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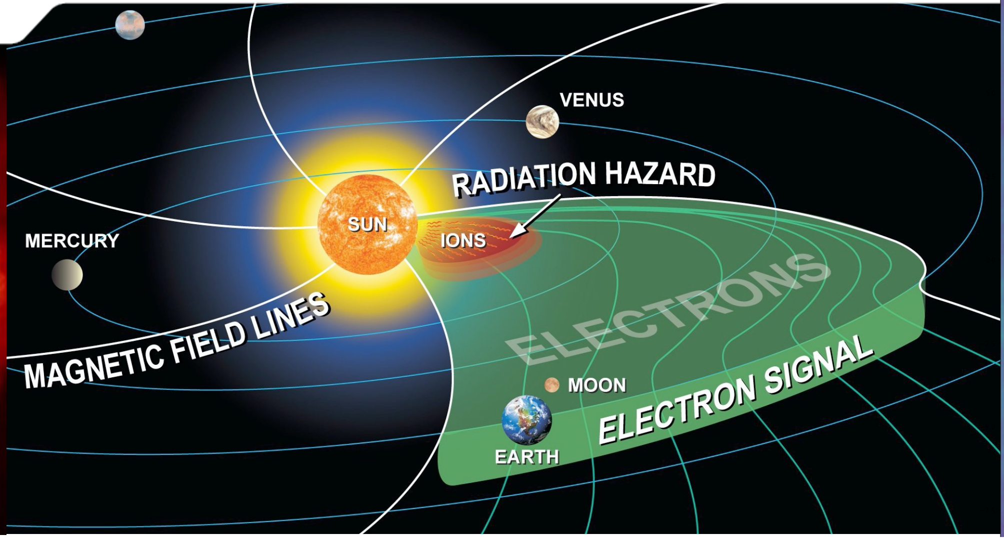

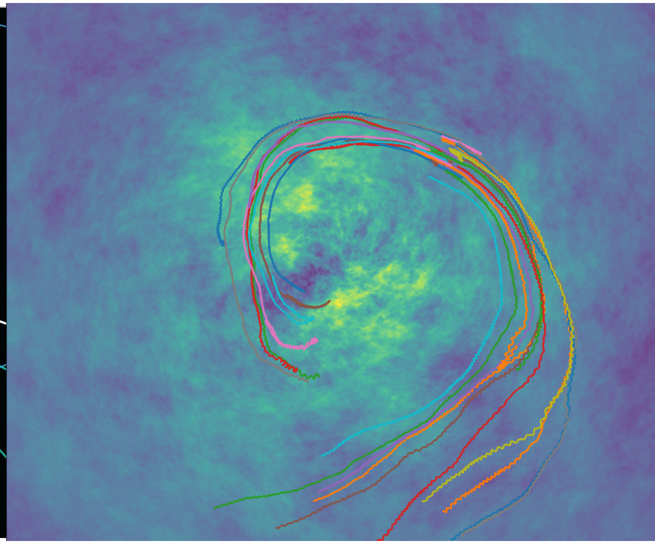

Fast charged particle transport in artificially generated magnetic turbulence (Project F1)

Frederic Effenberger, Ruhr-University Bochum, Germany

CRC Kick-Off, June 2, 2022

Overview

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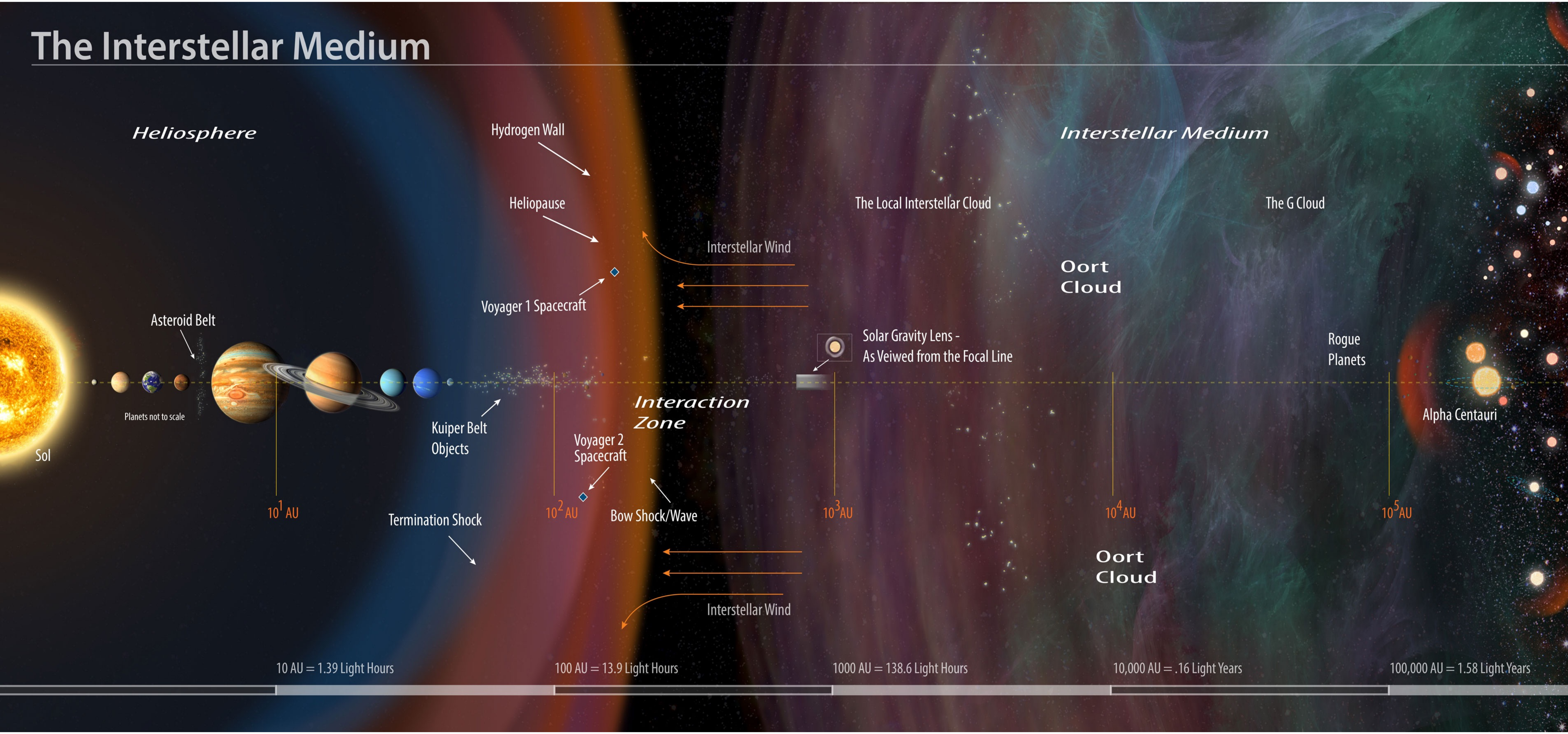
- **Introduction: Why consider energetic particle transport?**
- **How to study particle transport?**
  - Full-Orbit calculations
  - Synthetic turbulence
- **State of the Art**
  - Some previous works
  - Our first steps
- **Summary**

