Gravitational Lensing -How to make dark matter visible

and what we learn about cosmology with this tool



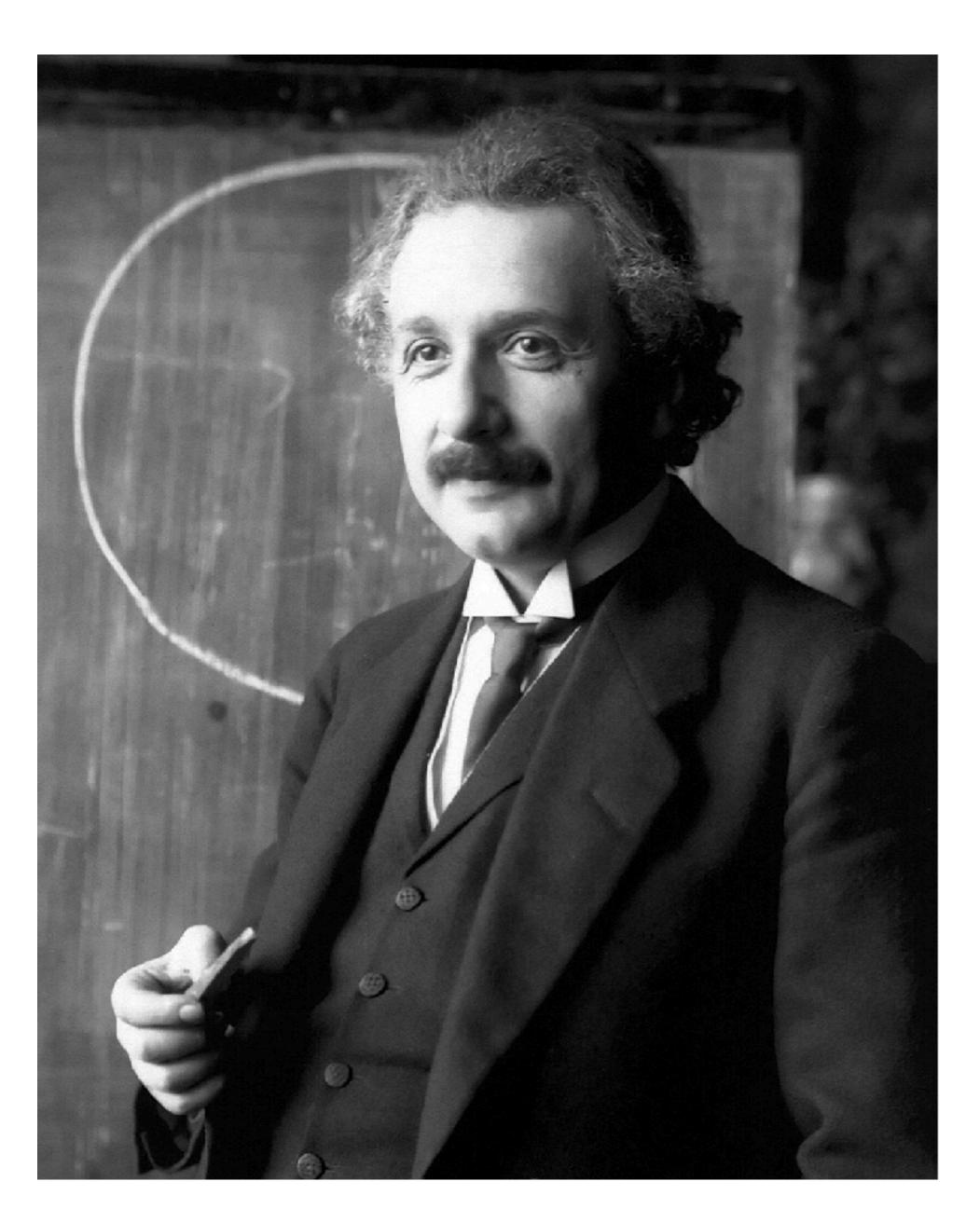


Established by the European Commission





Hendrik Hildebrandt, Ruhr University Bochum "Crossing the Desert", 21st May 2021



ANNALEN DER PHYSIK. VIERTE FOLGE. BAND 49.

1. Die Grundlage der allgemeinen Relativitätstheorie; von A. Einstein.

Die im nachfolgenden dargelegte Theorie bildet die denkbar weitgehendste Verallgemeinerung der heute allgemein als "Relativitätstheorie" bezeichneten Theorie; die letztere nenne ich im folgenden zur Unterscheidung von der ersteren "spezielle Relativitätstheorie" und setze sie als bekannt voraus. Die Verallgemeinerung der Relativitätstheorie wurde sehr erleichtert durch die Gestalt, welche der speziellen Relativitätstheorie durch Minkowski gegeben wurde, welcher Mathematiker zuerst die formale Gleichwertigkeit der räumlichen Koordinaten und der Zeitkoordinate klar erkannte und für den Aufbau der Theorie nutzbar machte. Die für die allgemeine Relativitätstheorie nötigen mathematischen Hilfsmittel lagen fertig bereit in dem "absoluten Differentialkalkül", welcher auf den Forschungen von Gauss, Riemann und Christoffel über nichteuklidische Mannigfaltigkeiten ruht und von Ricci und Levi-Civita in ein System gebracht und bereits auf Probleme der theoretischen Physik angewendet wurde. Ich habe im Abschnitt B der vorliegenden Abhandlung alle für uns nötigen, bei dem Physiker nicht als bekannt vorauszusetzenden mathematischen Hilfsmittel in möglichst einfacher und durchsichtiger Weise entwickelt, so daß ein Studium mathematischer Literatur für das Verständnis der vorliegenden Abhandlung nicht erforderlich ist. Endlich sei an dieser Stelle dankbar meines Freundes, des Mathematikers Grossmann, gedacht, der mir durch seine Hilfe nicht nur das Studium der einschlägigen mathematischen Literatur ersparte, sondern mich auch beim Suchen nach den Feldgleichungen der Gravitation unterstützte.

$$G_{\mu\nu} = \frac{8\pi G}{c^4} T_{\mu\nu}$$

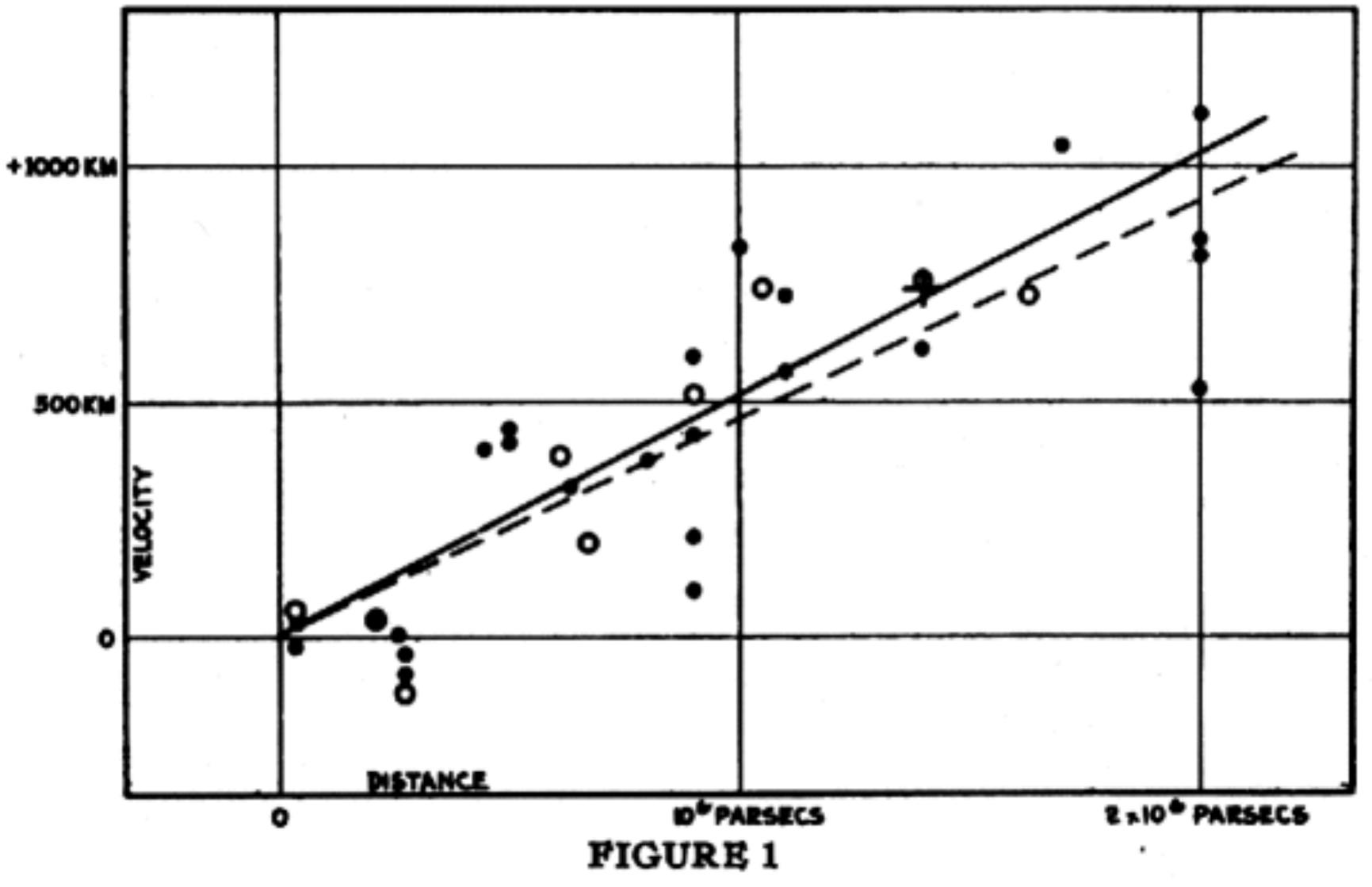
+ Robertson-Walker metric => evolving Universe (FLRW)

$G_{\mu\nu} + \Lambda g_{\mu\nu} = \frac{8\pi G}{c^4} T_{\mu\nu}$



$$v = H_0 D$$

$$H_0 \approx 70 \frac{\text{km}}{\text{s Mpc}}$$



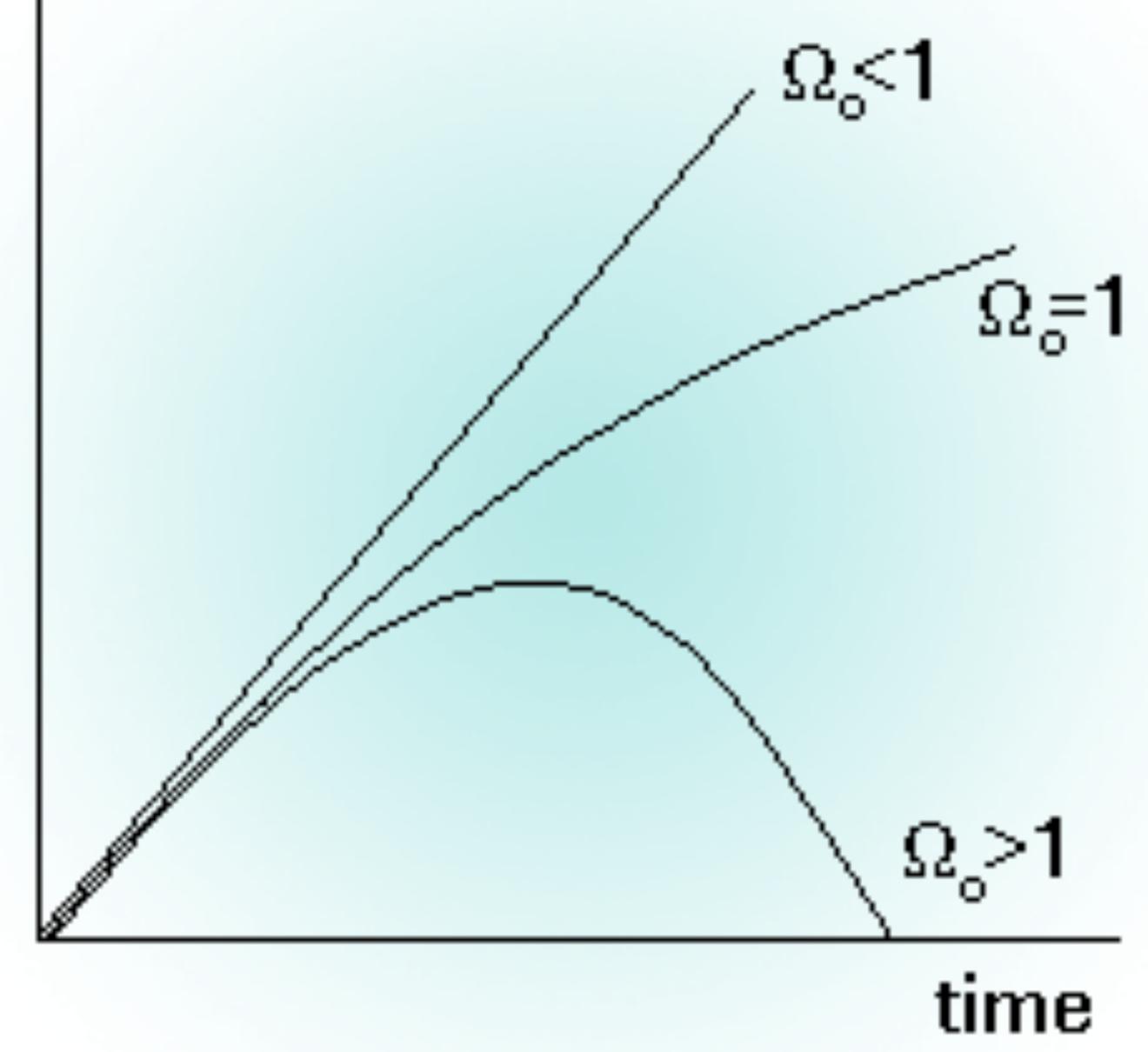
Velocity-Distance Relation among Extra-Galactic Nebulae.

Cosmological redshift Z

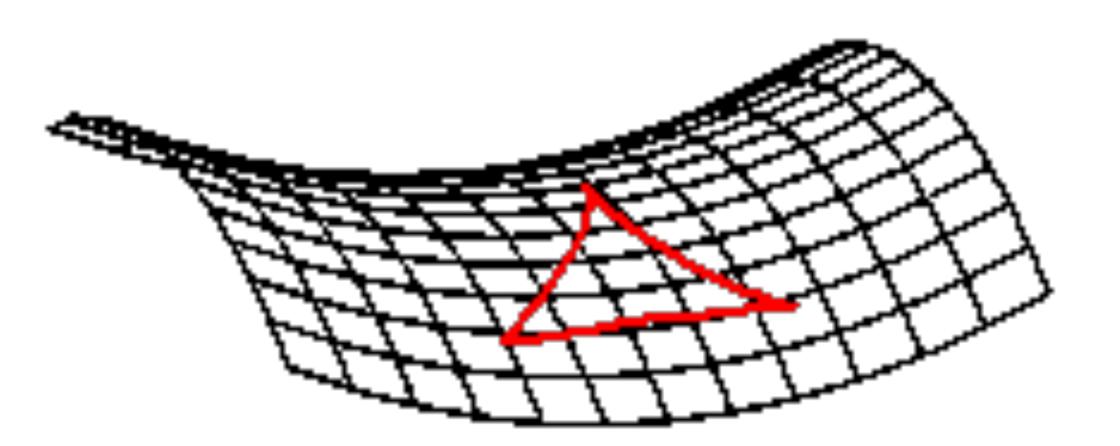
$$\frac{\lambda_{\text{obs}}}{\lambda_{\text{em}}} = (1 + z) = \frac{1}{a}$$

Expansion depends on contents

- Normal matter
- Electromagnetic radiation (only important in the early universe)

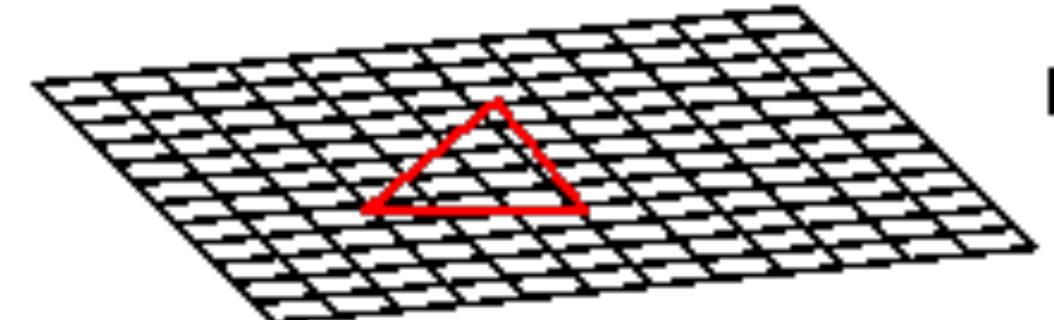


$\Omega_0 = 1$ critical density ~ 10 atoms per m³



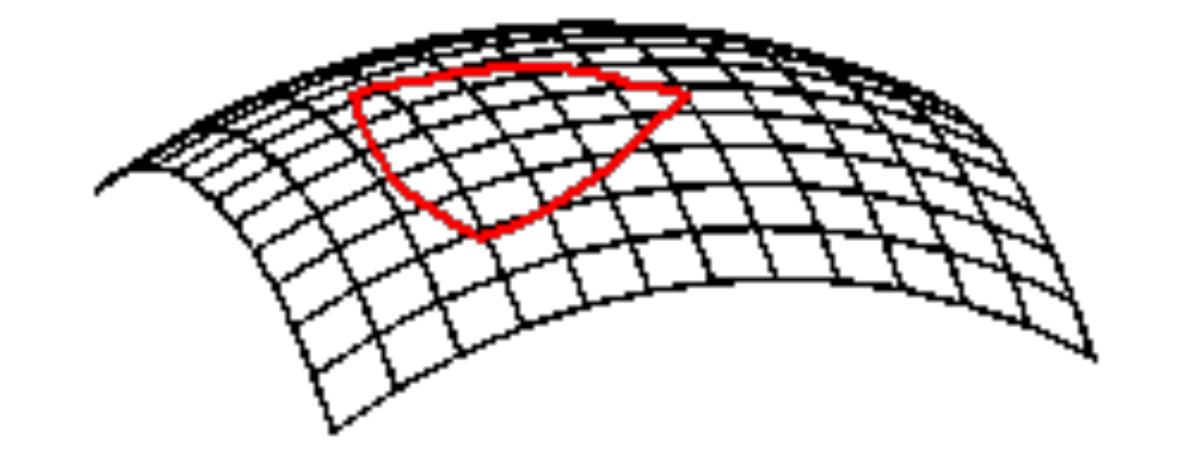
Open universe : looks like a horse saddle

