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High-energy Lepton Propagation

Alexander Sandrock

National Research Nuclear University MEPhI[1]

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[1]Now at Bergische Universität Wuppertal

Introduction

Leptons

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- standard model fermions
- come in three families: e, μ, τ
- neutrinos and charged leptons
- particles and antiparticles
- only electromagnetic and weak interaction

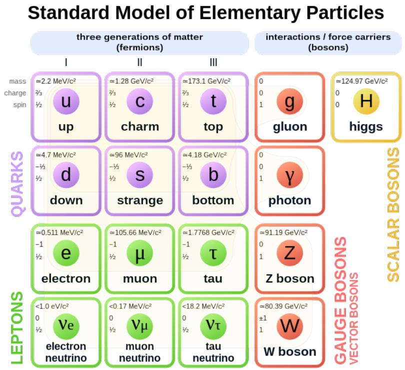

Figure: Standard Model of Elementary Particles [1]

Charged leptons

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- Charged leptons loose energy electromagnetically via
  - ionization: $\ell^\pm + {}^A_Z N \rightarrow \ell^\pm + {}^A_Z N^+ + e^-$
  - bremsstrahlung: $\ell^\pm + {}^A_Z N \rightarrow \ell^\pm + {}^A_Z N + \gamma$
  - pair production: $\ell^\pm + {}^A_Z N \rightarrow \ell^\pm + {}^A_Z N + e^+ + e^-$
  - photonuclear interaction: $\ell^\pm + N \rightarrow \ell^\pm + X$
- Average energy loss per distance

$$\left\langle -\frac{dE}{dx} \right\rangle = \frac{N_\mathrm{A}}{A} \rho \int E v \frac{d\sigma}{dv} dv \simeq a(E) + b(E)E \tag{1}$$

- Muons and taus decay

Neutrinos

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- Neutrinos interact only via the weak interaction
  - Exchange of Z-bosons (NC): $\overset{(-)}{\nu_\ell} + N \to \overset{(-)}{\nu_\ell} + X$
  - Exchange of W-bosons (CC): $\overset{(-)}{\nu_\ell} + N \to \ell^{\pm} + X^{\mp}$
- Mass eigenstates and interaction eigenstates do not coincide $\to$ neutrinos oscillate into different lepton family

$$P_{\alpha\to\beta} = \left| \sum_i U^*_{\alpha i} U_{\beta i} e^{-i m_i^2 L/2E} \right|^2 \tag{2}$$

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Ionization

Ionization

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Ionization is a cover name for several processes

- excitation $\mu^{\pm} + {}^{A}_{Z}N \rightarrow \mu^{\pm} + {}^{A}_{Z}N^{*}$
- ionization in the strict sense $\mu^{\pm} + {}^{A}_{Z}N \rightarrow \mu^{\pm} + {}^{A}_{Z}N^{+} + e^{-}$
- emission of δ-electrons $\mu^{\pm} + {}^{A}_{Z}N \rightarrow \mu^{\pm} + {}^{A}_{Z}N^{+} + e^{-}$

The first two processes are low-energy atomic physics processes (eV-scale). The dominant contribution for high-energy particles are δ-electrons.

Bethe formula

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- energy loss for relativistic particles

$$\left\langle -\frac{dE}{dx} \right\rangle = K \frac{Z}{A} \frac{1}{\beta^2} \left[\frac{1}{2} \ln \frac{2 m_e c^2 \beta^2 \gamma^2 T_{\max}}{I^2} - \beta^2 - \frac{\delta(\beta\gamma)}{2} \right], \tag{3}$$
$$T_{\max} = \frac{2 m_e c^2 \beta^2 \gamma^2}{1 + 2\gamma m_e/\mu + (m_e/\mu)^2}.$$

- Accuracy at small energies not worse than 1%

Density effect

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- The correction δ describes the density effect [2], which is connected to the polarisation of the medium at large energies
- Asymptotically

$$\delta \to \ln \frac{\hbar\omega_p}{I} + \ln \beta\gamma - \frac{1}{2} \qquad (4)$$

- The Sternheimer parametrization of δ is not worse than 10% at small energies and asymptotically exact at high energies.

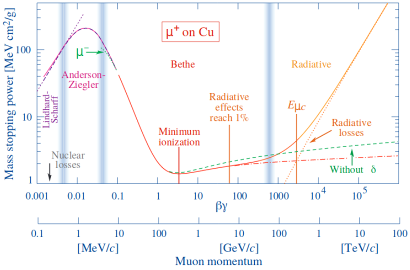

Figure: $\langle -dE/dx \rangle$ for muons in copper [3]

Radiative corrections

In the calculation of corrections to bremsstrahlung of muons scattered by atomic electrons, also the radiative corrections to ionization was obtained in logarithmic approximation [4]

$$\Delta \left| \frac{dE}{dx} \right| = \frac{NZ}{A} m \alpha r_e^2 \left(\ln \frac{2E}{\mu} - \frac{1}{3} \ln \frac{2\epsilon_{\text{max}}}{m} \right) \ln^2 \frac{2\epsilon_{\text{max}}}{m}. \quad (5)$$

The mean loss is increased by several percent due to these corrections.

