

Fast charged particle transport in artificially generated magnetic turbulence (Project F1)

Frederic Effenberger, Ruhr-University Bochum, Germany CRC Kick-Off, June 2, 2022

Overview

- **Introduction: Why consider energetic particle transport?**
- **How to study particle transport?**
	- Full-Orbit calculations
	- Synthetic turbulence
- **State of the Art**
	- Some previous works
	- Our first steps
- **Summary**

The Heliosphere & ISM

Particle Transport in the Heliosphere

How do particles travel from Sun to Earth?

ŕ,

A Primer on Focused Solar Energetic Particle Transport Basic physics and recent modelling results

Jabus van den Berg *·* Du Toit Strauss *·* Frederic Effenberger

1D Solar Energetic Particle Modelling of SEPs since the anisotropy is caused primarily by focusing. The simplest form **Solar Energetic Particle Modelli** Stochastic calculus is a study area with several works dealing with its mathematical formal-

RUB

Basic 1D focused transport equation or energy losses in the presence of very strong losses ism and application to a variety of problems, including Gardiner (1985), van Kampen (1992), Basic 1D focused transport equation and $\overline{2}$

$$
\frac{\partial f}{\partial t} + \frac{\partial}{\partial s} \left[\mu v f \right] + \frac{\partial}{\partial \mu} \left[\frac{(1 - \mu^2) v}{2L(s)} f \right] = \frac{\partial}{\partial \mu} \left[D_{\mu \mu} \frac{\partial f}{\partial \mu} \right]
$$

$$
D_{\mu\mu}^{\rm QLT}(\mu) = D_0 (1 - \mu^2) |\mu|^{q-1}
$$

 ϵ be found in THE ϵ

If S and M represents the stochastic variables corresponding to s and μ , respectively, then the two first order SDEs equivalent to the Roelof equation (Eq. 9) are If S and M represents the stochastic variables corresponding to s and μ , respectively, then

$$
dS = \mu v dt
$$

\n
$$
dM = \left[\frac{(1 - \mu^2)v}{2L(s)} + \frac{\partial D_{\mu\mu}}{\partial \mu} \right] dt + \sqrt{2D_{\mu\mu}} dW_{\mu}(t),
$$
\n
$$
\hat{S}_{\perp 1}
$$

where $dW_{\mu}(t)$ is a Wiener process. These SDEs are solved using the Euler-Maruyama scheme, $\begin{bmatrix} 1 & M & \end{bmatrix}$ μ mentum see See Section 2.4 for details. See Section 2.4 μ

$$
S(t + \Delta t) = S(t) + M(t)v\Delta t
$$

$$
M(t + \Delta t) = M(t) + \left[\frac{(1 - M^2(t))v}{2L(S(t))} + \frac{\partial D_{\mu\mu}}{\partial \mu} \Big|_{\mu = M(t)} \right] \Delta t + \sqrt{2D_{\mu\mu}(M(t))\Delta t} \Lambda,
$$

Galactic Cosmic Ray Transport IC RAV I ranshort range up to 1 Tev considered in the anisotropy can Γ 547, Γ **Galactic Cosmic Ray Transport** & \blacksquare and orbital flux variations are discussed and conclusion are discusse Galactic Cosmic Ray Trans ϵ and ϵ are ϵ

Importance of Anisotropic Transport, Diffusion Tensor be regarded to first order as independent of energy, thus followi ransport, Diffusion Tensor

bulgalence, and given the Galactic cosmic ray incorporation that α Anisotropic diffusion of Galactic cosmic ray protons tate azimuthal distribution **and their steady-state azimuthal distribution** and their steady-state azimuthal distributiv

F. Effenberger, H. Fichtner, K. Scherer, and I. Büsching

Institut für Theoretische Physik IV, Ruhr Universität Bochum, 44780 Bochum, Germany e-mail: fe@tp4.rub.de theory and the properties of the propagation of conversion of the propagation of cosmany e-mail: fe@tp4.rub.de