 $\Omega_0 < 1$



Flat universe $\Omega_0=1$

Closed universe: looks like the surface of a sphere

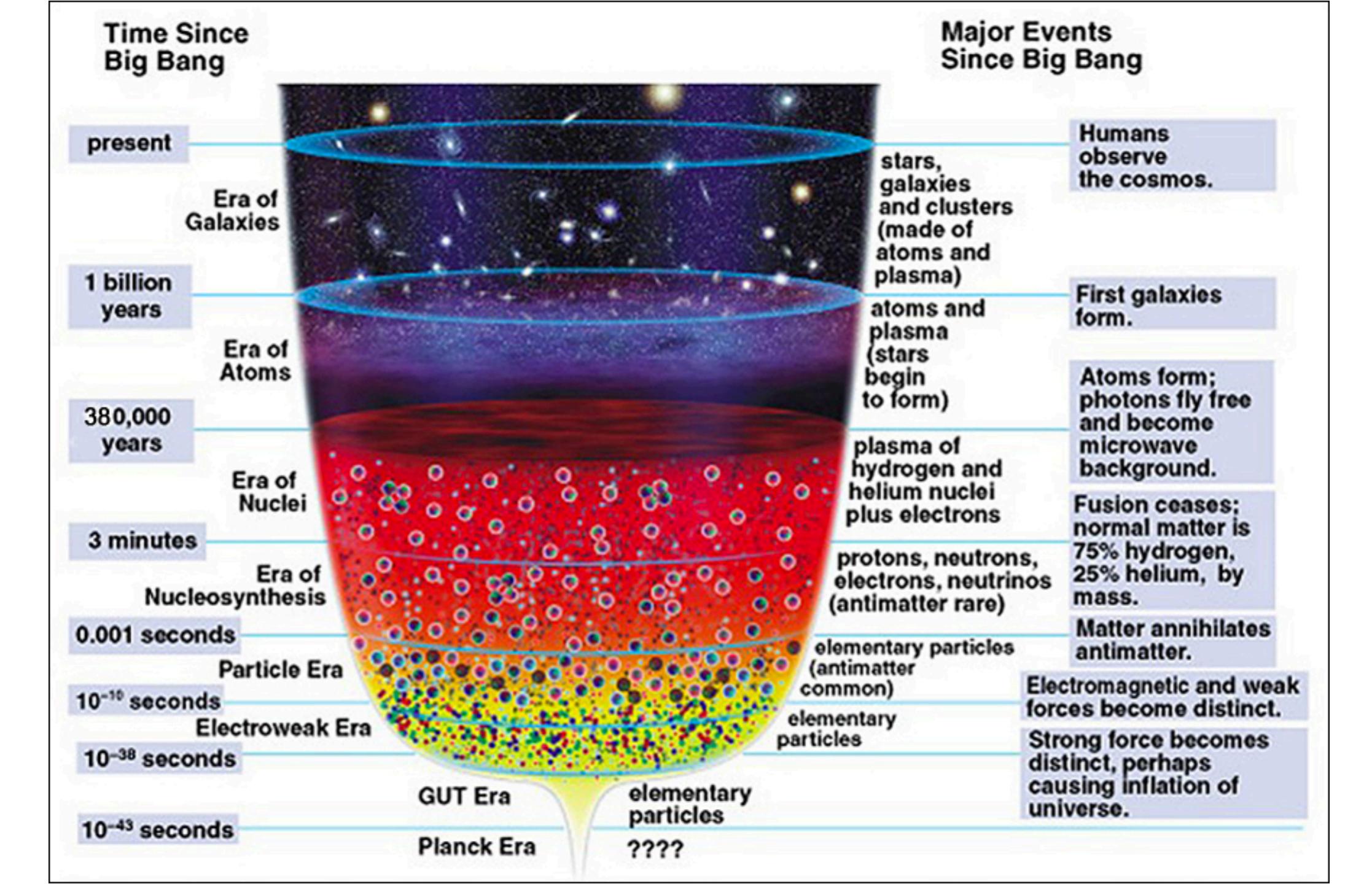


 $\Omega_0 > 1$

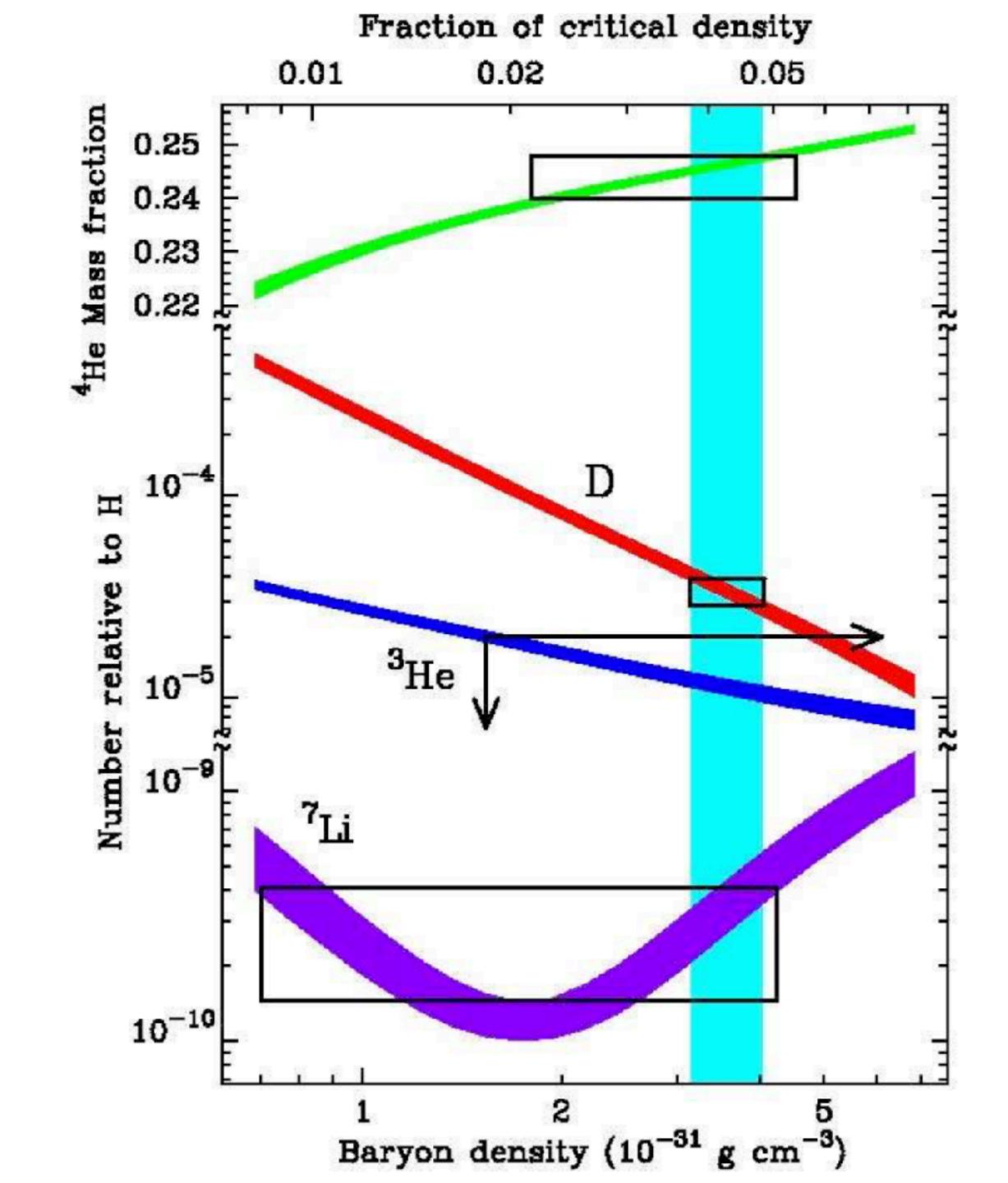


Hot big bang

$T \propto a^{-1} = (1 + z)$



Normal (baryonic) matter



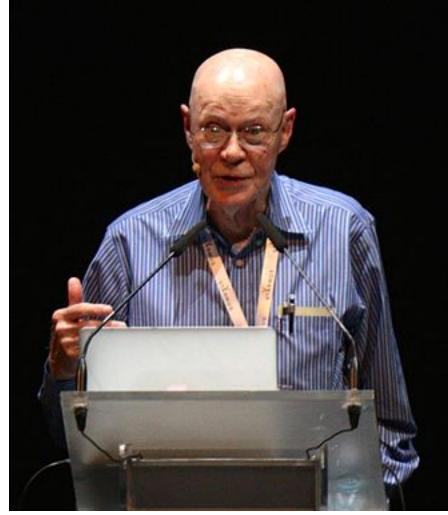
Normal (baryonic) matter

$$\Omega_{b} = 0.05$$

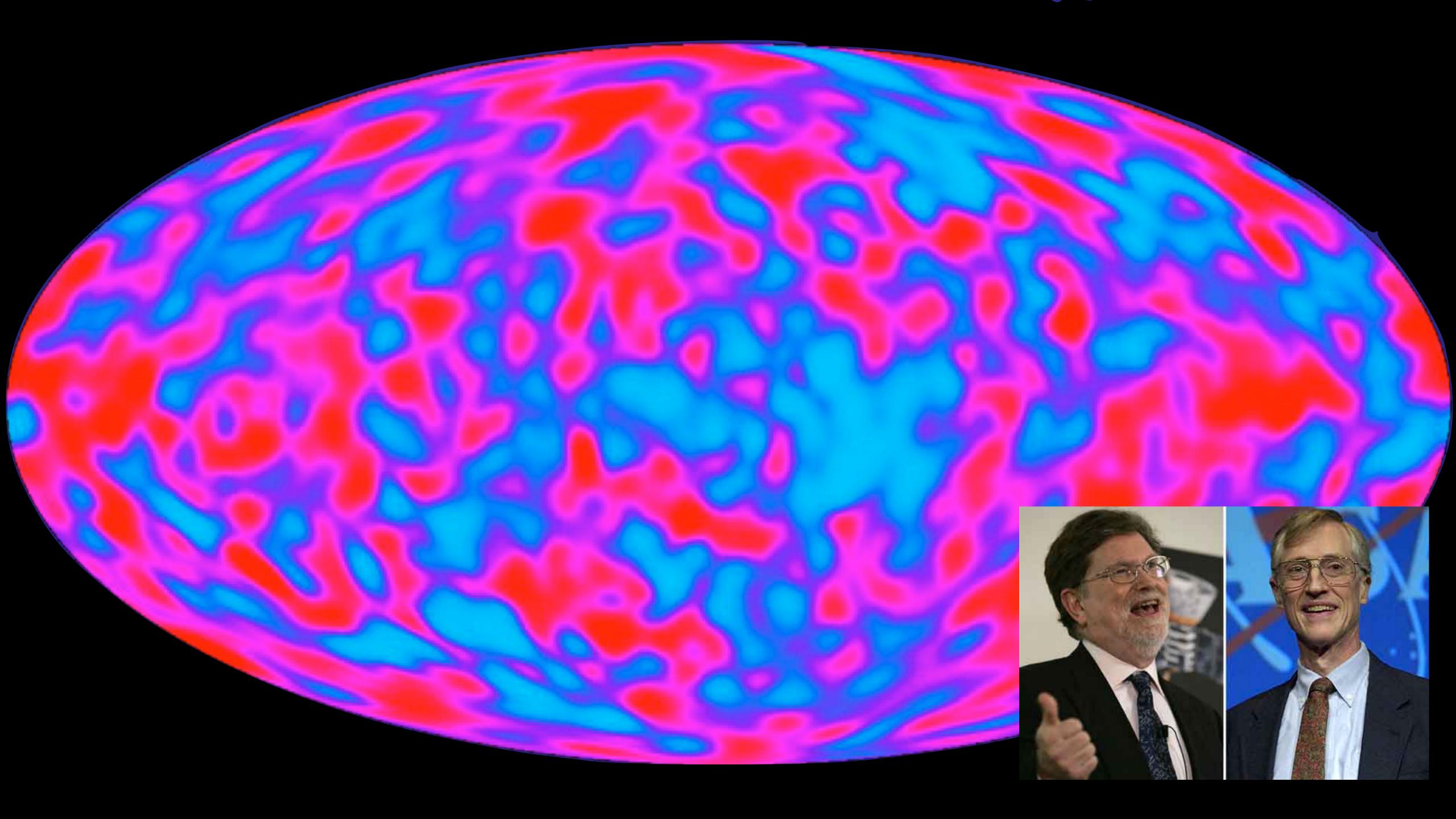
Recombination / CMB

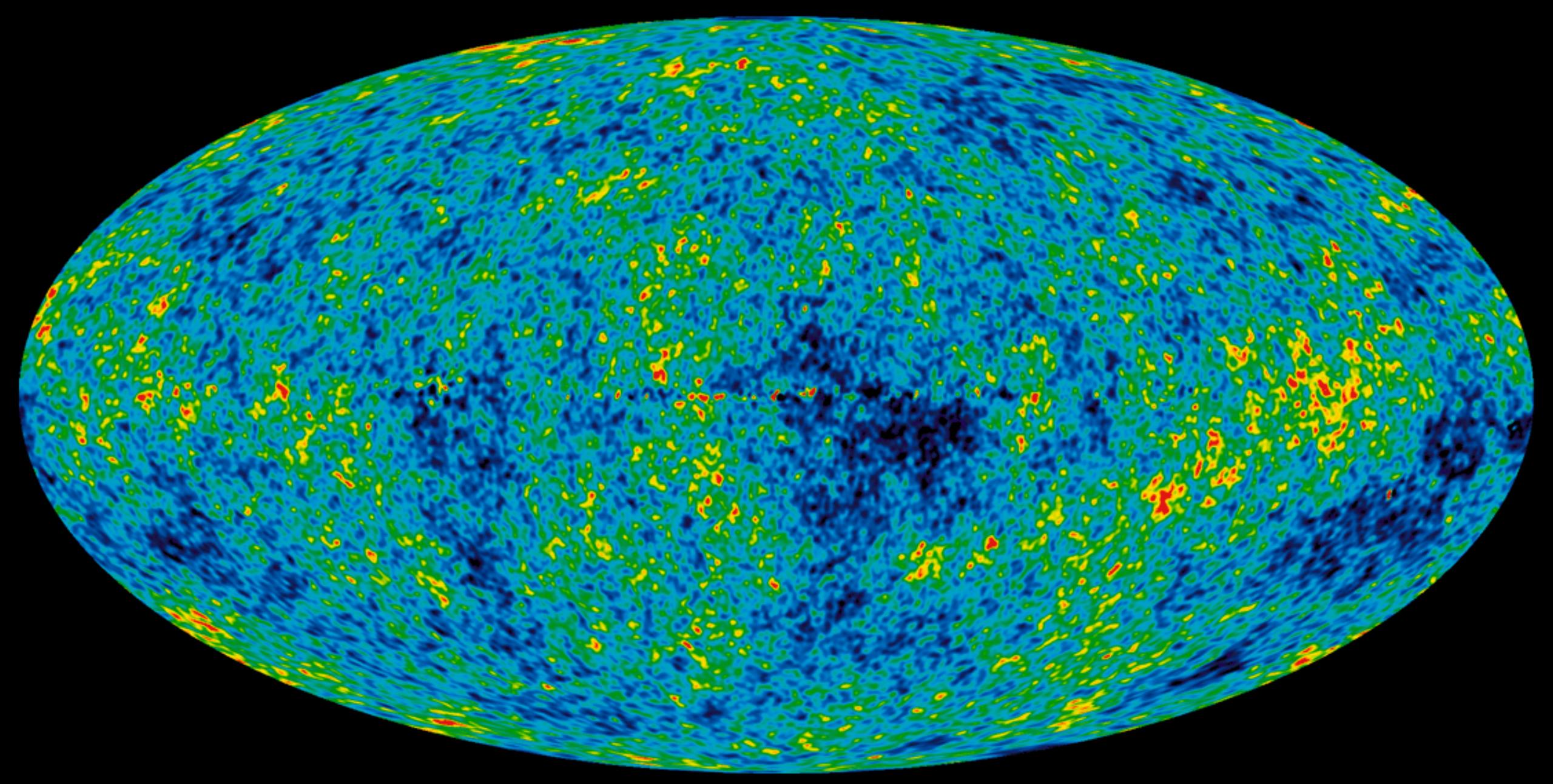
- 400,000 years after big bang temperature low enough for atoms to form
- Before: opaque plasma
- After: transparent neutral gas
- Light emitted from this time (z~1100) can still be seen today
- Black body radiation with temperature of 2.7K
- Tiny fluctuations in the temperature -> seeds of structure formation