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Bremsstrahlung

Bremsstrahlung

For relativistic particles, the bremsstrahlung cross-section can be written as [5, 6]

$$v\frac{d\sigma}{dv} = 4Z^2\alpha\left(r_e\frac{m}{\mu}\right)^2\left[(2-2v+v^2)\Phi_1(\delta)-\frac{2}{3}(1-v)\Phi_2(\delta)\right], \qquad \delta = \frac{\mu^2 v}{2E(1-v)}. \tag{6}$$

Screening functions

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In the absence of screening for a point-like nucleus we have

$$\Phi_1 = \Phi_2 = \ln \frac{\mu}{\delta} - \frac{1}{2}, \tag{7}$$

for complete screening of a point-like nucleus

$$\Phi_1 = \ln \left(\frac{\mu}{m} B Z^{-1/3} \right), \quad \Phi_2 = \Phi_1 - \frac{1}{6}, \tag{8}$$

where the constant B is ≈ 183 and $\ln(BZ^{-1/3})$ is the radiation logarithm. An analytic interpolation describing also intermediate screening was found by [7, 8].

Formfactors

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- Fourier transformation of charge distribution
- Nuclear formfactor: extended charge distribution inside nucleus
- Atomic formfactor: screening of nuclear charge by atomic electrons

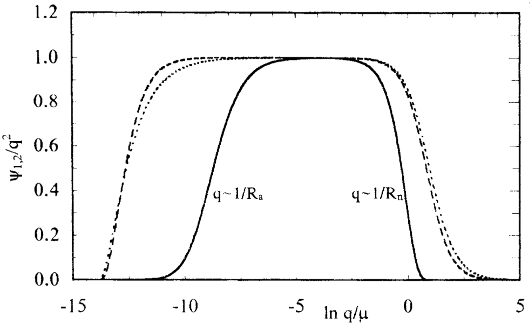

Figure: q-dependence of the bremsstrahlung cross section [9]

Nuclear formfactor

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- The Compton wavelength of a muon is comparable to nuclear dimensions.
- The nuclear formfactor effectively cuts off large momentum transfers.
- The cross-section is decreased by $\sim 10\%$.
- $\Phi_i \to \Phi_i - \Delta_i$, where

$$\begin{aligned} \Delta_1 - \Delta_2 &\approx \frac{1}{6}, \\ \Delta_1 &\approx \ln \frac{\mu}{q_c} + 1 \end{aligned} \tag{9}$$

  for heavy nuclei.
- The calculations by [9] and [10, 11] on the basis of different models of the nuclear formfactor differ somewhat, but [9] describes numerical calculations better.

Interaction with atomic electrons

- Electrons not only screen the nucleus, but are also targets.
- The recoil of electrons during scattering changes the situation compared to atomic nuclei
- An approximate formula is given by

$$
\begin{aligned}
v\frac{d\sigma}{dv} &= 4\alpha Z\left(r_e\frac{m}{\mu}\right)^2\left(\frac{4}{3}(1-v)+v^2\right)\Phi_{\text{in}}(\delta), \\
\Phi_{\text{in}}(\delta) &= \ln\frac{\mu/\delta}{\mu\delta/m^2+\sqrt{e}} - \ln\left(1+\frac{m}{\delta B' Z^{-2/3}}\right), \qquad (10)\\
B' &= 1429.
\end{aligned}
$$

Quasielastic target excitation

- excitation of nuclear levels
- Assuming the nuclear wavefunction as a non-symmetrized product of nucleon wave functions, the correction assumes the form [10]

$$\Delta_i^{\text{inel}} = \frac{1}{Z}\Delta_i. \tag{11}$$

Inelastic target excitation

- Interaction with separate nucleons
- Effect of a few percent
- Better considered not as a nucleon correction to bremsstrahlung, but as a radiative correction to nuclear interaction

Radiative corrections

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- Calculated recently [12, 13]
- Described by a universal function $f(v)$ in the equivalent photon approximation
- The energy loss increases by $\sim 2\%$