The Heliosphere & ISM



Particle Transport in the Heliosphere

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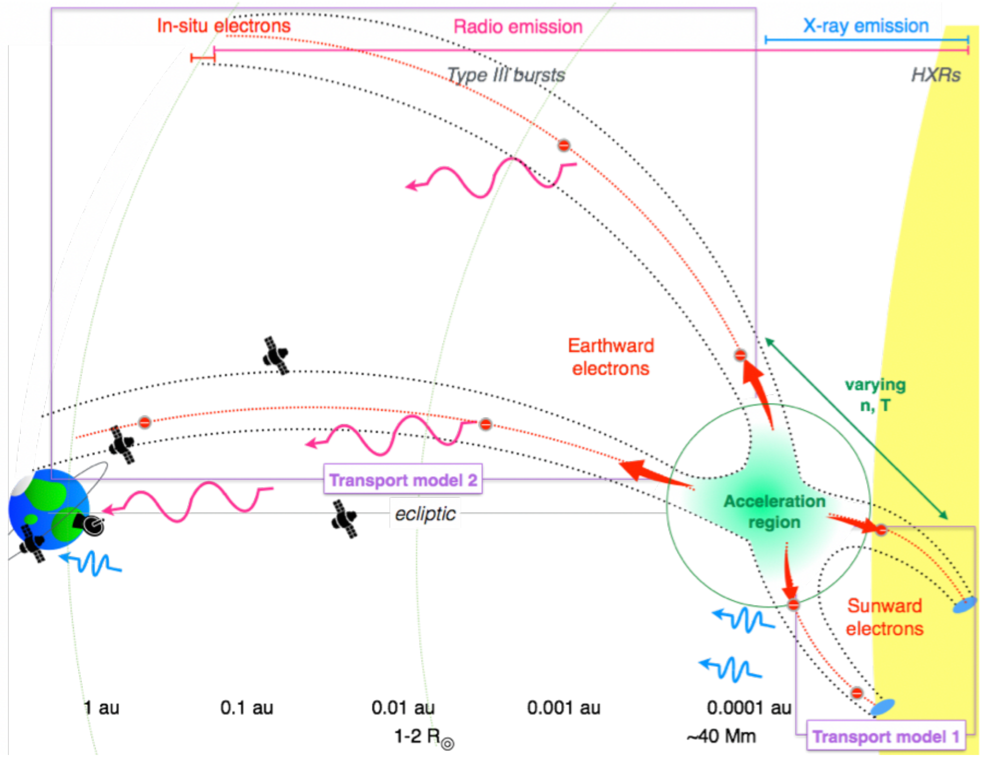
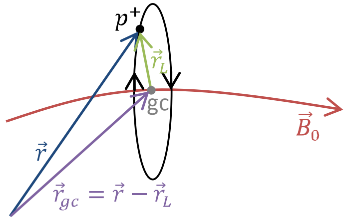
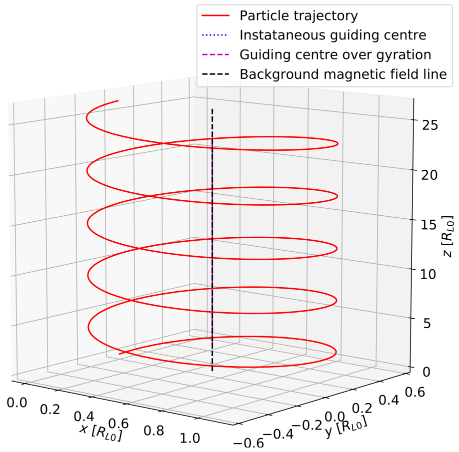
How do particles travel from Sun to Earth?



