu}_e)$ $(\sim 38.7\,\%)$

    $\pi^\pm \rightarrow \mu^\pm + \nu_\mu(\bar{\nu}_\mu)$ $(\sim 100\,\%)$

Muons are the tracers of hadronic interactions!



- Neutral pions transfer hadronic energy to electromagnetic cascade via

  $\pi^0 \rightarrow \gamma\gamma$ $(\sim 100\,\%)$

Basics: Muon Production

- Main decay channels for muon production:

- Pions: "Conventional"

$\pi^{\pm} \to \mu^{\pm} + \nu_{\mu}(\bar{\nu}_{\mu})$ ($\sim 100\,\%$)

- Kaons:

$K^{\pm} \to \mu^{\pm} + \nu_{\mu}(\bar{\nu}_{\mu})$ ($\sim 63.5\,\%$)

$K_L^0 \to \pi^{\pm} + \mu^{\mp} + \nu_{\mu}(\bar{\nu}_{\mu})$ ($\sim 27.0\,\%$)

$\pi^{\pm} \to \mu^{\pm} + \nu_{\mu}(\bar{\nu}_{\mu})$ ($\sim 100\,\%$)

$K_L^0 \to \pi^{\pm} + e^{\mp} + \nu_{e}(\bar{\nu}_{e})$ ($\sim 38.7\,\%$)

$\pi^{\pm} \to \mu^{\pm} + \nu_{\mu}(\bar{\nu}_{\mu})$ ($\sim 100\,\%$)

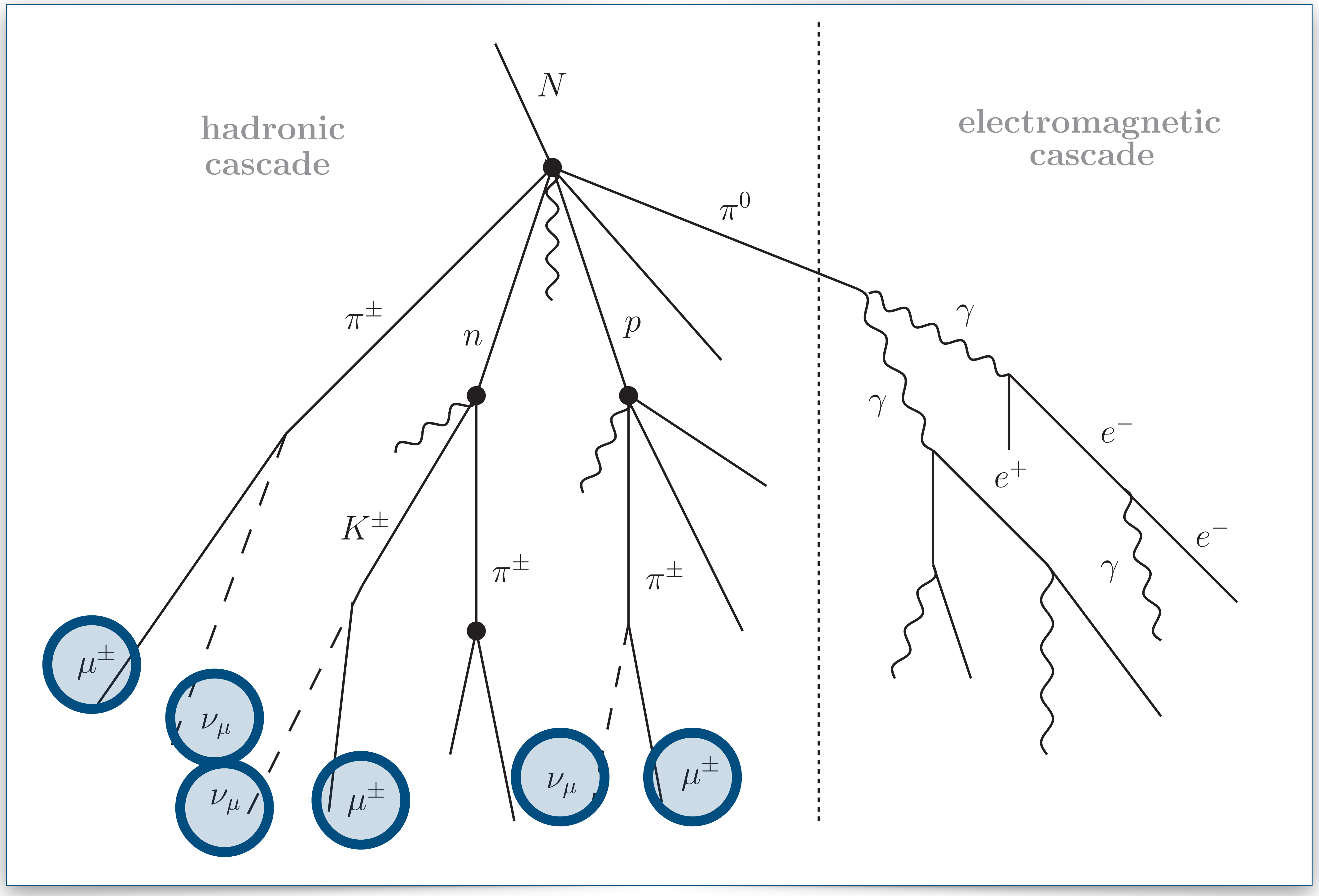
- "Prompt":

$D^{\pm} \to \mu^{\pm} + X$ ($\sim 17.6\,\%$)

$D^{0} \to \mu^{\pm} + X$ ($\sim 6.7\,\%$)

others… later more…

Muons are the tracers of hadronic interactions!



