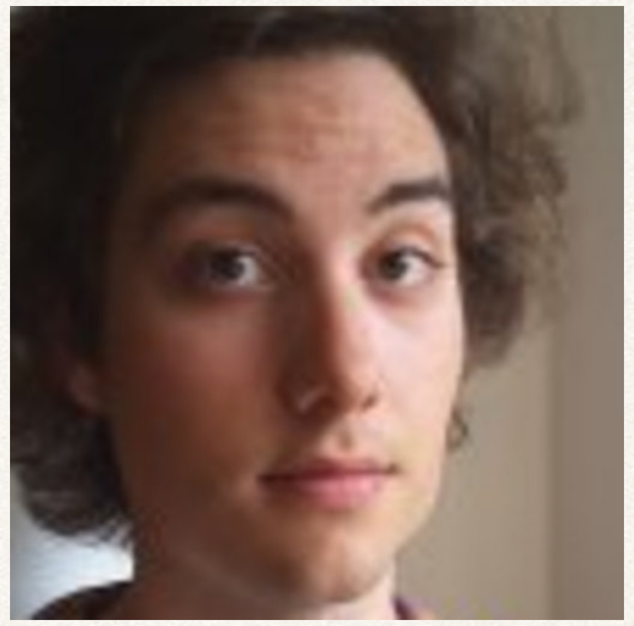
Relativistic plasma dynamics and turbulence

Who are we?

Sorry, we are not Astros !!!

Maria Elena (F2)

Jürgen (F2)



Mike (F2)

Rainer (F1/F2)

Horst (F2/A1/A4)

Frederic (F1)

Jeremiah (F1)

Jan (external, F1)

Relativistic plasma dynamics and turbulence

Relativistic plasmas (F2: Maria Elena, Rainer, Jürgen)

▸ The equations	Vlasov, 2 fluid, MHD
▸ A question of scales	example magnetotail
▸ Tools at TPI	iPIC, MuPhy, racoon
▸ What is the rel. numerical community doing?	explicit PIC, MHD
▸ What do we need?	rel. iPIC, 2 fluid, MHD
▸ Some scenarios	Alves et al. (2018), Sironi et al. (2014)
▸ Workplan years 1+2	
▸ Connection to F1, A6, A7 and questions	

PostDoc: Mike Wilbert (01.07.2022)
PhD: NN

Relativistic plasma dynamics and turbulence

Synthetic turbulence and transport	(F1: Horst, Rainer)
▸ Intro to turbulence	K41, intermittency, KI, GS, Boldyrev
▸ Why?	correlations
▸ What is the community doing?	mostly Fourier based
▸ Tools at TPI/IV?	Gaussian superposition, MF bridge
▸ What do we need?	AMR for synthetic fields —> wavelets
▸ Workplan years 1+2	
▸ Connection to F2, A1-A5 and questions	

PostDoc:	Frederic Effenberger	(01.01.2022)
PhD:	Jeremiah Lübke	(01.07.2022)

The equations

distribution function $f_s = f_s(x^\mu, p^\mu)$, $s =$ electron, protons, positrons

Liouville Theorem (no collisions): $\frac{\mathrm{d}f}{\mathrm{d}\tau} = \frac{\mathrm{d}x^\mu}{\mathrm{d}\tau}\frac{\partial f}{\partial x^\mu} + \frac{\mathrm{d}p^\mu}{\mathrm{d}\tau}\frac{\partial f}{\partial p^\mu} = 0$

using $\frac{\partial}{\partial x^\mu} = \left(\frac{\partial}{\partial ct}, \nabla_{\boldsymbol{x}}\right), \quad \frac{\partial}{\partial p^\mu} = \left(\frac{\partial}{\partial p^0}, \nabla_p\right) \quad \Longrightarrow \quad \frac{\mathrm{d}f}{\mathrm{d}t} = \frac{\partial f}{\partial t} + \frac{\mathrm{d}\boldsymbol{x}}{\mathrm{d}t}\frac{\partial f}{\partial \boldsymbol{x}} + \frac{\mathrm{d}p^\mu}{\mathrm{d}t}\frac{\partial f}{\partial p^\mu} = 0$

finally use energy-momentum shell: $p^\mu g_{\mu\nu} p^\nu = p^\mu p_\mu = -(mc)^2 \Longrightarrow p^0 = \sqrt{\boldsymbol{p}^2 + (mc)^2}$

$$\frac{\partial}{\partial t} f + \frac{\boldsymbol{p}}{m\gamma}\nabla_{\boldsymbol{x}} f + q\left(\boldsymbol{E} + \frac{\boldsymbol{p}}{m\gamma} \times \boldsymbol{B}\right) \cdot \nabla_{\boldsymbol{p}} f = 0$$

Vlasov + Maxwell (6D)

makes Vlasov numerics difficult

$\gamma(v)^2 = (1 - (v/c)^2)^{-1}$

taking moments $\Longrightarrow$ relativistic 5 moment equations (for each species s):

$$\frac{\partial}{\partial t}\gamma\rho + \nabla_{\boldsymbol{x}} \cdot \rho\boldsymbol{u} = 0$$ mass

$$\frac{\partial}{\partial t}\left((\epsilon + p)\gamma^2 - p\right) + \nabla_{\boldsymbol{x}} \cdot (\epsilon + p)\gamma\boldsymbol{u} = \frac{q}{m}\rho\boldsymbol{u} \cdot \boldsymbol{E}$$ energy

$$\frac{\partial}{\partial t}\frac{\epsilon + p}{c^2}\gamma\boldsymbol{u} + \nabla_{\boldsymbol{x}} \cdot \left(\frac{\epsilon + p}{c^2}\boldsymbol{u} \otimes \boldsymbol{u} + p\right) = \frac{q}{m}\rho(\gamma\boldsymbol{E} + \boldsymbol{u} \times \boldsymbol{B})$$ momentum

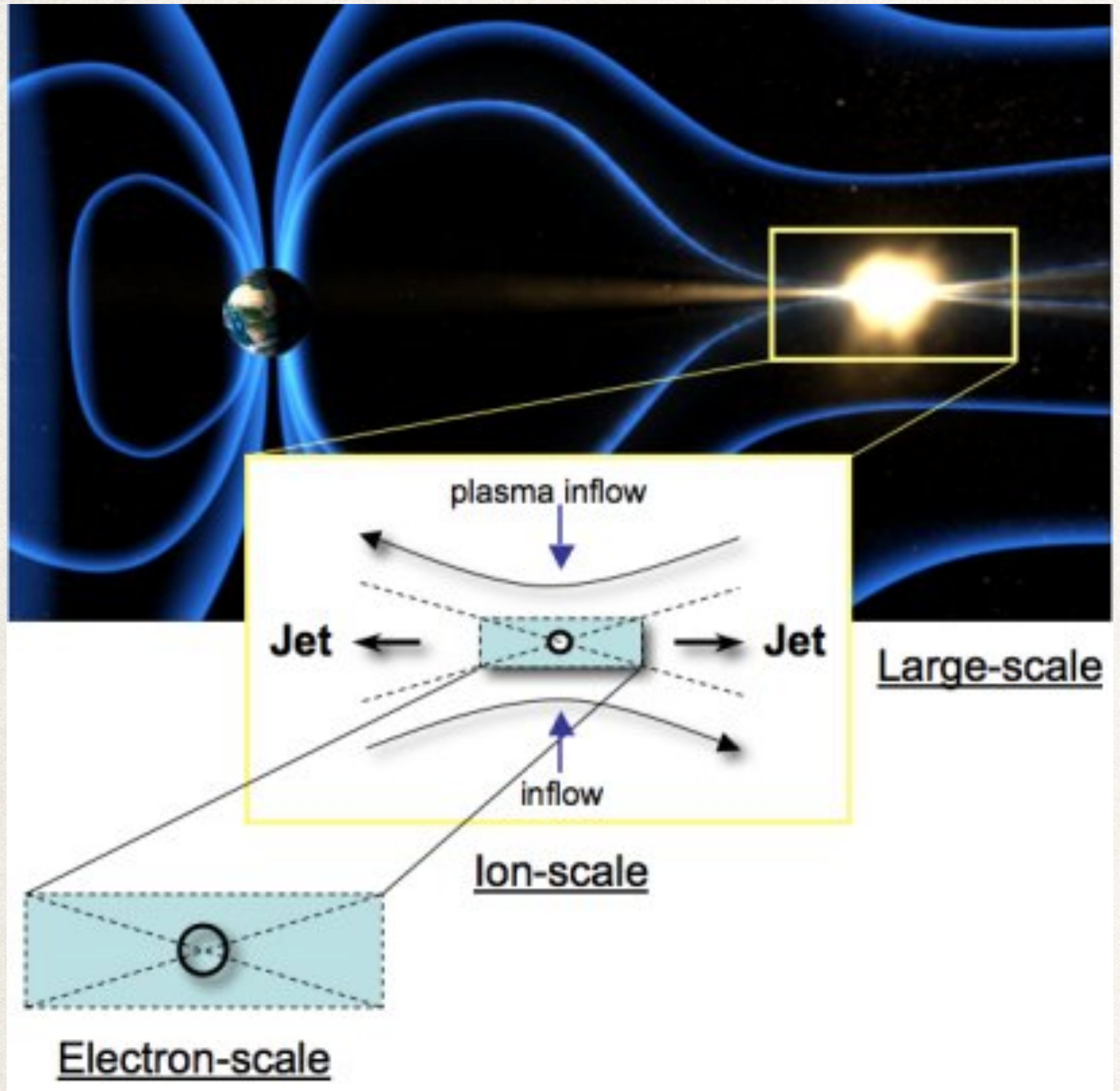
$\epsilon = \rho c^2 + \epsilon_{therm}\,,\ \epsilon_{therm} = \frac{p}{\Gamma - 1},\ \Gamma = \frac{4}{3}$, p: scalar pressure, $\epsilon_{therm} = \int d^3p' \frac{(p')^2}{p'^0} f_R$

good news: Franz Wilfahrt implemented these 2 species rel. 5 moment equations in his MA thesis based on Balsara, Amano et al. (2016)

conservation laws $\Longrightarrow$ relativistic MHD

good news: Eduard Warkentin implemented RMHD in his BA thesis based on Komissarov (2007)