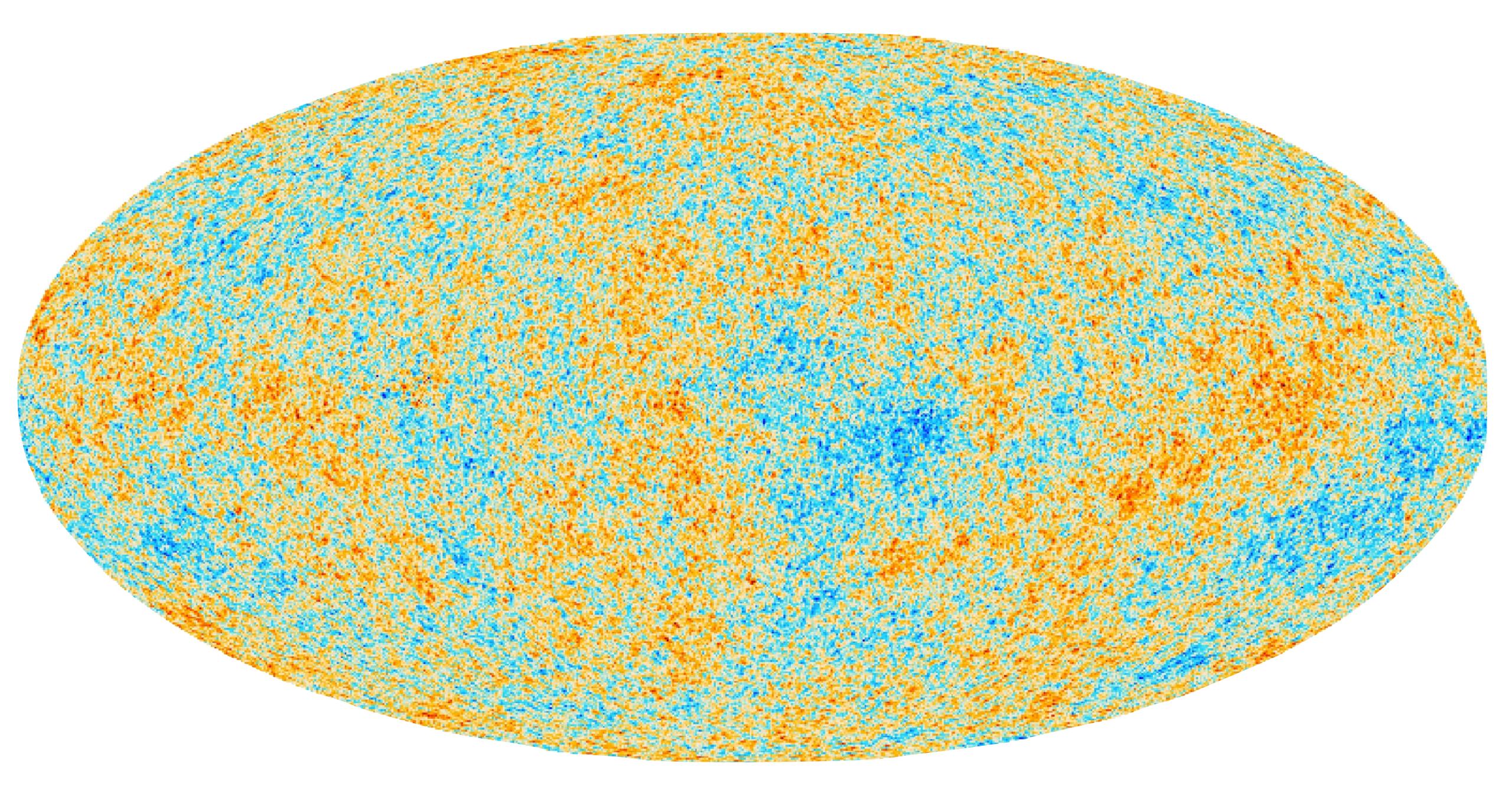




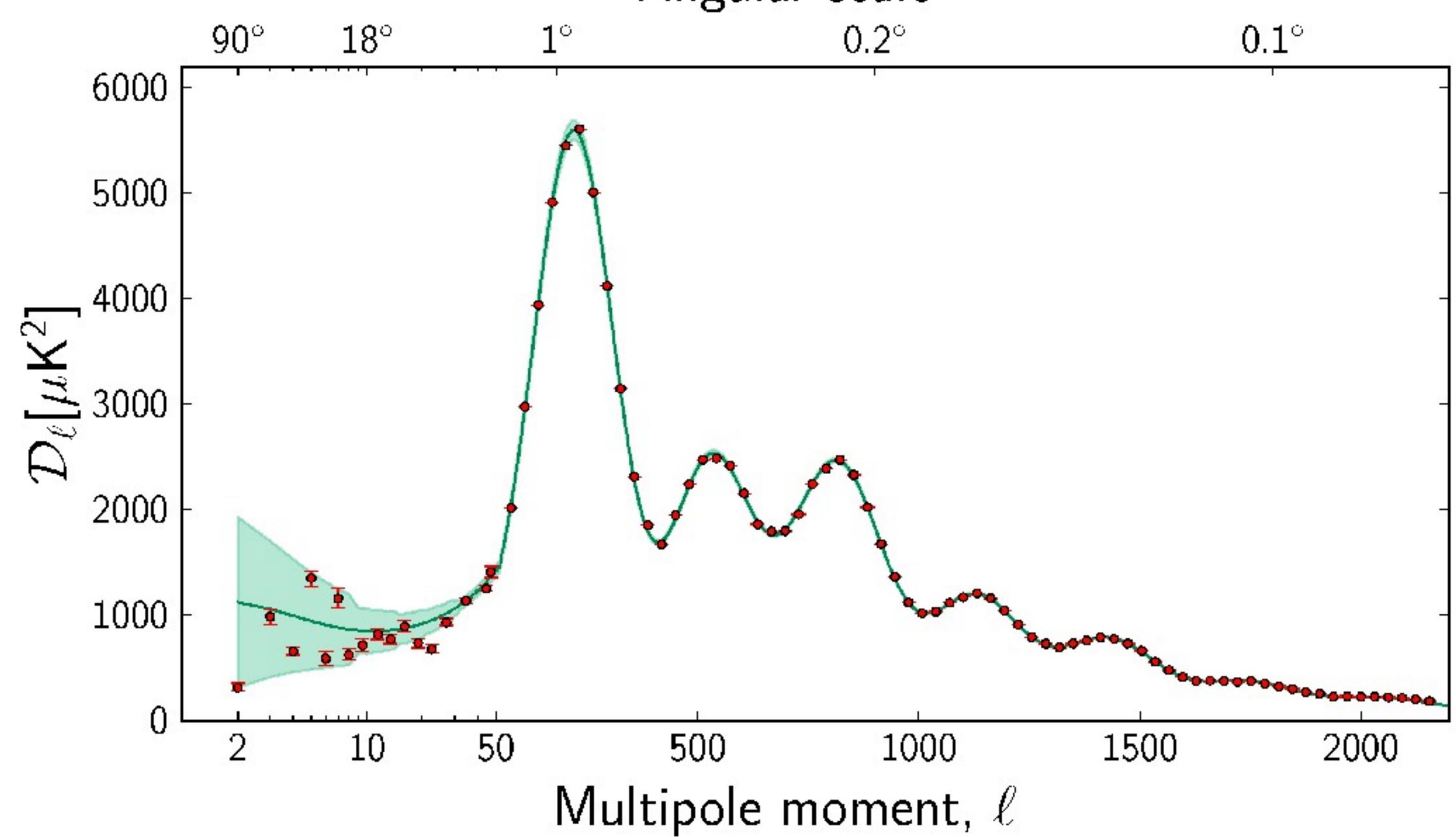






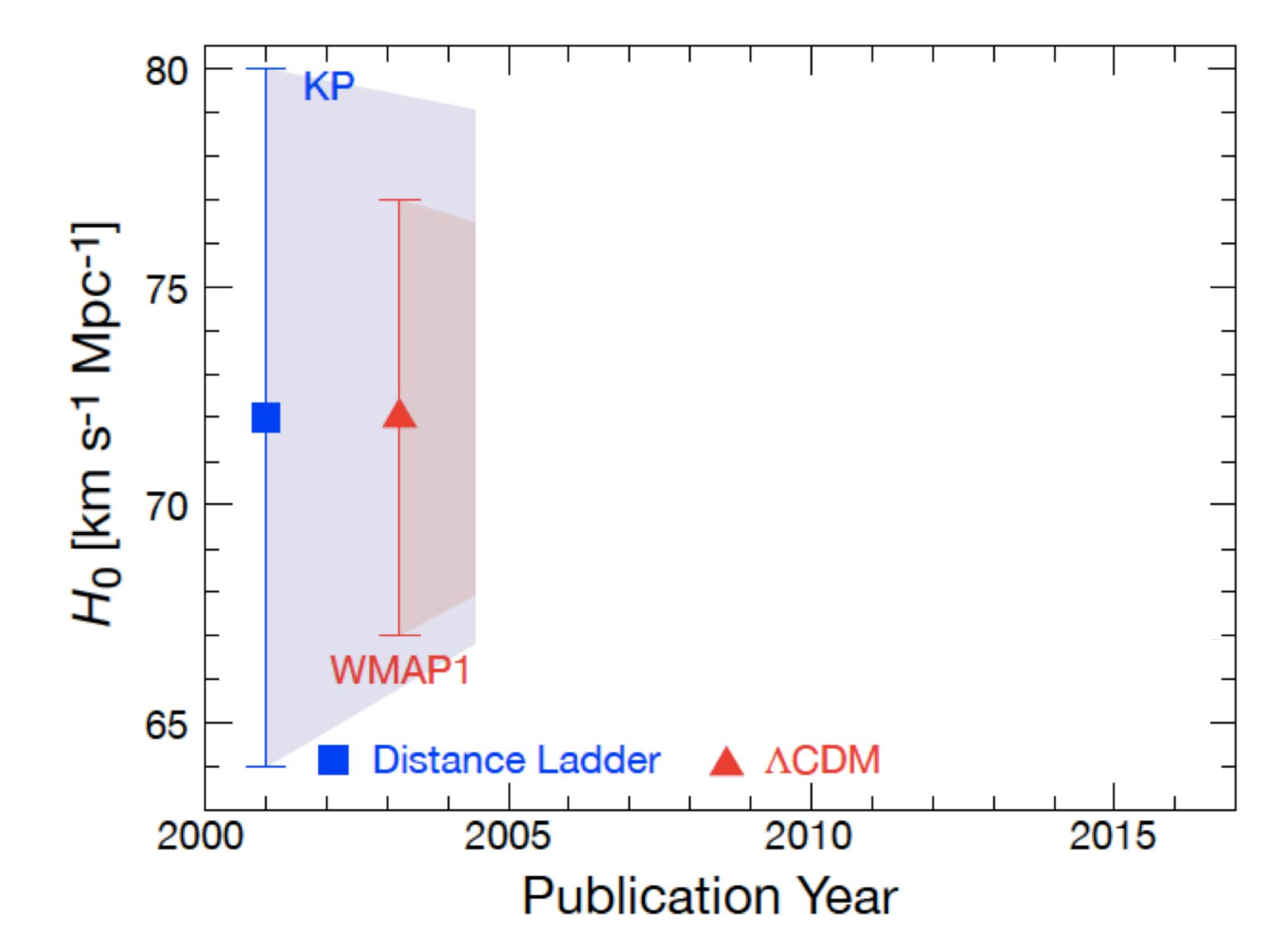


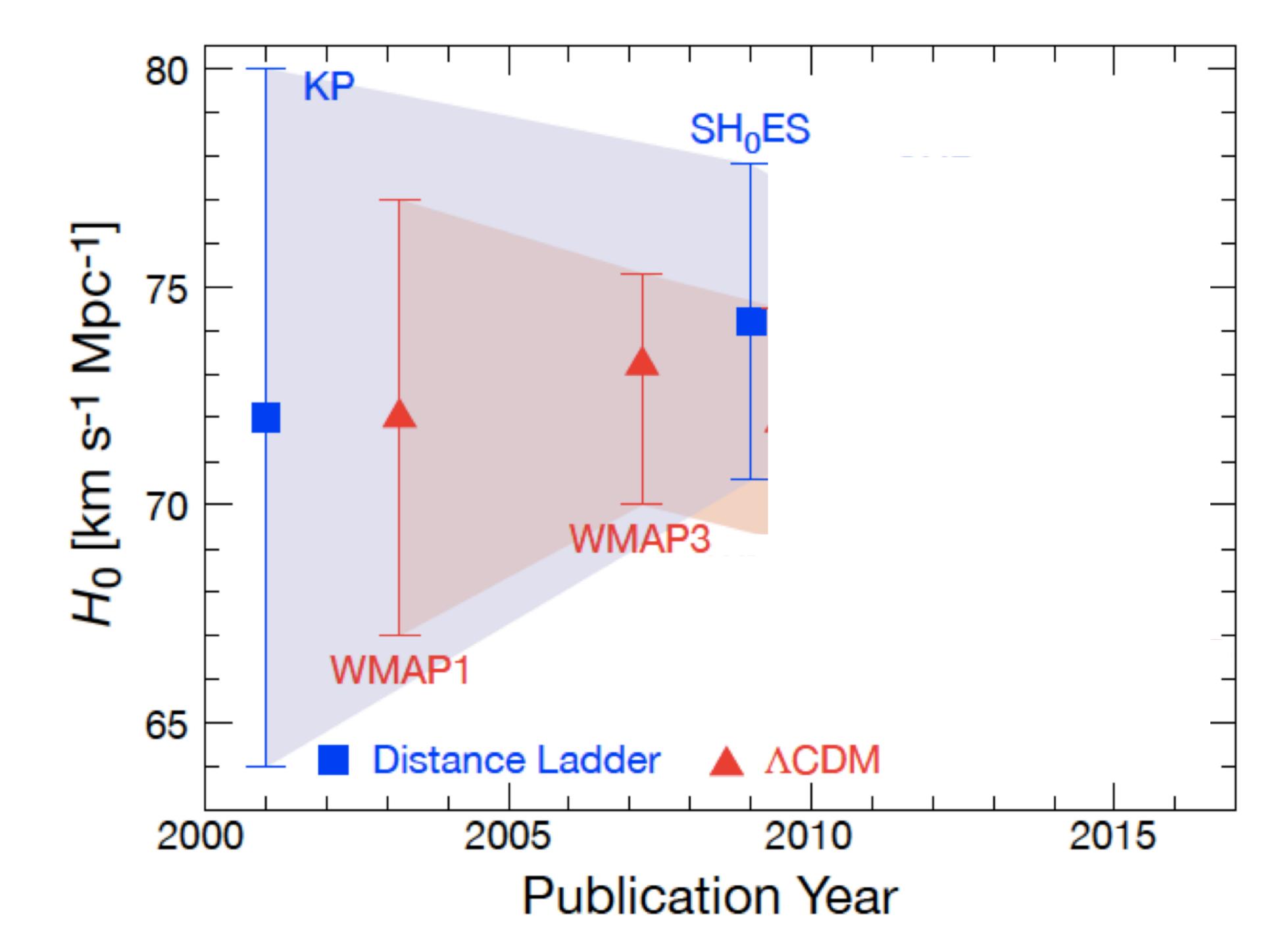
Angular scale

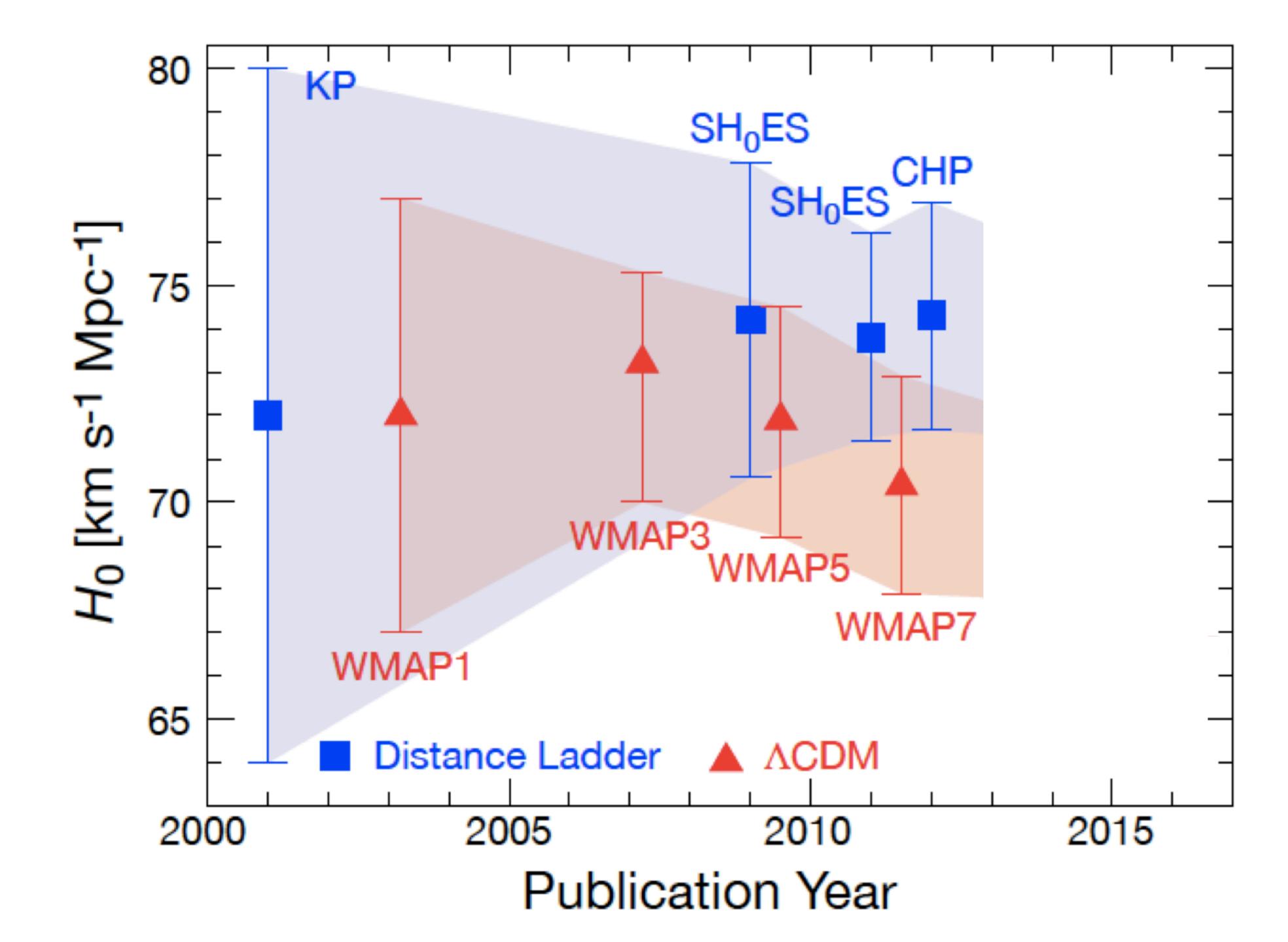


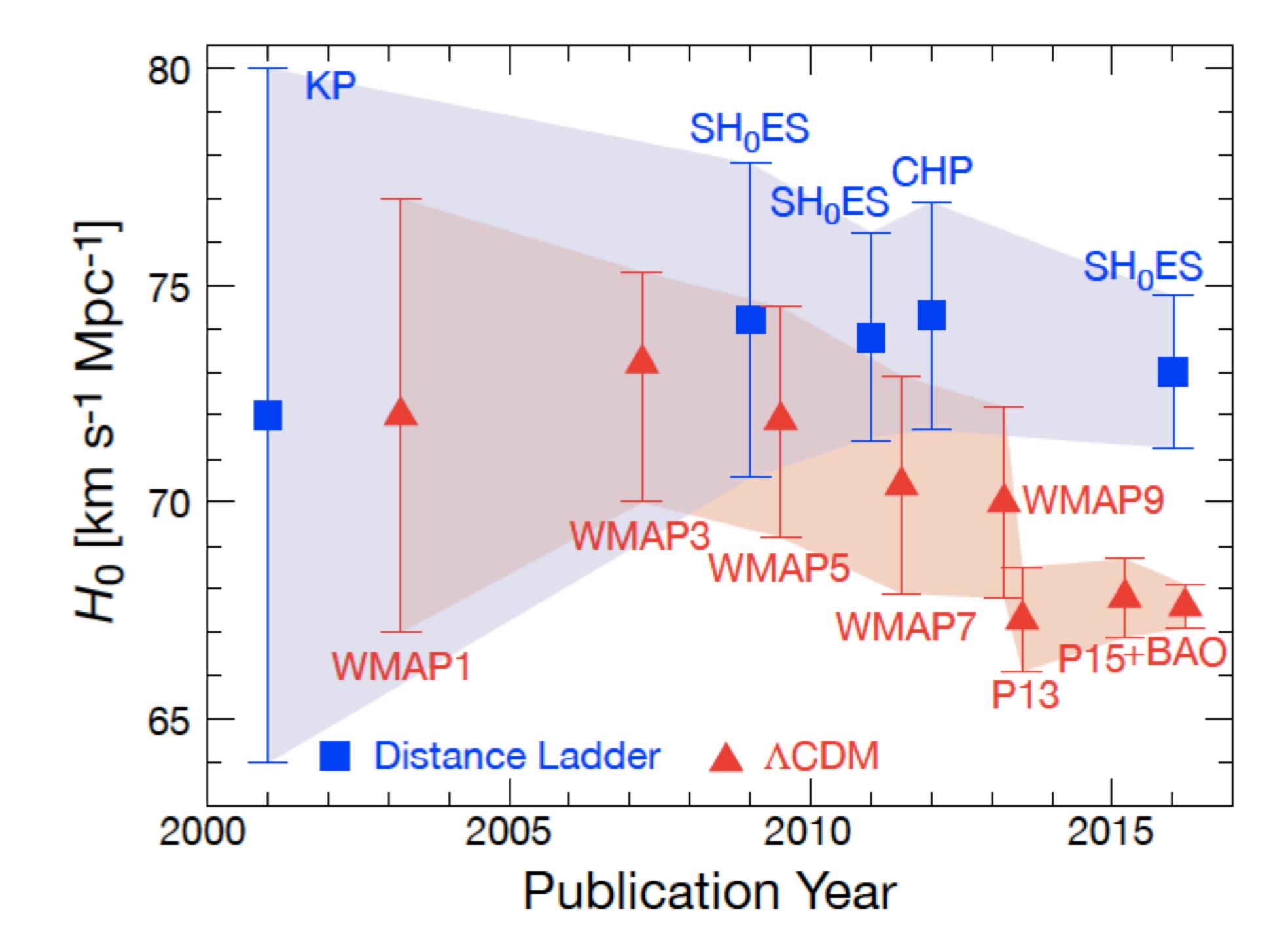
Parameter	TT+lowP 68 % limits	TT+lowP+lensing 68 % limits	TT+lowP+lensing+ext 68 % limits	TT,TE,EE+lowP 68 % limits	TT,TE,EE+lowP+lensing 68 % limits	TT,TE,EE+lowP+lensing+ext 68 % limits
$\Omega_{ m b} h^2 \ldots \ldots$	0.02222 ± 0.00023	0.02226 ± 0.00023	0.02227 ± 0.00020	0.02225 ± 0.00016	0.02226 ± 0.00016	0.02230 ± 0.00014
$\Omega_{ m c}h^2$	0.1197 ± 0.0022	0.1186 ± 0.0020	0.1184 ± 0.0012	0.1198 ± 0.0015	0.1193 ± 0.0014	0.1188 ± 0.0010
$100\theta_{\mathrm{MC}}$	1.04085 ± 0.00047	1.04103 ± 0.00046	1.04106 ± 0.00041	1.04077 ± 0.00032	1.04087 ± 0.00032	1.04093 ± 0.00030
au	0.078 ± 0.019	0.066 ± 0.016	0.067 ± 0.013	0.079 ± 0.017	0.063 ± 0.014	0.066 ± 0.012
$\ln(10^{10}A_{\rm s})\ldots\ldots\ldots$	3.089 ± 0.036	3.062 ± 0.029	3.064 ± 0.024	3.094 ± 0.034	3.059 ± 0.025	3.064 ± 0.023
$n_{\rm s}$	0.9655 ± 0.0062	0.9677 ± 0.0060	0.9681 ± 0.0044	0.9645 ± 0.0049	0.9653 ± 0.0048	0.9667 ± 0.0040
$\overline{H_0 \ldots \ldots \ldots \ldots}$	67.31 ± 0.96	67.81 ± 0.92	67.90 ± 0.55	67.27 ± 0.66	67.51 ± 0.64	67.74 ± 0.46
Ω_{Λ}	0.685 ± 0.013	0.692 ± 0.012	0.6935 ± 0.0072	0.6844 ± 0.0091	0.6879 ± 0.0087	0.6911 ± 0.0062
Ω_{m}	0.315 ± 0.013	0.308 ± 0.012	0.3065 ± 0.0072	0.3156 ± 0.0091	0.3121 ± 0.0087	0.3089 ± 0.0062
$\Omega_{-}L^{2}$	0.1426 + 0.0020	-0.1415 ± 0.0019	0 1413 + 0 0011	-0.1427 ± 0.0014	0.1422 + 0.0012	0 14170 - 0 00007
$\Omega_{\mathrm{m}}h^{3}$	0.09597 ± 0.00045	0.09591 ± 0.00045	0.09593 ± 0.00045	0.09601 ± 0.00029	0.09596 ± 0.00030	0.09598 ± 0.00029
σ_8	0.829 ± 0.014	0.8149 ± 0.0093	0.8154 ± 0.0090	0.831 ± 0.013	0.8150 ± 0.0087	0.8159 ± 0.0086
$\sigma_8\Omega_{ m m}^{0.5}$	0.466 ± 0.013	0.4521 ± 0.0088	0.4514 ± 0.0066	0.4668 ± 0.0098	0.4553 ± 0.0068	0.4535 ± 0.0059