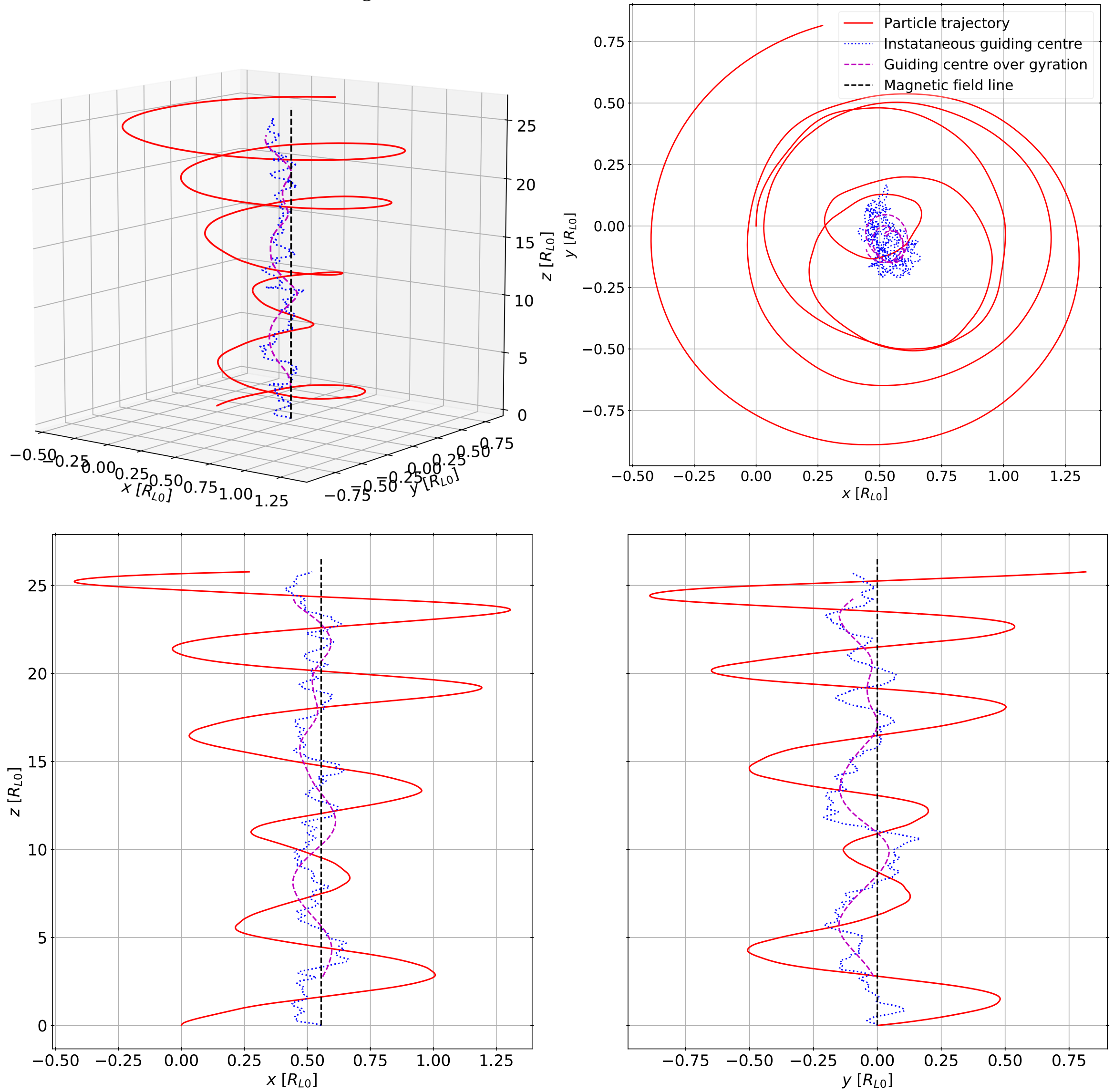




A Primer on Focused Solar Energetic Particle Transport

Basic physics and recent modelling results

Jabus van den Berg · Du Toit Strauss · Frederic Effenberger



1D Solar Energetic Particle Modelling

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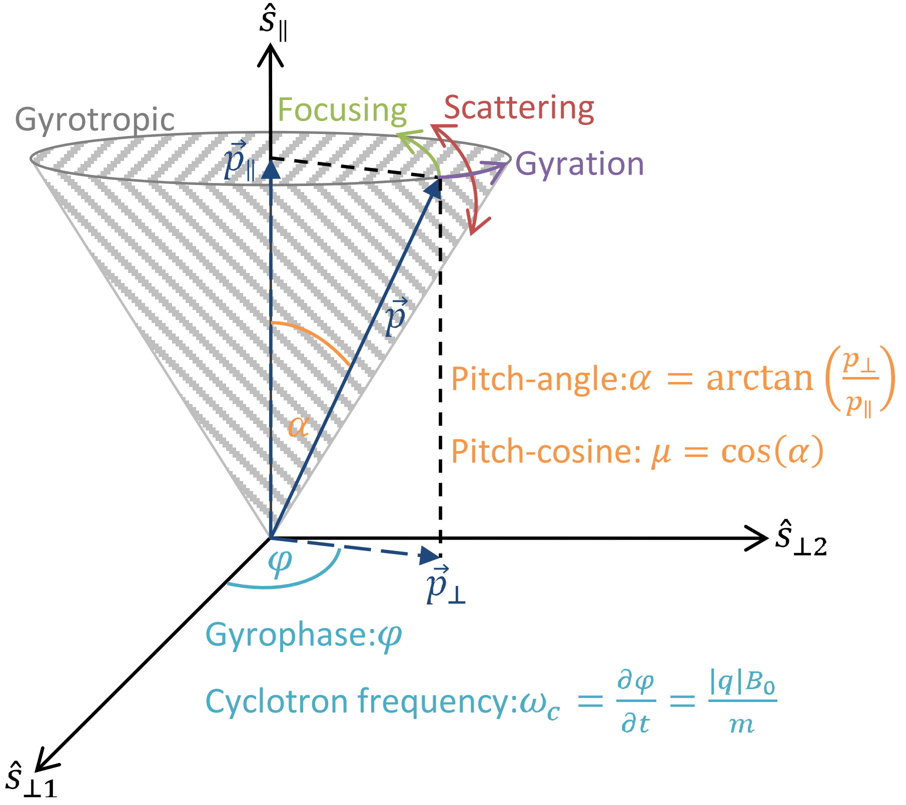
Basic 1D focused transport equation

$$\frac{\partial f}{\partial t} + \frac{\partial}{\partial s}\left[\mu v f\right] + \frac{\partial}{\partial \mu}\left[\frac{(1-\mu^2)v}{2L(s)} f\right] = \frac{\partial}{\partial \mu}\left[D_{\mu\mu}\frac{\partial f}{\partial \mu}\right]$$

$$D_{\mu\mu}^{\mathrm{QLT}}(\mu) = D_0(1-\mu^2)|\mu|^{q-1}$$



If S and M represents the stochastic variables corresponding to s and μ, respectively, then the two first order SDEs equivalent to the Roelof equation (Eq. 9) are

$$\mathrm{d}S = \mu v\,\mathrm{d}t$$

$$\mathrm{d}M = \left[\frac{(1-\mu^2)v}{2L(s)} + \frac{\partial D_{\mu\mu}}{\partial \mu}\right]\mathrm{d}t + \sqrt{2D_{\mu\mu}}\mathrm{d}W_\mu(t),$$

where $\mathrm{d}W_\mu(t)$ is a Wiener process. These SDEs are solved using the Euler-Maruyama scheme,

$$S(t+\Delta t) = S(t) + M(t)v\Delta t$$

$$M(t+\Delta t) = M(t) + \left[\frac{(1-M^2(t))v}{2L(S(t))} + \left.\frac{\partial D_{\mu\mu}}{\partial \mu}\right|_{\mu=M(t)}\right]\Delta t + \sqrt{2D_{\mu\mu}(M(t))\Delta t}\Lambda,$$

Galactic Cosmic Ray Transport

Importance of Anisotropic Transport, Diffusion Tensor

Anisotropic diffusion of Galactic cosmic ray protons and their steady-state azimuthal distribution

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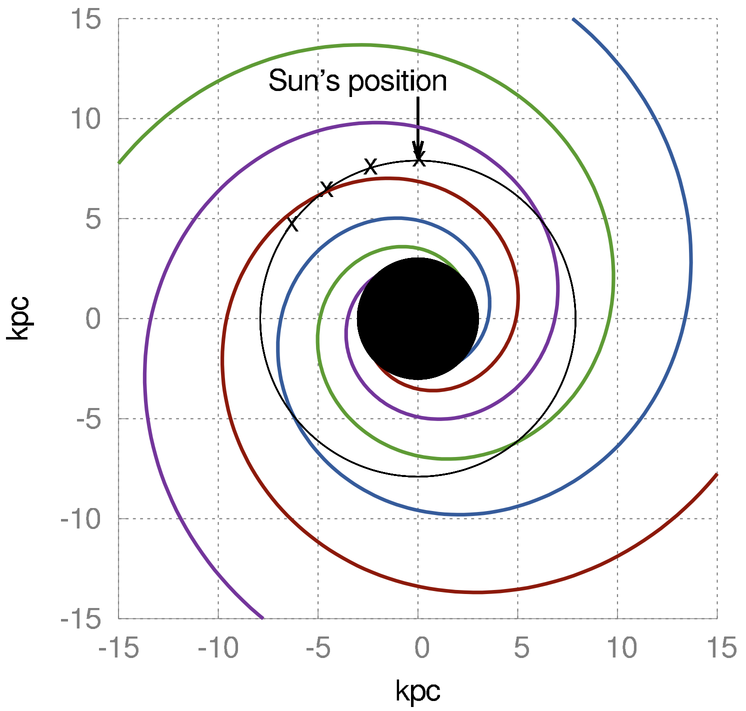
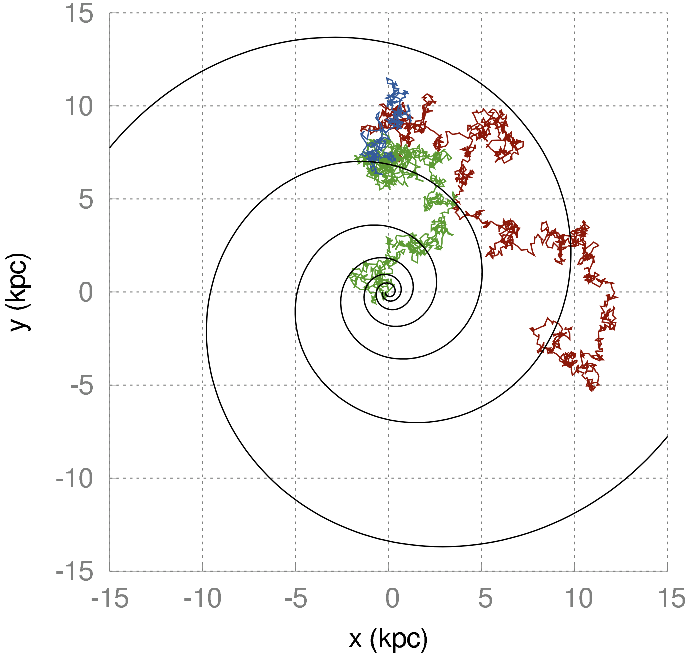
Received 10 August 2012 / Accepted 3 October 2012

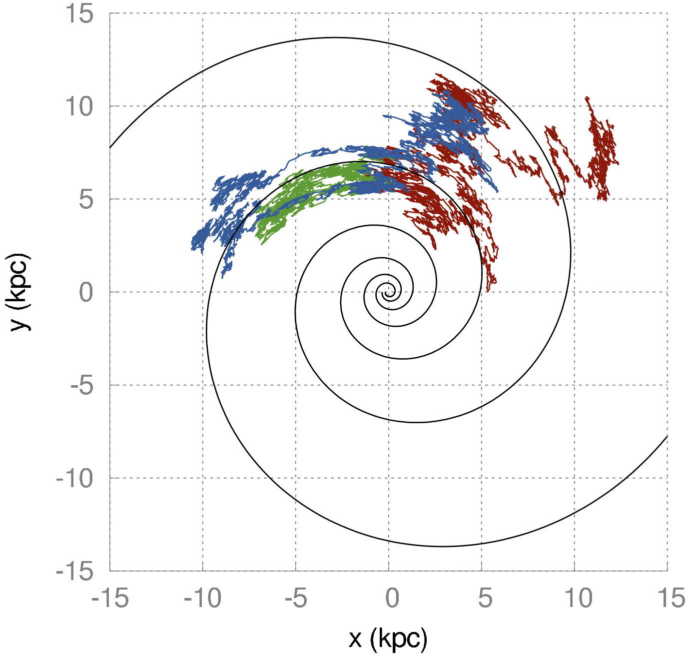
$$\frac{\partial N}{\partial t} = \nabla \cdot (\hat{\kappa} \cdot \nabla N - \boldsymbol{u} N) - \frac{\partial}{\partial p}\left[\dot{p}N - \frac{p}{3}(\nabla \cdot \boldsymbol{u})N\right] + Q$$

$$\hat{\kappa}_{\mathrm{L}} = \begin{pmatrix} \kappa_{\perp 1} & 0 & 0 \\ 0 & \kappa_{\perp 2} & 0 \\ 0 & 0 & \kappa_{\parallel} \end{pmatrix}$$

$$\mathrm{d}x_i = A_i(x_i)\mathrm{d}s + \sum_j B_{ij}(x_i)\mathrm{d}W_j$$







Full-Orbit Simulation

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Solving the Newton-Lorentz Equations for many charged (Test) Particles