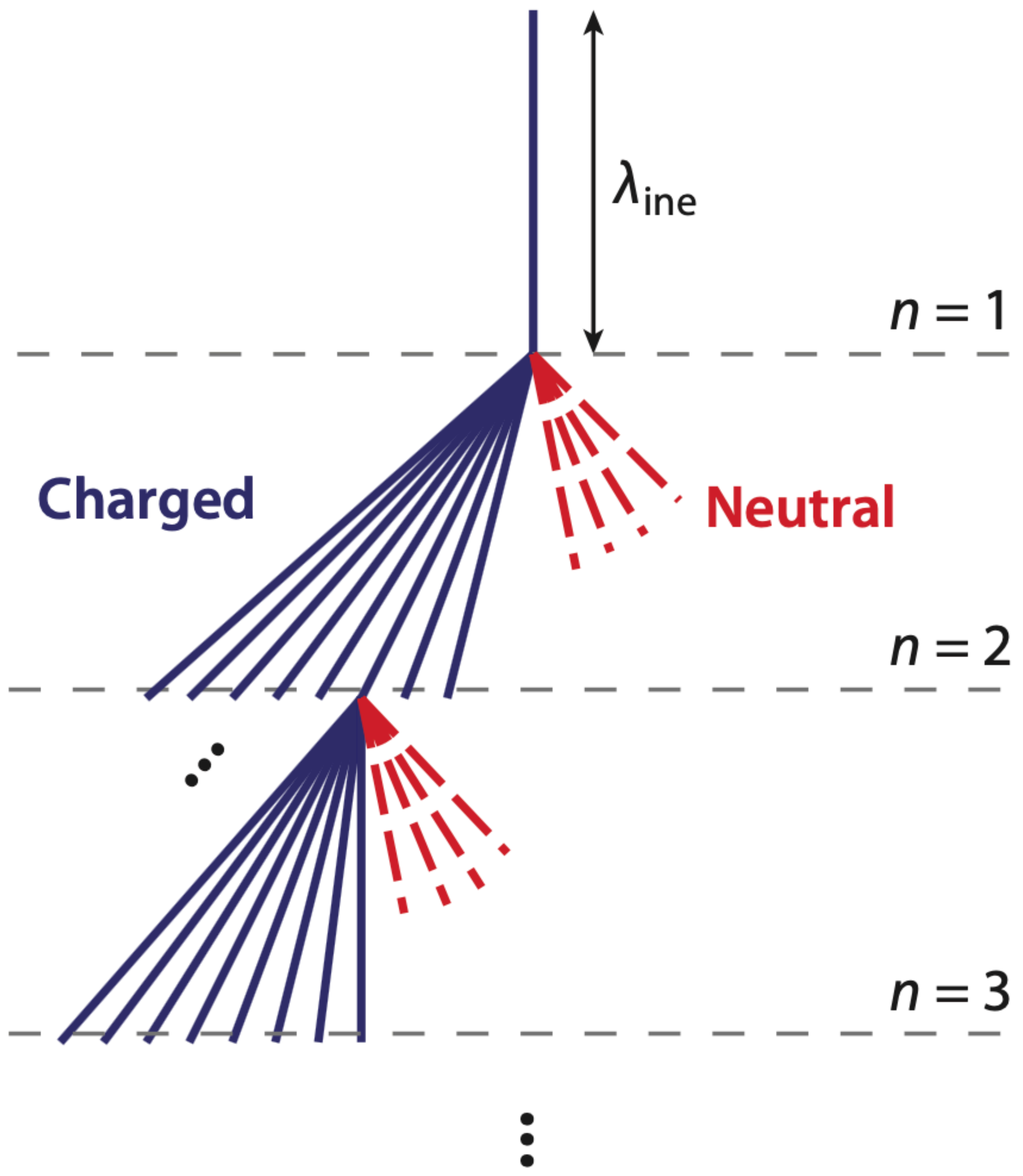
- Neutral pions transfer hadronic energy to electromagnetic cascade via

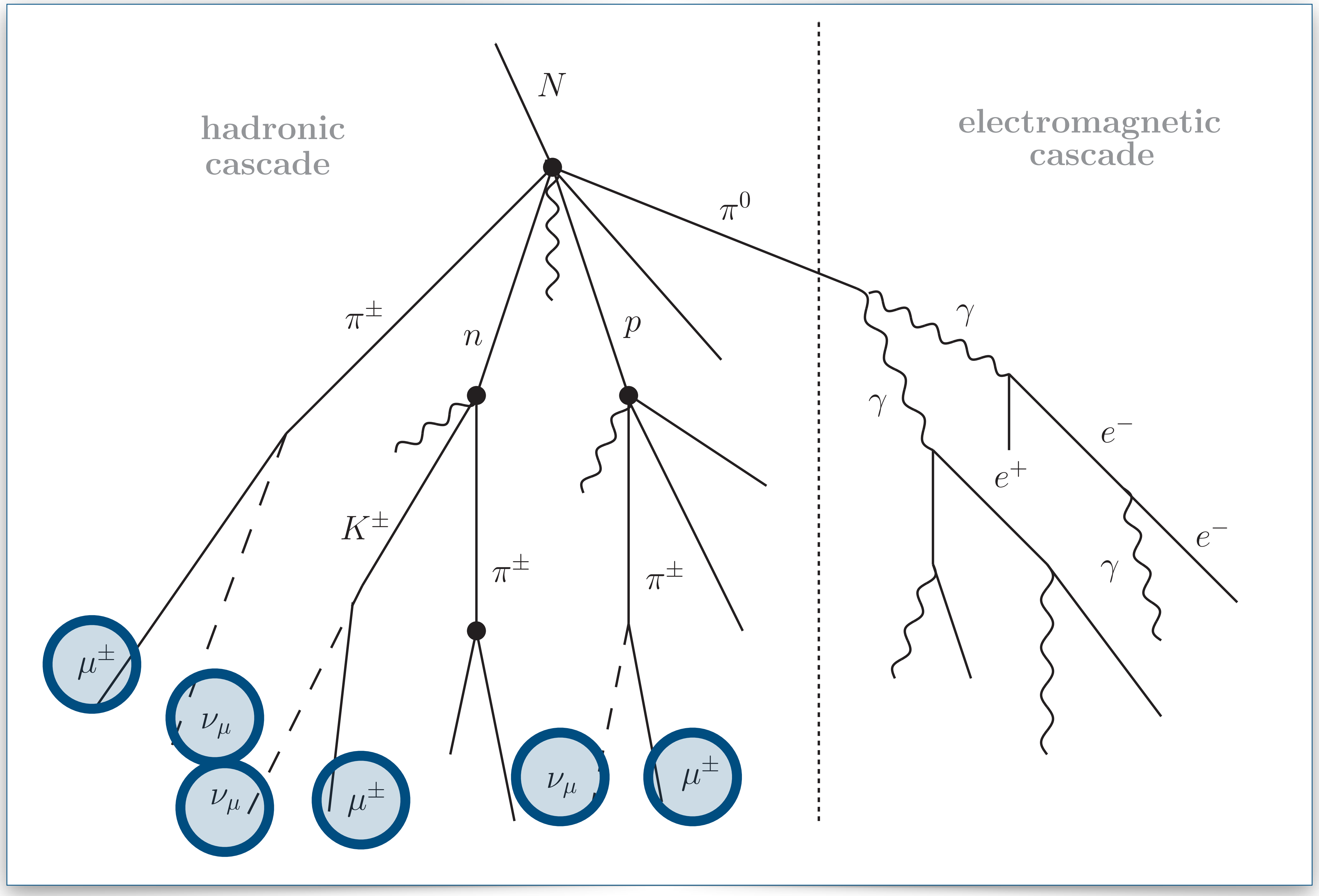
$\pi^0 \to \gamma\gamma$ ($\sim 100\,\%$)