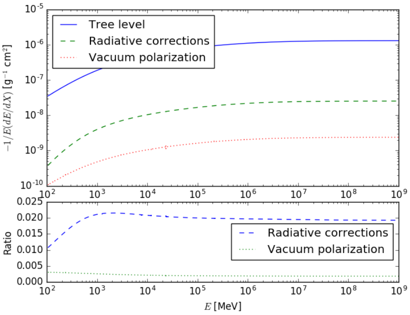

Figure: Bremsstrahlung energy loss of muons

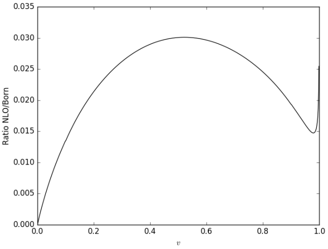

Figure: Universal function $f(v)$ describing the ratio between Born approximation and higher-order corrections

Diffractive corrections

- Photon emitted by nucleus, not by muon
- Interference effect dependent on sign of muon charge
- $\sim 0.1\%$; contrary to earlier calculations significantly overestimating the effect [14]

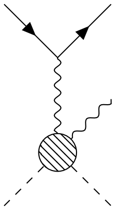

Figure: Feynman diagram for diffractive corrections

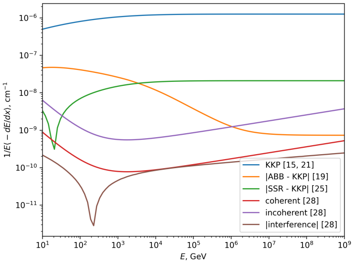

Figure: Muon energy loss in water with radiative and diffractive corrections [15]

LPM effect

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- at ultrahigh energies, multiple scattering perturbs interaction [16]
- the cross-section is decreased [17], in particular for photons of small energy compared to the muon energy

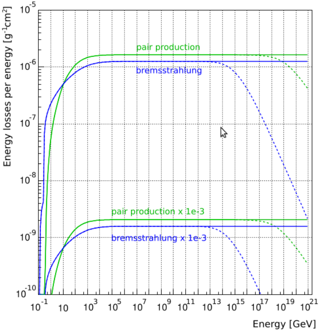

Figure: Energy loss in ice (above) and standard rock (below) [18]

Coulomb corrections

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- Born approximation: first term in expansion in $Z\alpha$
- Coulomb corrections: remaining terms of this expansion
- very important for electrons
- negligible for muons due to the influence of the extended nucleus $\lesssim 0.4\%$

Chemical bounds

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- The effect of molecular bounds was calculated by [19]
- small, $\lesssim 0.5\%$ for hydrogren and less for heavier nuclei

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Pair production

Electron-positron pair production

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- The leading-order cross section for complete screening and no screening was calculated by [20, 21].
- An analytic interpolation between these limiting cases was carried out by [22, 23].

$$\frac{d^2\sigma}{dv\ d\rho} = \frac{2}{3\pi}(Z\alpha r_e)^2\frac{1-v}{v}\left(\Phi_e + \frac{m^2}{\mu^2}\Phi_\mu\right), \qquad (12)$$

$$\Phi_{e,\mu} = L_{e,\mu}B_{e,\mu} + \frac{1}{2}\Delta_{e,\mu}$$

Nuclear and atomic formfactor corrections

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- The influence of the nuclear formfactor was investigated in [24]
  - unimportant for dE/dx
  - effect on $d\sigma/dv$ of the order of 1% for $v \gtrsim m/\mu$
- interaction with atomic electrons important [25], of the order of $1/Z$
- target excitation unimportant [26]
- Screening functions were parametrized more accurately in [13], leading to an effect of the order of 1% for $d\sigma/dv$, but $\lesssim 0.5\%$ for dE/dx

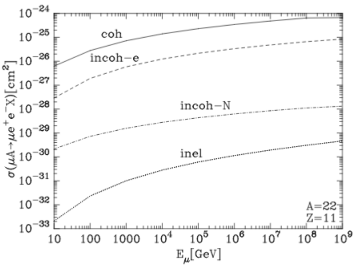

Figure: Total pair production cross-section in standard rock [26]

Screening functions

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- The functions $L_{e,\mu}$ are analogous to the function Φ_1 from bremsstrahlung
- A new expression for the cross section has been derived taking into account the difference between the analogues of $\Phi_{1,2}$ [13]
- difference to earlier works $\sim 0.5\%$ for dE/dx, $\sim 1\%$ for $d\sigma/dv$

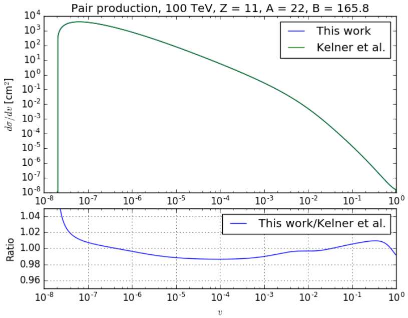

Figure: Differential cross section at 100 TeV

Muon pair production

- Calculated in [27]
- very small effect on the energy loss ($\sim 10^{-4}$ compared to e^+e^- pair production)
- potentially interesting as it converts a single muon to a (small) muon bundle