> Received 10 August 2012 / Accepted 3 October 2012 $\frac{1}{2012}$

$$
\frac{\partial N}{\partial t} = \nabla \cdot (\hat{\kappa} \cdot \nabla N - uN) - \frac{\partial}{\partial p} \left[\dot{p}N - \frac{p}{3} (\nabla \cdot u)N \right] + Q
$$
\n
$$
\hat{\kappa}_{\mathcal{L}} = \begin{pmatrix} \kappa_{\mathcal{L}1} & 0 & 0 \\ 0 & \kappa_{\mathcal{L}2} & 0 \\ 0 & 0 & \kappa_{\mathcal{V}} \end{pmatrix}
$$
\n
$$
\hat{\kappa}_{\mathcal{L}} = \begin{pmatrix} 0 & 0 & 0 \\ 0 & \kappa_{\mathcal{L}1} & 0 \\ 0 & 0 & \kappa_{\mathcal{V}} \end{pmatrix}
$$

$$
dx_i = A_i(x_i)ds + \sum_j B_{ij}(x_i)dW_j
$$

Full-Orbit Simulation standard method in this field, more recently a modified third-order symplectic integration \mathbf{r} standard method in this field, more recently a modified third-order symplectic integration. method was used as an alternative (see Arendt and Shallon 2019). This energy is a set \mathbb{R}^n conservation and showled provide an important important important important important in provide an important i stochastic acceleration due to turbulent electric fields is studied. **Full-Orbit Simulation**
method was used as an alternative (SNDS). This energy is an alternative (SNDS). This energy is an alternative (SNDS).

Runge-Kutta solver with an adaptive time step option. Although this can be seen as a

Solving the Newton-Lorentz Equations for many charged (Test) Particles $\frac{1}{2}$ order to solve the second-order Newton-Lorentz equation one can use a fourth-order $\frac{1}{2}$ *Runger Cuttaa solver with an additional seeding solving the Nutter step of the step of the seeding this can be seen as a seen a seeding the seeding t* **Solving the Newton-Lorentz Equations for many charged (Test) Particles**
 Solving the Newton-Lorentz Equations for many charged (Test) Particles

Dimensionless Time: $T = \Omega t$ $r = T - \Omega t$ and the dimensionless Time: and $T = \Omega t$ S Dimensionless Time $T = \Omega t$

R \overline{d}

 α is a important i Dimensionless Rigidity: $\mathbf{R} := \mathbf{v} / (\Omega \ell)$ Γ in simulations parameters are made to be dimensionless. For instance, all length scales Γ \mathbf{u} are divided by turbulence bendover \mathbf{u} . Furthermore, we define the dimensionless regions the dimensionless regions of \mathbf{u} . p article public pushing of the pushing \mathbf{y} , we are solving \mathbf{y}

 \mathcal{I} simulations parameters are made to be dimensionless. For instance, all length scales \mathcal{I} With gyro frequency Ω and typical length scale (bendover scale) ℓ with gyrd independent
 $\frac{1}{2}$ d₁ sc **R** *R* ϵ + **R** ϵ + **R** ϵ + **R** ϵ + **R** ϵ + $\$ With gyro frequency Ω and typical length scale (bendover scale) ℓ With gyro frequency Ω and typical length scale (bendover scale) ℓ With gyro frequency Ω and typical length scale (bendover scale) ℓ ∆*t q m*

running time via *T* = "*t* and the dimensionless rigidity vector via **R** := **v***/(*"!*)*. With these ne literatu where the turbulent field α is given by Eq. (10.1). The relation between position between position between position and the relation of the **In the literature, often RK methods are used.** We use the Boris Push method for energy conservation: \mathbf{r} notice due can eliminate that we can eliminate the electric field by defining by defining by defining \mathbf{r}