Heitler-Matthews Model of EAS

- Limitations of the Heitler-Matthews model:
  - Hadronic interactions produce other particles in addition to pions
  - All particles per generation are assumed to receive the same energy fraction
  - The hadronic interaction length and the hadron multiplicity are not constant but weakly energy dependent
  - The atmosphere does not have constant density which has an impact on the critical energy ξ_C
  - Random processes are replaced by the average process and extensions of the basic model are needed to describe intrinsic shower fluctuations
- To calculate the EAS development accurately very complex coupled differential equations, the Cascade Equations, have to be solved…. hard…

Cascade Equations





Cascade Equations

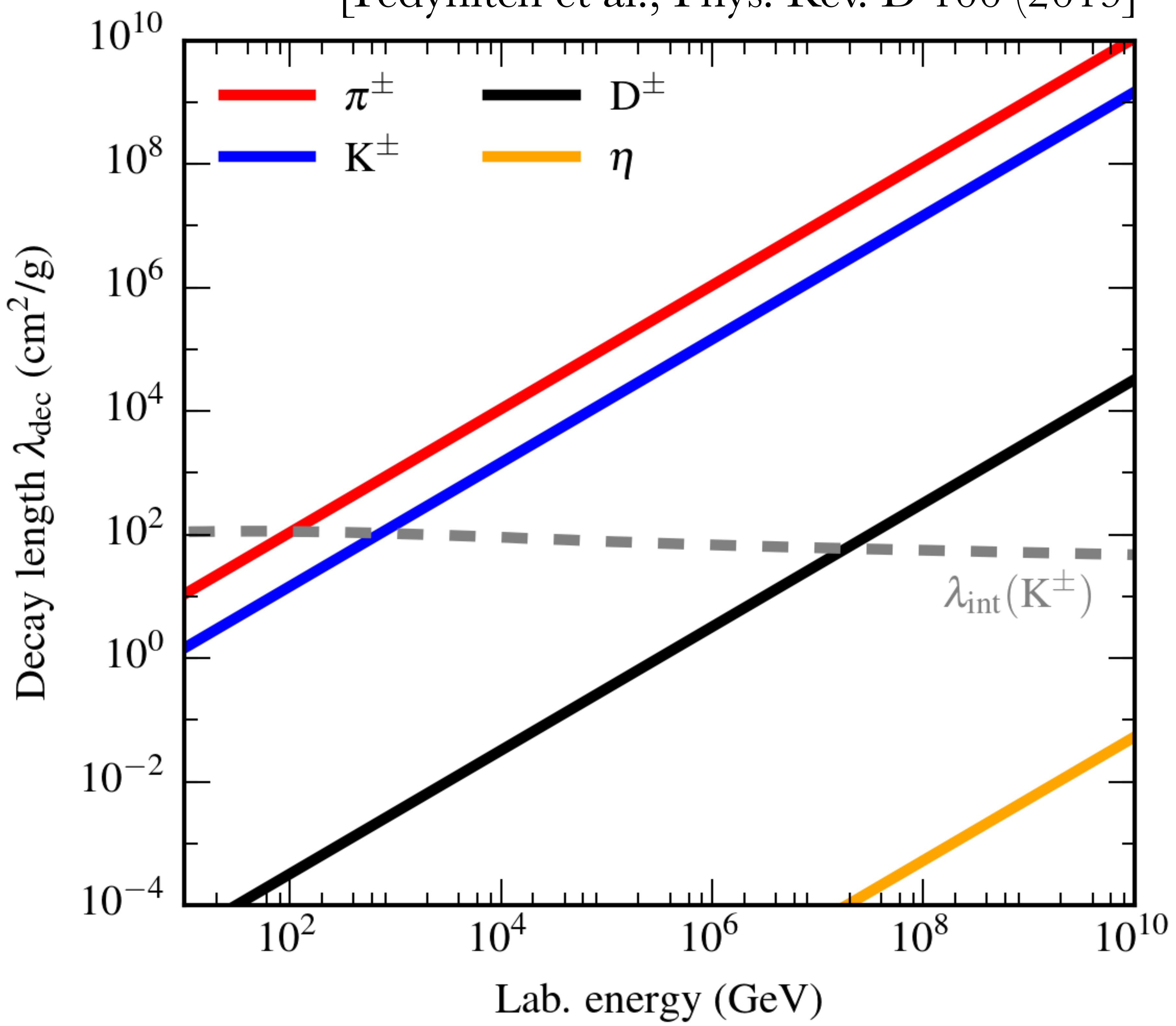
- If a particle h decays or re-interacts in the atmosphere depends on its
  - decay length:

$$\lambda_{\mathrm{dec},h}(E_h, X) = \rho \cdot l_{\mathrm{dec}} = c \cdot \tau_h \cdot \beta \cdot \gamma \cdot \rho(X)$$

  - interaction length:

$$\lambda_{\mathrm{int},h}(E_h, X) = \frac{\langle m_{\mathrm{target}} \rangle}{\sigma_{\mathrm{int}}} = \frac{\rho(X)}{\sum_A \sigma_{hA}(E_h) \cdot n_A(X)}$$

[Fedynitch et al., Phys. Rev. D 100 (2019)]