$\sigma_{\rm e}O^{0.25}$	0.621 ± 0.013	<u> </u>	0 6066 + 0 0070	0 622 + 0 011	0 (001 - 000/7	0.6002 0.0066
$z_{ m re}$	$9.9^{+1.8}_{-1.6}$	$8.8^{+1.7}_{-1.4}$	$8.9^{+1.3}_{-1.2}$	$10.0^{+1.7}_{-1.5}$	$8.5^{+1.4}_{-1.2}$	$8.8^{+1.2}_{-1.1}$
$10^9 A_{\rm s}$	$2.198^{+0.076}_{-0.085}$	2.139 ± 0.063	2.143 ± 0.051	2.207 ± 0.074	2.130 ± 0.053	2.142 ± 0.049
$10^9 A_{\rm s} e^{-2\tau} \dots \dots$	1.880 ± 0.014	1.874 ± 0.013	1.873 ± 0.011	1.882 ± 0.012	1.878 ± 0.011	1.876 ± 0.011
Age/Gyr	13.813 ± 0.038	13.799 ± 0.038	13.796 ± 0.029	13.813 ± 0.026	13.807 ± 0.026	13.799 ± 0.021
Z*	1090.09 ± 0.42	1089.94 ± 0.42	1089.90 ± 0.30	1090.06 ± 0.30	1090.00 ± 0.29	1089.90 ± 0.23
r_*	144.61 ± 0.49	144.89 ± 0.44	144.93 ± 0.30	144.57 ± 0.32	144.71 ± 0.31	144.81 ± 0.24
$100\theta_*$	1.04105 ± 0.00046	1.04122 ± 0.00045	1.04126 ± 0.00041	1.04096 ± 0.00032	1.04106 ± 0.00031	1.04112 ± 0.00029
Zdrag · · · · · · · · · · · · · · · · · · ·	1059.57 ± 0.46	1059.57 ± 0.47	1059.60 ± 0.44	1059.65 ± 0.31	1059.62 ± 0.31	1059.68 ± 0.29
$r_{\rm drag}$	147.33 ± 0.49	147.60 ± 0.43	147.63 ± 0.32	147.27 ± 0.31	147.41 ± 0.30	147.50 ± 0.24
k_{D}	0.14050 ± 0.00052	0.14024 ± 0.00047	0.14022 ± 0.00042	0.14059 ± 0.00032	0.14044 ± 0.00032	0.14038 ± 0.00029
$z_{\rm eq}$	3393 ± 49	3365 ± 44	3361 ± 27	3395 ± 33	3382 ± 32	3371 ± 23
k_{eq}	0.01035 ± 0.00015	0.01027 ± 0.00014	0.010258 ± 0.000083	0.01036 ± 0.00010	0.010322 ± 0.000096	0.010288 ± 0.000071
$100\theta_{\rm s,eq}$		0.4529 ± 0.0044	0.4533 ± 0.0026	0.4499 ± 0.0032	0.4512 ± 0.0031	0.4523 ± 0.0023
5,04				0 = 0.00 <i>E</i> =	0.1512 = 0.0051	0.1323 ± 0.0023

Planck 2015 results XIII.

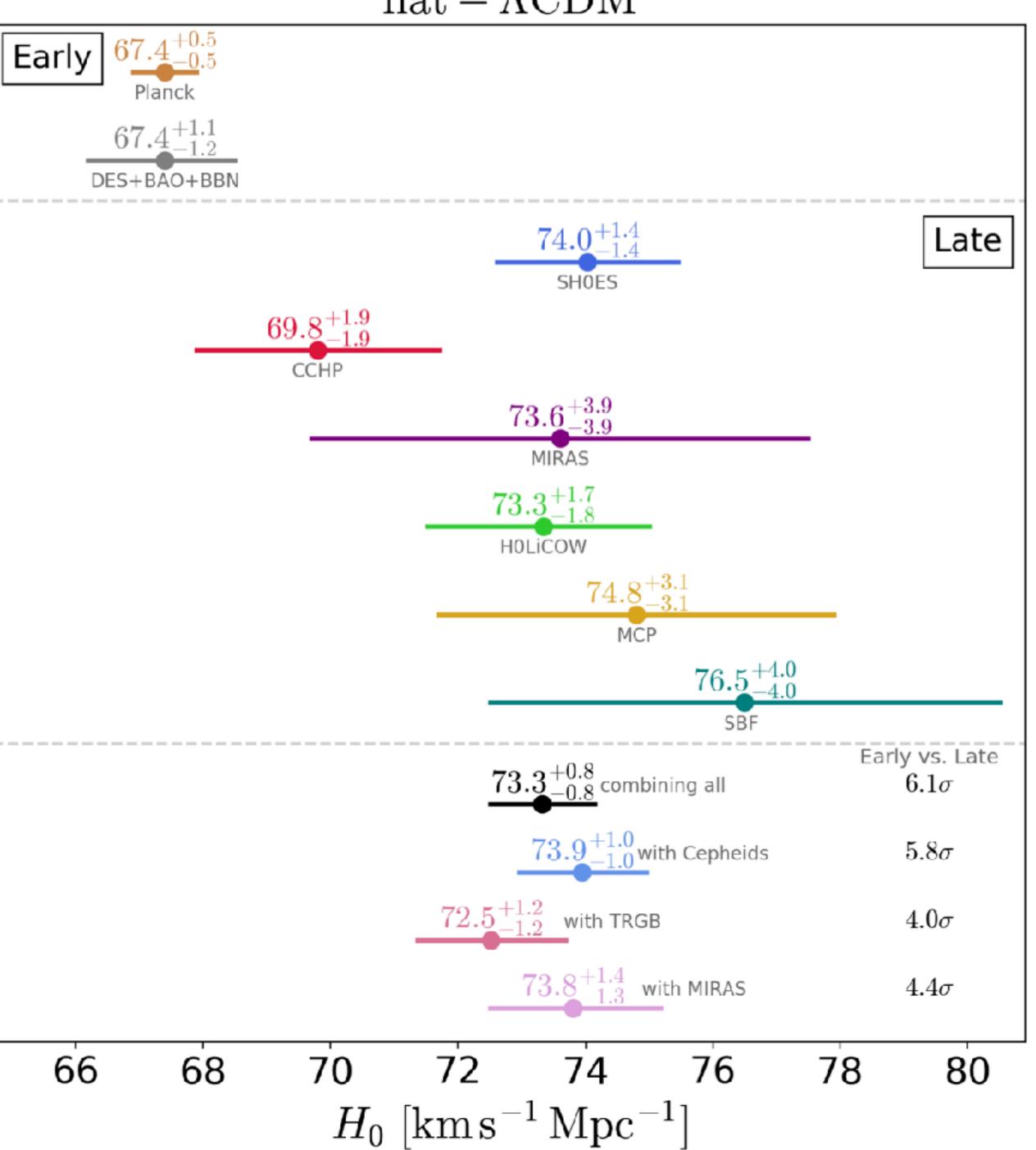








$flat - \Lambda CDM$

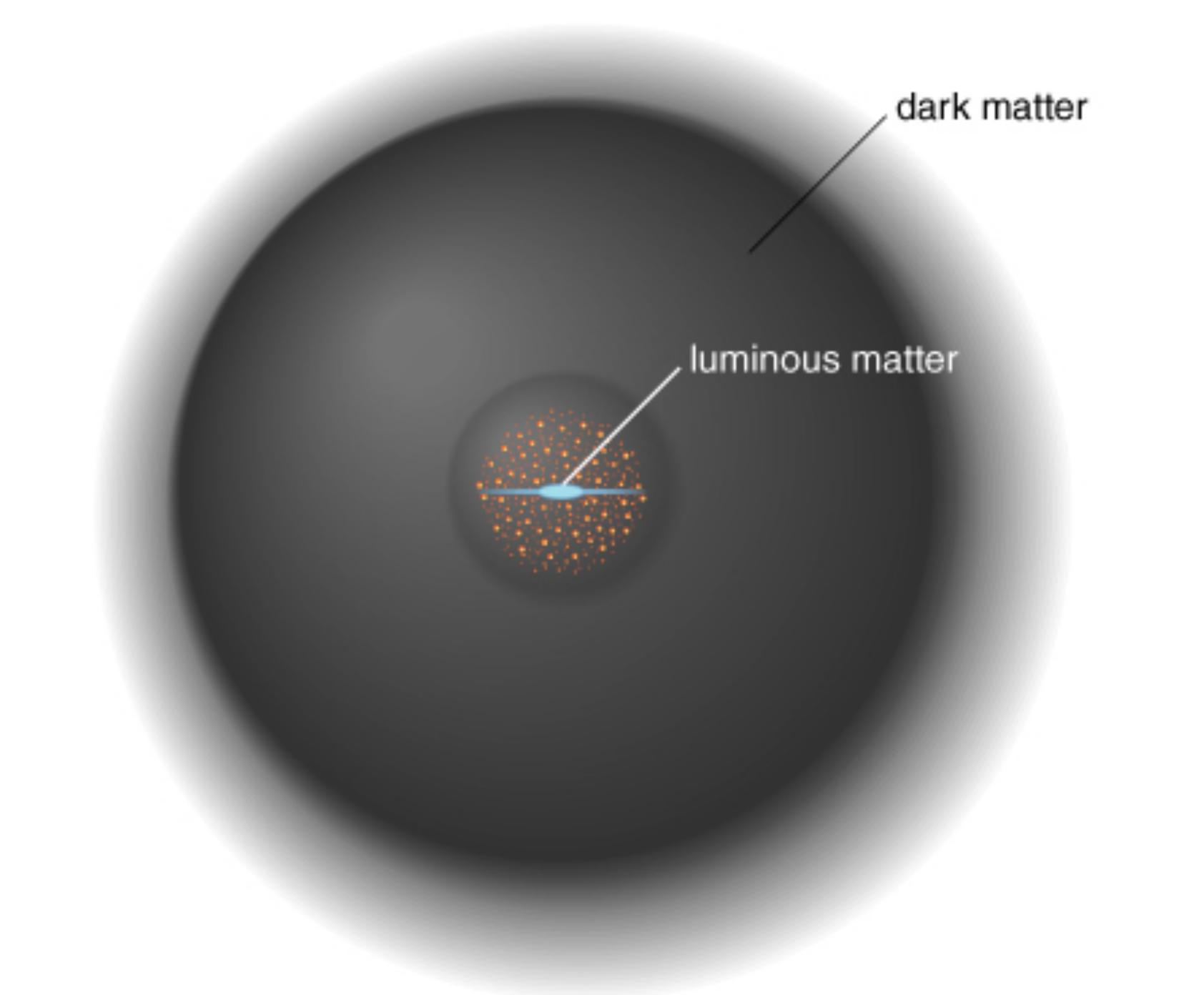


Normal (baryonic) matter

Open Universe?







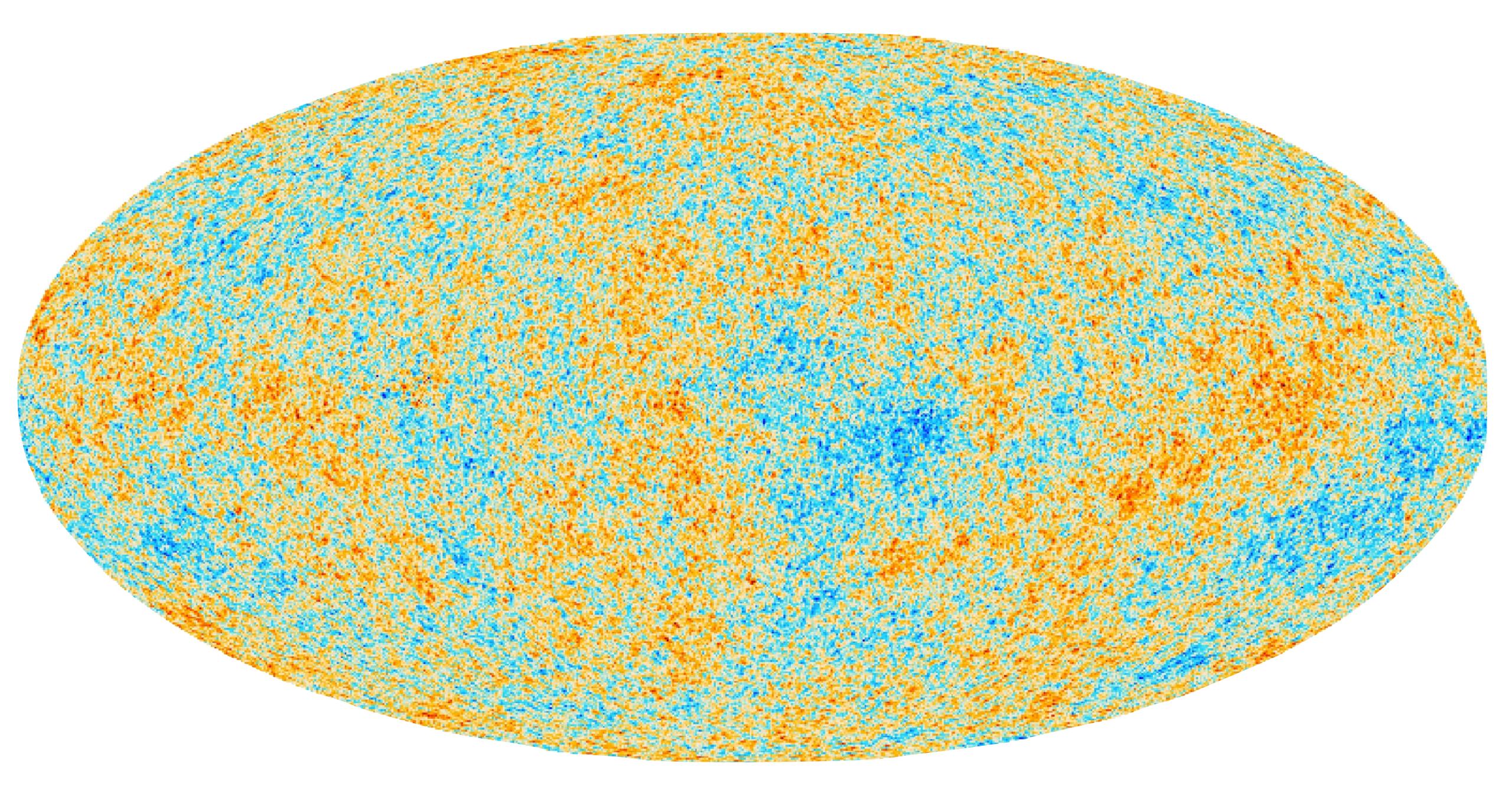
Expansion depends on contents

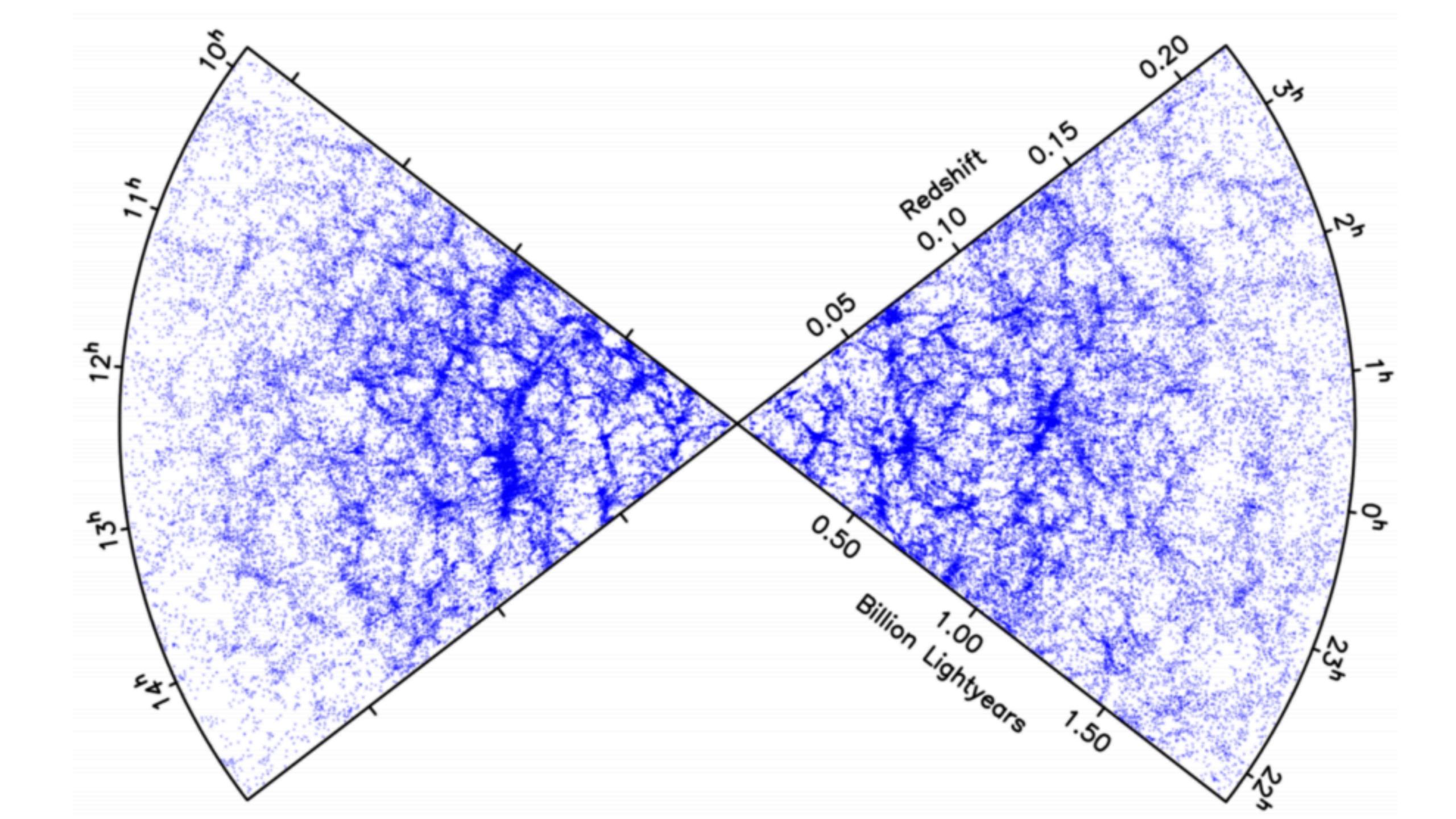
- Normal matter
- Electromagnetic radiation
- Dark matter