Dimensionless Time: $T = \Omega t$

Dimensionless Rigidity: $\mathbf{R} := \mathbf{v}/(\Omega \ell)$

With gyro frequency Ω and typical length scale (bendover scale) ℓ

$$\frac{d}{dT}\mathbf{R} = \mathbf{R} \times \left(\mathbf{e}_z + \frac{\delta \mathbf{B}(\mathbf{x})}{B_0} \right)$$

$$\frac{d}{dT}\frac{\mathbf{x}}{\ell} = \mathbf{R}.$$

In the literature, often RK methods are used.
We use the Boris Push method for energy conservation:

$$\frac{\mathbf{v}^{n+1/2} - \mathbf{v}^{n-1/2}}{\Delta t} = \frac{q}{m}\left[\mathbf{E} + \frac{\mathbf{v}^{n+1/2} + \mathbf{v}^{n-1/2}}{2} \times \mathbf{B} \right]$$

$$\mathbf{v}^{n-1/2} = \mathbf{v}^{-} - \frac{q\mathbf{E}}{m}\frac{\Delta t}{2} \qquad \mathbf{v}^{n+1/2} = \mathbf{v}^{+} + \frac{q\mathbf{E}}{m}\frac{\Delta t}{2}$$

$$\frac{\mathbf{v}^{+} - \mathbf{v}^{-}}{\Delta t} = \frac{q}{2m}\left(\mathbf{v}^{+} + \mathbf{v}^{-} \right) \times \mathbf{B}$$

Synthetic Turbulence

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Requirements for an Advanced Synthetic Turbulence Model

i) The synthetic magnetic fields have to be divergence free: $\nabla \cdot \vec{B} = 0$.

ii) The synthetic fields need to be homogeneous.

iii) The synthetic fields should reproduce a predefined energy spectrum (Kolmogorov, 1941; Iroshnikov, 1963; Kraichnan, 1965; Boldyrev, 2005).

iv) There should be no restriction other than computational ones for a maximum Reynolds number.

v) The spectrum should be anisotropic, which means that it should have different exponents perpendicular and parallel to a local guide field (Goldreich & Sridhar, 1995; Boldyrev, 2005).

vi) The generation of the synthetic fields must be local and adaptive in space.

vii) The synthetic turbulence should exhibit intermittency, as prescribed by a given intermittency model.

viii) The synthetic turbulent fields should exhibit an increment PDF that is negatively skewed.

ix) Synthetic turbulence should be constructed as a multifractal Brownian bridge.

Synthetic Turbulence

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Magnetostatic Turbulence (following Shalchi 2020 review)

$$\mathbf{B}(\mathbf{x}, t) = B_0\mathbf{e}_z + \delta\mathbf{B}(\mathbf{x}, t).$$

$$\delta\mathbf{B}(\mathbf{x}) = \sqrt{2}\delta B \sum_{n=1}^{N} A(k_n)\boldsymbol{\xi}_n \cos[\mathbf{k}_n \cdot \mathbf{x} + \beta_n]$$

$$\mathbf{k}_n = k_n \mathbf{e}_{k,n}$$

$$\mathbf{e}_{k,n} = \begin{pmatrix} \sqrt{1-\eta_n^2}\cos\phi_n \\ \sqrt{1-\eta_n^2}\sin\phi_n \\ \eta_n \end{pmatrix}$$

$$\boldsymbol{\xi}_n = \begin{pmatrix} -\sin\phi_n\cos\alpha_n + \eta_n\cos\phi_n\sin\alpha_n \\ \cos\phi_n\cos\alpha_n + \eta_n\sin\phi_n\sin\alpha_n \\ -\sqrt{1-\eta_n^2}\sin\alpha_n \end{pmatrix}$$

$$A^2(k_n) = G(k_n)\Delta k_n \left(\sum_{m=1}^{N} G(k_m)\Delta k_m\right)^{-1}$$

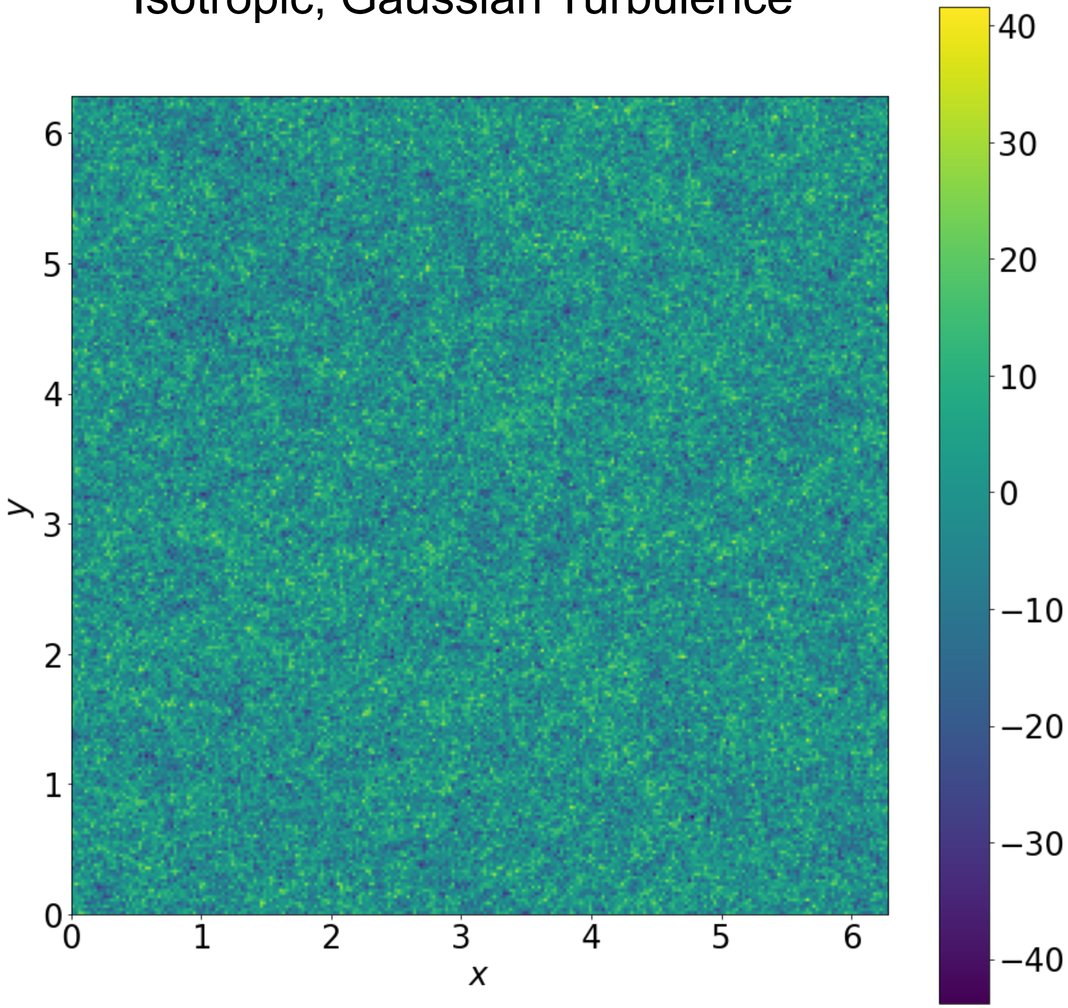
$$G(k_n) = \frac{k_n^q}{(1+k_n^2)^{(s+q)/2}}.$$