LPM effect

- the same effect as in bremsstrahlung, applied to the $\gamma^* Z \to e^+ e^- Z$ subprocess
- significant effect on dE/dx only at energies $\gtrsim 10^{24}$ eV [17]

Coulomb corrections

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- analytical expression for point-like nuclei [28, 29]
- numerical results show that a nuclear formfactor decreases the correction for very heavy elements [30]
- for dE/dx: in standard rock $\sim 0.5\%$, in lead $\sim 9\%$
- for $d\sigma/dv$: in standard rock $\sim 1\%$ for $v \gtrsim m/\mu$

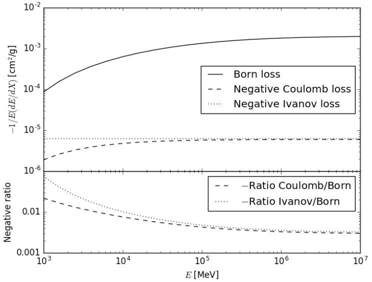

Figure: Energy loss in standard rock [30]

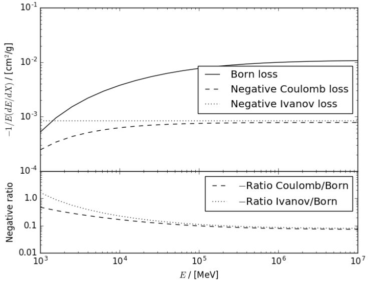

Figure: Pair production energy loss in lead [30]

Radiative corrections, double pair production

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- radiative corrections increase the cross section by $\sim 2\%$
- double pair production: logarithmically increasing loss, $\sim 0.5\%$ at PeV energies

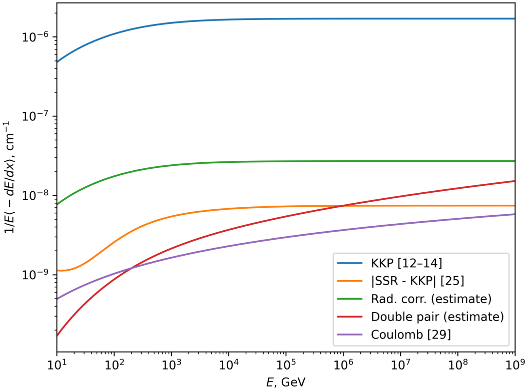

Figure: Pair production energy loss and higher-order corrections.

Diffractive corrections

- very small correction to the μ-diagrams, negligible
- no dependence on sign of muon charge

Nuclear interaction

Nuclear interaction

- Inelastic interaction with nucleons at energies $E \lesssim 10^{15}$ eV gives a contribution of $\sim$ 10%–20% to the energy loss
- the contribution rises with energy

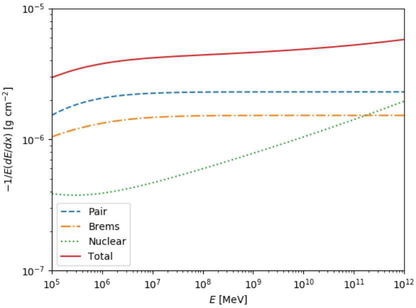

Figure: Energy losses in standard rock, divided by energy.

Limiting cases

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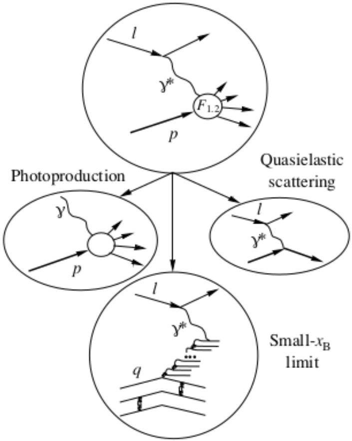

Figure: Diagrams of limiting cases of inelastic interaction [31]

Models of nuclear interaction

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- not a purely electromagnetic process
- predominantly nonperturbative QCD
- process with largest uncertainty

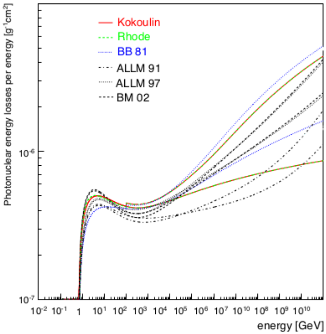

Figure: Different models of muon energy loss by nuclear interaction [18]