Dark Matter



Cosmic structure formation





Structure formation

- Small fluctuations around the mean cosmic density present at early times
- Overdensities/underdensities grow over time due to gravitational instability
- Can be described mathematically with perturbation theory
- Growth depends on cosmological parameters
- High-density regions need to be simulated
- Structure growth is a very rich resource of cosmological information

Dark Matter

$$\Omega_{dm} = 0.25$$

Dark Matter

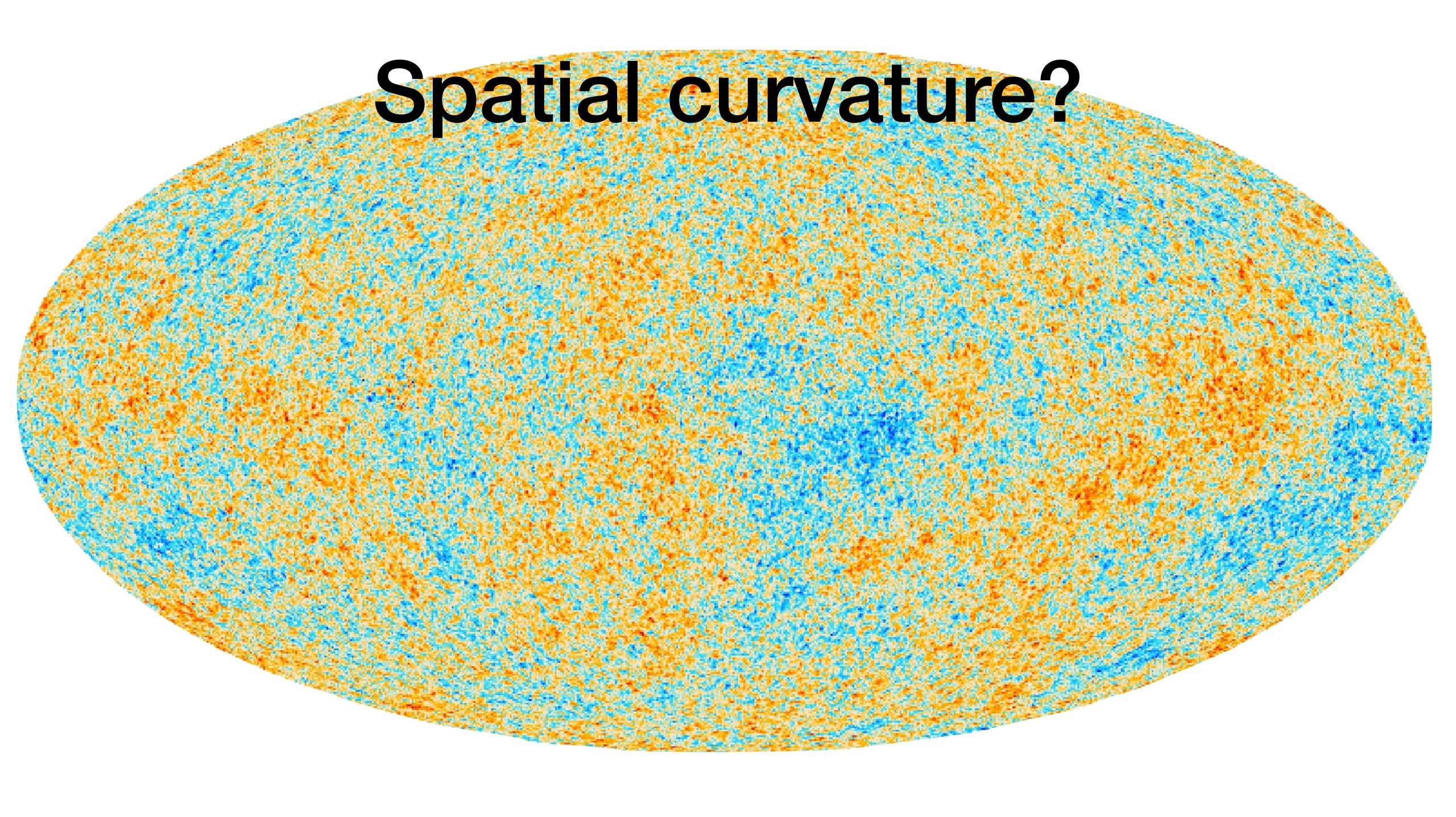
- Collisionless
- Dissipationless
- Cold
- Just weak interaction and gravity
- WIMPs? Axions? Sterile Neutrinos?

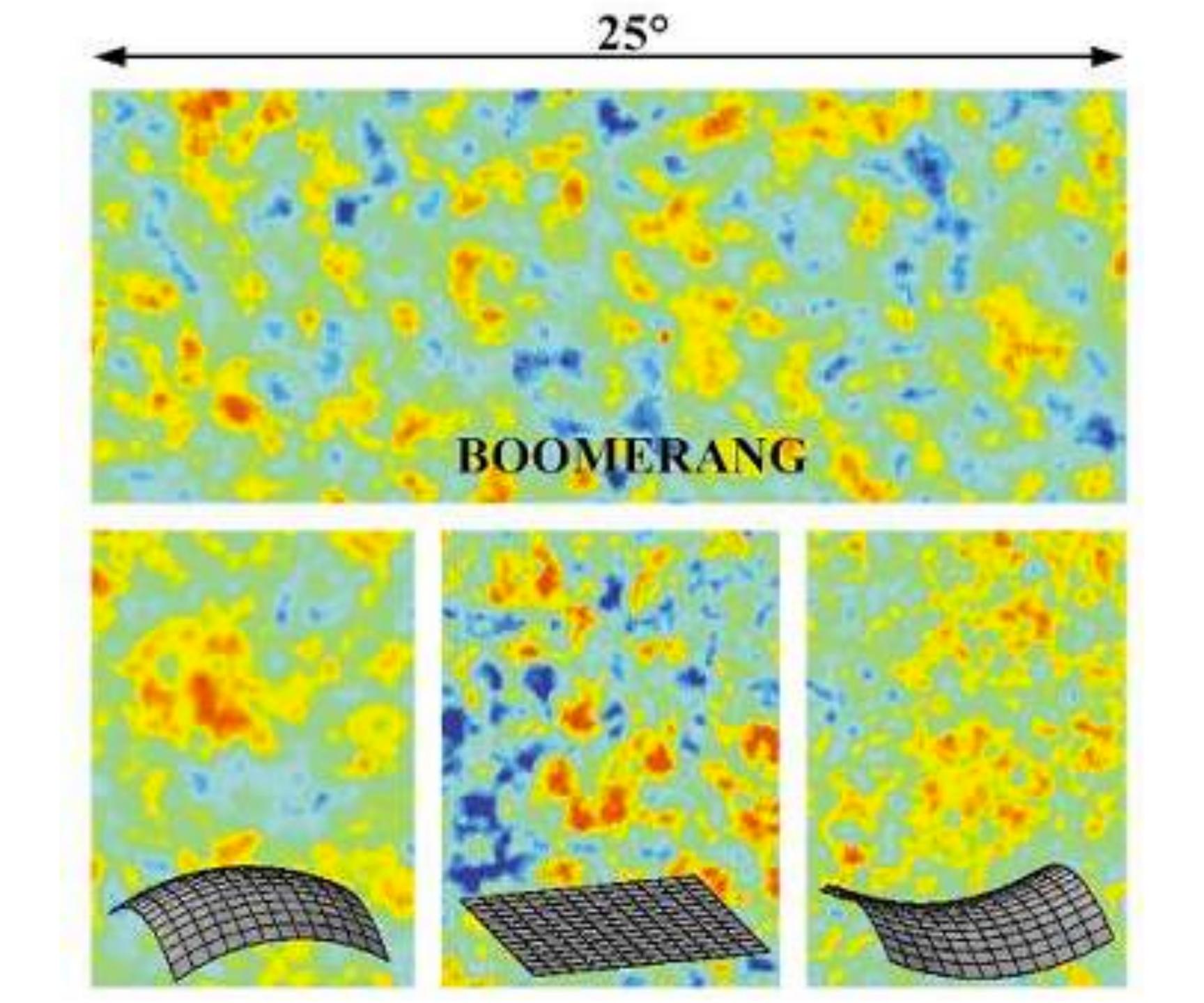
• Alternative: Modification of general relativity

Total matter density

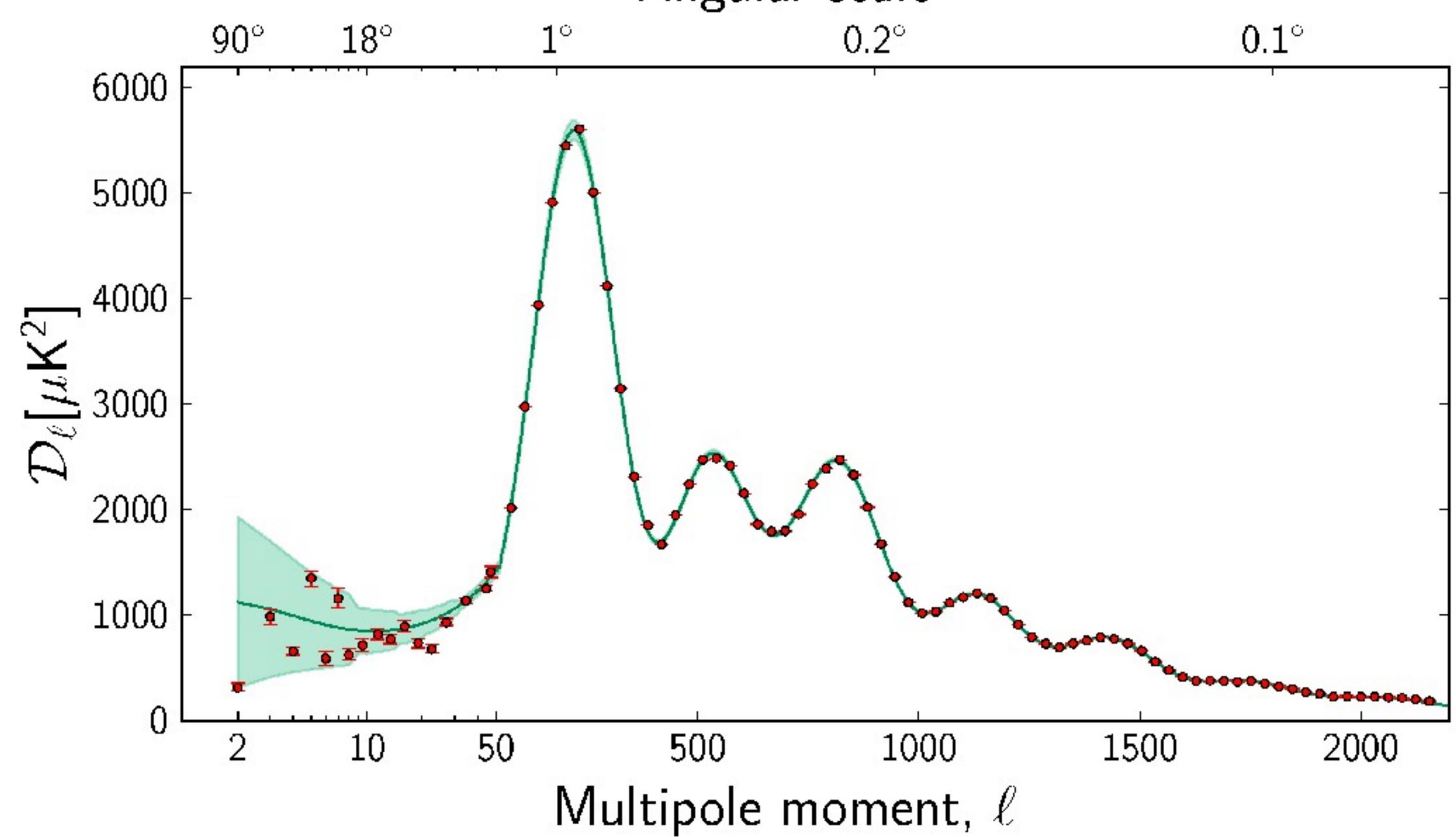
$$\Omega_{\rm m} = \Omega_{\rm b} + \Omega_{\rm dm} = 0.3$$

Open Universe?





Angular scale



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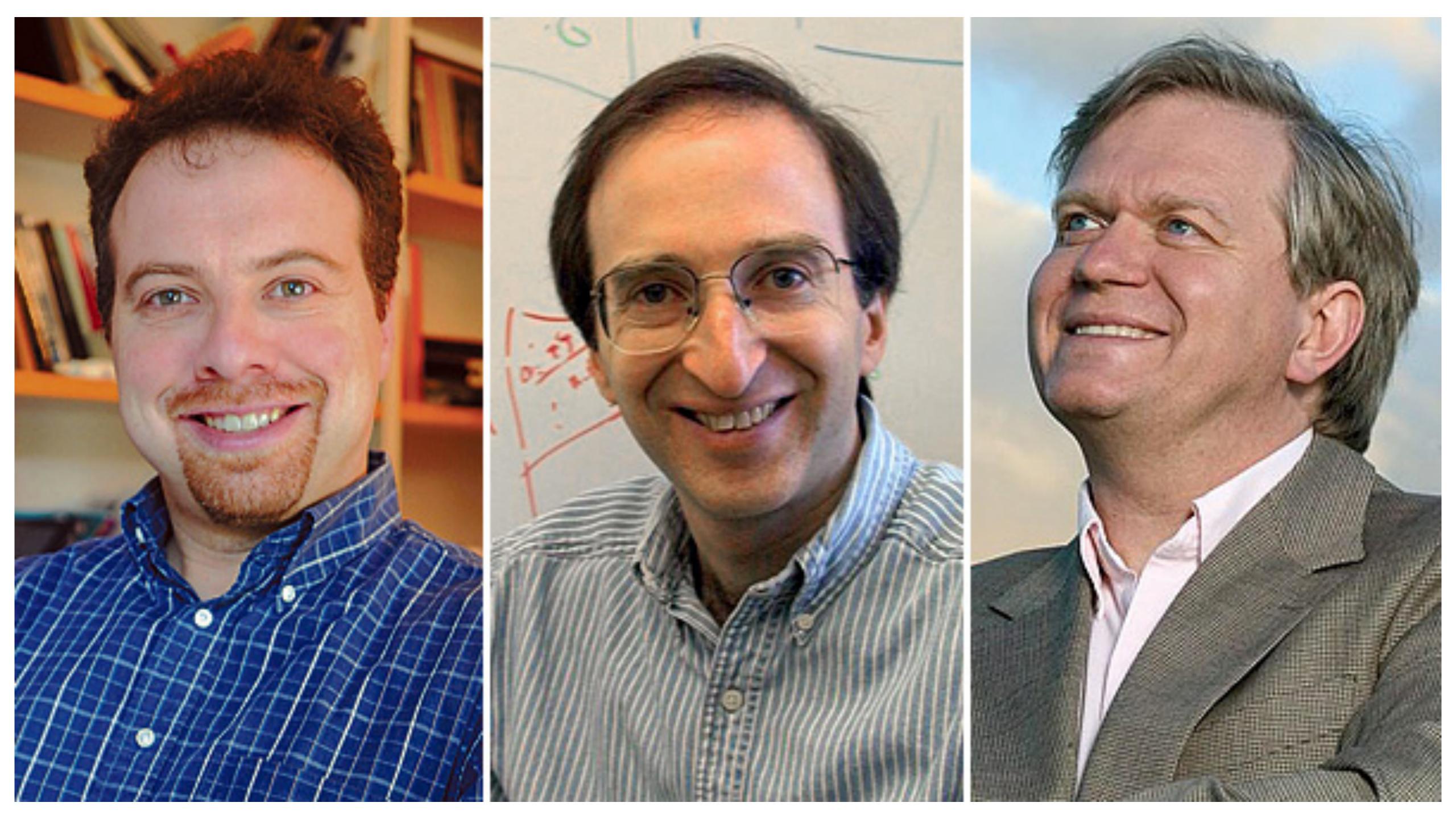
Planck 2015 results XIII.