Turbulence model	η_n	α_n	Φ_n	Wave numbers	q
Slab	1	0	Random	$k_n = \ell_\parallel k_\parallel$	0
Two-dimensional	0	0	Random	$k_n = \ell_\perp k_\perp$	2 or 3
Isotropic	Random	Random	Random	$k_n = \ell_0 k$	3
Noisy slab model	0	0	Random	$k_n = \ell_\perp k_\perp$, $k_m = \ell_\parallel k_\parallel$	0
NRMHD	0	0	Random	$k_n = \ell_\perp k_\perp$, $k_m = \ell_\parallel k_\parallel$	3

Synthetic Turbulence

$$\delta\mathbf{B}(\mathbf{x}) = \sqrt{2}\delta B \sum_{n=1}^{N} A(k_n)\boldsymbol{\xi}_n \cos[\mathbf{k}_n \cdot \mathbf{x} + \beta_n]$$

Isotropic, Gaussian Turbulence



Example Results for the Diffusion Coefficient

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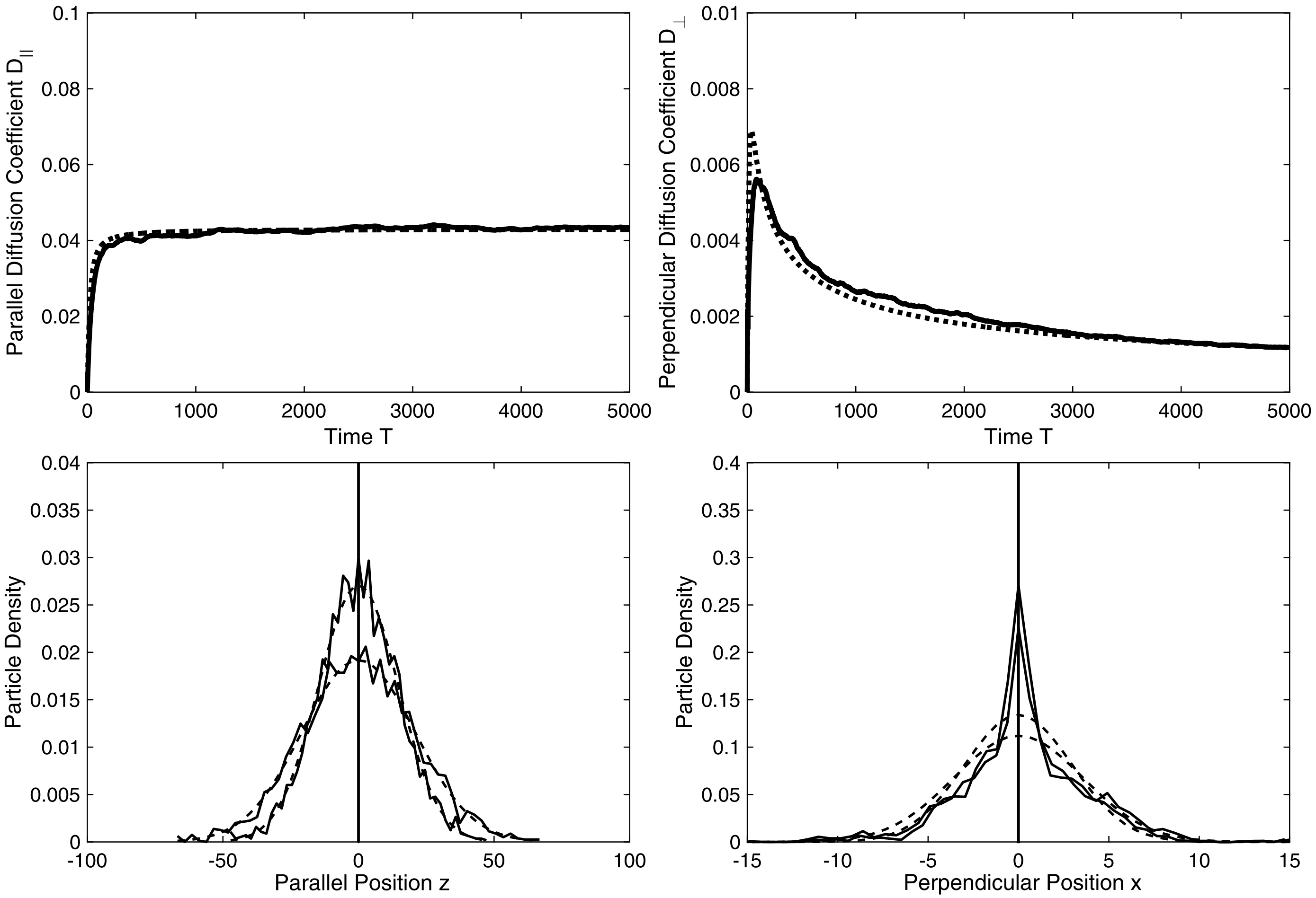
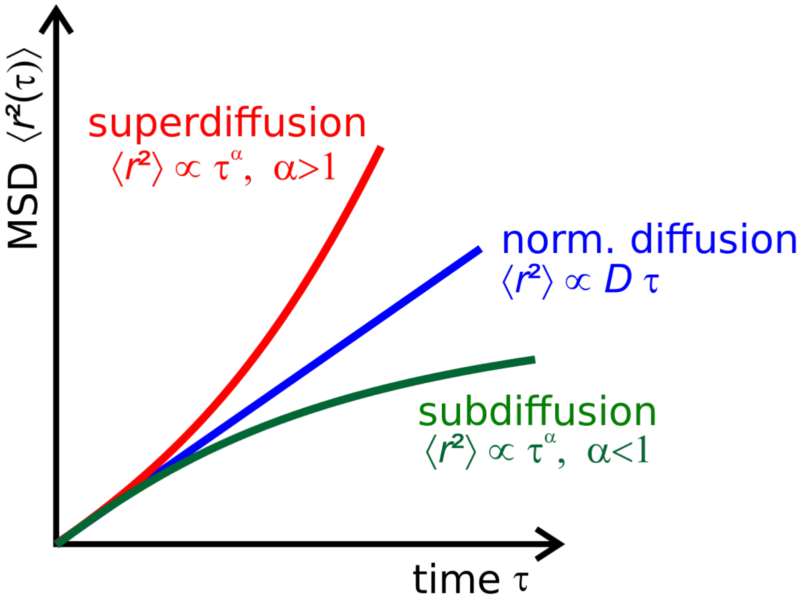


Fig. 16 Diffusion coefficients and distribution functions for pure slab turbulence, a magnetic rigidity of $R = 0.1$, and a magnetic field ratio of $\delta B_{slab}^2 / B_0^2 = 1$. The used parameters T, R, $K_{\|}$, and $D_{\|}$ are defined in

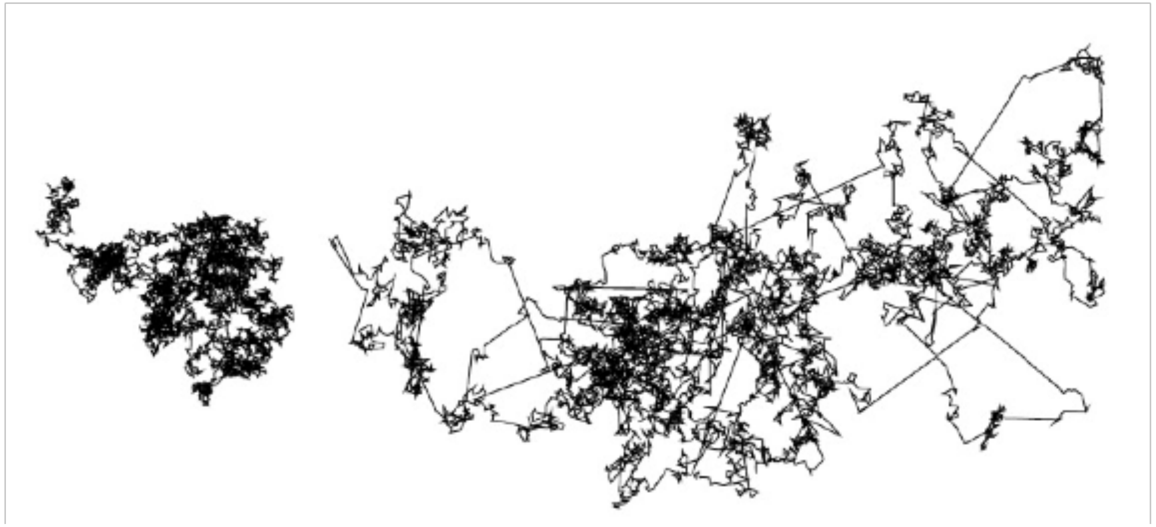
Shalchi 2020

Anomalous Diffusion

Gaussian	Anomalous	
$(\Delta x)^2 \propto t$	$(\Delta x)^2 \propto t^\zeta$	Superdiffusion: $1 < \zeta < 2$ Subdiffusion: $0 < \zeta < 1$