A question of scales



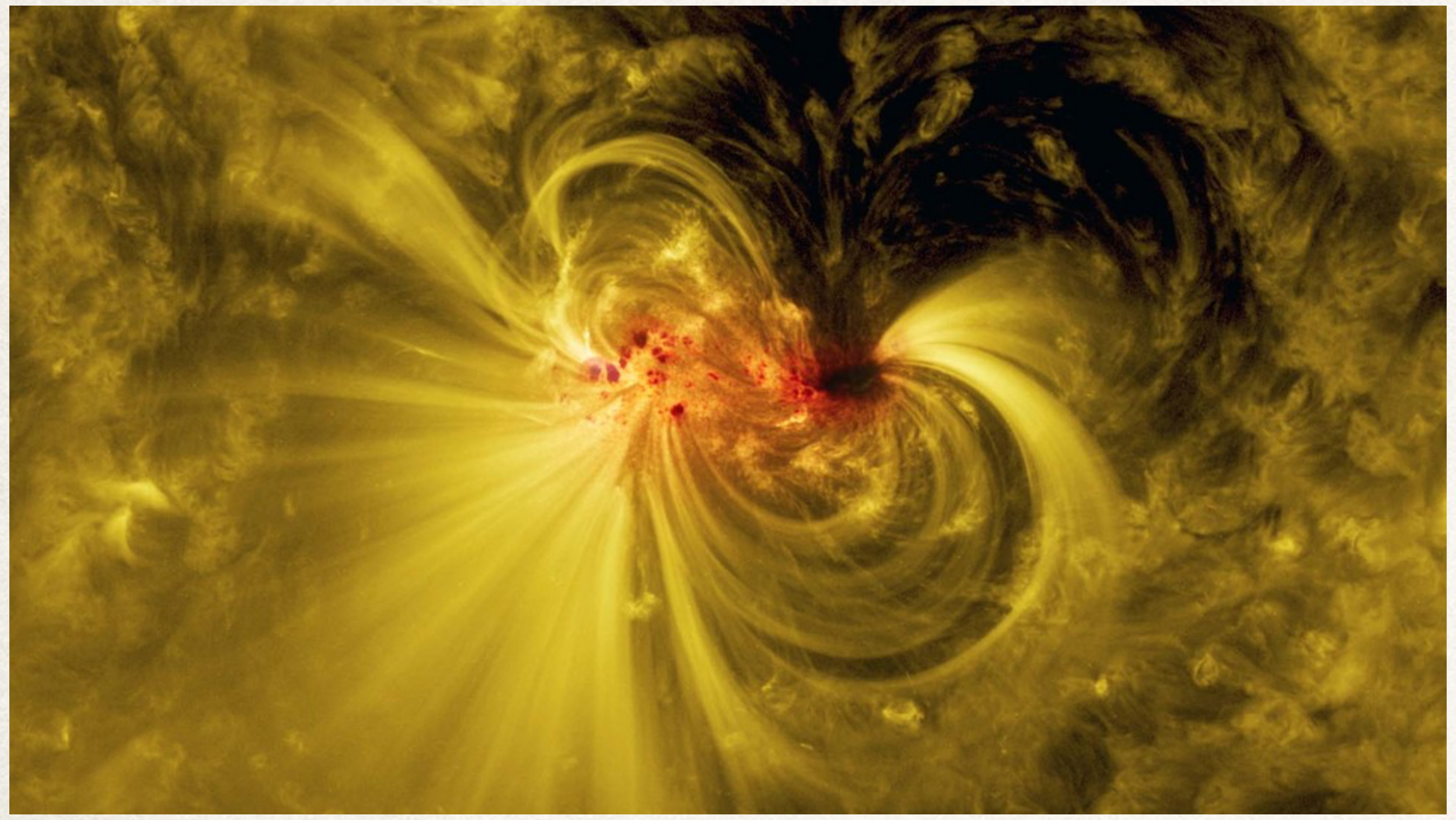
from G. Lapenta ISSS10

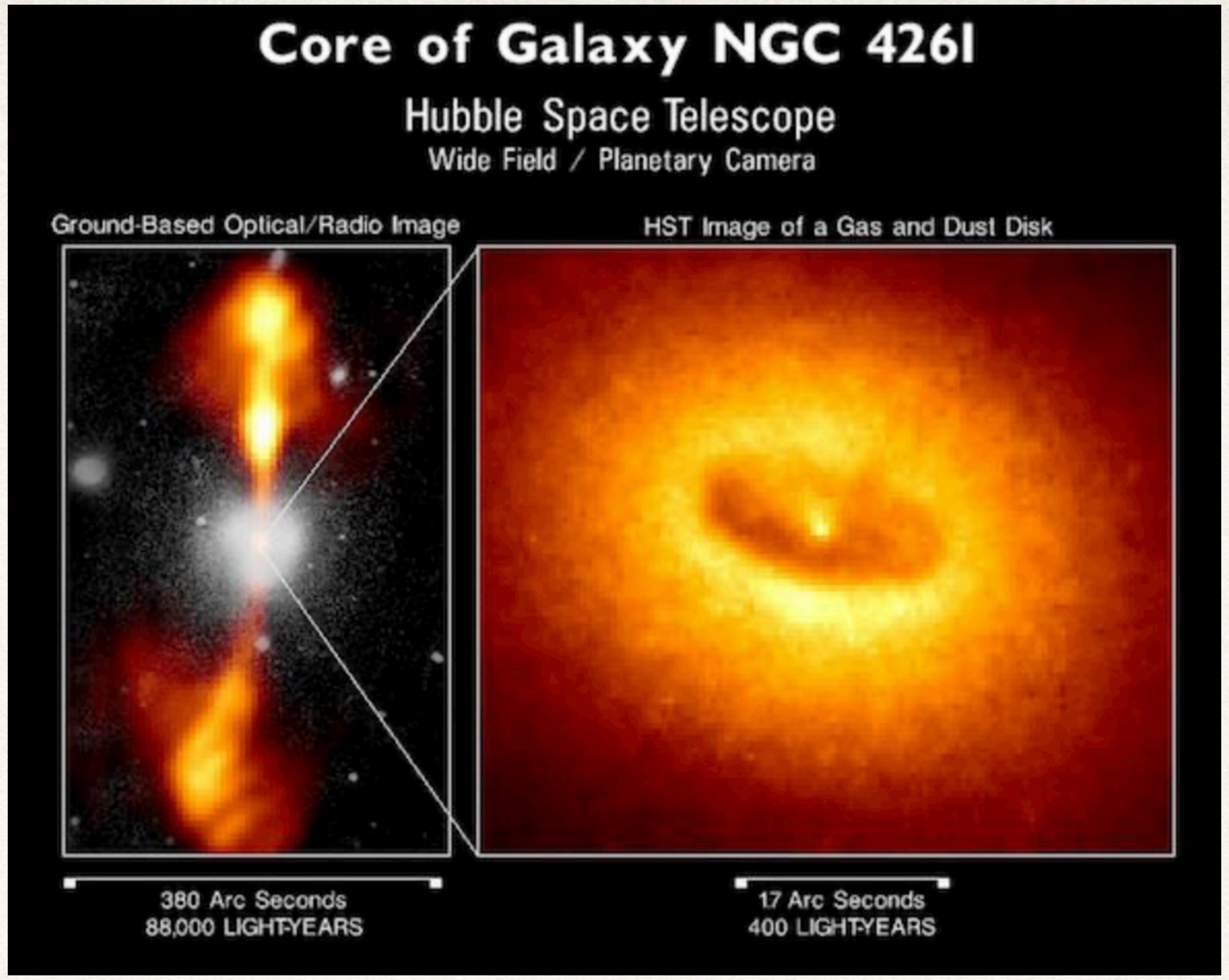
		spatial scales	time scales
global scale:		10^6 km	hours
system scale:		10^5 km	minutes
ion scales ρ_i, d_i:		10^3 km	seconds
electron scales	d_e:	10 km	10^{-3} s
	ρ_e:	1 km	10^{-4} s
	λ_e:	100 m	10^{-5} s

MHD
2 T MHD
5/10 moment
Vlasov

Ohm
Maxwell

A question of scales





Credits:
Hubble Space Telescope

Solar corona:

Ion skin depth: ~10 m

System scales: $\sim \frac{R_s}{100} = \sim 10^6$ m

Scale separation: $\sim 10^5$

Astrophysical jets:

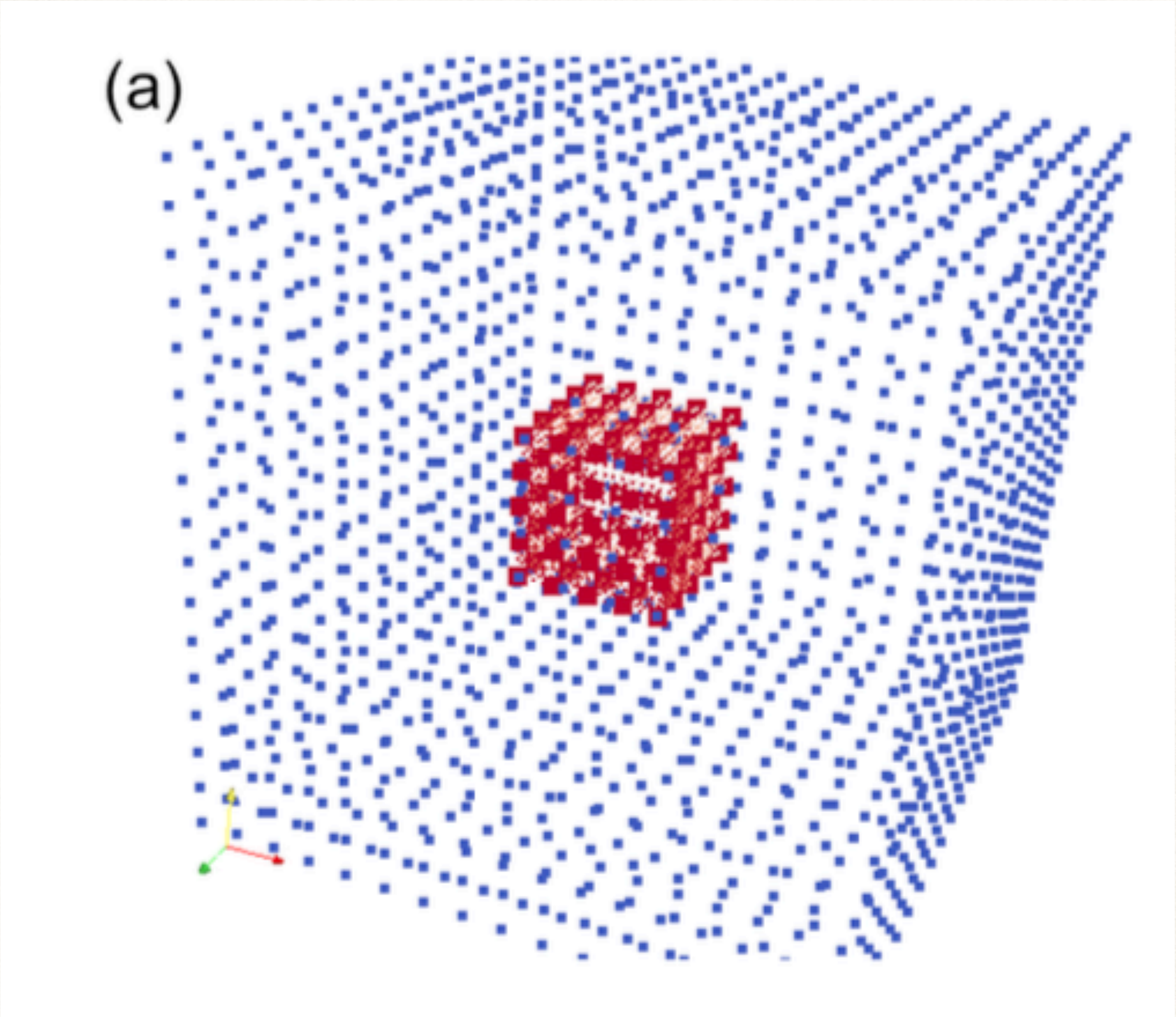
Ion skin depth: ~10^4 m

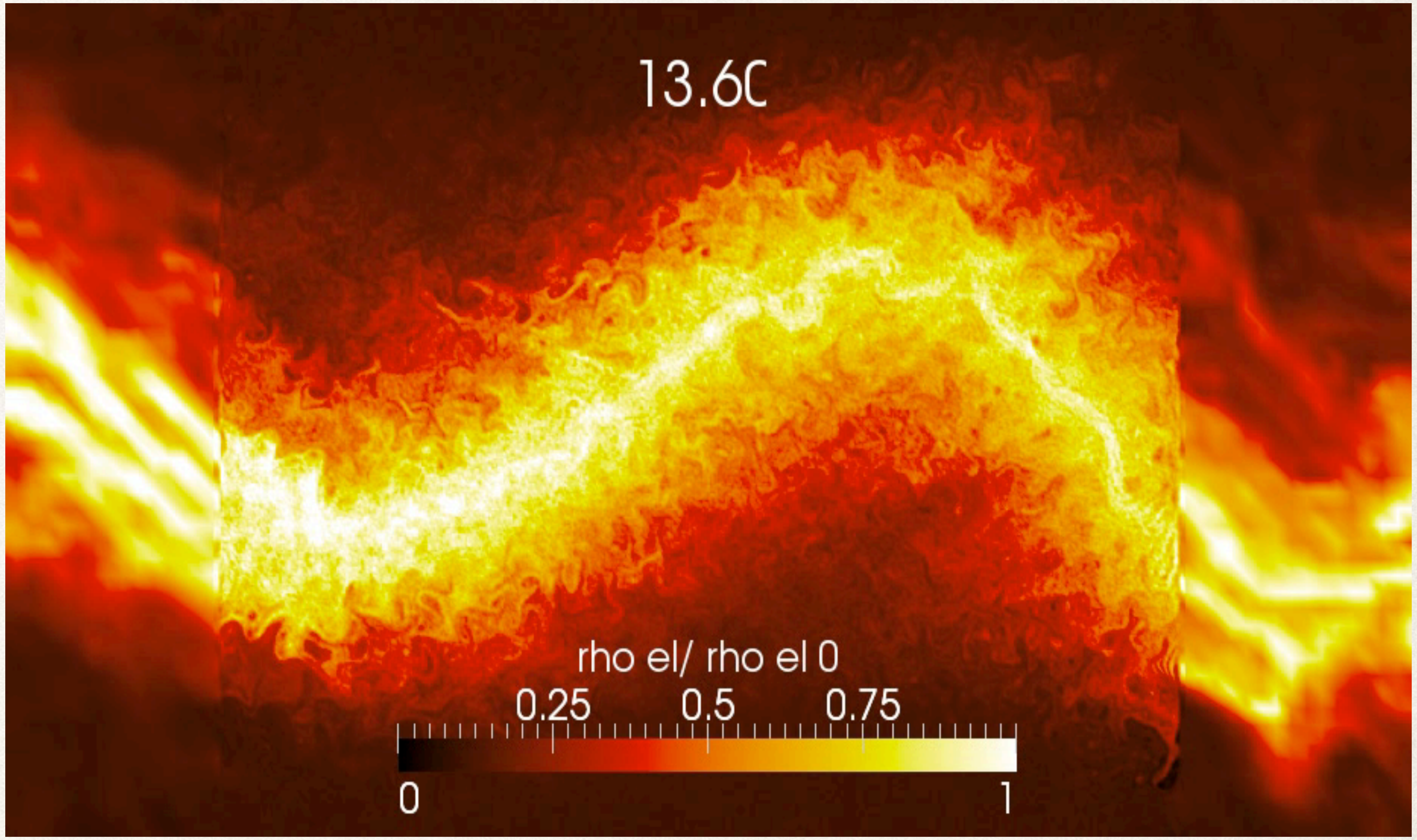
Length scale of reconnection: ~10^{14} m [Petropoulou et al, 2016]

Jet length: ~10^{19} m [Porth & Kommissarov, 2015]