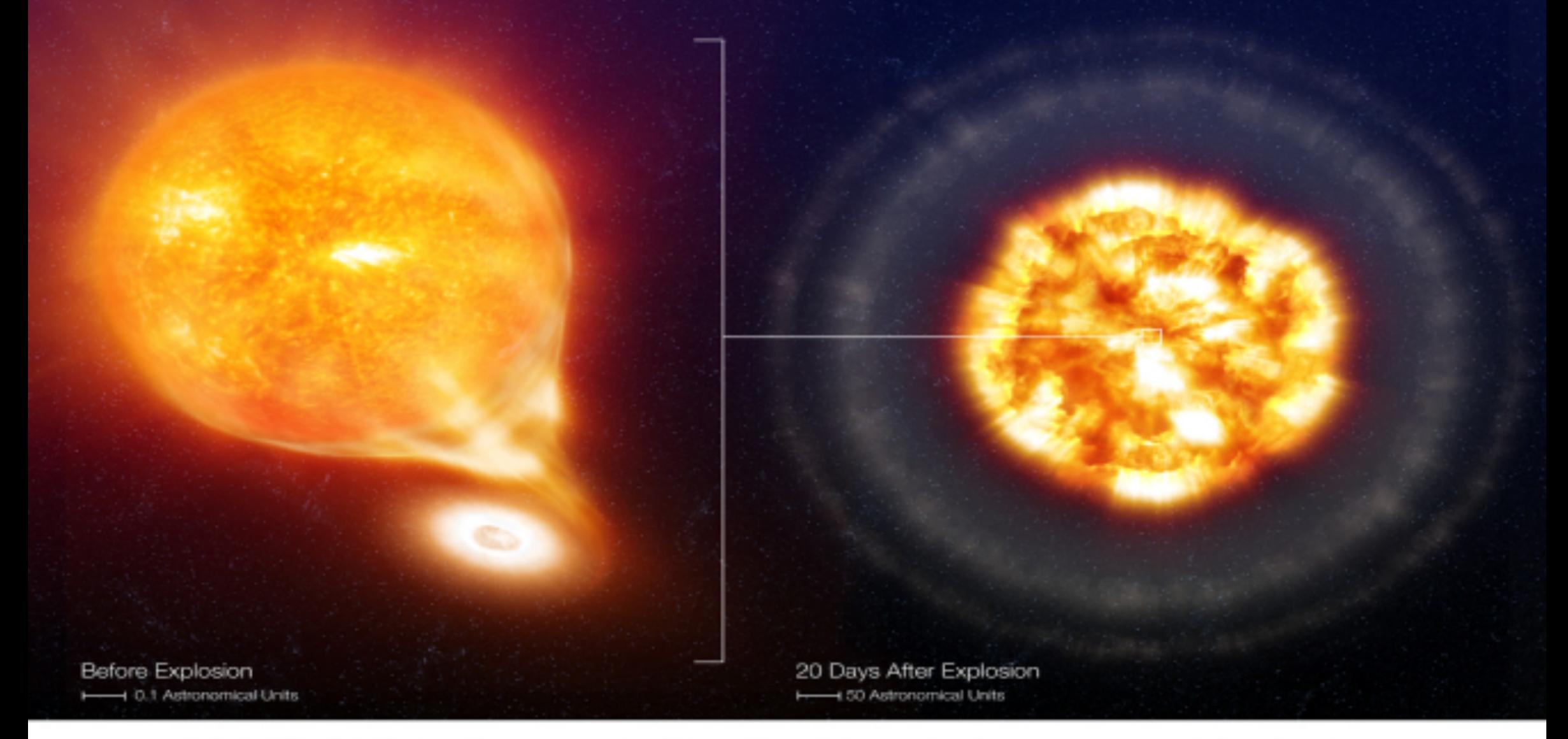
$\Omega_0 = 1 + / - 0.01$

Expansion depends on contents

- Normal matter
- Electromagnetic radiation
- Dark matter

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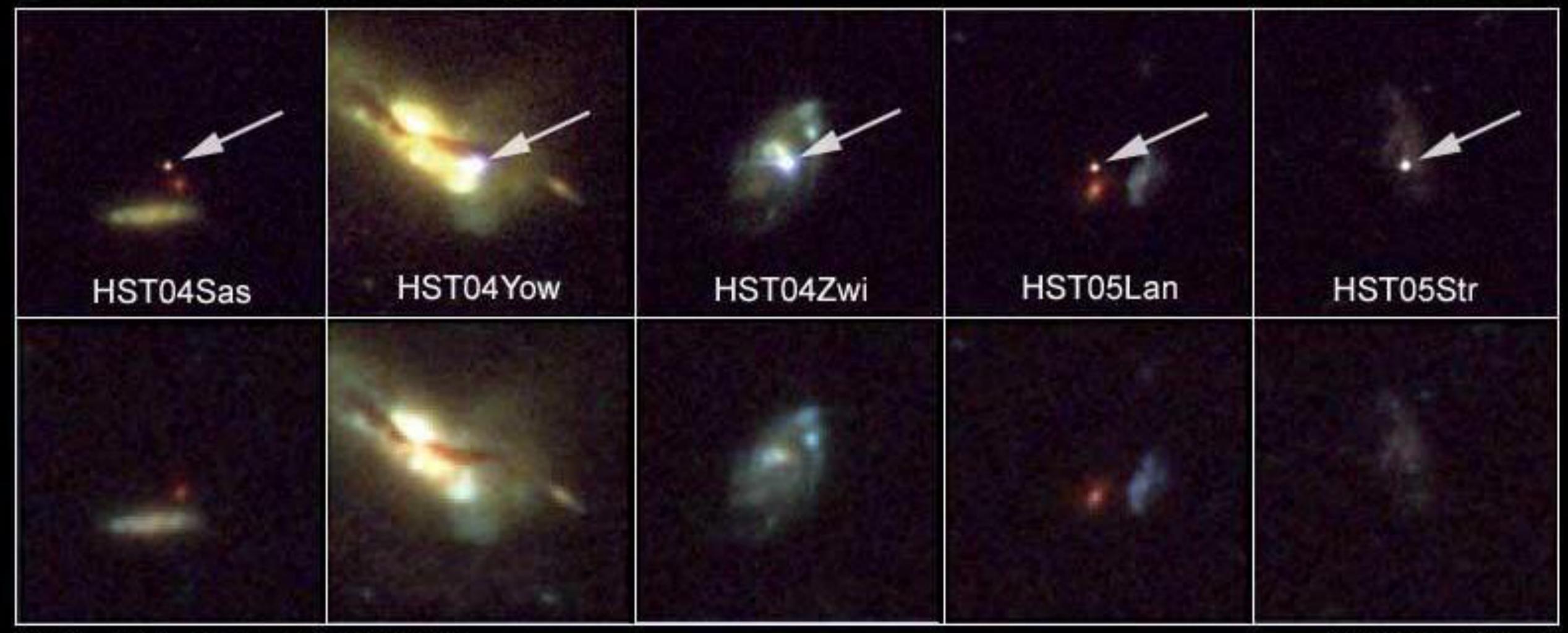
SN 2006X, before and after the Type Ia Supernova Explosion (Artist Impression)





Host Galaxies of Distant Supernovae

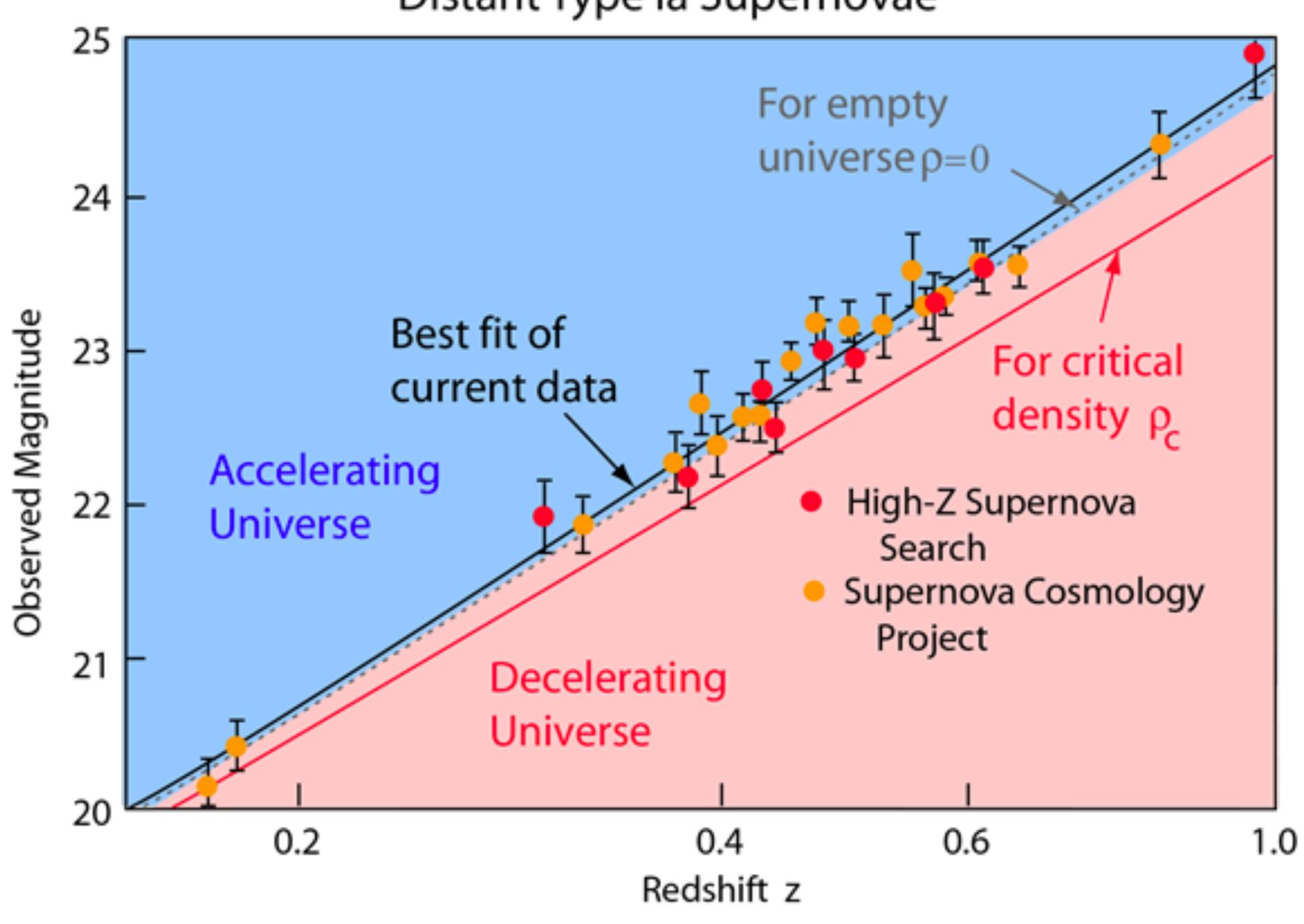
HST - ACS/WFC

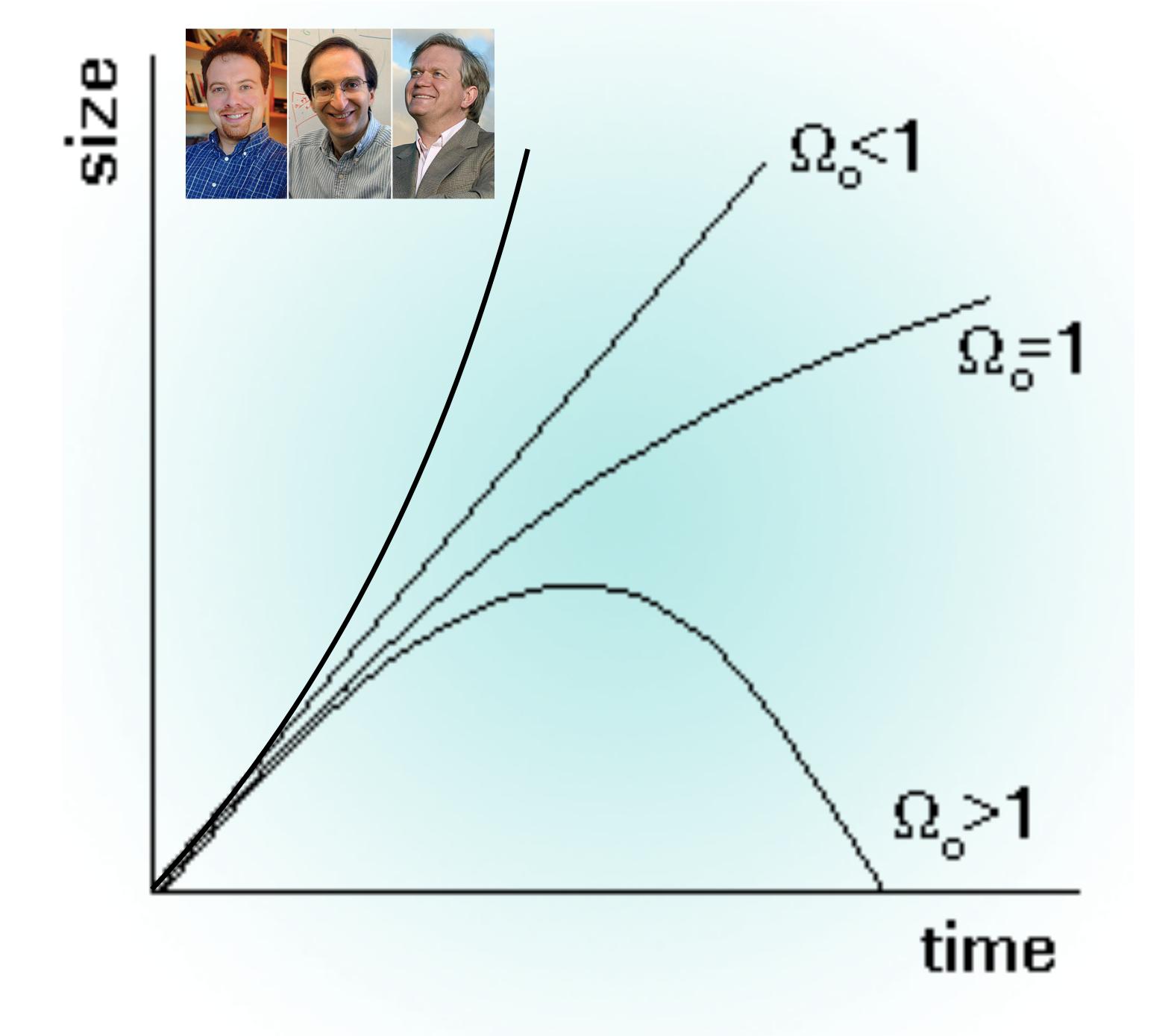


NASA, ESA, and A. Riess (STScI)

STScI-PRC06-52

Distant Type Ia Supernovae





Expansion depends on contents

- Normal matter
- Electromagnetic radiation
- Dark matter
- Cosmological constant / dark energy

Energy density of dark energy

Energy density of dark energy

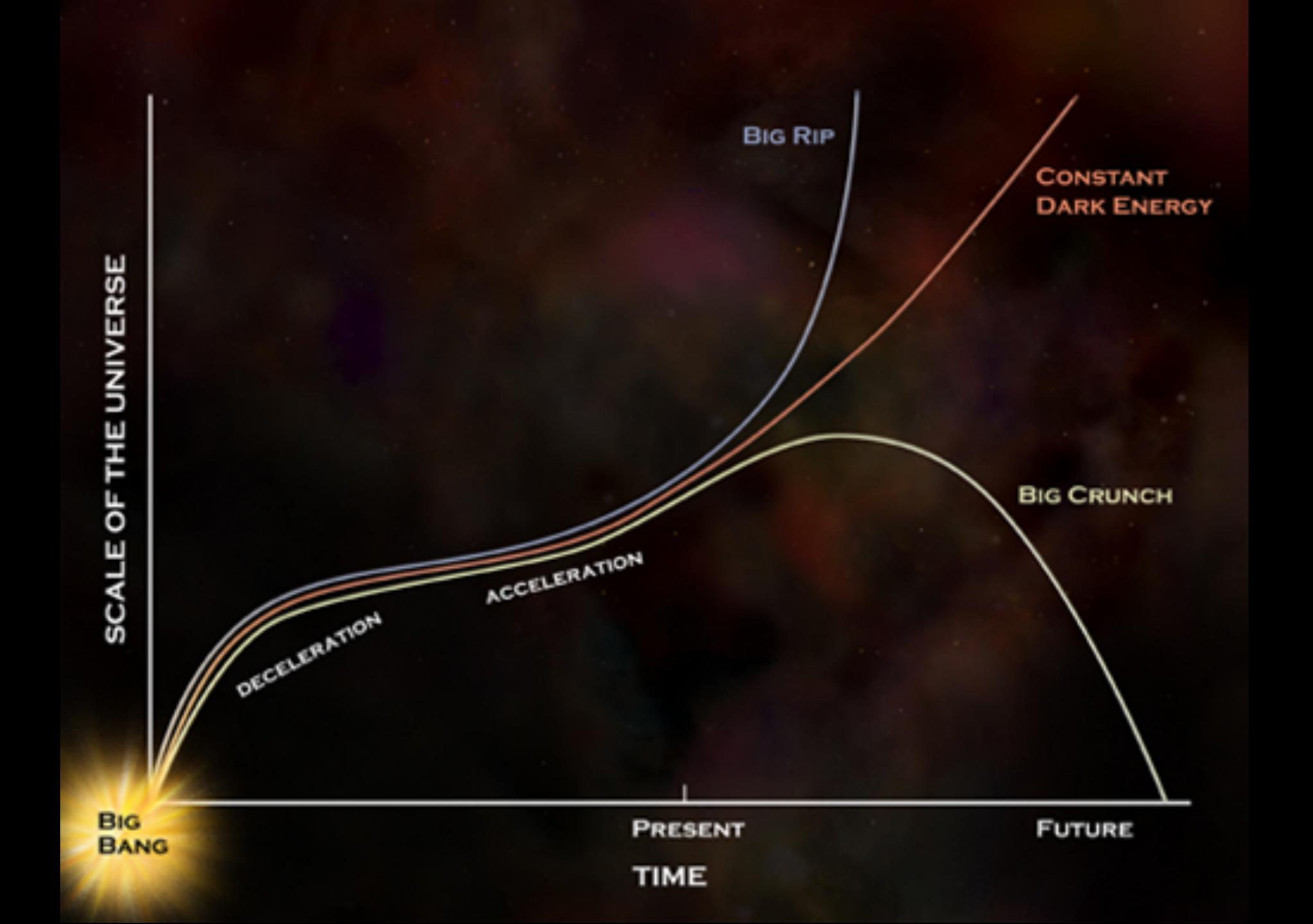
$\Omega_0 = \Omega_m + \Omega_{de} = 1$

Cosmological Constant?

Vacuum energy?

Exotic particles?

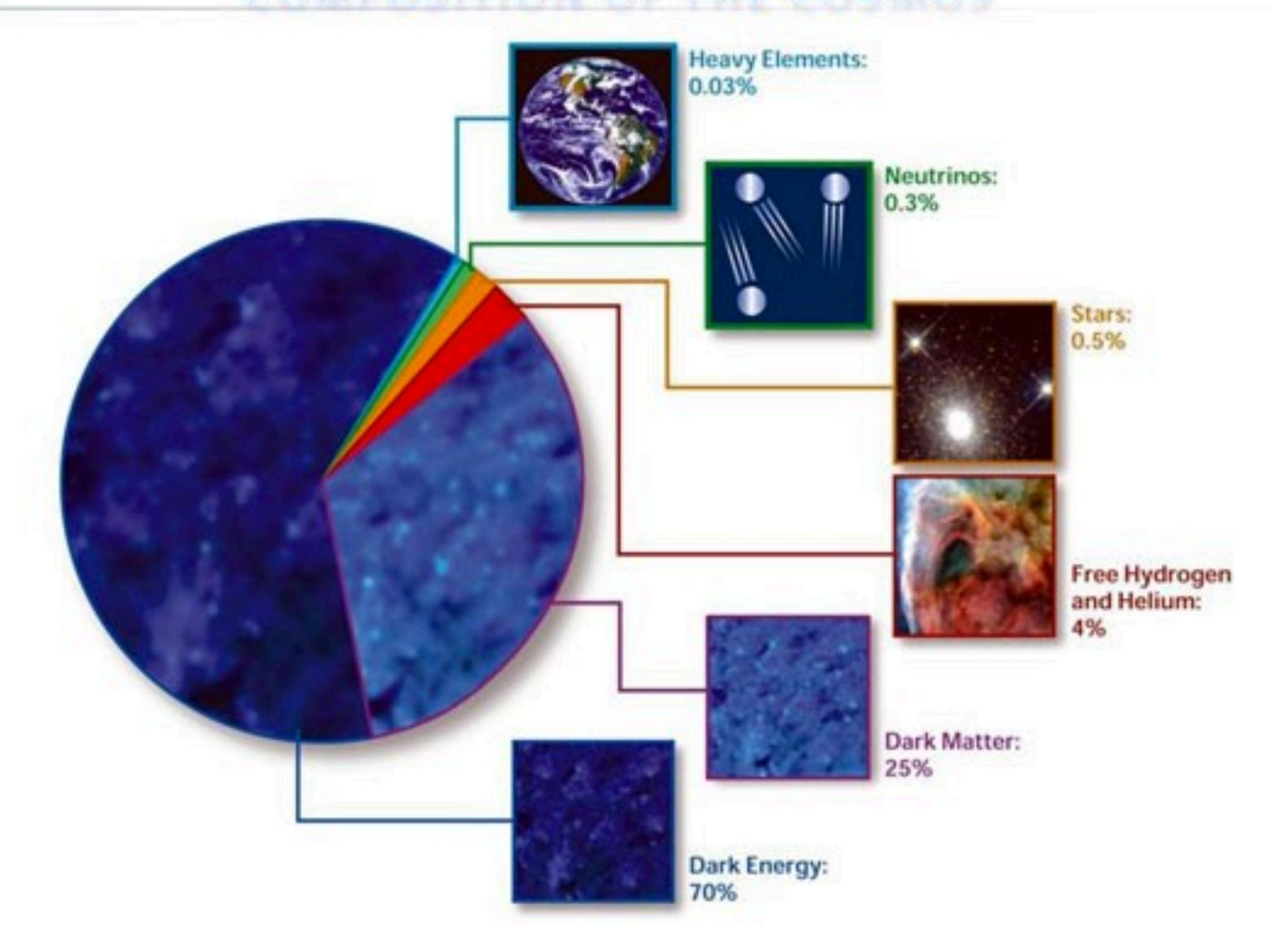
Modification of general relativity?