- Propagation described by coupled cascade equations:

$$\frac{d\Phi_h(E_h, X)}{dX} = -\left(\frac{1}{\lambda_{\mathrm{int},h}} - \frac{1}{\lambda_{\mathrm{dec},h}}\right) \cdot \Phi_h(E_h, X) + \sum_j \int \frac{E_j \cdot dN_j(E_h, E_j)}{E_h \cdot dE_j} \cdot \frac{\Phi_j(E_j)}{\lambda_{\mathrm{int},j}} dE_j$$

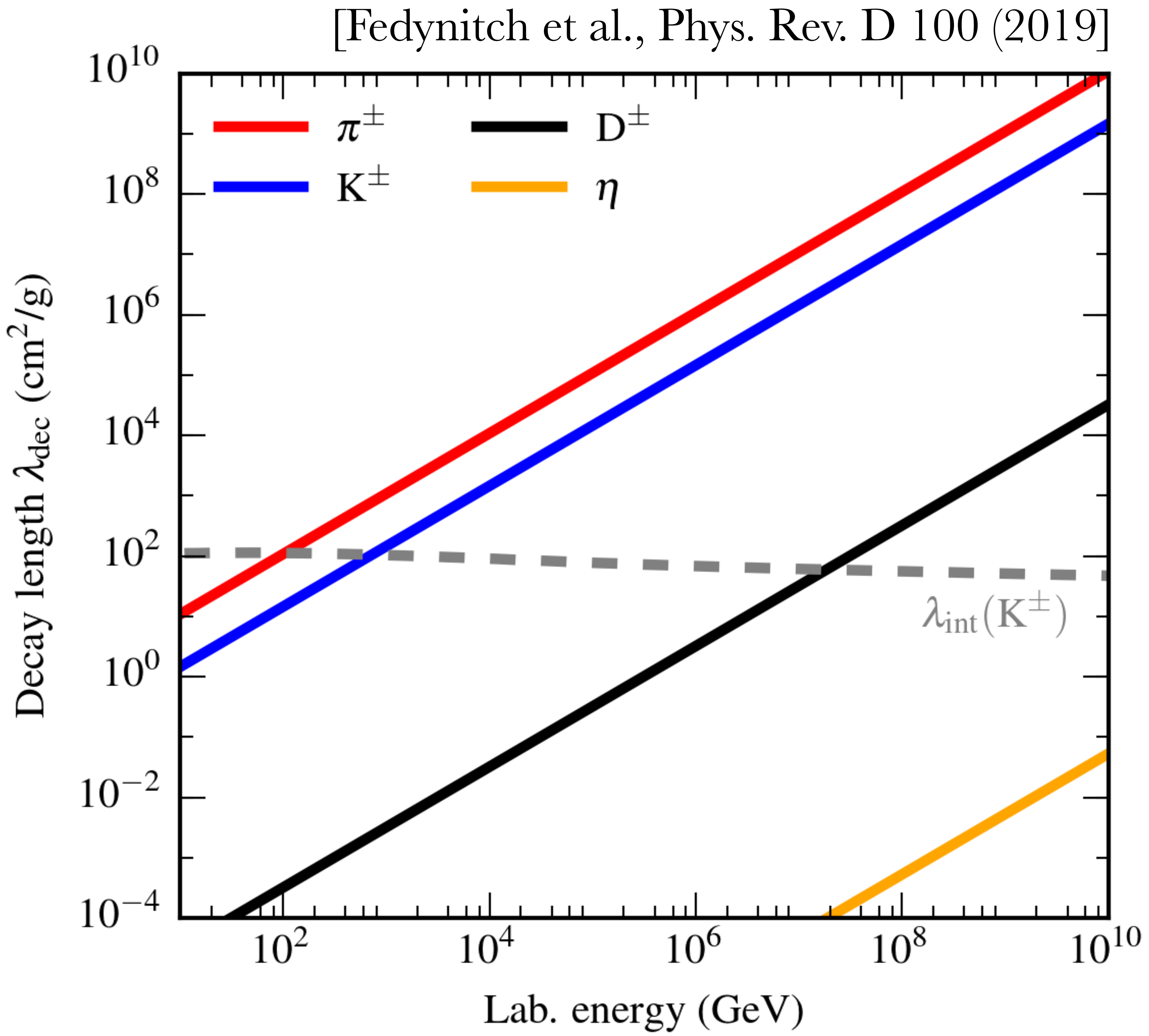
Cascade Equations

- If a particle h decays or re-interacts in the atmosphere depends on its
  - decay length:

$$\lambda_{\mathrm{dec},h}(E_h, X) = \rho \cdot l_{\mathrm{dec}} = c \cdot \tau_h \cdot \beta \cdot \gamma \cdot \rho(X)$$

  - interaction length:

$$\lambda_{\mathrm{int},h}(E_h, X) = \frac{\langle m_{\mathrm{target}} \rangle}{\sigma_{\mathrm{int}}} = \frac{\rho(X)}{\sum_A \sigma_{hA}(E_h) \cdot n_A(X)}$$



- Propagation described by coupled cascade equations:

$$\frac{d\Phi_h(E_h, X)}{dX} = -\left(\frac{1}{\lambda_{\mathrm{int},h}} - \frac{1}{\lambda_{\mathrm{dec},h}}\right) \cdot \Phi_h(E_h, X) + \sum_j \int \frac{E_j \cdot dN_j(E_h, E_j)}{E_h \cdot dE_j} \cdot \frac{\Phi_j(E_j)}{\lambda_{\mathrm{int},j}} dE_j$$