Scale separation: $\sim 10^{10}, 10^{15}$

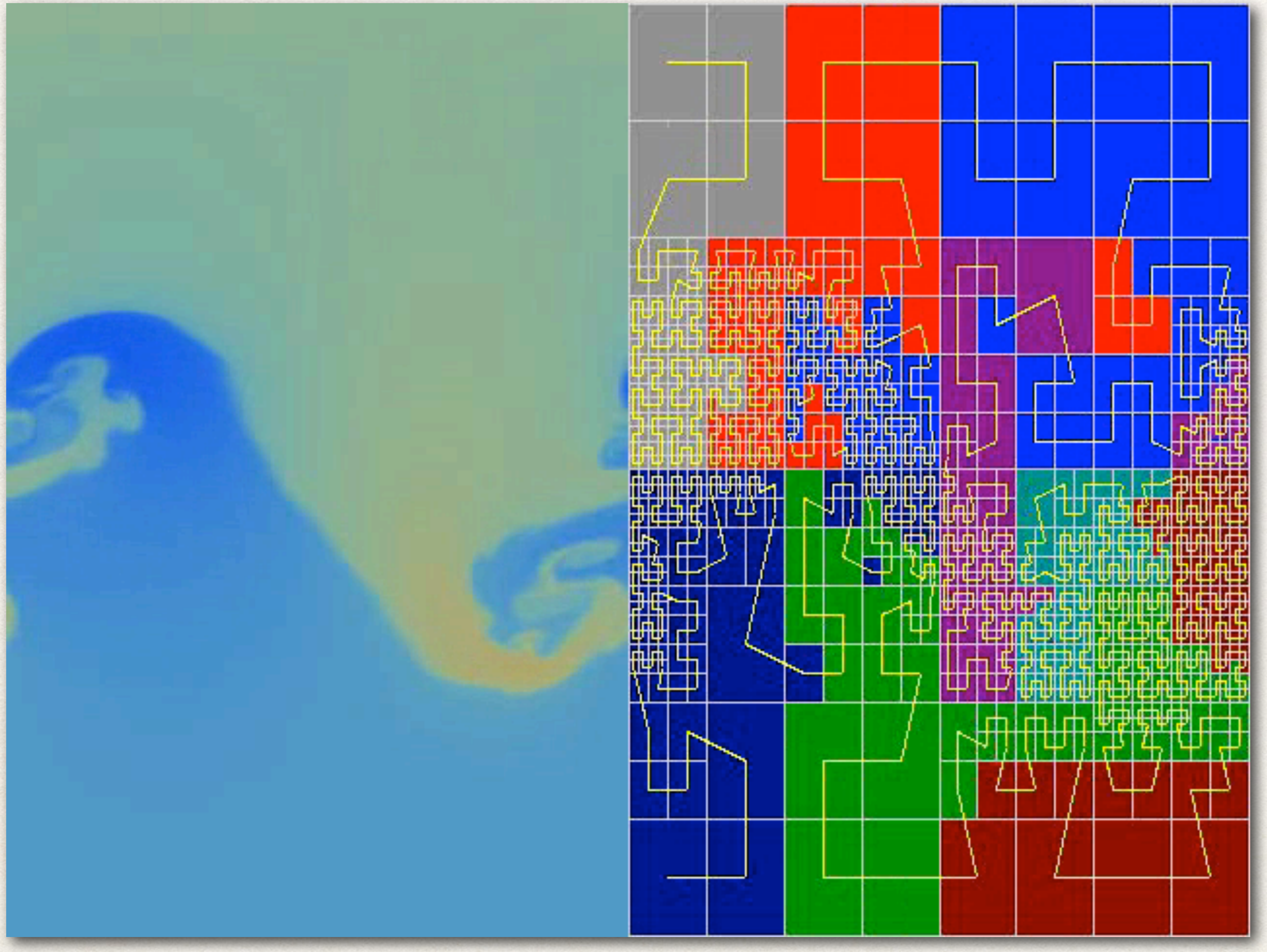
Tools at TPI? iPIC: implicit PIC, based on implicit moment method

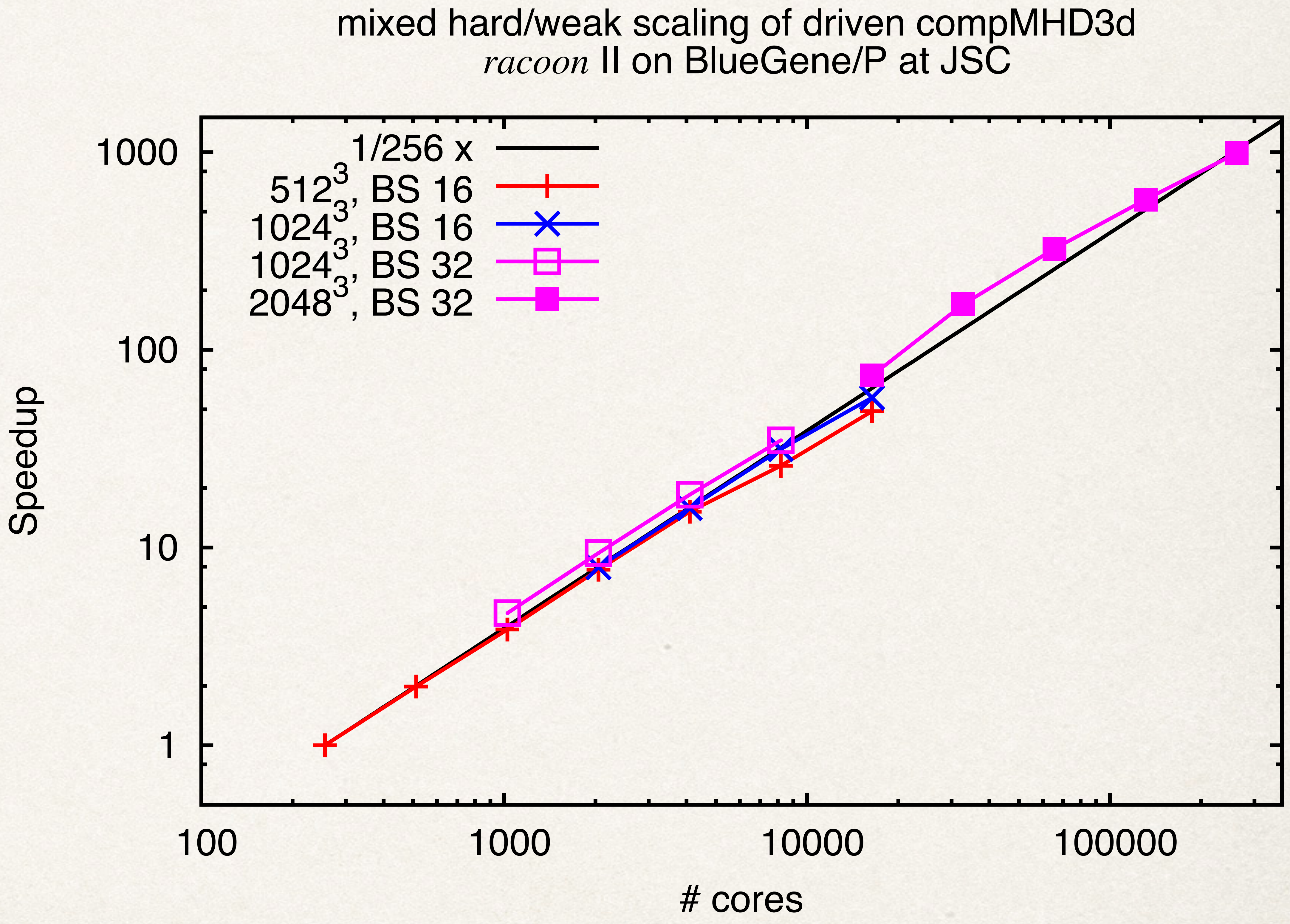




Innocenti et al, 2016: kinetic/ kinetic coupling, with semi-implicit fully kinetic codes

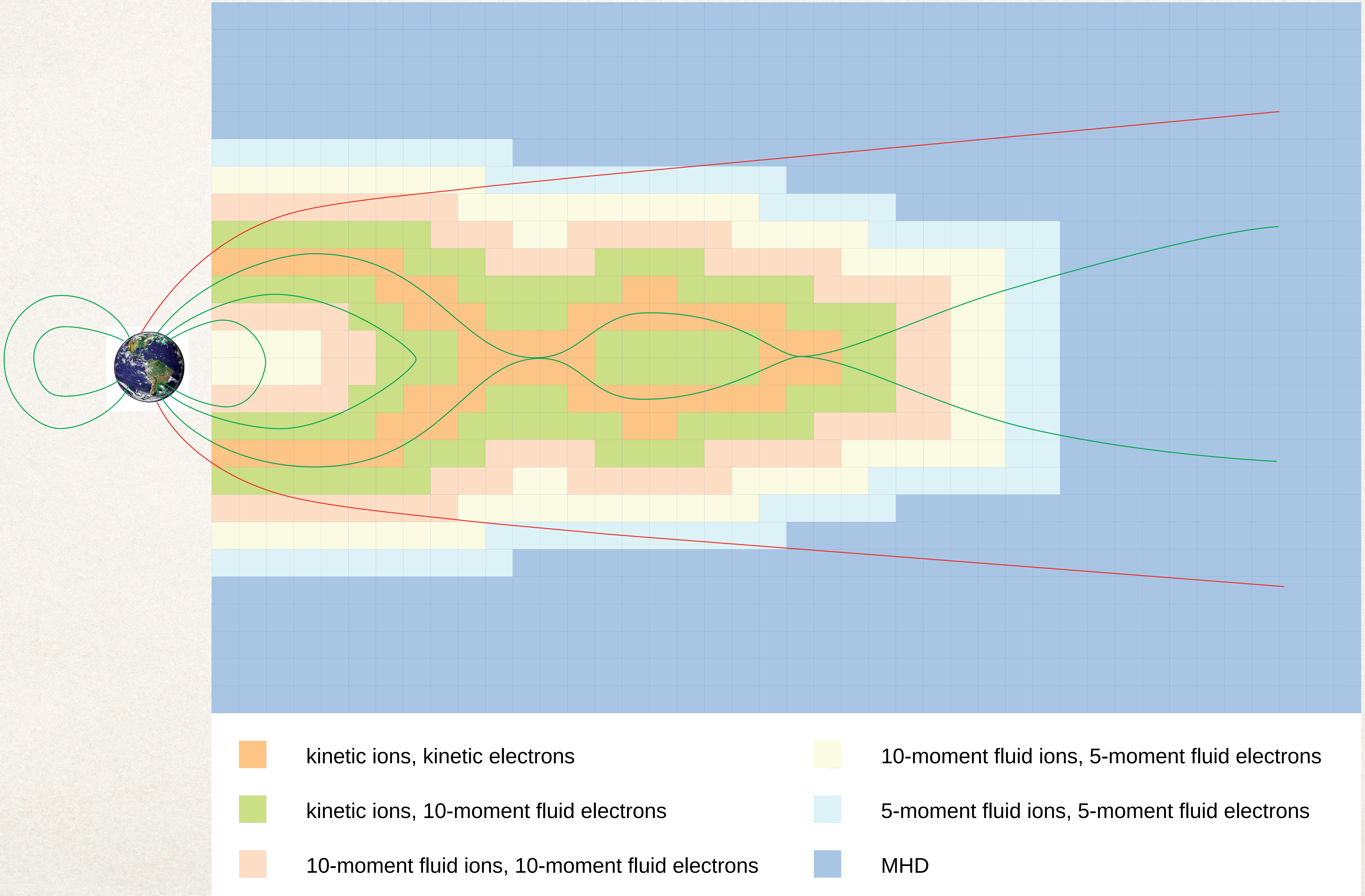
Tools at TPI? racoon: block structured adaptive mesh refinement



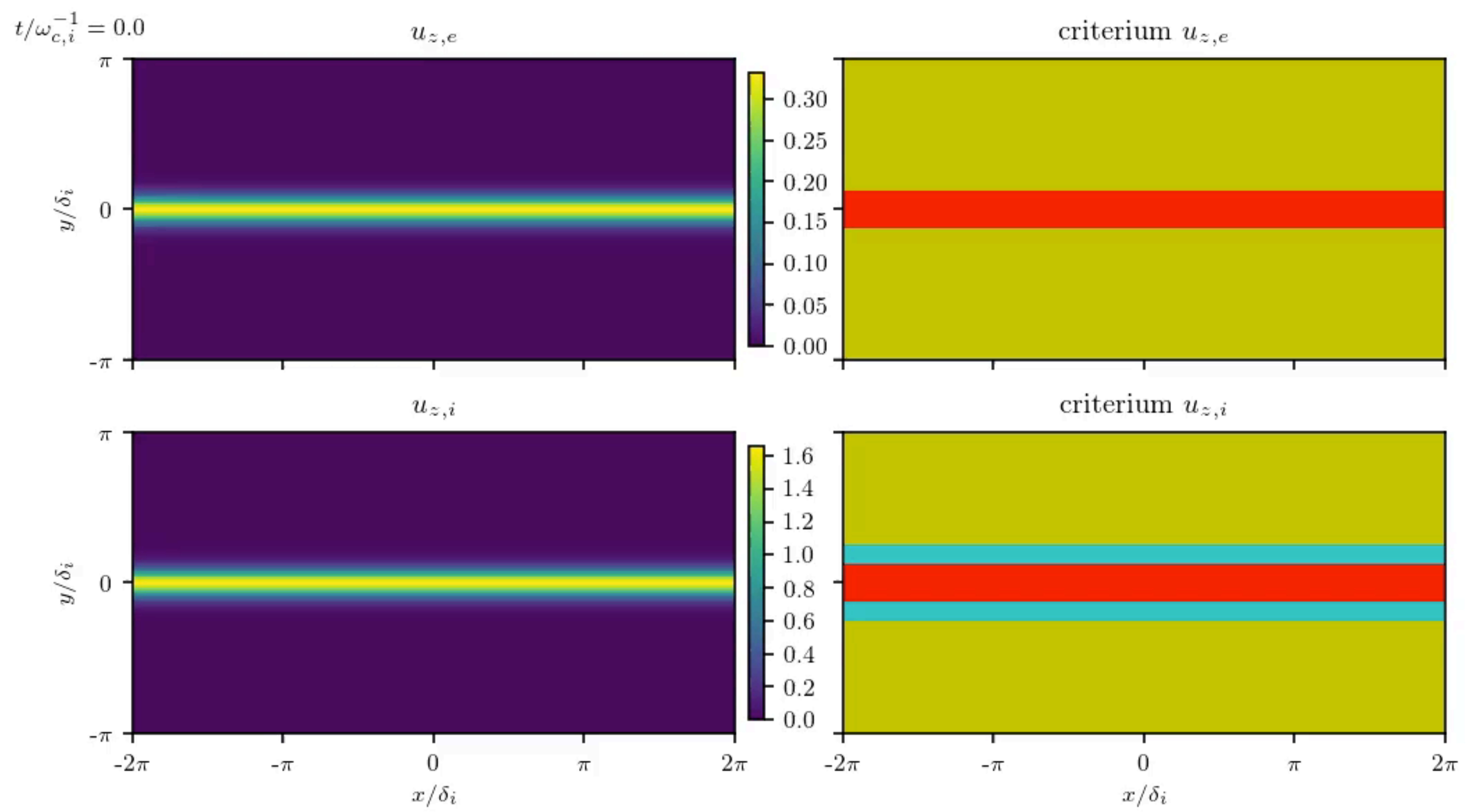


Tools at TPI?