$$
\frac{d}{dT} \mathbf{R} = \mathbf{R} \times \left(\mathbf{e}_z + \frac{\delta \mathbf{B}(\mathbf{x})}{B_0} \right)
$$
\n
$$
\frac{\mathbf{v}^{n+1/2} - \mathbf{v}^{n-1/2}}{\Delta t} = \frac{q}{m} \left[\mathbf{E} + \frac{\mathbf{v}^{n+1/2} + \mathbf{v}^{n-1/2}}{2} \times \mathbf{B} \right]
$$
\n
$$
\frac{d}{dT} \frac{\mathbf{x}}{\ell} = \mathbf{R}.
$$
\n
$$
\frac{\mathbf{v}^+ - \mathbf{v}^-}{\Delta t} = \frac{q}{m} (\mathbf{v}^+ + \mathbf{v}^-) \times \mathbf{B}
$$

Synthetic Turbulence

Requirements for an Advanced Synthetic Turbulence Model

- **i) The synthetic magnetic fields have to be divergence free:** ∇ **· B**⃗ **= 0.**
- **ii) The synthetic fields need to be homogeneous.**

iii) The synthetic fields should reproduce a predefined energy spectrum (Kolmogorov, 1941; Iroshnikov, 1963; Kraichnan, 1965; Boldyrev, 2005).

iv) There should be no restriction other than computational ones for a maximum Reynolds number.

v) The spectrum should be anisotropic, which means that it should have different exponents perpendicular and parallel to a local guide field (Goldreich & Sridhar, 1995; Boldyrev, 2005).

vi) The generation of the synthetic fields must be local and adaptive in space.

vii) The synthetic turbulence should exhibit intermittency, as prescribed by a given intermittency model.

viii) The synthetic turbulent fields should exhibit an increment PDF that is negatively skewed.

ix) Synthetic turbulence should be constructed as a multifractal Brownian bridge.

Synthetic Turbulence

phases, and orientations of the turbulent simulations of the turbulent simulations. In the turbulent simulations of the turbulent simulations of the turbulent simulations. In the turbulent simulatio magnetice field values edge of the properties of these stochastic magnetic fields is important in several applications s of the theory of the theory of the such and consider \sim Isotropic Random Random Random *kn* = \$0*k* 3 \blacktriangleright synthetic Turbulence staller. This means, for the parameter of the parameter $\boldsymbol{\mathsf{F}}$ in Eq. (10.4) is not the parameter $\boldsymbol{\mathsf{F}}$ In a very similar manner we can generate two-dimensional turbulence. In this case WYHUIGHU TUI DUIGHUG
Mennetostatis Turkulanes (fellowing Gladelsi 2020 noview)

Magnetostatic Turbulence (following Shalchi 2020 review) really *knowledge* in the amplitude function \overline{a} of \overline{b} and \overline{b} **e** *Magnetostatic Turbulence (following Shalchi 2020 review)* and N

$$
\mathbf{B}(\mathbf{x},t) = B_0 \mathbf{e}_z + \delta \mathbf{B}(\mathbf{x},t). \qquad \delta \mathbf{B}(\mathbf{x}) = \sqrt{2} \delta B \sum_{n=1}^N A(k_n) \xi_n \cos[\mathbf{k}_n \cdot \mathbf{x} + \beta_n]
$$

$$
\mathbf{k}_{n} = k_{n} \mathbf{e}_{k,n}
$$
\n
$$
\left(\sqrt{1 - \eta_{n}^{2}} \cos \phi_{n}\right)
$$
\n
$$
- \sin \phi_{n} \cos \phi_{n}
$$

$$
\mathbf{e}_{k,n} = \begin{pmatrix} \sqrt{1 - \eta_n^2} \cos \phi_n \\ \sqrt{1 - \eta_n^2} \sin \phi_n \\ \eta_n \end{pmatrix} \qquad \qquad \xi_n = \begin{pmatrix} -\sin \phi_n \cos \alpha_n + \eta_n \cos \phi_n \sin \alpha_n \\ \cos \phi_n \cos \alpha_n + \eta_n \sin \phi_n \sin \alpha_n \\ -\sqrt{1 - \eta_n^2} \sin \alpha_n \end{pmatrix}
$$