re-interactions

Cascade Equations

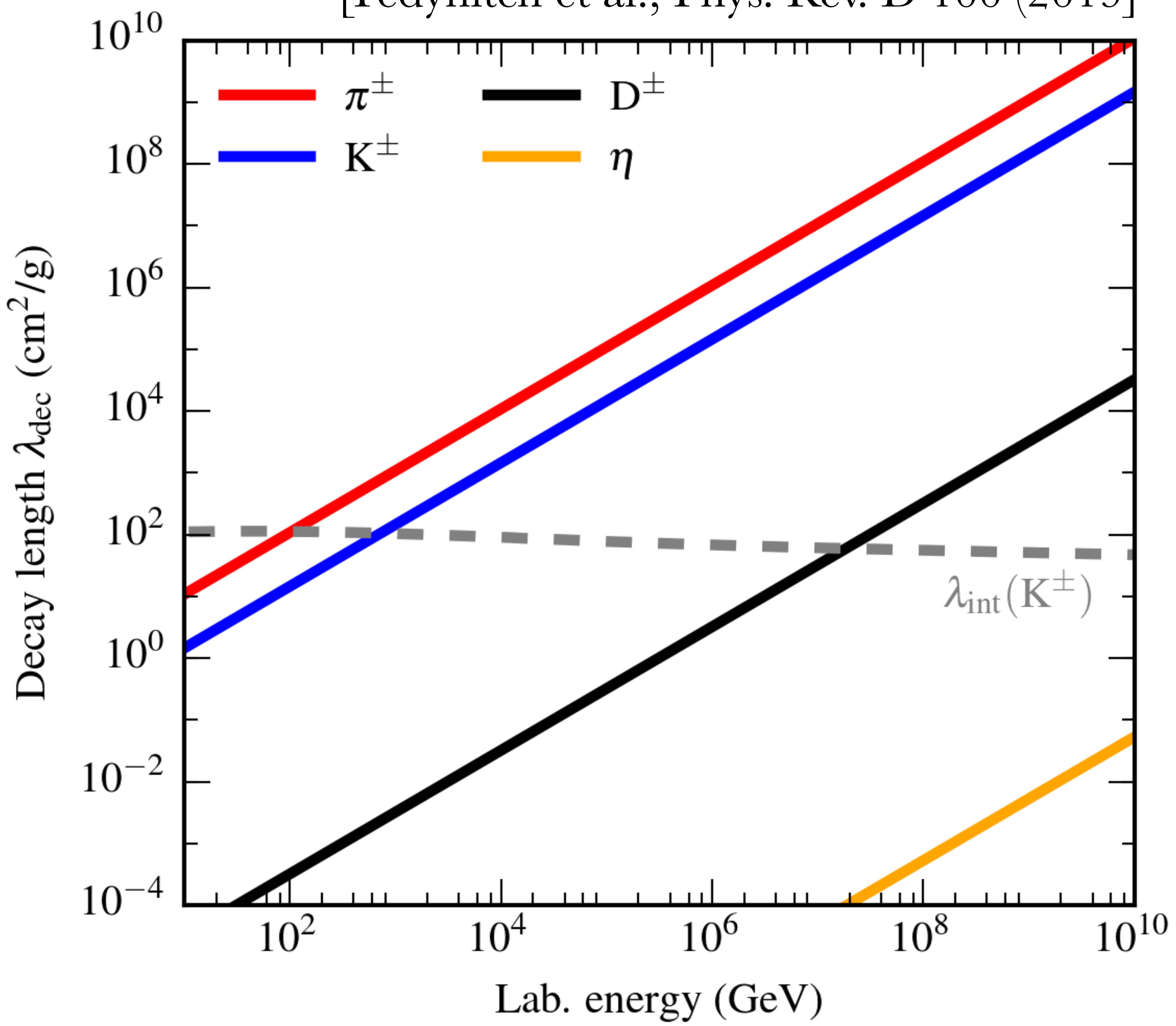
- If a particle h decays or re-interacts in the atmosphere depends on its
  - decay length:

$$\lambda_{\mathrm{dec},h}(E_h, X) = \rho \cdot l_{\mathrm{dec}} = c \cdot \tau_h \cdot \beta \cdot \gamma \cdot \rho(X)$$

  - interaction length:

$$\lambda_{\mathrm{int},h}(E_h, X) = \frac{\langle m_{\mathrm{target}} \rangle}{\sigma_{\mathrm{int}}} = \frac{\rho(X)}{\sum_A \sigma_{hA}(E_h) \cdot n_A(X)}$$

[Fedynitch et al., Phys. Rev. D 100 (2019)]



- Propagation described by coupled cascade equations:

$$\frac{d\Phi_h(E_h, X)}{dX} = -\left(\frac{1}{\lambda_{\mathrm{int},h}} - \frac{1}{\lambda_{\mathrm{dec},h}}\right) \cdot \Phi_h(E_h, X) + \sum_j \int \frac{E_j \cdot dN_j(E_h, E_j)}{E_h \cdot dE_j} \cdot \frac{\Phi_j(E_j)}{\lambda_{\mathrm{int},j}} dE_j$$

re-interactions — decays

Cascade Equations

- If a particle h decays or re-interacts in the atmosphere depends on its
  - decay length:

    $$\lambda_{\text{dec},h}(E_h, X) = \rho \cdot l_{\text{dec}} = c \cdot \tau_h \cdot \beta \cdot \gamma \cdot \rho(X)$$

  - interaction length:

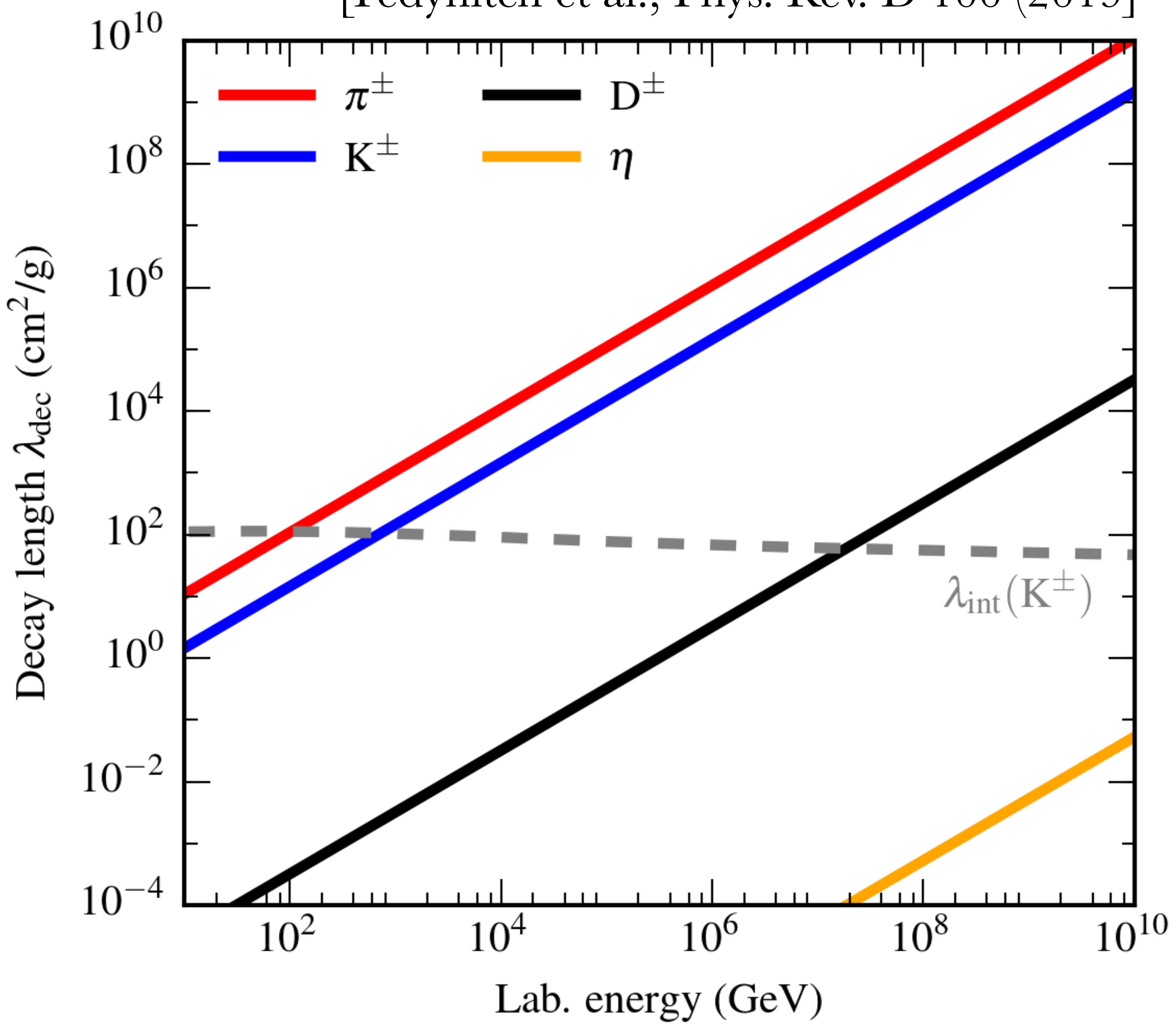
    $$\lambda_{\text{int},h}(E_h, X) = \frac{\langle m_{\text{target}} \rangle}{\sigma_{\text{int}}} = \frac{\rho(X)}{\sum_A \sigma_{hA}(E_h) \cdot n_A(X)}$$

- Propagation described by coupled cascade equations:

$$\frac{d\Phi_h(E_h, X)}{dX} = -\left(\frac{1}{\lambda_{\text{int},h}} - \frac{1}{\lambda_{\text{dec},h}}\right) \cdot \Phi_h(E_h, X) + \sum_j \int \frac{E_j \cdot dN_j(E_h, E_j)}{E_h \cdot dE_j} \cdot \frac{\Phi_j(E_j)}{\lambda_{\text{int},j}} dE_j$$

re-interactions — decays — production of new particles

[Fedynitch et al., Phys. Rev. D 100 (2019)]



Electromagnetic Cascade

- Reminder:
  - Energy loss of e^+/e^-:
    $$\frac{dE}{dX} = -\alpha(E) - \frac{E}{X_0}$$
  - Radiation length:
    $$X_0 \approx 36\,\mathrm{g/cm^2}$$
  - Critical energy:
    $$\xi_C = \alpha X_0 \approx 85\,\mathrm{MeV}$$
- Cascade equations (electromagnetic cascades):

$$\frac{d\Phi_e(E)}{dE} = -\frac{\sigma_e}{\langle m_{\mathrm{air}}\rangle}\Phi_e(E) + \int_E^\infty -\frac{\sigma_e}{\langle m_{\mathrm{air}}\rangle}\Phi_e(\tilde{E})P_{e\to e}(\tilde{E},E)d\tilde{E} + \int_E^\infty -\frac{\sigma_\gamma}{\langle m_{\mathrm{air}}\rangle}\Phi_\gamma(\tilde{E})P_{\gamma\to e}(\tilde{E},E)d\tilde{E} + \alpha\frac{\partial\Phi_e(E)}{\partial E}$$

[Rossi & Greisen, Rev. Mod. Phys. 13 (1940) 240]

$$\Rightarrow \quad X_{\max} \approx X_0 \ln(E_0/\xi_C) \quad \text{and} \quad N_{\max} \approx \frac{0.31}{\sqrt{\ln(E_0/\xi_C) - 0.33}} E_0/\xi_C$$

Shower Age and Greisen Formula

- Longitudinal Profile:
  - Greisen (1956):