MuPhy: multiphysics simulations of collisionless plasmas



Tools at TPI? MuPhy: multiphysics simulations of collisionless plasmas



What is the rel. numerical community doing?

Kommissarov (2001)
Zanotti et al. (2015)
Del Zanna et al. (2016)
Bromberg et al. (2019)
Athena++: Stone

rel. MHD

very large scales

very little in between:
our chance

Sironi, Spitkovsky (2014)
Sironi et al. (2015)
Werner et al. (2015)
Werner, Uzdensky (2017)
Alves et al. (2018)
Davelaar et al. (2020)
Meli et. al. (2020)

explicit PIC (Tristan), partially 2D

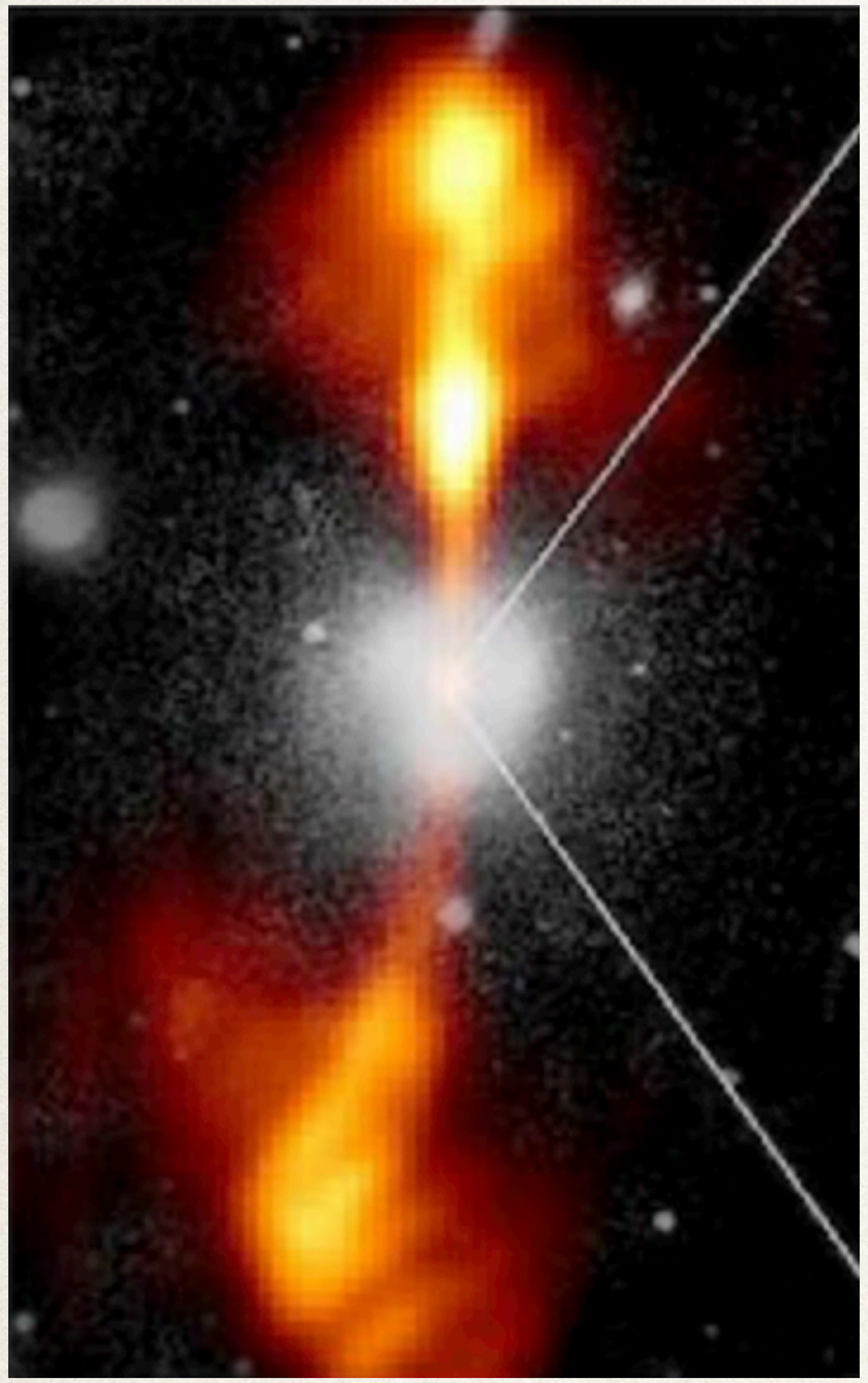
very small scales

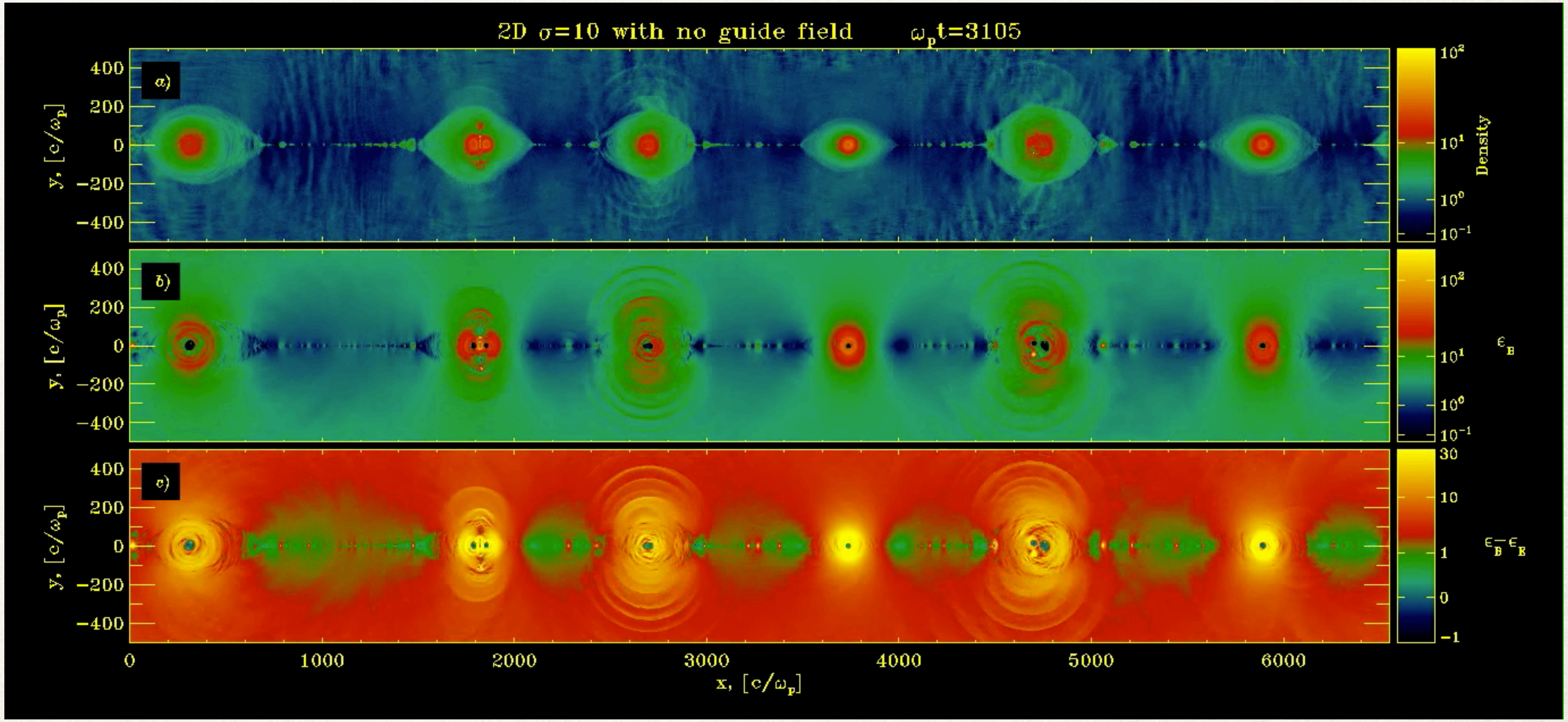
What do we need (challenges) ?

- iPIC — non-relativistic —> relativistic: relativistic particle pushers, implicit moment method
  advantage: implicit schemes allow time steps and spatial resolution far above kinetic scales
- MuPhy — implement 2/3 fluid 5 moment/Maxwell solver
  integrate master thesis Franz Wilfahrt in MuPhy
- racoon — relativistic MHD: Kurganov-Tadmor FV, cell centered fields, divergence cleaning
  difficult: non-ideal terms
- connect iPIC, MuPhy, racoon
- calculate spectra