$$
A^{2}(k_{n}) = G(k_{n}) \Delta k_{n} \left(\sum_{m=1}^{N} G(k_{m}) \Delta k_{m} \right)^{-1}
$$

$$
G(k_n) = \frac{k_n^q}{(1 + k_n^2)^{(s+q)/2}}.
$$

 ${\bf e}_{k,n} =$

 \mathbf{I} \mathbf{I}

"*^N*

N

 \mathbf{p}_{0}

where *N* is the number of wave modes. The quantity *A(kn)* denotes the amplitude function

RUB

Synthetic Turbulence

Isotropic, Gaussian Turbulence

RUB Example Results for the Diffusion Coefficient

Fig. 16 Diffusion coefficients and distribution functions for pure slab turbulence, a magnetic rigidity of $R = 0.1$, and a magnetic field ratio of $\delta B_{slab}^2/B_0^2 = 1$. The used parameters *T*, *R*, *K*_{||}, and *D*_{||} are defined in

Shalchi 2020

Anomalous Diffusion

Super or Subdiffusion regimes? ! 2006. The American Astronomical Society. All rights reserved. Printed in U.S.A.

RUB are integrated with a high-precision fifth-order Runge-Kutta

SUPERDIFFUSIVE AND SUBDIFFUSIVE TRANSPORT OF ENERGETIC PARTICLES IN SOLAR WIND ANISOTROPIC MAGNETIC TURBULENCE

G. Zimbardo, P. Pommois, and P. Veltri

Physics Department, University of Calabria, Arcavacata di Rende, 87036 Rende, Italy; zimbardo@fis.unical.it, pommois@fis.unical.it, veltri@fis.unical.it *Received 2005 June 6; accepted 2006 January 26; published 2006 February 13*

FIG. 1. – Anomalous transport exponents γ_i (top pane 10^{-3} , and $\rho/\lambda_{\text{min}} = 1.28 \times 10^{-2}$. Magnetic turbulence is axisymmetric, and the ratio of correlation lengths varies from $l_x/l_z = 10$ (far left) to 3, 1, 0.33, and 0.1 (far right). Results for the x-, y-, and z-directions are indicated by solid, dashed, and dotted lines, respectively. FIG. 1. Anomalous transport exponents γ_i (top panels) and flatnesses F_i (bottom panels) are plotted as a function of time for $\delta B/B_0 = 0.5$, $\rho/\lambda = 3.2 \times 10^{-3}$ me for $\delta B/B_0 = 0.5$, $\rho/\lambda = 3.2 \times 10$ (*far left*) to 3, 1, 0.33, and 0.1 g ! 2 in the case of superdiffusive regime (Le´vy random walk) *ⁱ*

Influence of Intermittency? Shukurov 2017 The Astrophysical Journal Letters, 834:L16 (5pp), 2017 April 10 https://doi.org/ © 2017. The American Astronomical Society. All rights reserved.

RUB

 C ss k

Cosmic Rays in Intermittent Magnetic Fields

Anvar Shukurov¹, Andrew P. Snodin², Amit Seta¹, Paul J. Bushby¹, and Toby S. Wood¹ ¹ School of Mathematics and Statistics, Newcastle University, Newcastle Upon Tyne NE1 7RU, UK; a.seta1@ncl.ac.uk, amitseta90@gmail.com ² Department of Mathematics, Faculty of Applied Science, King Mongkut's University Received 2017 February 8; revised 2017 March 12; accepted 2017 March 31; published 2017 April 12