Subdiffusion (extended waiting times)



Superdiffusion (Lévy-Flights)

Super or Subdiffusion regimes?

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SUPERDIFFUSIVE AND SUBDIFFUSIVE TRANSPORT OF ENERGETIC PARTICLES IN SOLAR WIND ANISOTROPIC MAGNETIC TURBULENCE

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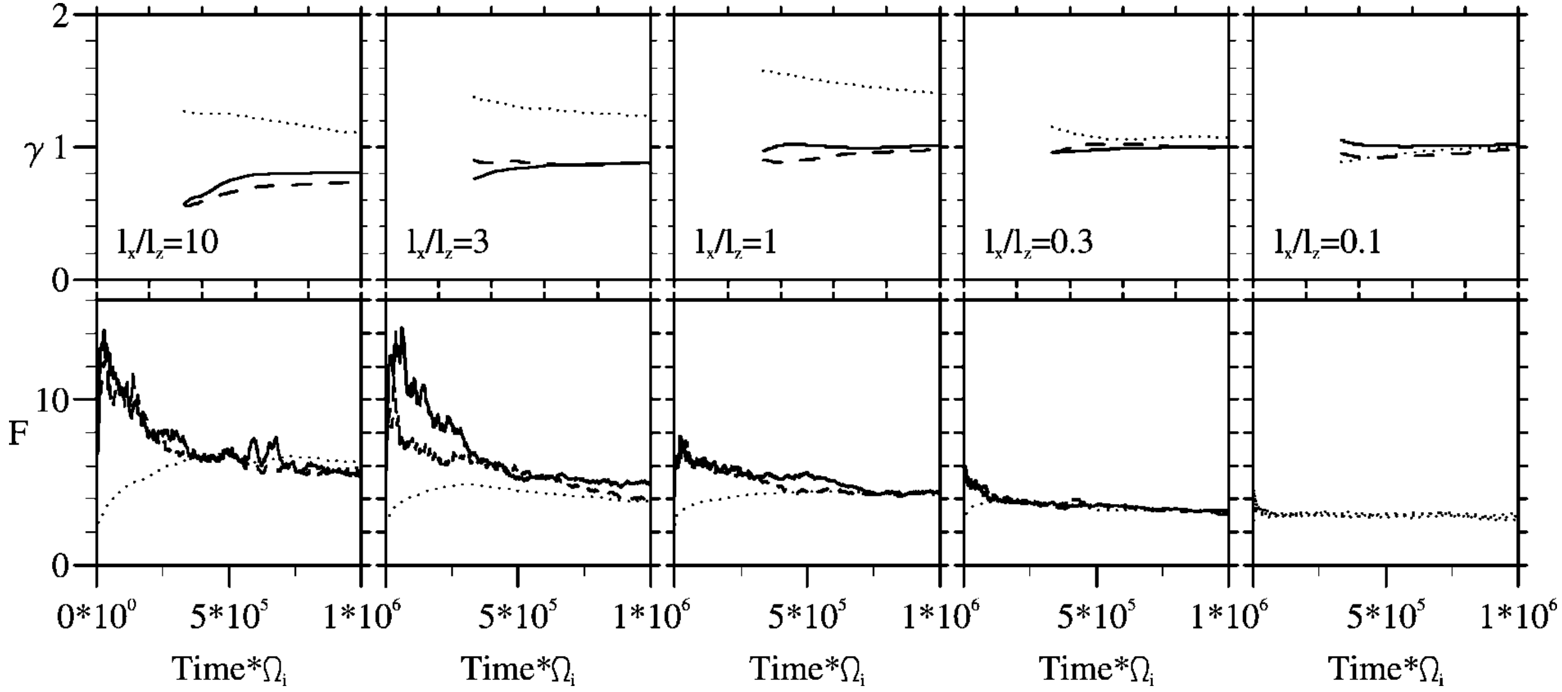

$$\langle \Delta x_i^2 \rangle = 2\kappa_i t^{\gamma_i}$$

Fig. 1.—Anomalous transport exponents γ_i (*top panels*) and flatnesses F_i (*bottom panels*) are plotted as a function of time for $\delta B/B_0 = 0.5$, $\rho/\lambda = 3.2 \times 10^{-3}$, and $\rho/\lambda_{min} = 1.28 \times 10^{-2}$. Magnetic turbulence is axisymmetric, and the ratio of correlation lengths varies from $l_x/l_z = 10$ (*far left*) to 3, 1, 0.33, and 0.1 (*far right*). Results for the *x*-, *y*-, and *z*-directions are indicated by solid, dashed, and dotted lines, respectively.

Influence of Intermittency? Shukurov 2017

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Cosmic Rays in Intermittent Magnetic Fields

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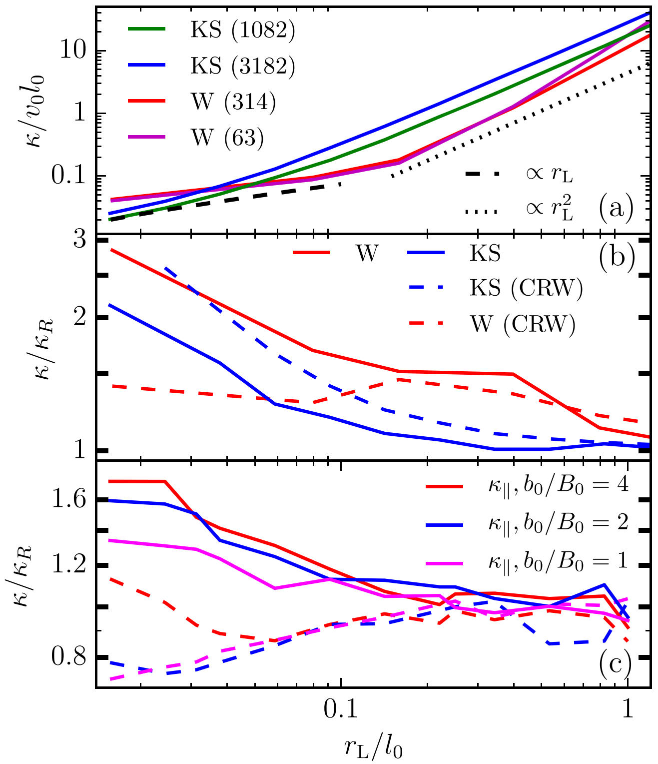
2. Magnetic Field Produced by Dynamo Action

We generate intermittent, statistically isotropic, fully three-dimensional random magnetic fields $\boldsymbol{b}$ by solving the induction equation with a prescribed velocity field $\boldsymbol{u}$:

$$\frac{\partial \boldsymbol{b}}{\partial t} = \nabla \times (\boldsymbol{u} \times \boldsymbol{b}) + R_{\mathrm{m}}^{-1}\nabla^2 \boldsymbol{b}, \qquad \nabla \cdot \boldsymbol{b} = 0,$$