Some scenarios

- simplify geometry: focus on electron heating and acceleration

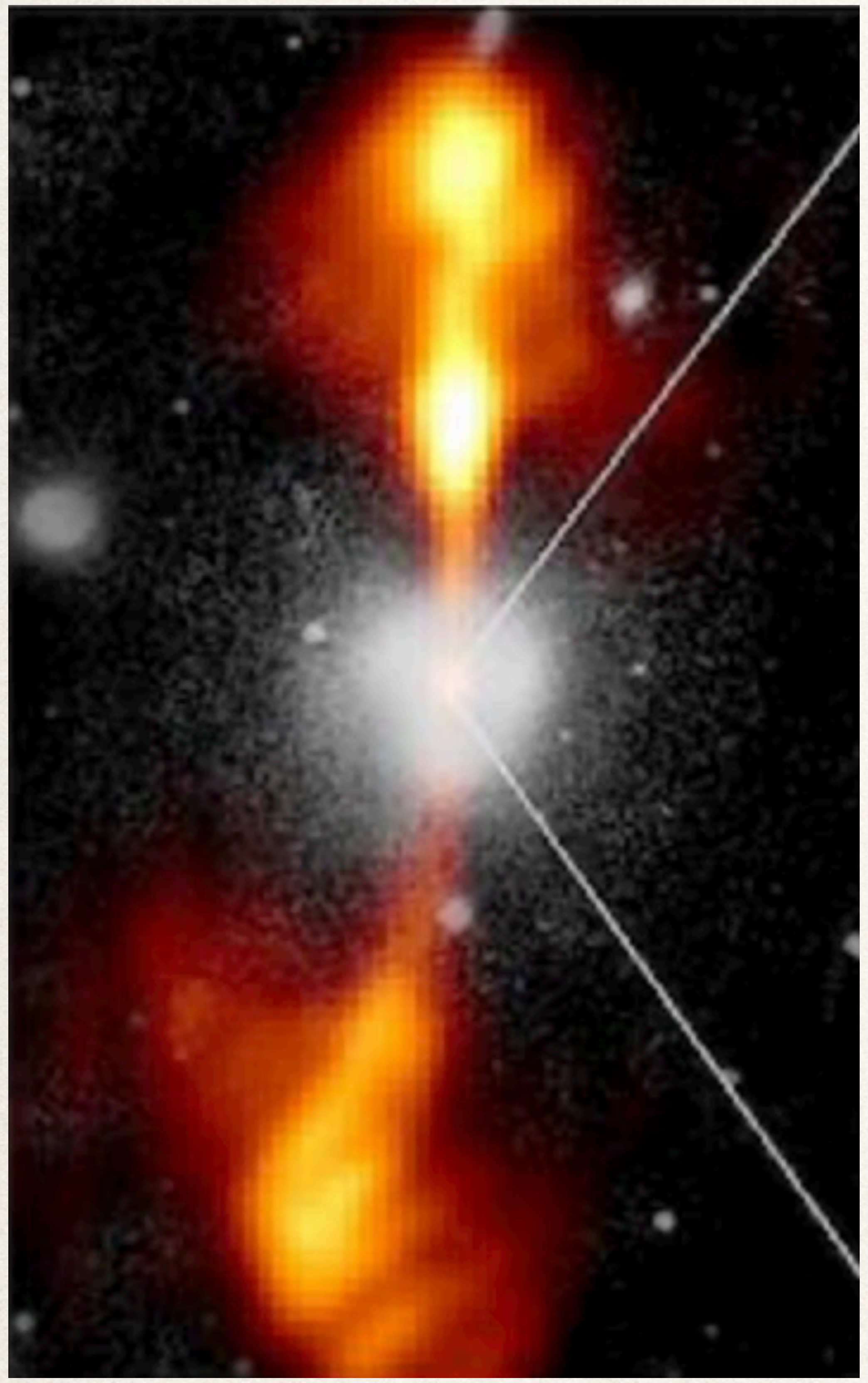


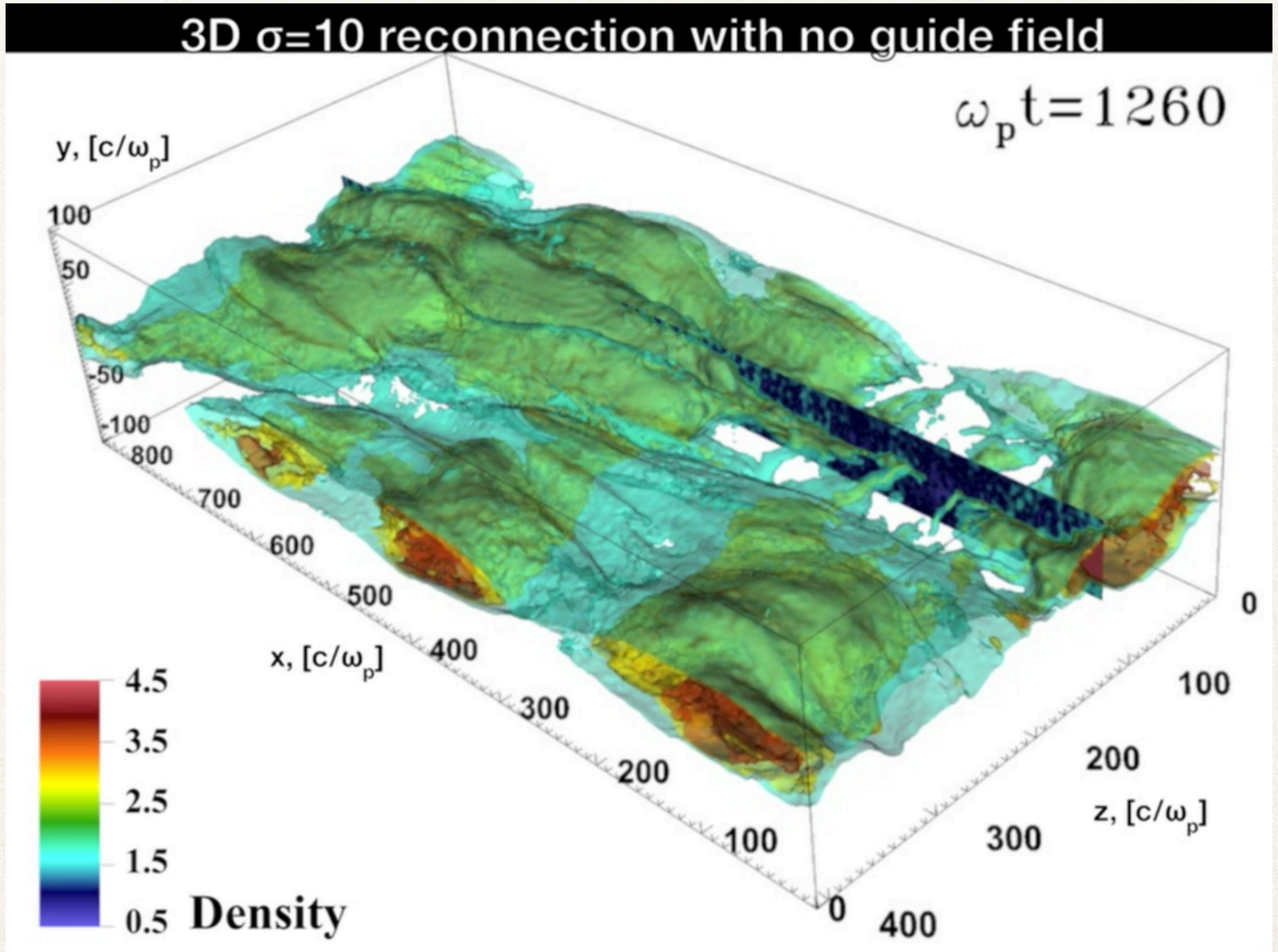


2D fully kinetic relativistic simulation of plasmoid instability: Sironi et al, 2014

Some scenarios

- simplify geometry: focus on electron heating and acceleration

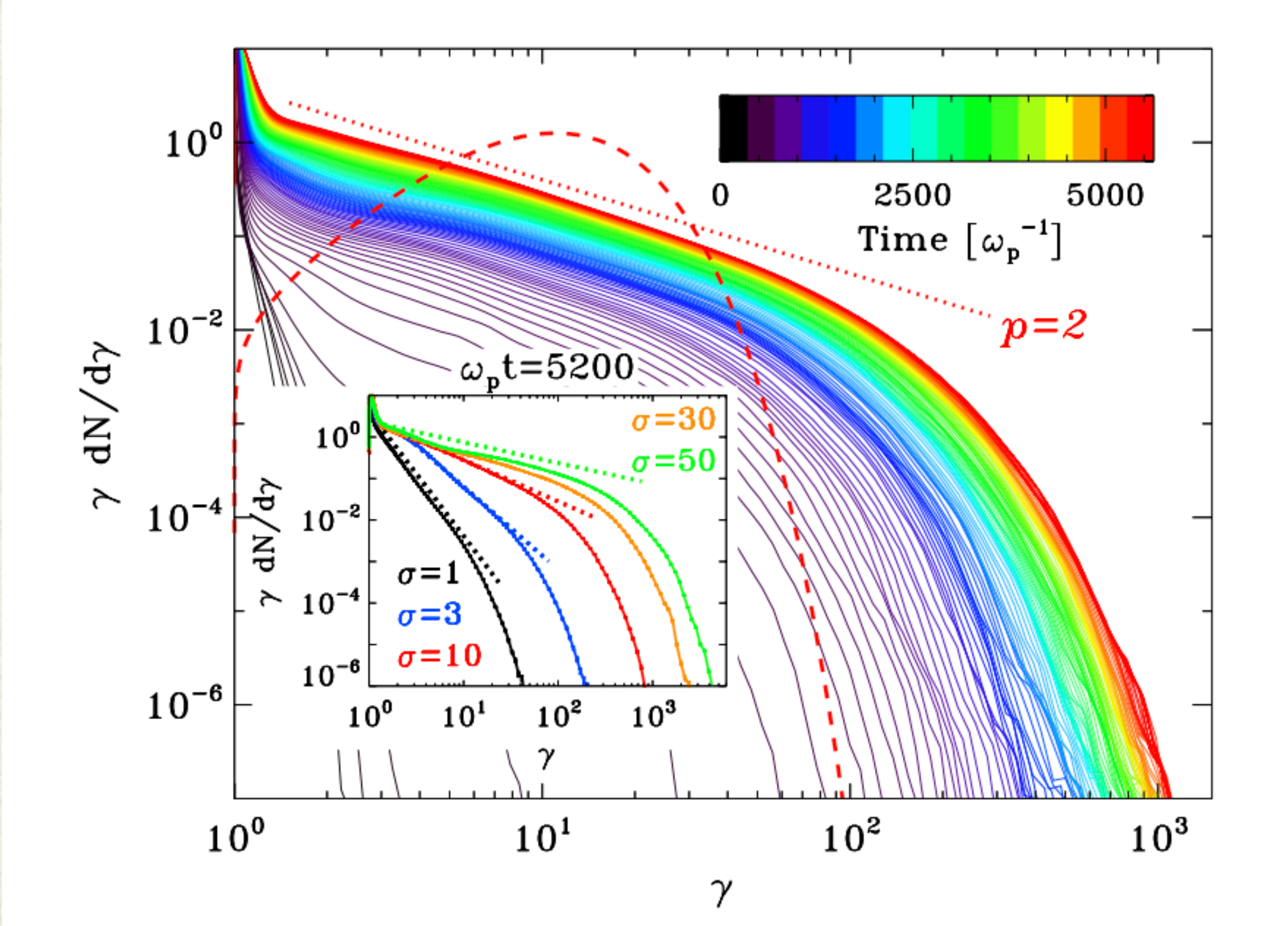


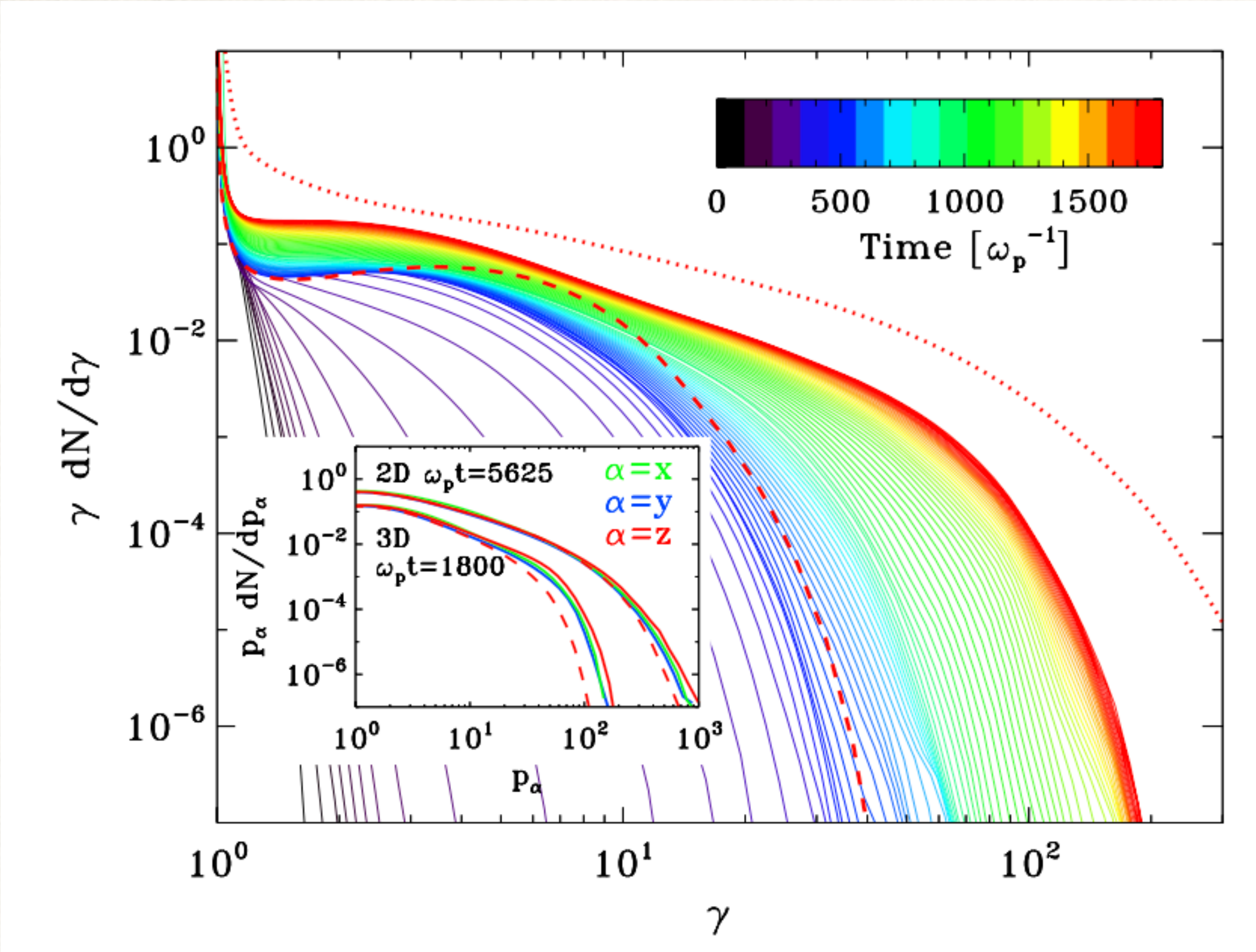


3D fully kinetic relativistic simulation of plasmoid instability: Sironi et al, 2014