Vector meson dominance

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- photons and vector mesons (ρ, ω, φ and their excited states) have identical quantum numbers
- the photon converts to a virtual vector meson, the meson interacts hadronically with the nucleus
- formulae of Bezrukov and Bugaev are often used [32]
- structure functions are proportional to photoabsorption cross-section $\sigma_{\gamma p}$ in this model
- applicable for small momentum transfer $Q^2 \lesssim$ few GeV^2

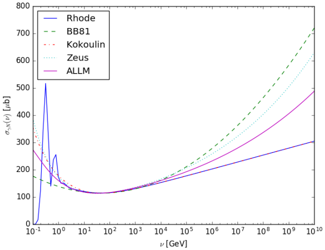

Figure: Different parametrizations of the photoabsorption cross-section $\sigma_{\gamma p}$

Perturbative contribution

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- hard interactions with high momentum transfer Q^2 (deepy inelastic scattering)
- described by color dipole model
- contribution rises with energy

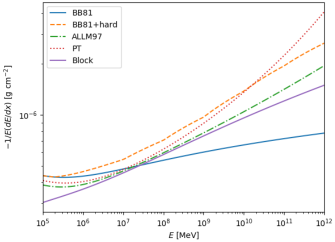

Figure: Perturbative and nonperturbative contributions to energy loss

Regge theory

- phenomenological approach to scattering problem
- uses analytical properties of scattering amplitudes at complex values of orbital momenta
- new degree of freedom (quasi-particles): reggeons, pomerons
- Regge trajectory corresponds to a family of particles
- e. g. Abramowicz, Levin, Levy & Maor [33, 34]

Nuclear corrections

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- nuclear shadowing
  - $\sigma_{\gamma A} < A\sigma_{\gamma p}$
  - effect: $\sim 20\%$ [35]
- EMC effect [36–38]
- Fermi motion of nucleons [37]

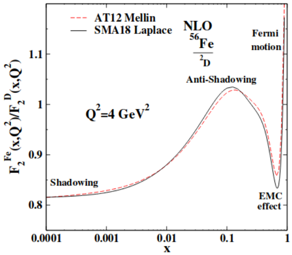

Figure: Nuclear effects at $Q^2 = 4\,\mathrm{GeV}^2$ [38]

Weak interaction

- at large momentum transfers Z bosons can contribute
- also interference between γ and Z
- effect on $dE/dx \lesssim 10^{-4}$ [36, 39]

Radiative corrections

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- bremsstrahlung during nuclear interaction, together with vertex correction and vacuum polarization
- calculated within the VMD model
- dE/dx increases by $\sim 3\%$

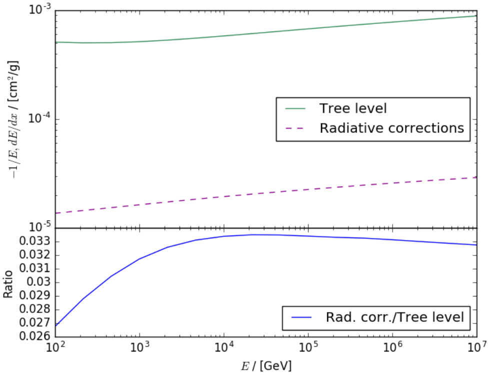

Figure: Energy loss by nuclear interaction with radiative corrections [40]

Nuclear interaction experimental data

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- data from fixed-target experiments
- data from *ep* collider HERA
- total combined HERA data only available recently

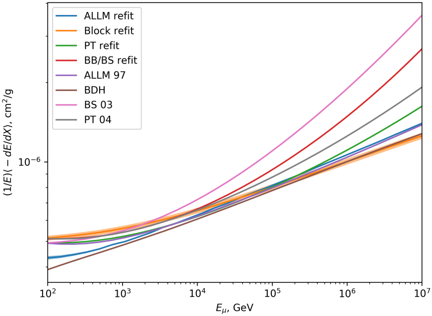
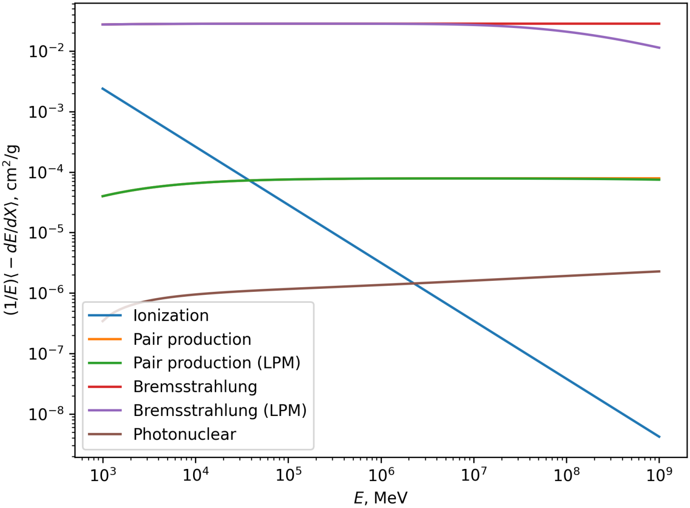
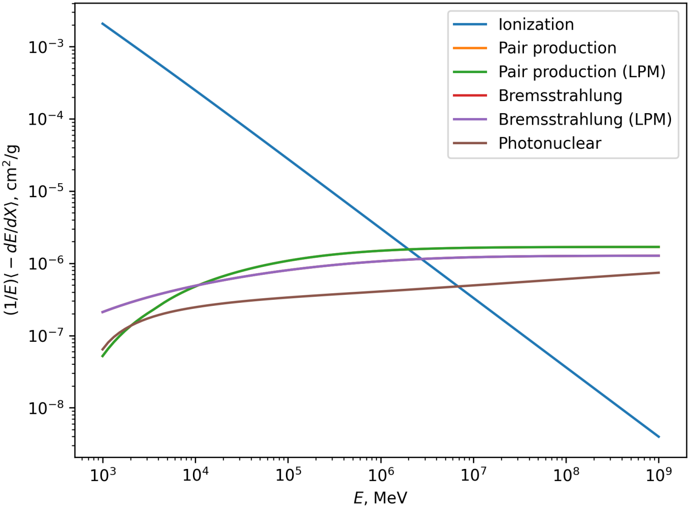