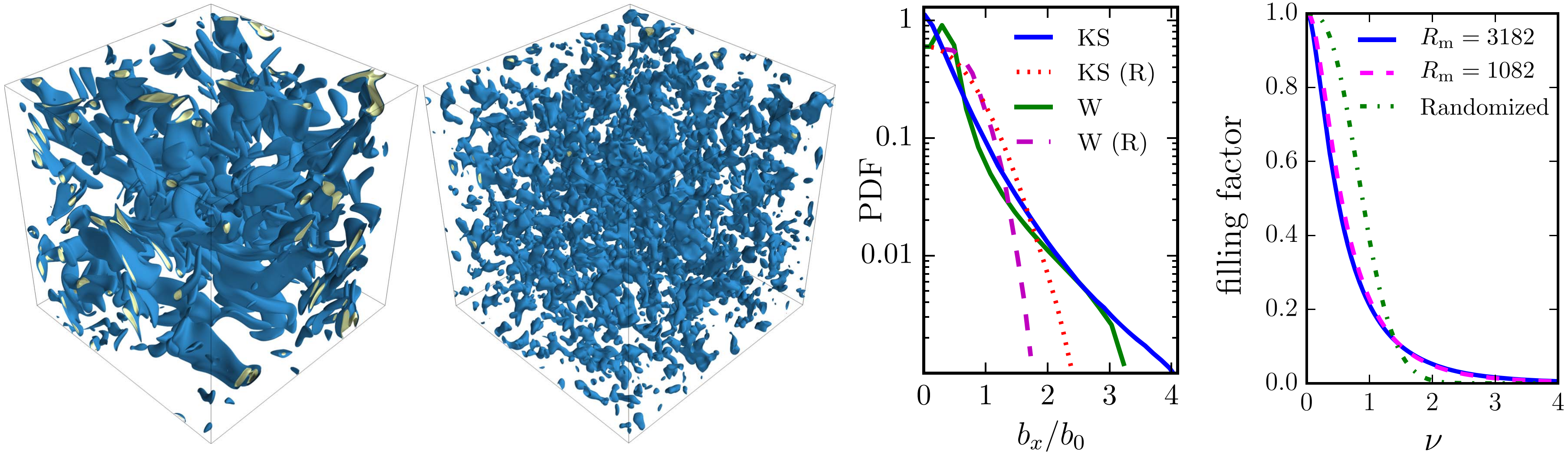


Figure 1. Isosurfaces of magnetic field strength $b^2/b_0^2 = 2.5$ (blue) and $b^2/b_0^2 = 5$ (yellow) with b_0 the rms magnetic field, for magnetic field generated by the KS flow (3) at $R_{\rm m} = 1082$ (left) and for the same magnetic field after Fourier phase randomization as described in the text (second from left). Magnetic field generated by the W flow (2) is similarly affected (not shown). The second from right panel shows the PDFs of a magnetic field component b_x for the original (KS, W: solid) and randomized (KS (R), W (R): dashed) magnetic fields obtained with both velocity fields (only $b_x > 0$ is shown as the PDFs are essentially symmetric about $b_x = 0$). The randomized fields have almost perfectly Gaussian statistics, whereas magnetic intermittency leads to heavy tails. The panel on the right shows the fractional volume within magnetic structures where $b \geqslant \nu b_0$, with b_0 the rms field strength, as a function of ν for the intermittent magnetic field produced by the flow (3) (solid for $R_{\rm m} = 3182$ and dashed for $R_{\rm m} = 1082$) and its Gaussian counterpart (dashed–dotted for $R_{\rm m} = 3142$ and 1082) obtained by Fourier phase randomization; the filling factor of the randomized fields is independent of $R_{\rm m}$.

Teaser (Parker background field)

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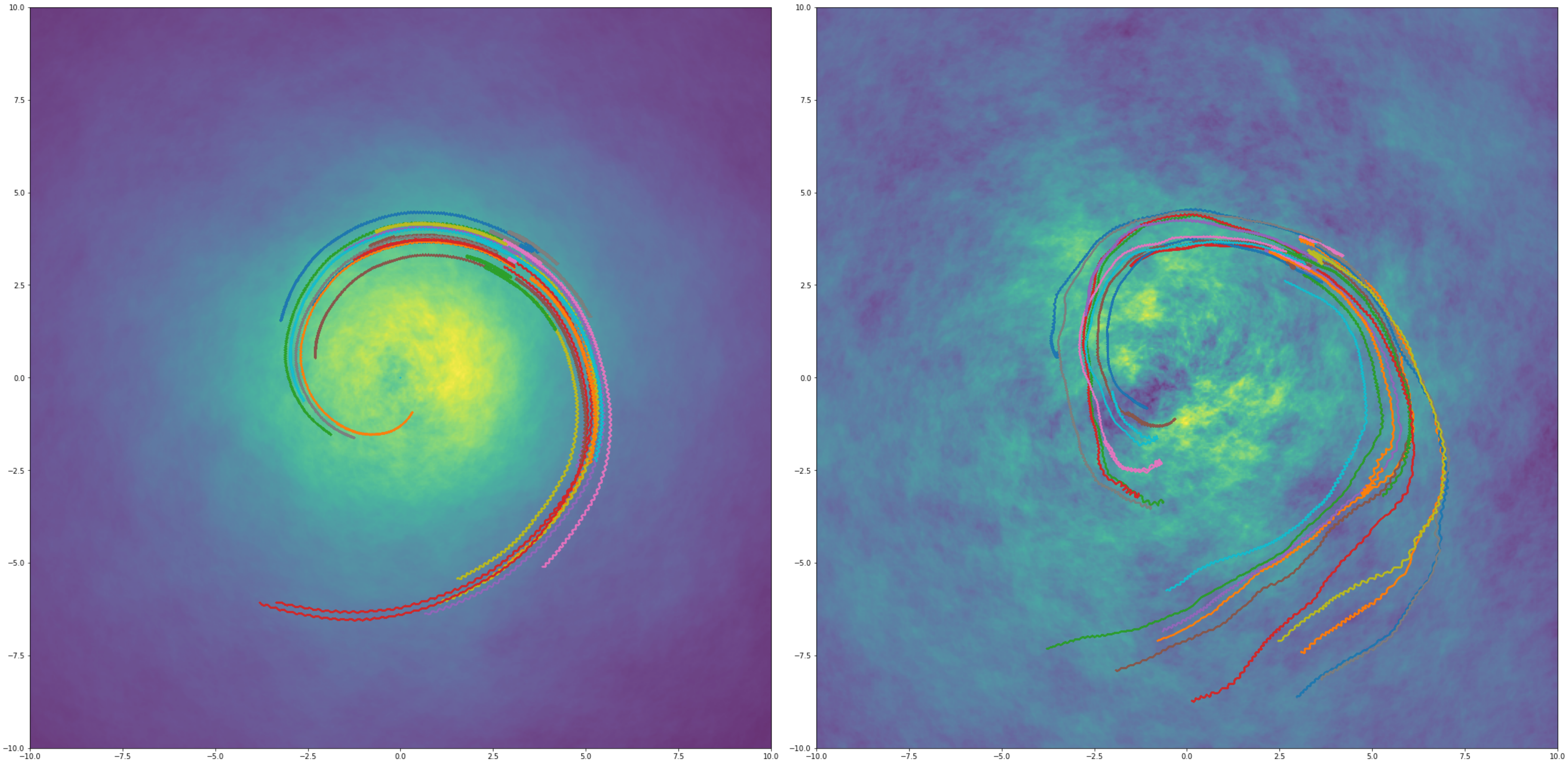