Some scenarios

- simplify geometry: focus on electron heating and acceleration

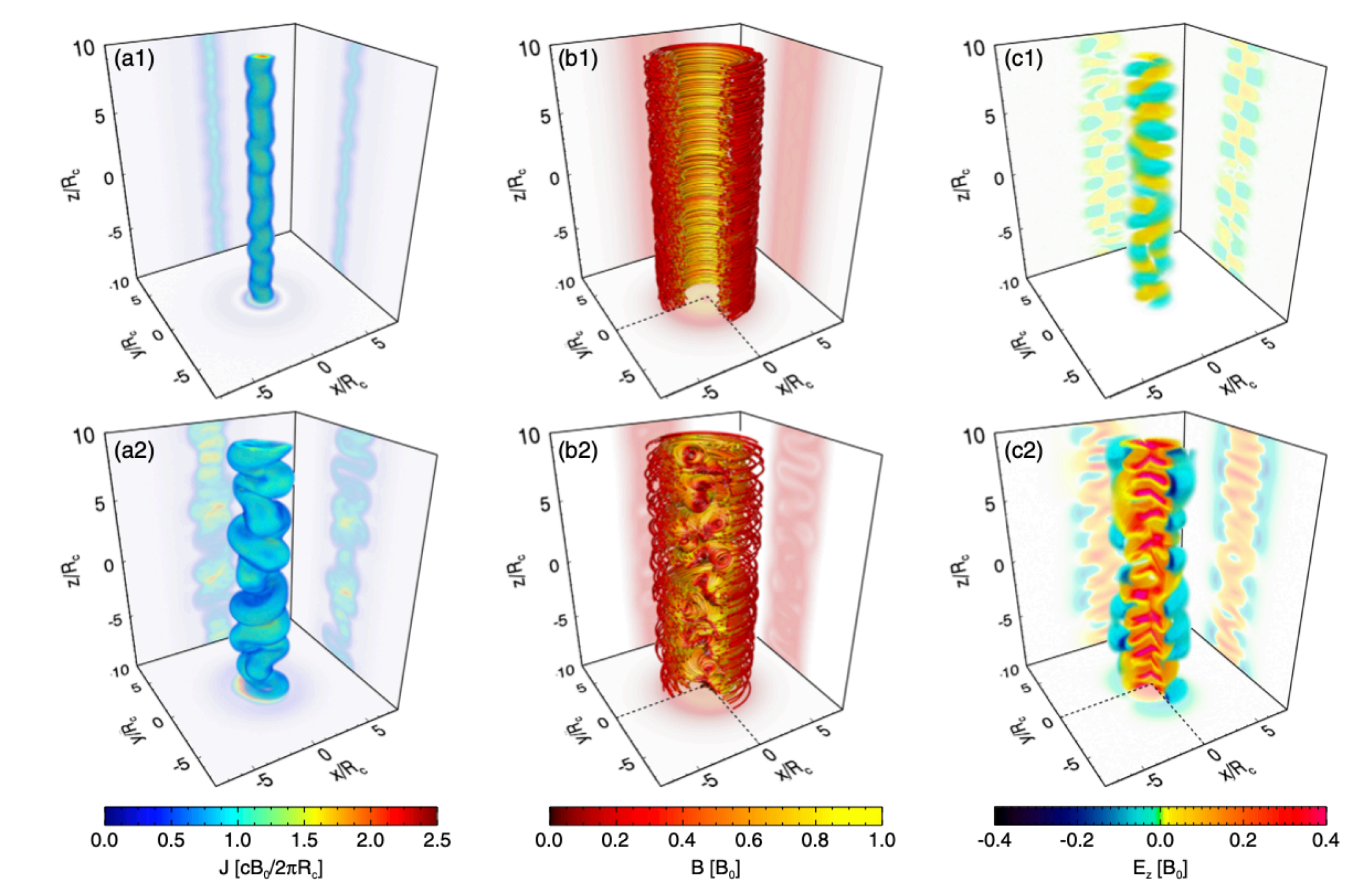




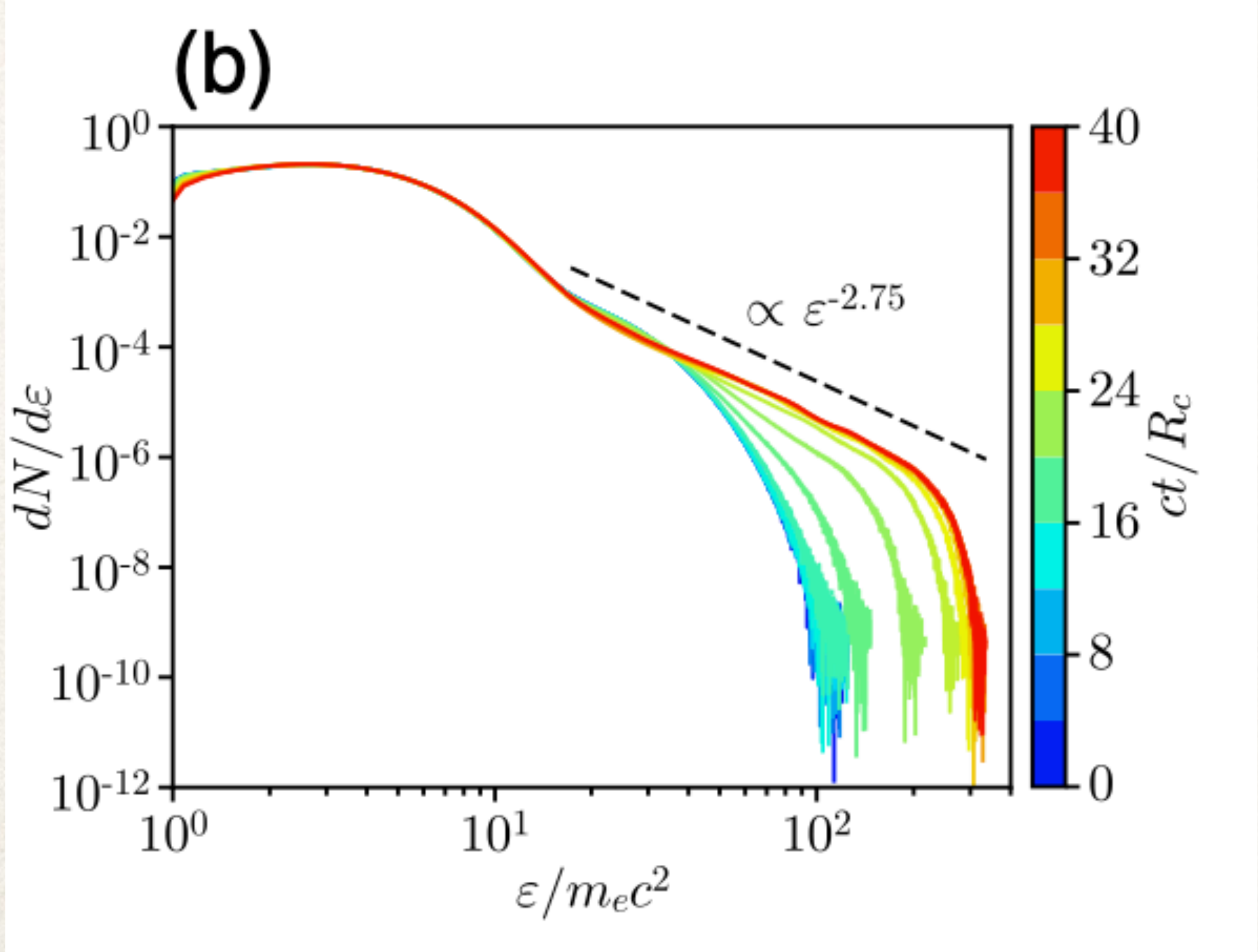
- Fully kinetic, relativistic simulations of plasmoid instability form non-thermal electron spectra, more easily than non relativistic simulations
- Electron acceleration is attributed to the reconnecting electric field (Sironi et al, 2014, Melzani et al, 2014) or to first-order Fermi acceleration (Guo et al, 2014, 2015)
- The larger the magnetization, the easier it is for electron to capture converted magnetic energy
- The spectral slope depend on the magnetization, at least at relatively low magnetization
- Spectral slopes depend **on 2D vs 3D** geometry, presence or absence of guide field
- No consensus on dependence of the slopes on system size (preliminary results are limited by box sizes)
- Positrons presence influence plasmoid production and dimension, hence electron spectra

Some scenarios

- ignore large/small scale interaction, focus on either the small or the larges scales, using more realistic

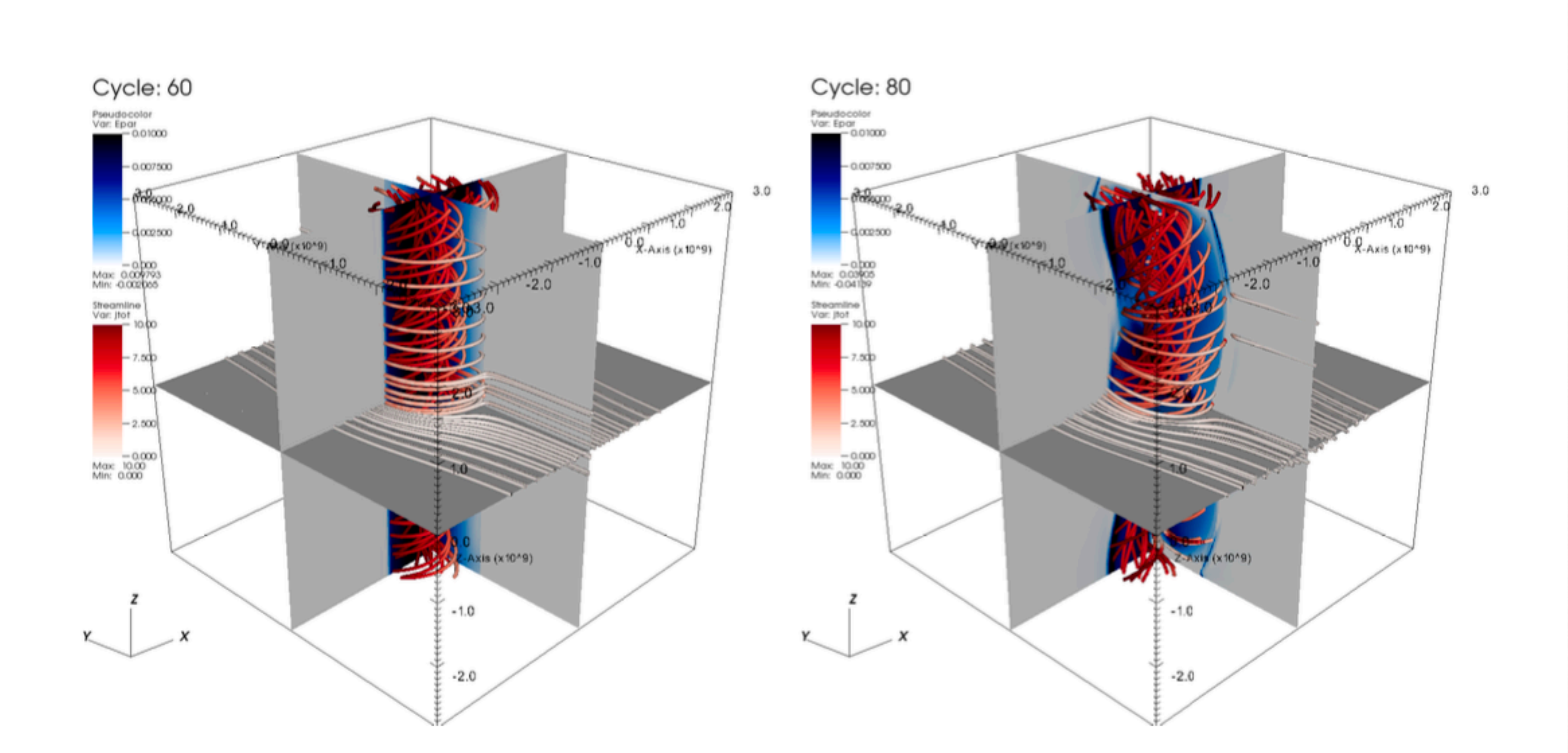


Alves et al 2018: fully **kinetic** simulation of relativistic, kinking plasma column -> kinking instability -> formation of loci of non-thermal electron energization



Some scenarios

- ignore large/small scale interaction, focus on either the small or the larges scales, using more realistic



Ripperda et al, 2018: relativistic MHD simulation of kinking jet
Box: 6L x 6L x 6L, with L= 10^7 m

We cannot ignore the large scales!!!

$\Longrightarrow$ combine large scales (racoon) with medium and small scales (iPIC, MuPHY)

Workplan year 1+2

- Implementation and testing of a relativistic test particle pusher (PIC)
  test particle module using electric/magnetic fields from iPIC, racoon and MuPhy
  Vay (2008), Higuera, Cary (2017), Petri (2017), Ripperda et al. (2018), Burby ()
  Mike and Frederic
- Implementation and testing of relativistic 2 fluid model
  Balsara et al. (2016)
  Franz (until Sept. 2022) and Mike, Rainer
- Implementation and testing of relativistic MHD model
  Jürgen
- Simulations using the relativistic fluid models with test particles
  Mike

Deliverables: these first energy spectra, e^+/e^- vs. p/e^-spectra, including a prediction of the spectra index and cutoff energies will be provided to projects A6 and A 7.