Figure: Photonuclear energy loss according to the literature and refits of popular models [41]

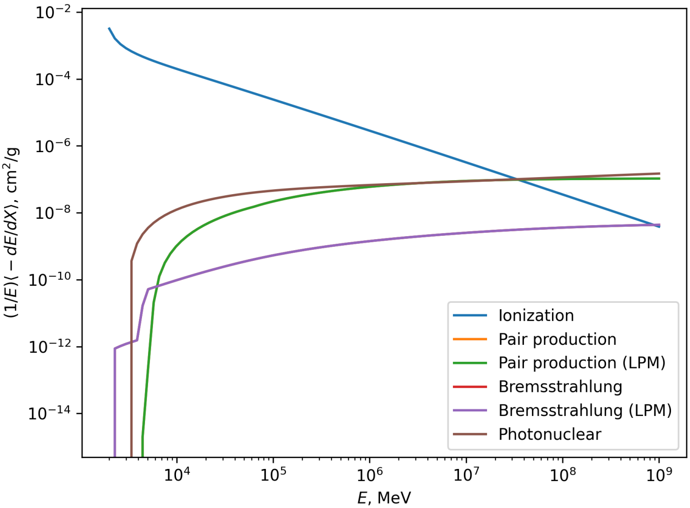
Electron energy loss



Muon energy loss



Tau lepton energy loss



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Propagation of particles

Propagation of particles

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- interaction length

$$\lambda_{\text{int}} = \frac{A}{N_A \rho \sigma} \tag{13}$$

- probability to traverse a distance λ without interaction

$$P(x) = \frac{1}{\lambda_{\text{int}}} e^{-\lambda/\lambda_{\text{int}}} \tag{14}$$

- sufficient for calculation of attenuation factors

Analytical calculations

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- Charged lepton interactions are typically not catastrophic, the lepton propagates further and produces secondary particles, thus losing energy
- simplest approximation: loss happens continuously with $-dE/dx = a + bE$ with constant a, b
- Range of lepton with initial energy E

$$R_{\langle -dE/dx \rangle} = \frac{1}{b} \ln \frac{a + bE}{a}. \tag{15}$$

- surface energy E_0 of muon with energy E after traversing a layer of matter with thickness h

$$E_0 = \exp(bh) \frac{a + bE}{b} - \frac{a}{b} \tag{16}$$

- spectrum at depth h for a surface spectrum $dN/dE = N_0 E^{-\gamma}$:

$$\frac{dN}{dE} = N_0 \exp(-\gamma bh) \left\{ E + \frac{a}{b} [1 - \exp(-bh)] \right\}^{-\gamma} \tag{17}$$

Necessity of Monte-Carlo simulations

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- energy loss is a stochastic process $\rightarrow$ fluctuations around the average energy loss
- effect of fluctuations becomes more pronounced at higher energies due to radiative processes
- example for monoenergetic muons: average range $\langle R \rangle$ is smaller than $R_{\langle -dE/dx \rangle}$