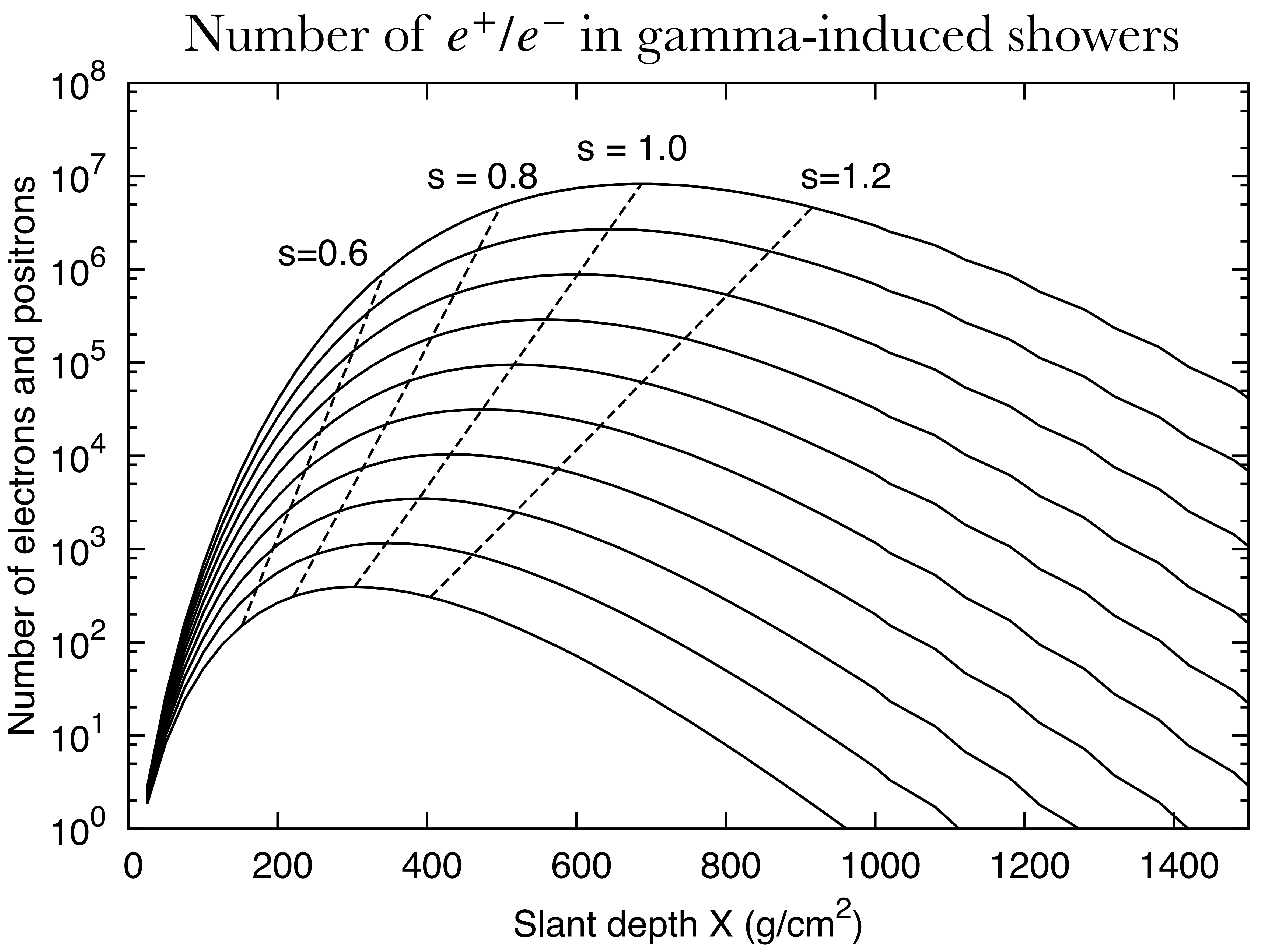
$$N_e(X) = \frac{0.31}{\sqrt{\ln(E_0/\xi_C)}} \exp\left[\frac{X}{X_0}\left(1 - \frac{3}{2}\ln s\right)\right]$$

  - Shower age:

$$s = \frac{3X}{X + 2X_{\max}}$$

- Energy spectrum:

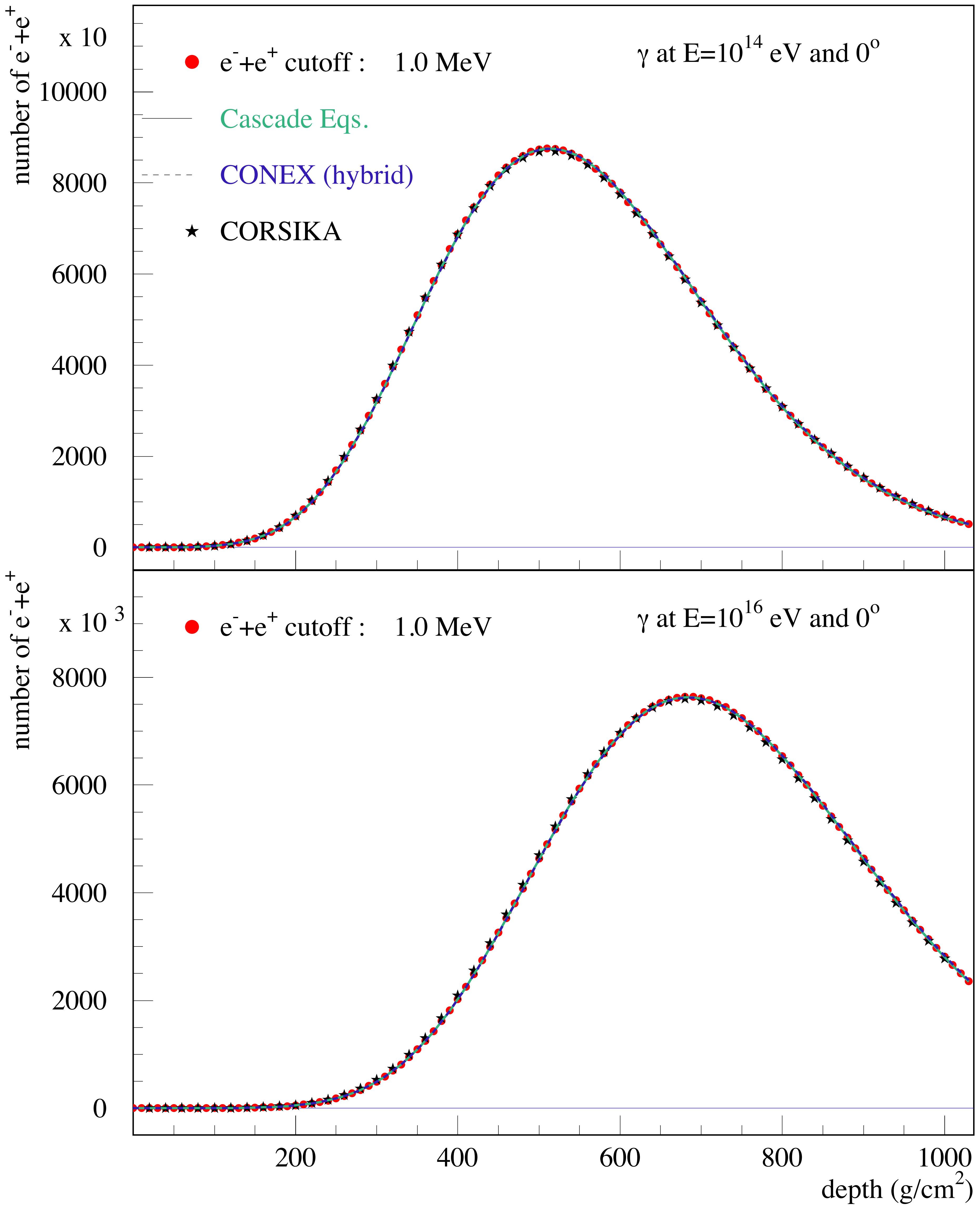
$$\frac{dN_e}{dE} \sim \frac{1}{E^{1+s}}$$