2. Magnetic Field Produced by Dynamo Action

 $\frac{1}{\sqrt{2}}$ rays in turbulent magnetic fields in the scattering process driven by the scattering of the scatteri μ random magnetic fluctuations. Such fields are usually highly intermittent, consisting of intense $induction$ We generate intermittent, statistically isotropic, fully threedimensional random magnetic fields \boldsymbol{b} by solving the induction andomized a prescribed velocity field *u*:
equation with a prescribed velocity field *u*: $T_{\text{induction}}$ $\mathcal{L} = \mathcal{L} \mathcal{L} = \mathcal{L} \mathcal{L} \mathcal{L} = \mathcal{L} \mathcal{L} \mathcal{L} \mathcal{L} = \mathcal{L} \mathcal{L} \mathcal{L} \mathcal{L} \mathcal{L} = \mathcal{L} \mathcal{L} \mathcal{L} \mathcal{L} \mathcal{L} \mathcal{L} \mathcal{L} = \mathcal{L} \mathcal{$ We generate intermittent, statistically isotropic, for the same magnetic \hat{R} equation with a prescribed velocity field \boldsymbol{u} :

$$
\frac{\partial b}{\partial t} = \nabla \times (\boldsymbol{u} \times \boldsymbol{b}) + R_{\rm m}^{-1} \nabla^2 \boldsymbol{b}, \qquad \nabla \cdot \boldsymbol{b} = 0,
$$

Example 1. Isosurfaces of magnetic field strength $b^2/b_0^2 = 2.5$ (blue) and $b^2/b_0^2 = 5$ (yellow) with b_0 the rms magnetic $=$ ⁴ flow (3) at R_m = 1082 (left) and for the same magnetic field after Fourier phase randomization as described in the text (second from left). Magnetic field generated by

the W flow (2) is similarly effected (not a $t = 2$ the W flow (2) is similarly affected (not shown). The second from right panel shows the PDFs of a magnetic field contrarandomized (KS (R), W (R): dashed) magnetic fields obtained with both velocity fields (only $b_x > 0$ is shown as the PDFs are essentially symmetric about $b_x = 0$). randomized (KS (K), W (K): dashed) magnetic neids obtained with both velocity neids (only $b_x > 0$ is shown as the PDFs are essentially symmetric about $b_x = 0$).
The randomized fields have almost perfectly Gaussian statist volume within magnetic structures where $b \ge v b_0$, with b_0 the rms field strength, as a function of ν for the intermittent magnetic field produced by the flow (3) (solid For $R_m = 3182$ and dashed for $R_m = 1082$ and its Gaussian counterpart (dashed-dotted for $R_m = 3142$ and 1082) $\frac{C}{D}$ filling factor of the randomized fields is independent of R_m . $\frac{1}{1}$ correlation or the point $\frac{1}{1}$ therefore interpret (district dotted to \overline{D} $m₁$ Figure 1. Isosurfaces of magnetic field strength $b^2/b_0^2 = 2.5$ (blue) and $b^2/b_0^2 = 5$ (yellow) with b_0 the rms magnetic field, for magnetic field generated by the KS the W flow (2) is similarly affected (not shown). The second from right panel shows the PDFs of a magnetic field component b_x for the original (KS, W: solid) and reademized (KS, W: solid) and reademized (KS, W: solid) a for $R_m = 3182$ and dashed for $R_m = 1082$) and its Gaussian counterpart (dashed–dotted for $R_m = 3142$ and 1082) obtained by Fourier phase randomization; the $\frac{1}{2}$ function of $\frac{1}{2}$ interpretation $\frac{1}{2}$ $\frac{1}{2}$

Teaser (Parker background field)

RUB

Summary, next Steps

- **Particle transport in synthetic turbulent fields has been studied numerically in the past**
- **Often issues with limited resolution and energy conservation, only periodic boxes**
- **Almost always relying on random phase approximations -> no correlations and intermittency in synthetic turbulence**

Next (first) Steps

- **Study literature cases for isotropic and anisotropic synthetic turbulence as benchmark**
- **Extend to intermittent fields and study energy dependence of diffusion. Non-diffusive regimes?**
- **Work on embedding in large scale background field, e.g. Heliospheric Parker spiral field, Galactic field**
- **Consider embedding in large scale MHD turbulence, connect to CRPropa**

RUB