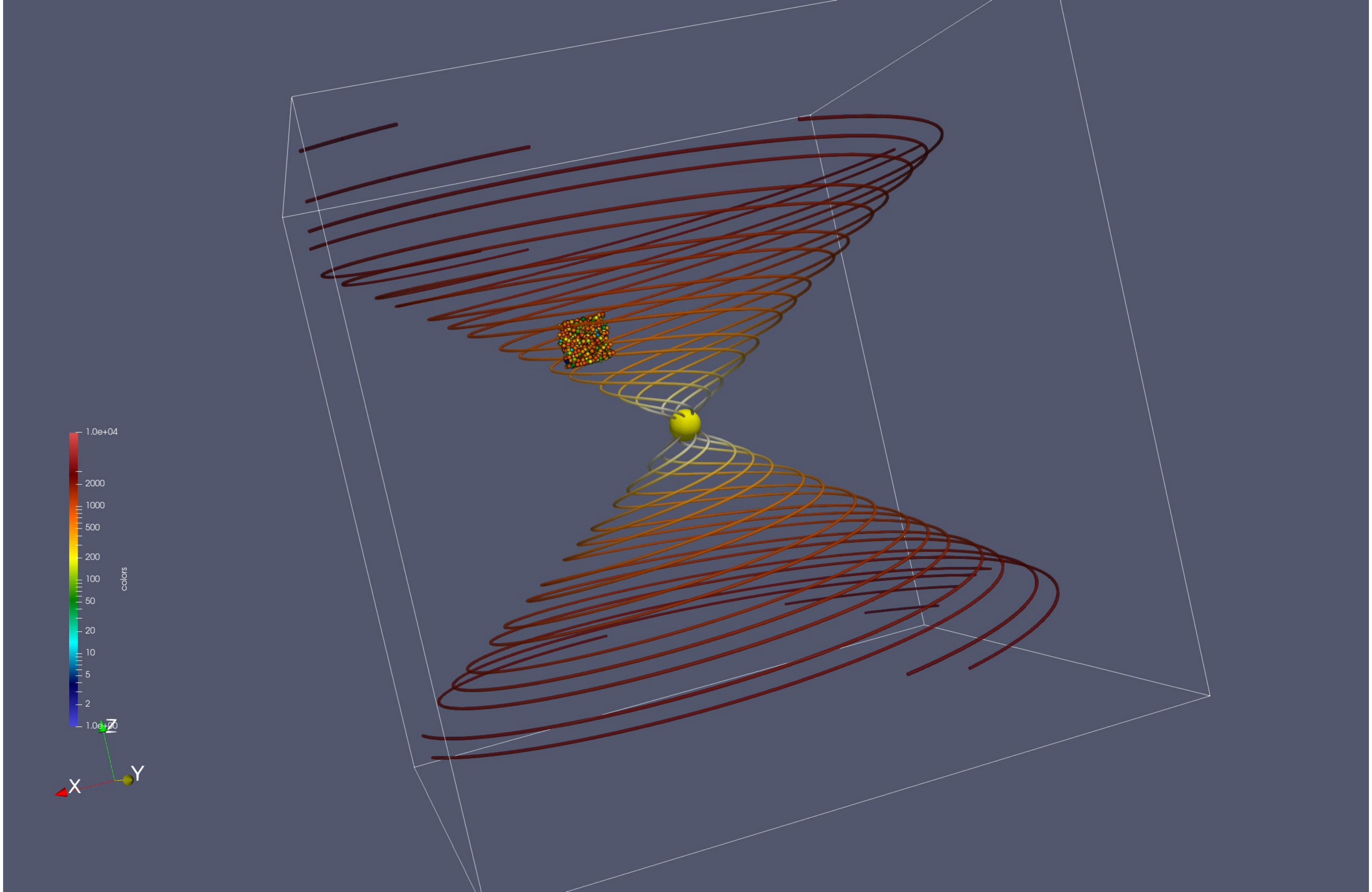
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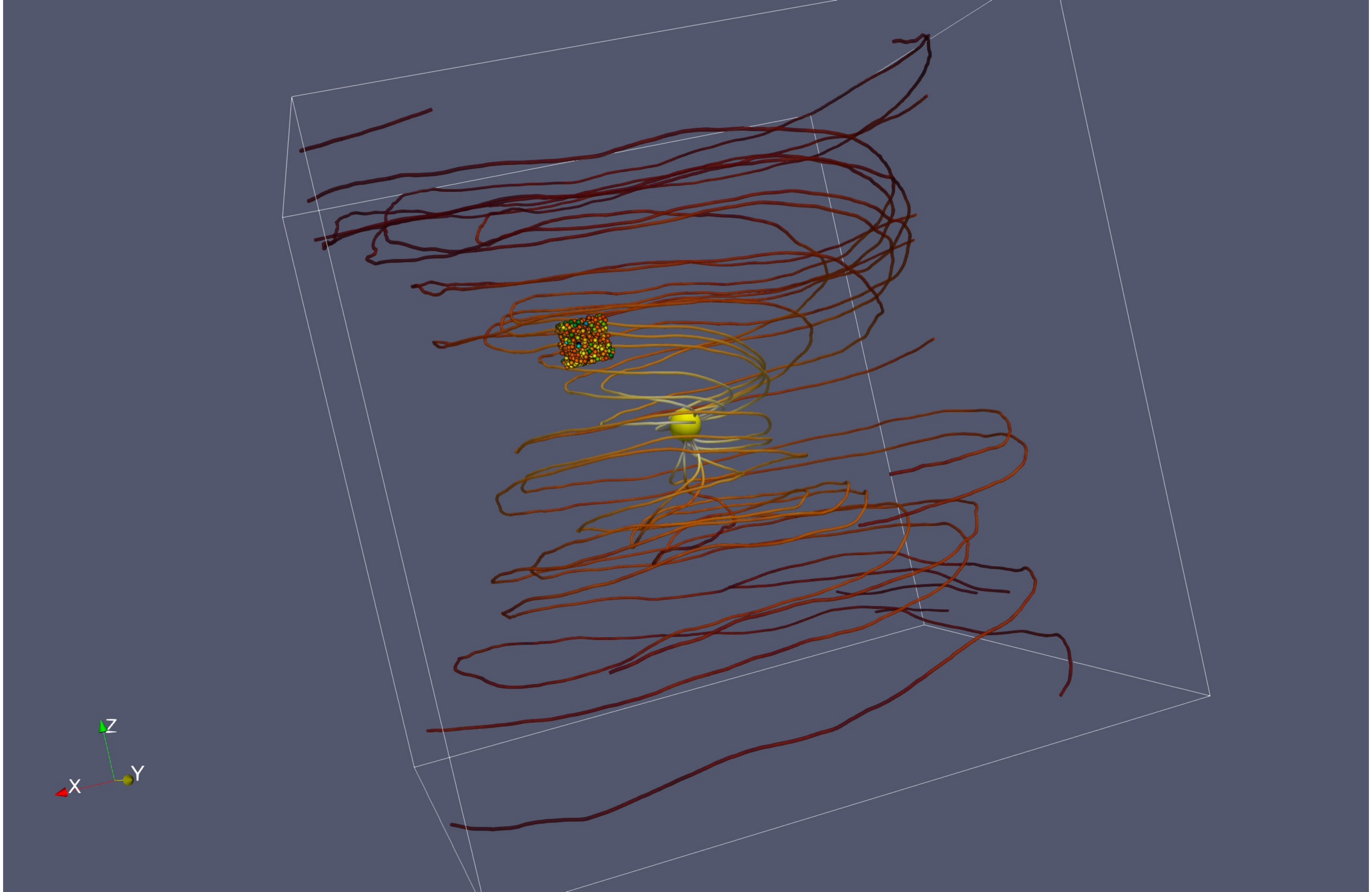
Z
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X

Summary, next Steps

- **Particle transport in synthetic turbulent fields has been studied numerically in the past**
- **Often issues with limited resolution and energy conservation, only periodic boxes**
- **Almost always relying on random phase approximations -> no correlations and intermittency in synthetic turbulence**

Next (first) Steps

- **Study literature cases for isotropic and anisotropic synthetic turbulence as benchmark**
- **Extend to intermittent fields and study energy dependence of diffusion. Non-diffusive regimes?**
- **Work on embedding in large scale background field, e.g. Heliospheric Parker spiral field, Galactic field**
- **Consider embedding in large scale MHD turbulence, connect to CRPropa**