Mean Longitudinal Profile

- <u>Calculation with cascade equations:</u>
  - Photons:
    - Pair production
    - Compton scattering
  - Electrons:
    - Bremsstrahlung
    - Moller scattering
  - Positrons:
    - Bremsstrahlung
    - Bhabha scattering

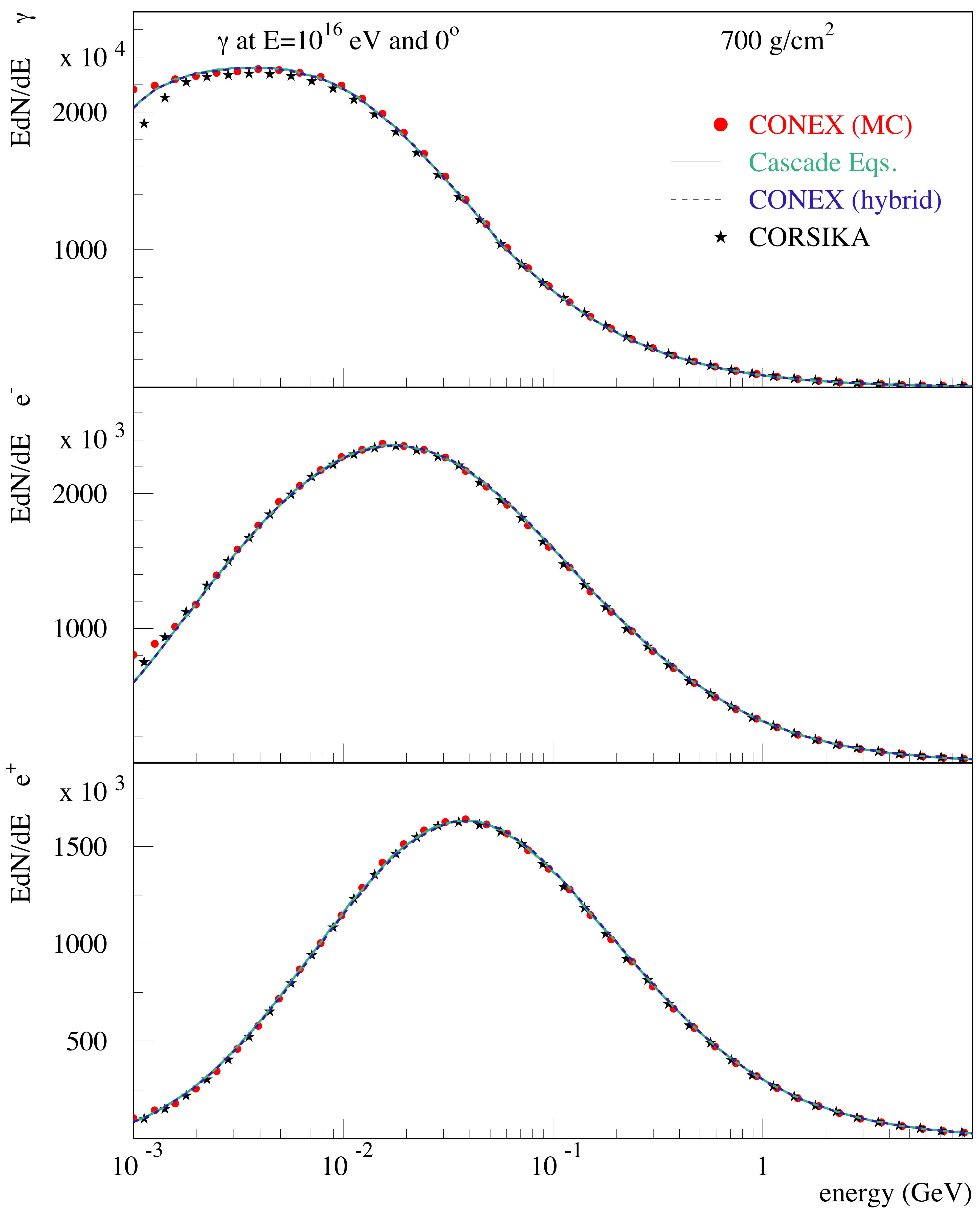
[Bergmann et al., Astropart.Phys. 26 (2007) 420]



Mean Longitudinal Profile

- Calculation with cascade equations:
  - Number of photons divergent (energy threshold applied)
  - Typical energy of e^+/e^-: $\xi_C \sim 80\,\mathrm{MeV}$
  - Electron excess 20%-30%
    - Pair production symmetric
    - Excess of electrons in target

[Bergmann et al., Astropart.Phys. 26 (2007) 420]



Mean Lateral Profile

- Lateral spread driven by Coulomb scattering:

$$\frac{dN}{d\Omega} = \frac{1}{64\pi}\frac{1}{\ln(191 \cdot Z^{-1/3})}\left(\frac{E_s}{E}\right)^2 \frac{1}{\sin^4(\theta/2)} \quad , \quad E_s \approx 21\,\mathrm{MeV}$$

- Resulting mean displacement of a particle in air:

$$r \sim \frac{E_s}{E}\frac{X_0}{\rho_{\mathrm{air}}} \equiv r_M$$

- Moliere unit r_M (78m at seas level)
- Nishimura-Kamata-Greisen (NKG) lateral distribution function (LDF):

$$\frac{dN_e}{rdr} = \left(\frac{r}{r_M}\right)^{s-2}\left(1 + \frac{r}{r_M}\right)^{s-4.5}$$

- For more details on the analytical description of EAS, see Tom Gaisser's book…