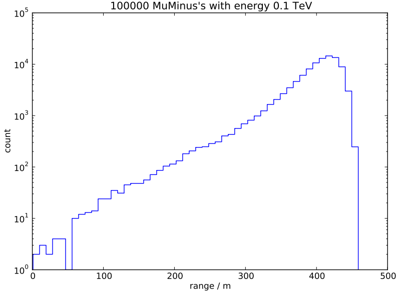

Figure: Range distribution of 100 GeV muons

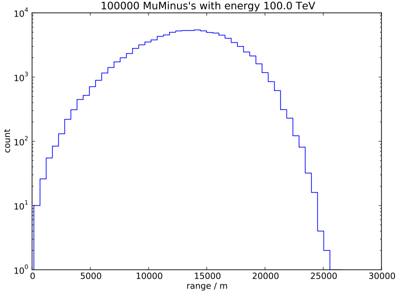

Figure: Range distribution of 100 TeV muons

Energy cuts

- Simulation of all energy losses impossible due to infrared divergence: Bremsstrahlung cross section diverges $d\sigma/dv \sim 1/v$ for $v \to 0 \Rightarrow$ infinitely many secondary particles, total cross section diverges
- Separate losses into soft and hard losses
  - soft losses: continuous treatment
  - hard losses: stochastic treatment
- Cutoff (relative v_{cut} or absolute e_{cut}) is an artificial scale; has to be chosen sufficiently small so as not to influence the simulation results

Propagation algorithm

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- Probability of stochastic hard loss over a distance dx

$$
\begin{aligned}
dP(E) &= dx \left.\frac{dN}{dx}\right|_{\text{hard}}, \\
\left.\frac{dN}{dx}\right|_{\text{hard}} &= \sum_{\text{processes}} \frac{N_{\text{A}}}{A} \rho \int_{v_{textcut}}^{v_{\mathbf{max}}} \frac{d\sigma}{dv}
\end{aligned}
\tag{18}
$$

Propagation algorithm

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- Probability to experience no hard losses over a finite distance $[x_i, x_f]$ and a hard loss between x_f and $x_f + dx$

$$
\begin{aligned}
&(1 - dP(E(x_i))) \cdots (1 - dP(E(x_f))) \cdot dP(E(x_f)) \\
&\approx \exp(-dP(E(x_i))) \cdots \exp(-dP(E(x_f))) \cdot dP(E(x_f)) \\
&\rightarrow^{dx \rightarrow 0} \exp\left(-\int_{E(x_i)}^{E(x_f)} dP(E(x))\right) dP(E(x_f)) \\
&= d\left[-\exp\left(\int_{E(x_i)}^{E(x_f)} \frac{\frac{dN}{dx}(E)\big|_{\text{hard}}}{-\frac{dE}{dx}\big|_{\text{soft}}} dE\right)\right] \\
&=: d(-\xi), \quad \xi \in (0, 1].
\end{aligned}
\tag{19}
$$

Propagation algorithm

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- $\Rightarrow$ one random number determines the energy E_f and distance $x_f - x_i$ of the next interaction

$$-\ln\xi = \int_{E_i}^{E_f} \frac{\frac{dN}{dx}(E)\big|_{\text{hard}}}{-\frac{dE}{dx}\big|_{\text{soft}}} dE \qquad (20)$$

- another random number determines which process and which relative energy loss v is chosen based on the differential cross-section $d\sigma/dv$

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Consequences and Applications

Neutrino event topologies in very large volume neutrino telescopes

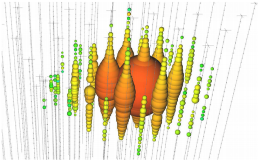

Figure: Cascade: ν_e CC events, $\nu_{e,\mu,\tau}$ NC events

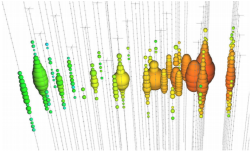

Figure: Track: μ, ν_μ CC events

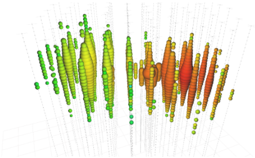

Figure: Double cascade/double bang: ν_τ CC events

Energy reconstruction of muon tracks in VLVνT

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- muons of high energy travel large distances, so they do not deposit all their energy inside the detector
- small pair production losses are well correlated to the energy, bremsstrahlung and photonuclear losses less well correlated $\rightarrow$ truncate large losses for energy reconstruction

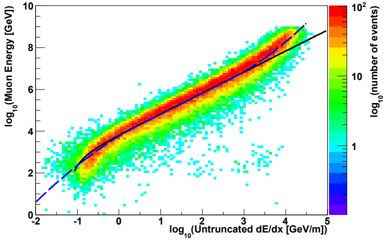

Figure: Untruncated energy loss per distance

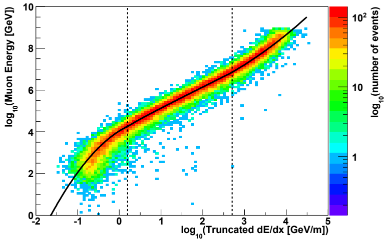

Figure: Truncated energy loss per distance

Selection of leading (quasi-single) muons based on energy loss characteristics

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- muons come in groups
- the muons loose energy independently of each other, smoothing out the energy loss pattern
- if a muon track has large energy losses, this cannot be the effect of multiple low-energy muons