Dark energy equation of state

- Equation of state: $P = w \rho c^2$
- Accelerating expansion for w < -1/3
- Cosmological constant: w = -1
- Parametrisation of time dependence: $w(a) = w_0 + w_a(1+a)$ mit a=1/(1+z)

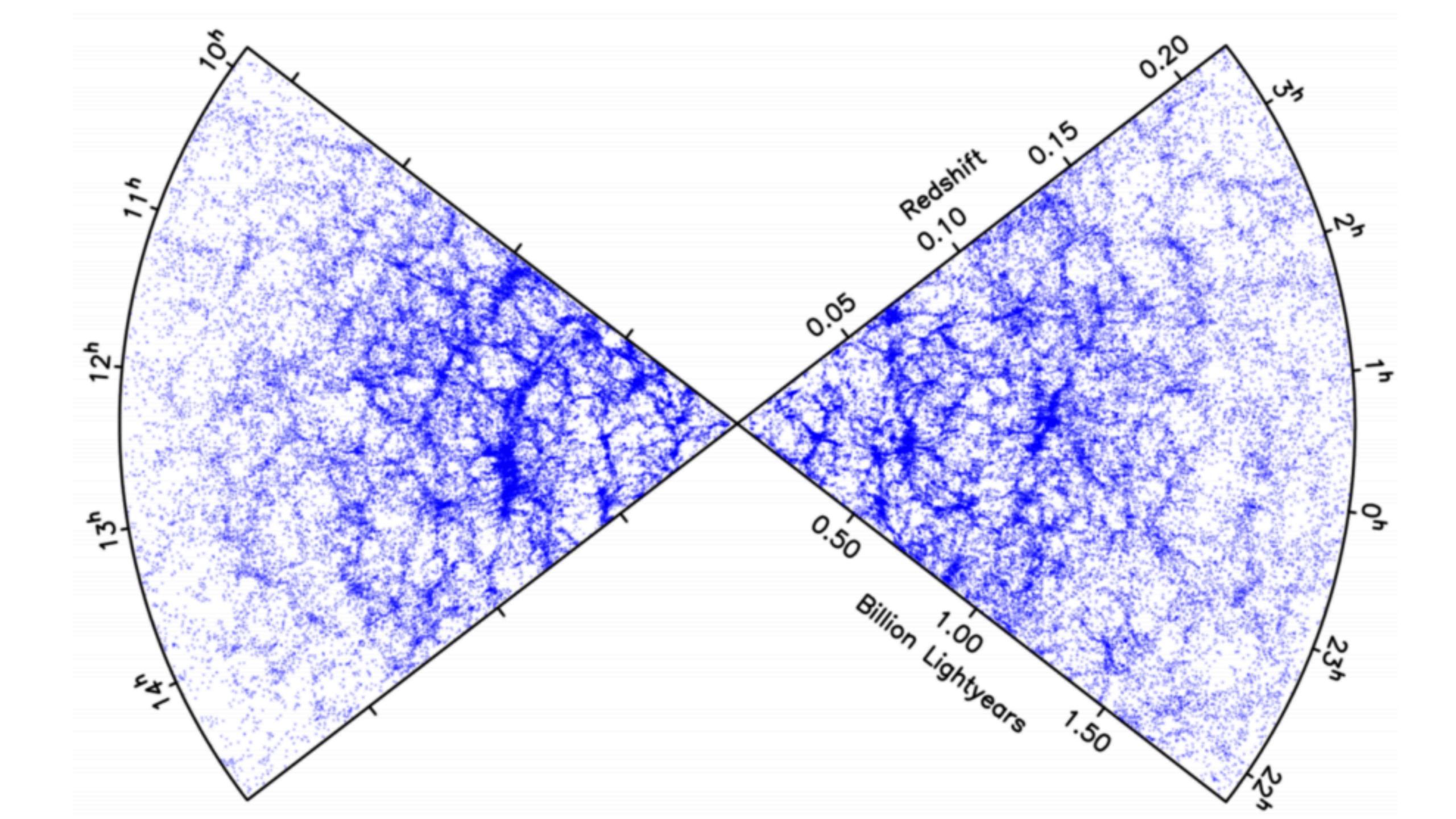
COMPOSITION OF THE COSMOS



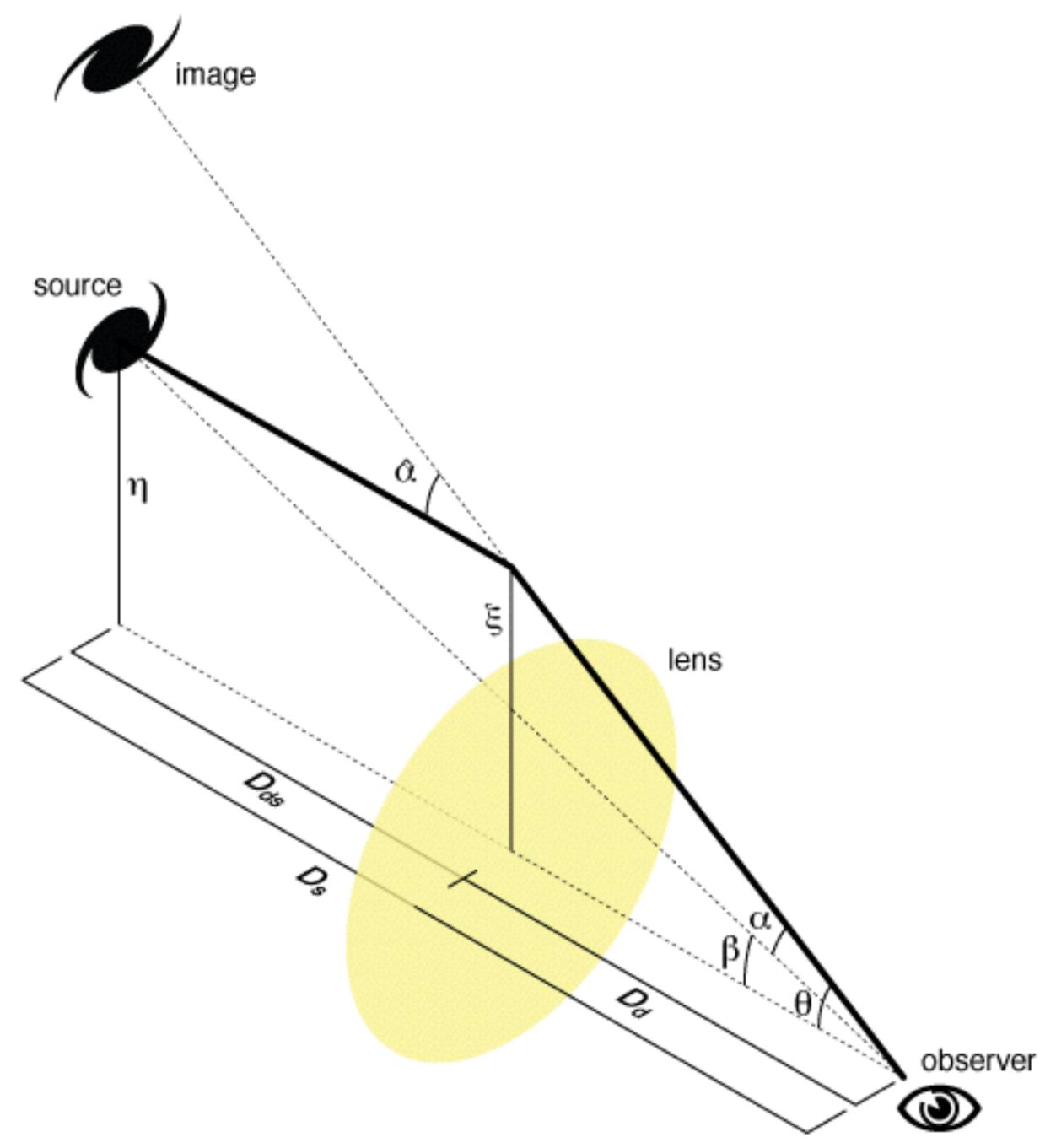
Observing dark energy

- Distance-redshift relation:
 - 1. Supernovae type la
 - 2. Baryon acoustic oscillations
- + Growth of structures:
 - 3. Galaxy cluster mass function
 - 4. Weak gravitational lensing