Connection to F1, A6, A7 and questions

F2 $\Longleftrightarrow$ F1: Relativistic test particle pusher (PIC)

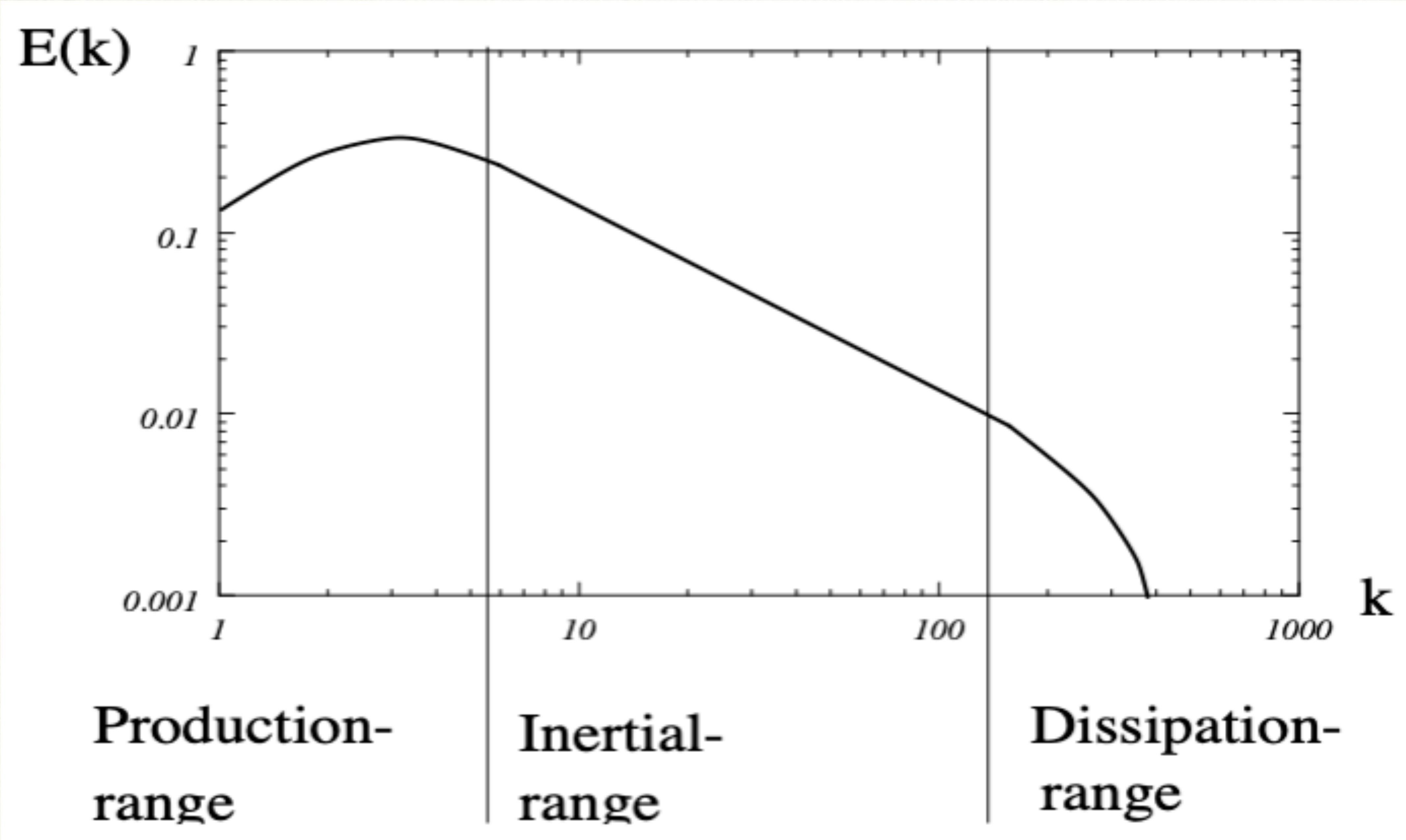
F2 $\Longleftrightarrow$ A6: F2 will provide particle spectra and cutoff energies and the ratio between magnetic
A7: and kinetic energy obtained from simulation of relativistic magnetic reconnection.

A few questions:

- What is the composition of the plasma ? Where are the protons and where are the positrons?
- What is relativistic ? bulk, rel. speeds between protons and electrons, …
- An idea of the gamma and magnetization of the jet?
- Which slope of the electron energy spectra is needed to explain observations?
- Maybe a couple of names of people who are known in their community for simulations, apart from Spitkovsky and Sironi that we know? So we can check what they do already
- How are pair plasma produced?
- If you have a feeling of what are the most important factors that we cannot afford not to have in the simulations to get decent slopes e.g. large scales, kinetic physics, magnetization larger than something...

Intro to turbulence and Why ?

- K41
  Navier-Stokes: $\partial_t \mathbf{u} + \mathbf{u} \cdot \nabla \mathbf{u} + \nabla p = \nu \Delta \mathbf{u}$
  energy dissipation $\epsilon = \nu \int |\nabla \mathbf{u}|^2 d\Omega$ is independent of $\nu \Longrightarrow \boldsymbol{\omega} = \nabla \times \mathbf{u}$ for $\nu \longrightarrow 0$



- cascade
- scaling-invariance:
  $\mathbf{r} \longrightarrow \lambda \mathbf{r}, \mathbf{u} \longrightarrow \lambda^h \mathbf{u}, t \longrightarrow \lambda^{1-h} t$
- local transfer
  ϵ does not depend on the scale

$\Longrightarrow h = 1/3$

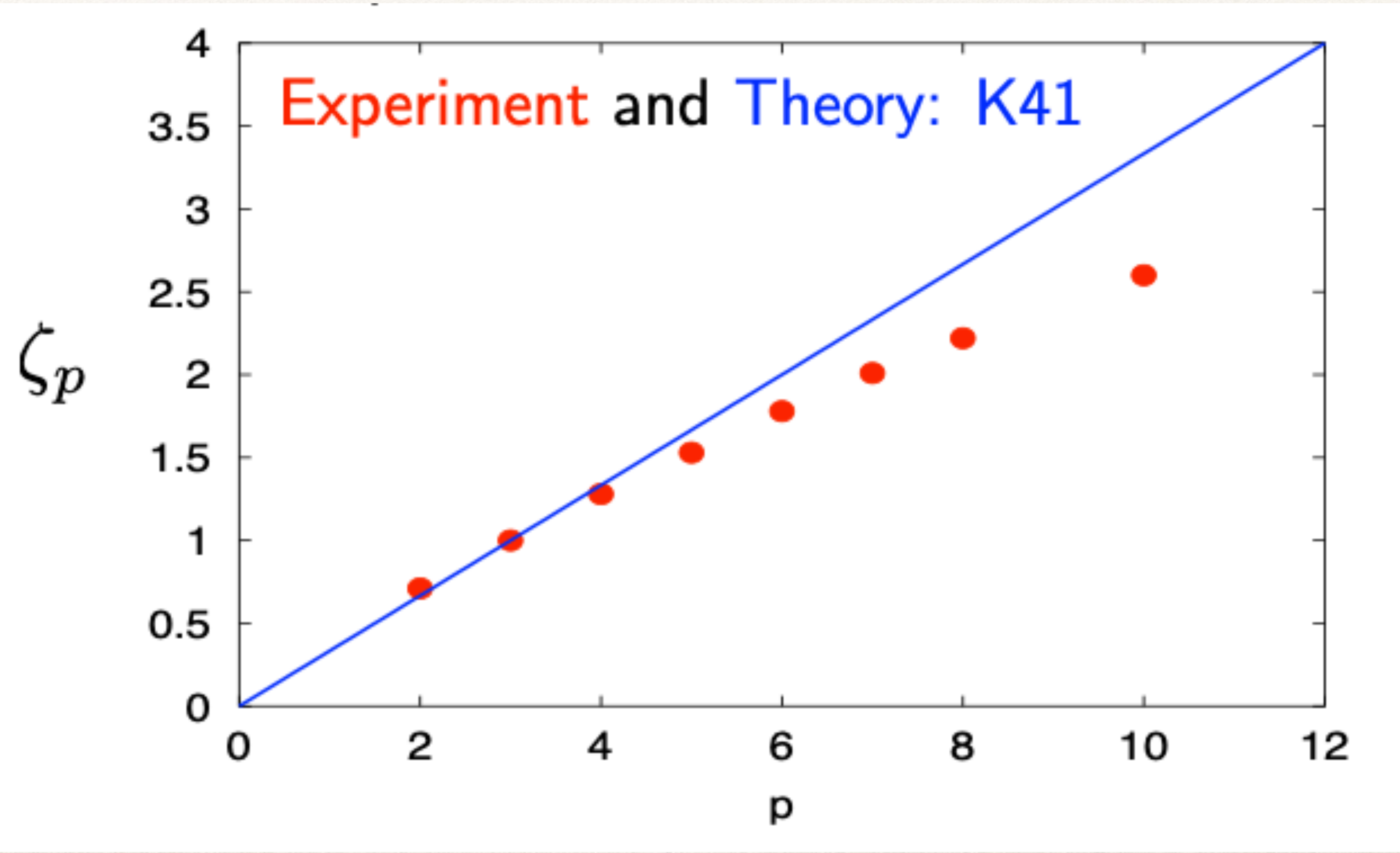
Intro to turbulence and Why ?

Structure functions: $\langle |\mathbf{u}(\mathbf{r}+\mathbf{l})-\mathbf{u}(\mathbf{r})|^p \rangle \propto l^{\zeta_p}\ ,\ \ \zeta_p = \dfrac{p}{3}$

Fourier transform for $p = 2$ $E(k) \sim k^{-5/3}$

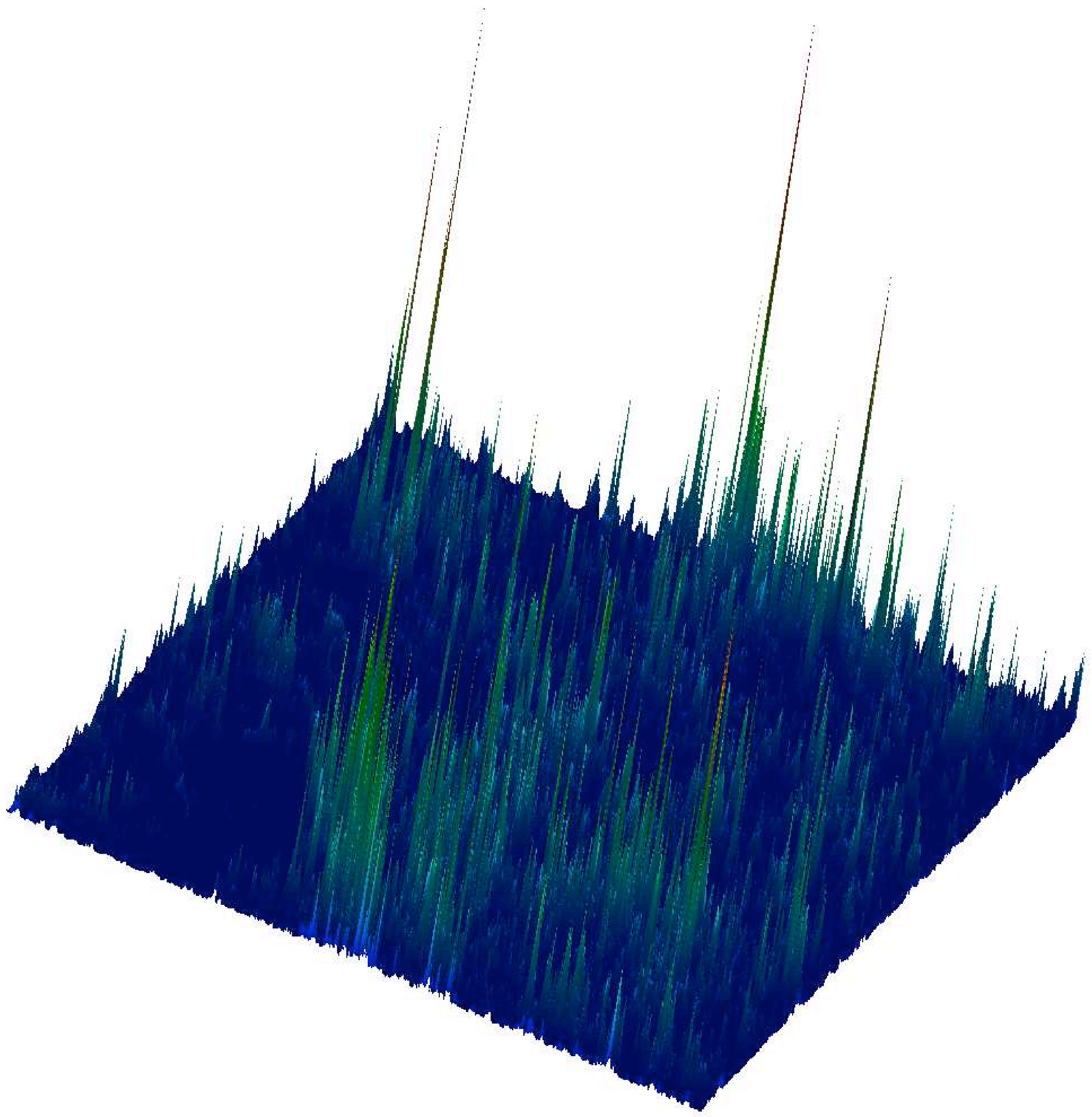
Kolmogorov 1941
Obukhov 1941
Weizsäcker 1948
Heisenberg 1948

What does the experiment show ?



Intro to turbulence and Why ?

Why ?



DNS 1024^3: Homann, Grauer (2006)

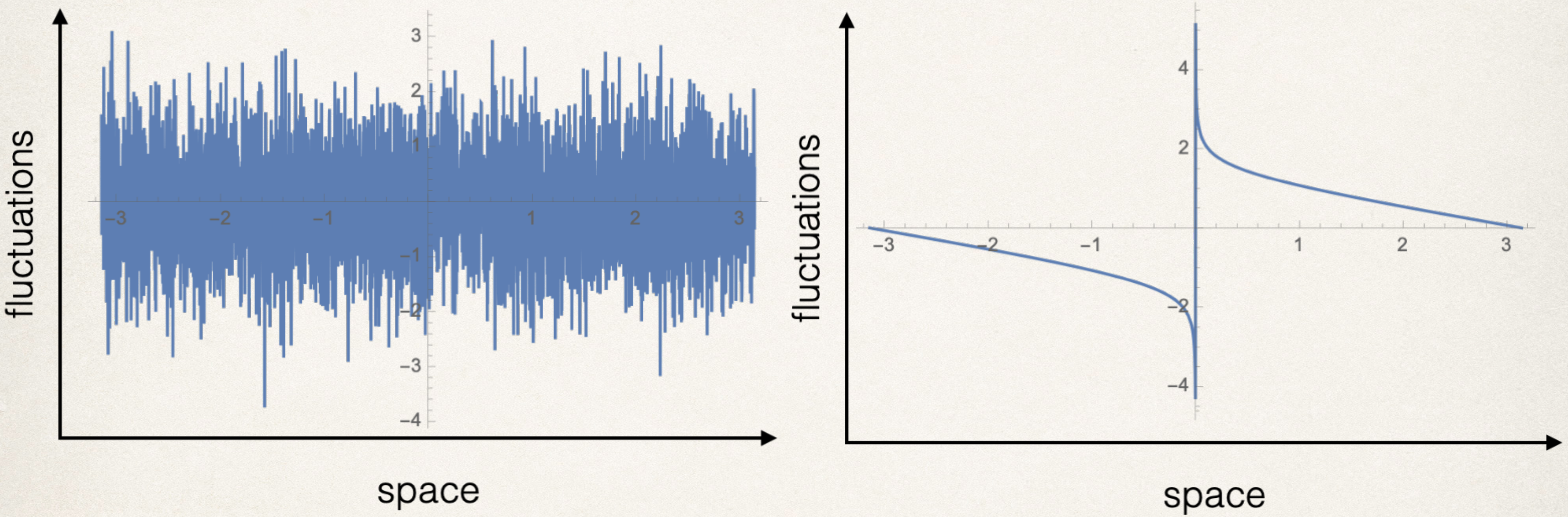
Structures imply:

order

correlations

non-Gaussian

Intro to turbulence and **Why** ?