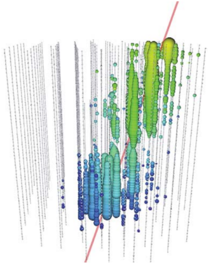

Figure: Simulated muon bundle event [42]

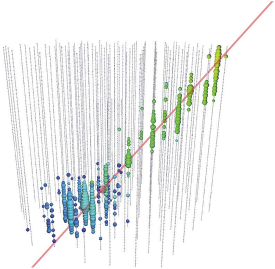

Figure: Simulated leading muon event [42]

Measuring muon cross sections in muon neutrino datasets

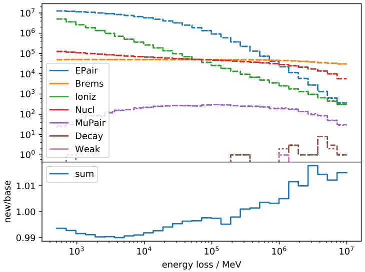

Figure: Energy loss distribution of 10^7 muons in ice [43]

Mean energy of muons in inclined air showers

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- in more densely instrumented detectors, such as the NEVOD-DECOR detector, the muon multiplicity can be measured directly
- using an estimator of the primary cosmic ray energy (local muon density), the mean energy of muons in the shower can be measured based on the energy losses
- this throws light on possible solutions to the so-called muon puzzle

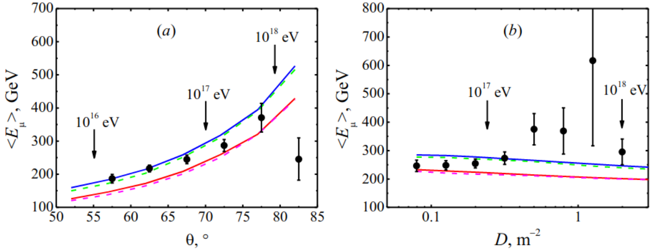

Figure: Mean energy of muons in inclined air showers in the NEVOD-DECOR detector [44]

Tau neutrino regeneration

DFG

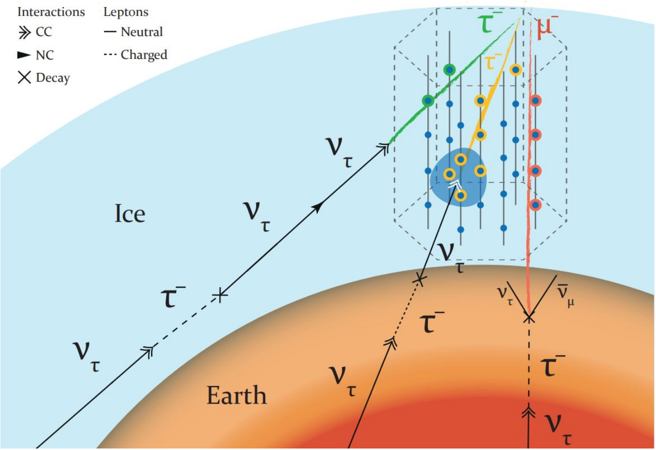

Figure: Schematic of tau neutrino regeneration [45]

Muography

DFG

- Muon flux variations trace changes in composition, integrated along particle track
- Muons are abundant penetrating particles, that can be used to investigate natural and artificial objects, e. g. volcanoes, blast furnaces or nuclear reactors

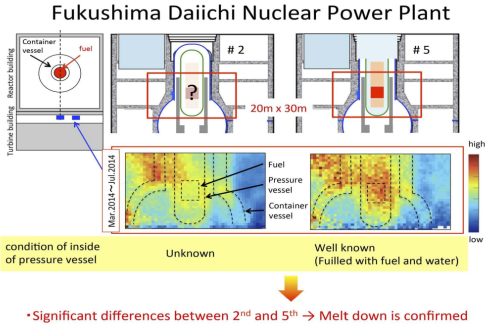

Figure: Muographic image of the nuclear reactor in Fukushima-Daichi [46]

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Summary

Summary

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- Lepton propagation is a central part of the simulation for practically every underground experiment, in particular VLVνT
- Accurate simulation of muon propagation is essential to muon energy reconstruction and thus to measuring muon and muon neutrino spectra
- Muon energy losses can shed light on the muon puzzle
- The propagation and decay of tau leptons opens the possibility to observe tau neutrinos at ultrahigh energies
- Muon propagation is the basis of muography applications

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