Two example fields possessing the **same** K41 energy spectrum: the spectrum may be the same, but particle transport in the two fields is not.

Intro to MHD turbulence

- Iroshnikov-Kraichnan: $E_k = C_{IK}\left(\epsilon V_A\right)^{1/2} k^{-3/2}$
- Goldreich-Sridhar: $E\left(k_\perp\right) \sim \epsilon^{2/3} k_\perp^{-5/3}\,,\; E\left(k_\parallel\right) \sim \epsilon^{3/2} v_A^{-5/2} k_\parallel^{-5/2}$

  strong anisotropy

- Boldyrev: 3 lengthscales (currentsheets) $\Longrightarrow \quad E\left(k_\perp\right) \propto k_\perp^{-3/2}$

  $\Longrightarrow \delta v_\lambda \propto \lambda^{1/(3+\alpha)}, \quad \xi \propto \lambda^{3/(3+\alpha)}, \quad l \propto \lambda^{2/(3+\alpha)}\,,\; \alpha = 1$

  very strong anisotropy

What is the community doing?

- pioneering work Giacalone, Jokipii (1999) and subsequent work
  Qin et al. (2002) , Tautz (2010), Tautz, Dosch (2013), Laitinen et al. (2012), Reichherzer et al. (2020)
  based on superposition of Fourier modes
- nice comparison: Dundovic et al. (2020)
- MHD simulations: Cohet, Marcowith (2016), Wisniewski et al. (2012)
- anisotropic: Pommois et al. (2007)
- intermittency: Alouani-Bibi, le Roux (2014) (q-Gaussian), Pucci et al. (2016) p-model
  Shukurov et al. (2017) (dynamo turbulence): intermittency effects particles for $E \lesssim 10^{10} GeV$
- Durrive et al. (2020) generalizing an approach from fluid dynamics (Pereira et al., 2016).
  Method is based on a generalized Biot-Savart kernel that takes into account the stretching of the vorticity encoded in the Cauchy-Green tensor. Fourier methods are used to calculate the integral.

Tools at TPI/IV

- MuPhy, racoon, Cronos: direct simulations
- CRPropa, Picard: Fokker-Planck cosmic ray transport
- construction of 3D multifractal fields using Fourier: working (fine tuning)
  construction of 3D multifractal fields using Wavelets: 1D (fine tuning)
- new method: coherent Gaussian superpositions

How does this work:
superstatistical mixture of
fractional Orstein-Uhlenbeck prozess with **same** noise

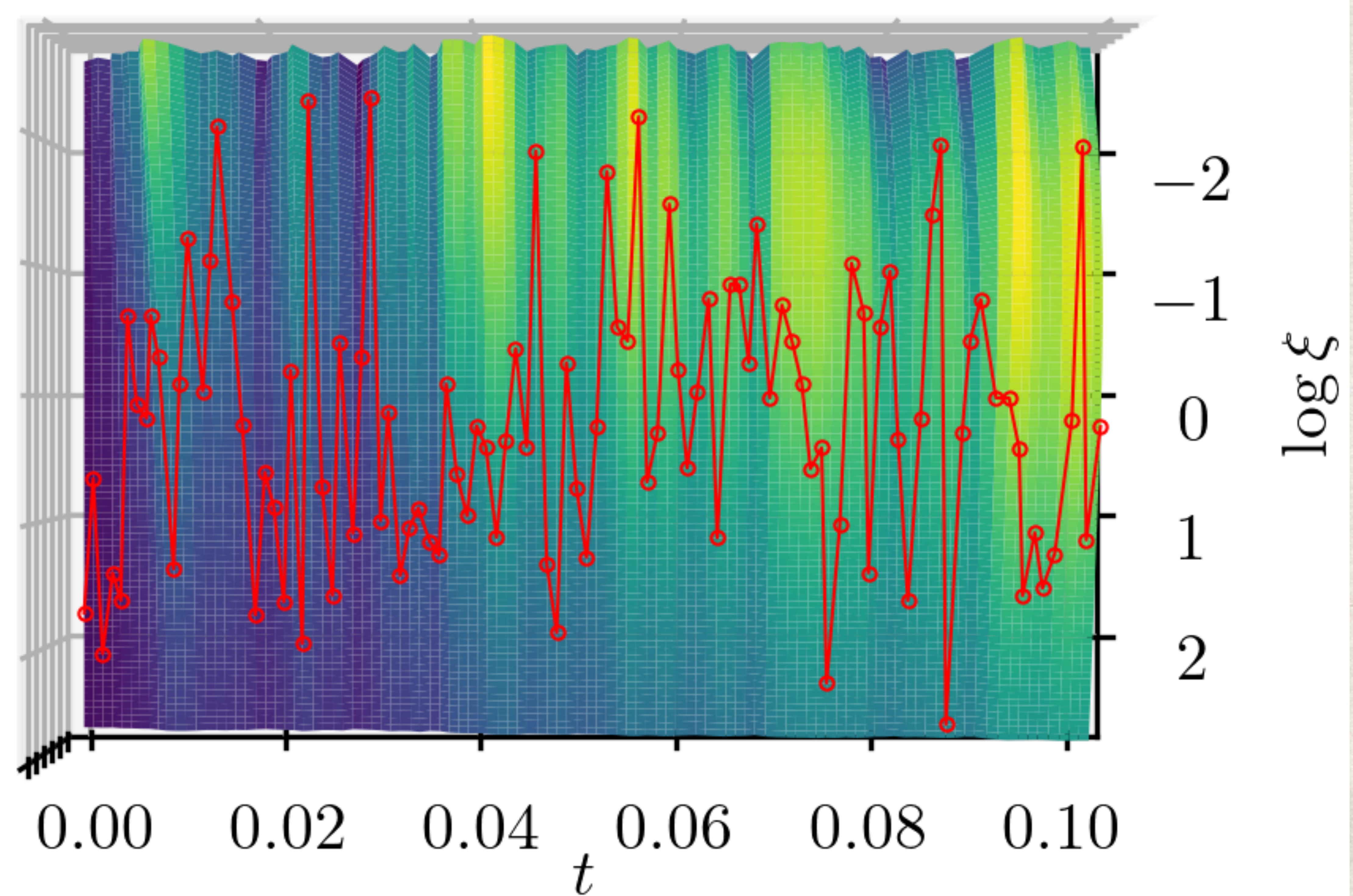
Langevin equation:

$$du^H(t) = -\frac{1}{T}u^H(t)dt + \sigma dB^H(t)$$

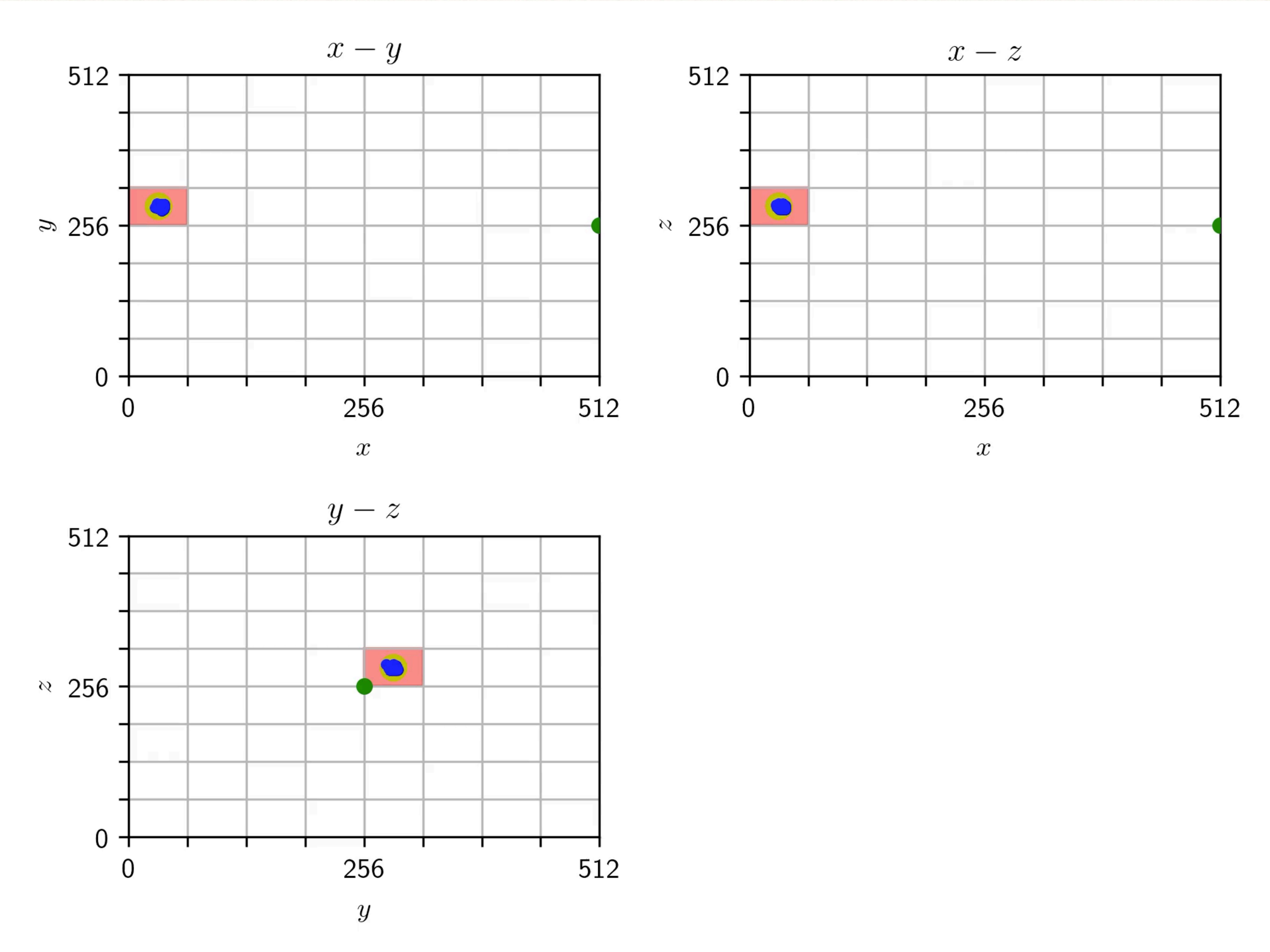
$B^H(t)$ = fractional Brownian motion

covariance $\langle B^H(t)B^H(s)\rangle = \frac{1}{2}\left(|t|^{2H} + |s|^{2H} - |t-s|^{2H}\right)$

Hurst exponent $H \in (0,1)$



- **sketch** of local hierarchical construction



What do we need?

- The synthetic magnetic fields have to be divergence free: $\nabla \cdot \mathbf{B} = 0$
- The synthetic fields should reproduce a predefined energy spectrum
(Kolmogorov, 1941; Iroshnikov, 1963, Kraichnan, 1965 ; Boldyrev, 2005)
- The spectrum should be anisotropic, which means that it should have different exponents perpendicular and parallel to a local guide field
(Goldreich & Sridhar, 1995; Boldyrev, 2005)
- The construction should be local and adaptive in space to produce *large scale* fields
$\Longrightarrow$ Fourier based methods excluded
- The synthetic turbulence should exhibit intermittency, as prescribed by a given intermittency model.
- The synthetic turbulent fields should exhibit an increment PDF that is negatively skewed to produce a cascade.
- Synthetic turbulence should be constructed as a multifractal Brownian bridge.

Workplan years 1+2

- construction of anisotropic spectra in the Gaussian case
  impact of anisotropy on transport
- fractional Gaussian bridges for embedding in large scale fields
- hierarchical non-local construction of multifractal fields
- multifractal bridges
- relativistic pusher (with F2)

work in progress by
Frederic and Jan

1D paper in progress by
Jeremiah, Jan, Rainer

Deliverables: First calculations of the diffusion tensor as input to projects A*

Connection to F2, A1-A5 and questions

F1 $\Longleftrightarrow$ F2: Relativistic test particle module

F1 $\Longleftrightarrow$ A1: Diffusion tensor in the Galactic center outflow

A2: Diffusion tensor in dwarf galaxies

A3: Diffusion tensor in the Galactic halo

A4: Turbulence in dynamical halos

A few questions:

- From A* we need the different parameters:
  large scales, kinetic/dissipation scales (skin depth), particle energies, magnetization, turbulence level, …

- What do you know from Astro on electrical fields ?

Summary F1,F2: We need some form of uncertainty quantification !

Thanks for your patience