Gravitational Lensing

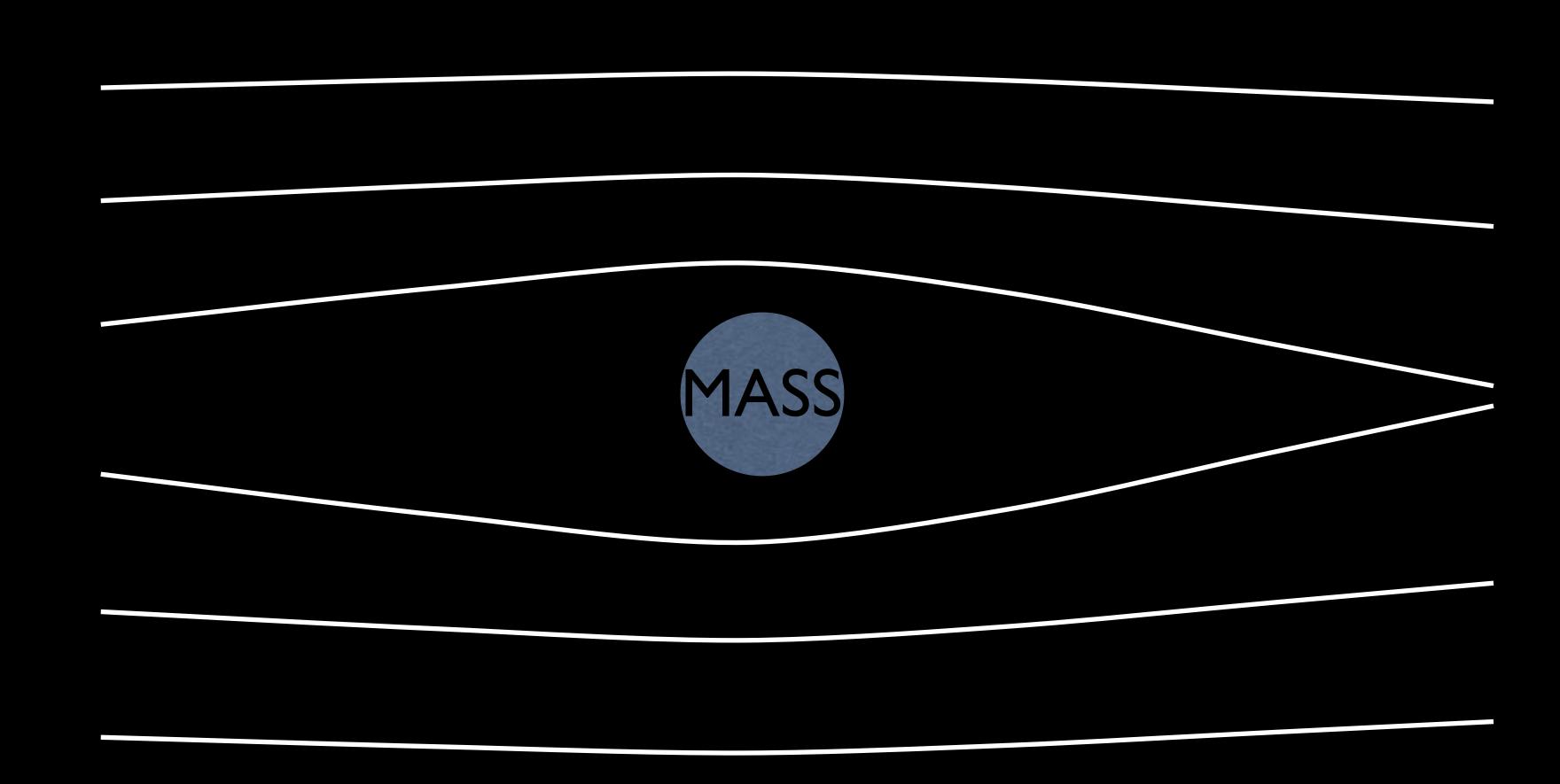




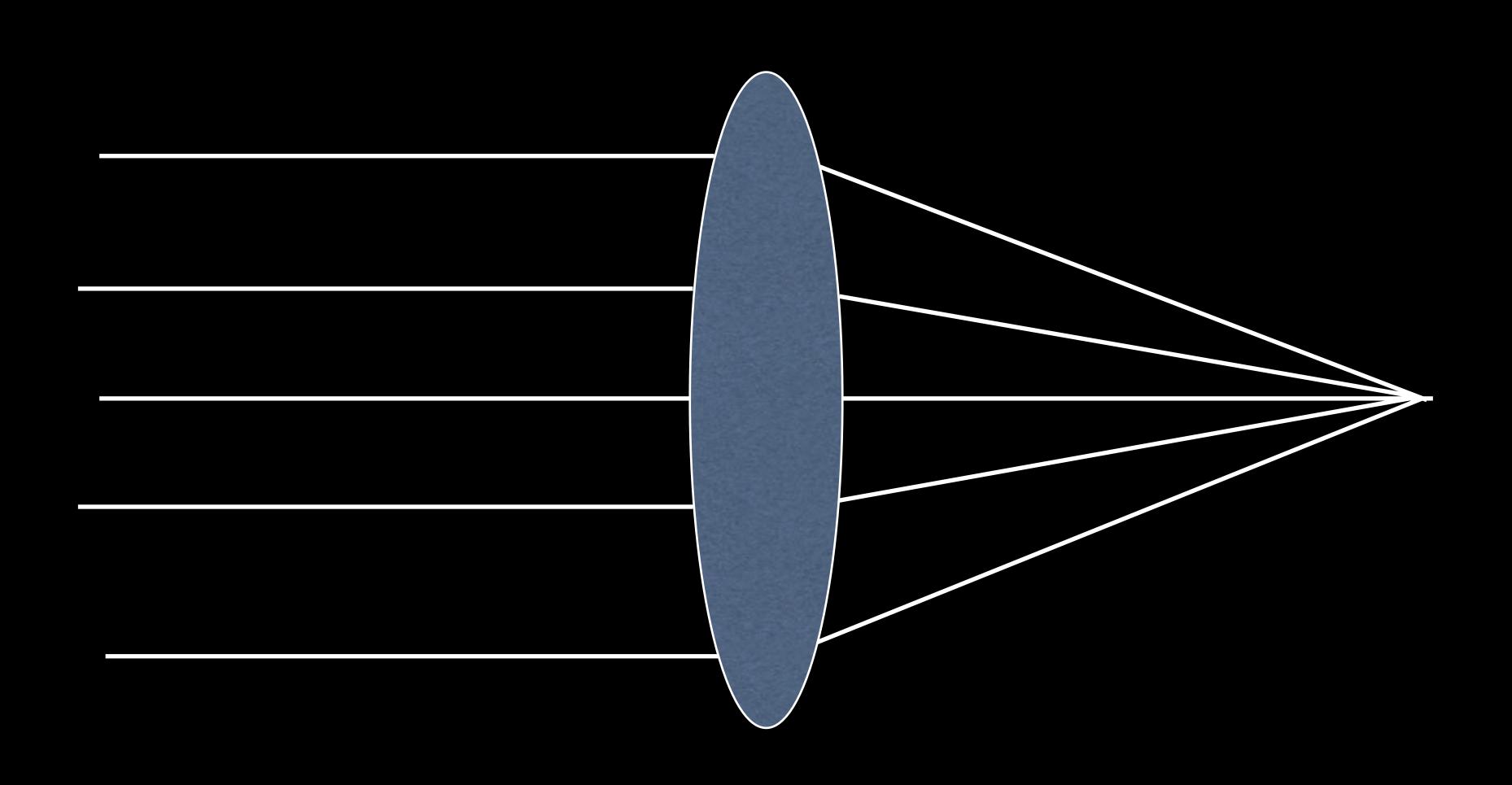


Credit: Michael Sachs

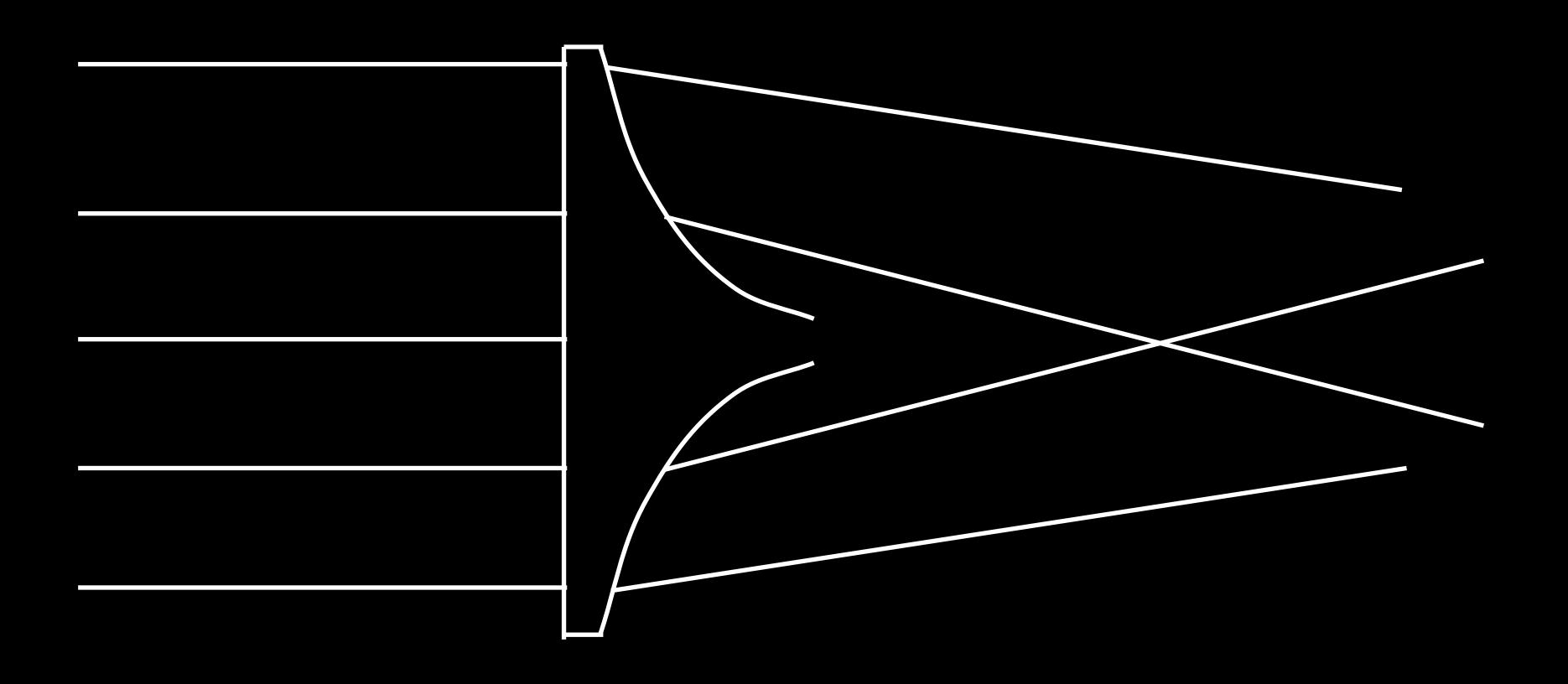
Gravitational lens



Optical lens

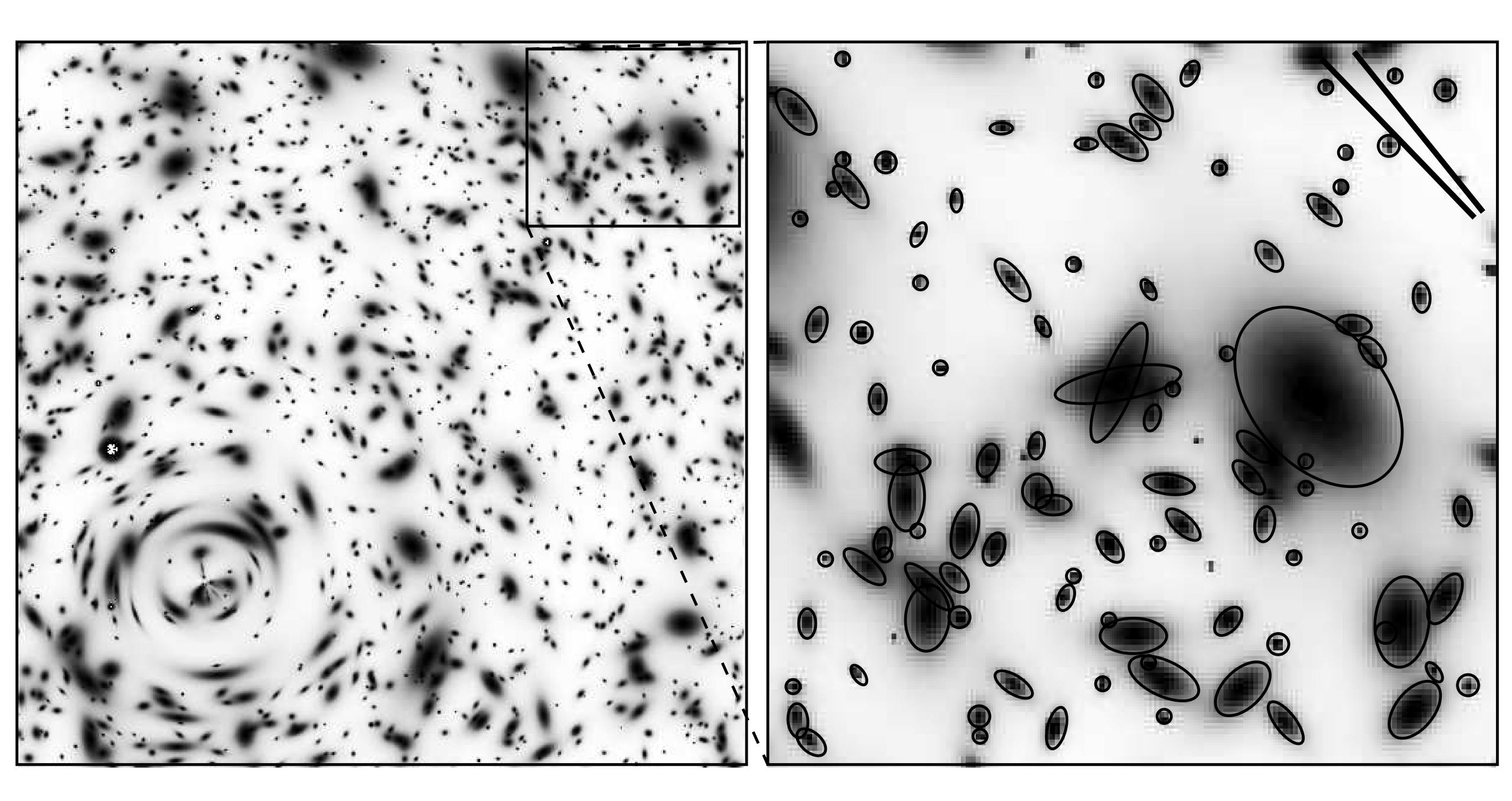


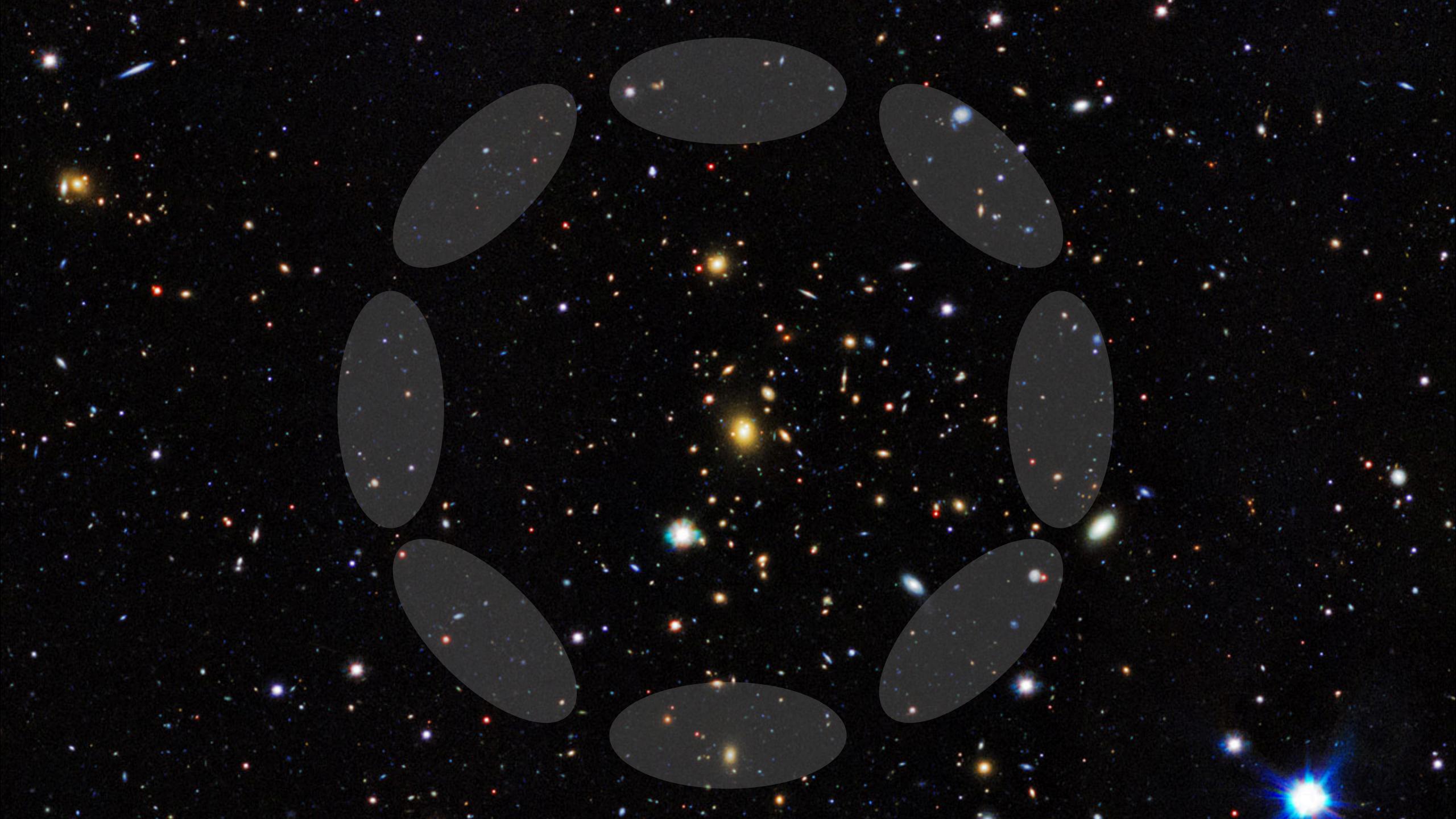
Gravitational lens analogue



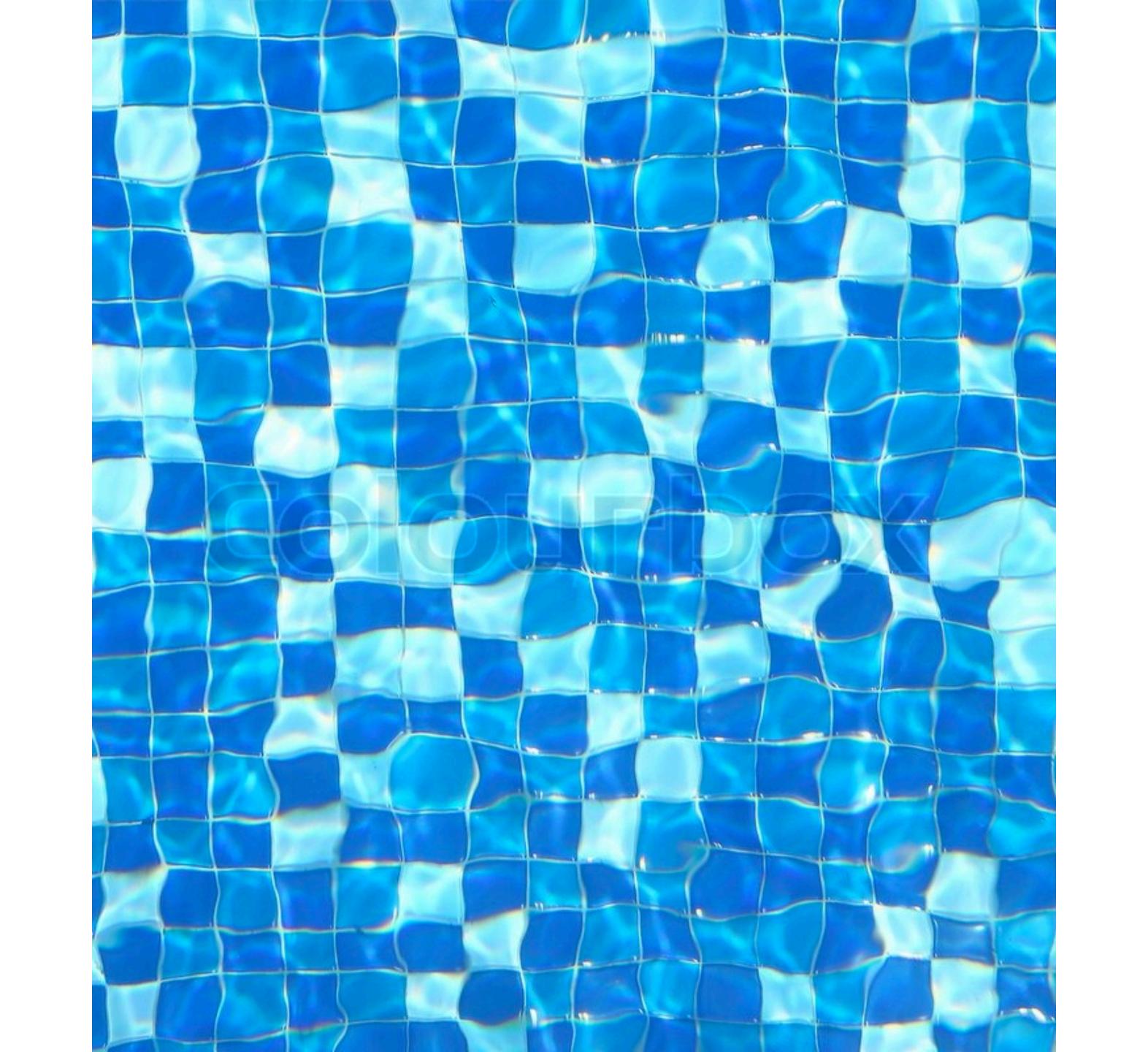
Spherical abberation!

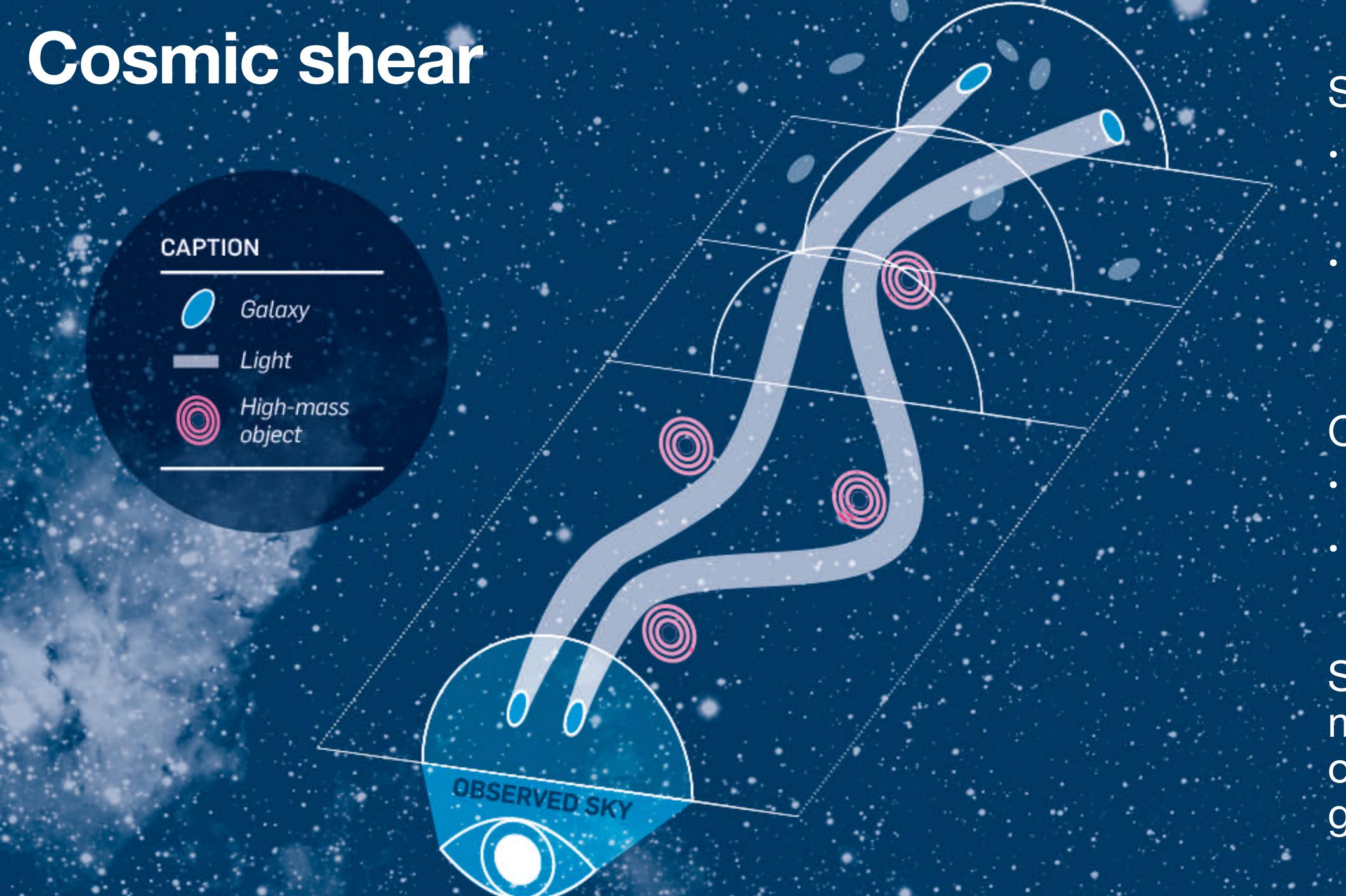












Sensitive to:

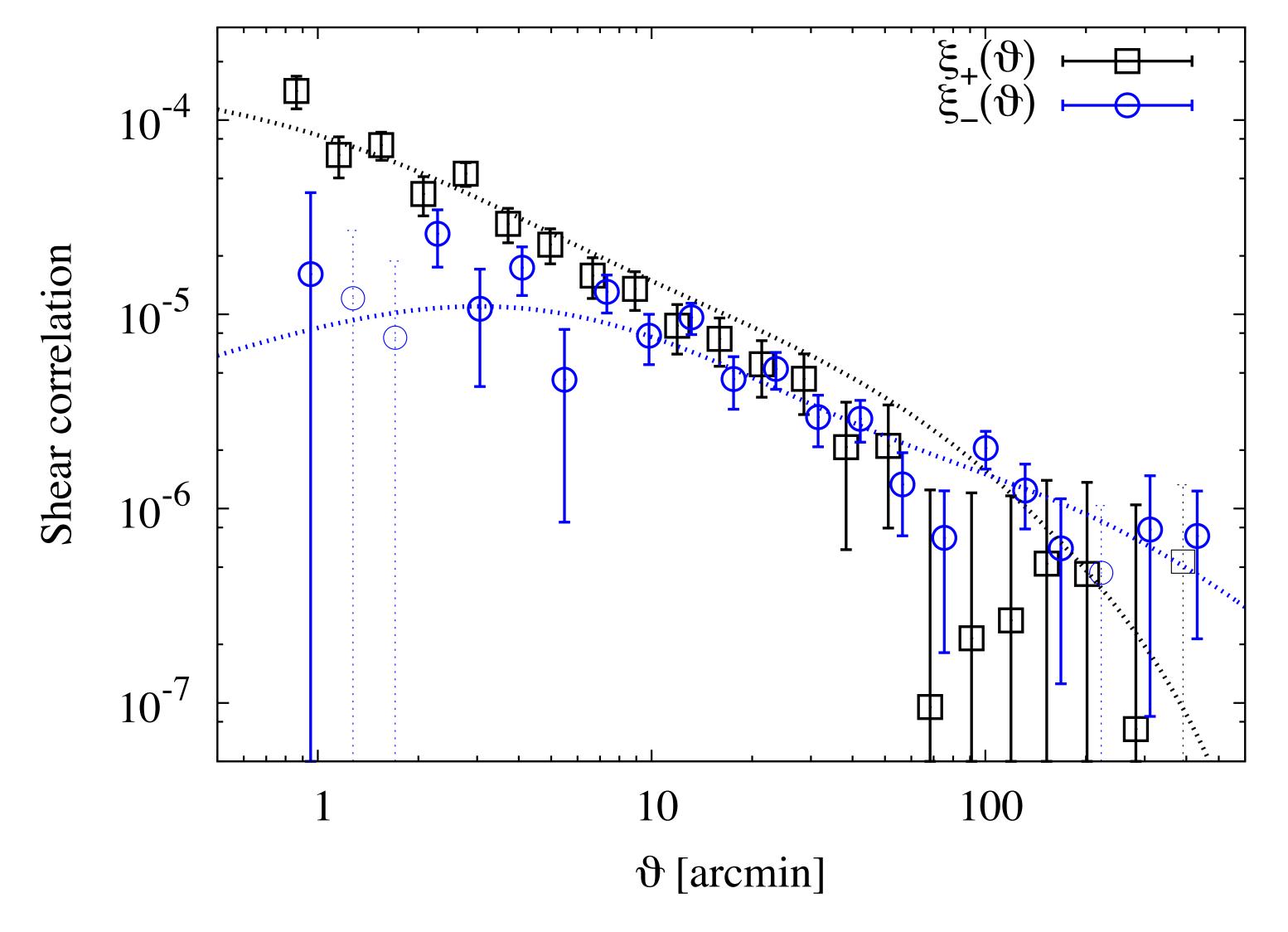
- Matter distribution
- Geometry

Observables:

- Ellipticities
- Photo-z

Statistical measurement of many galaxies

2pt shear correlation functions



Very directly related to the matter power spectrum P_{δ} .

Observation—theory

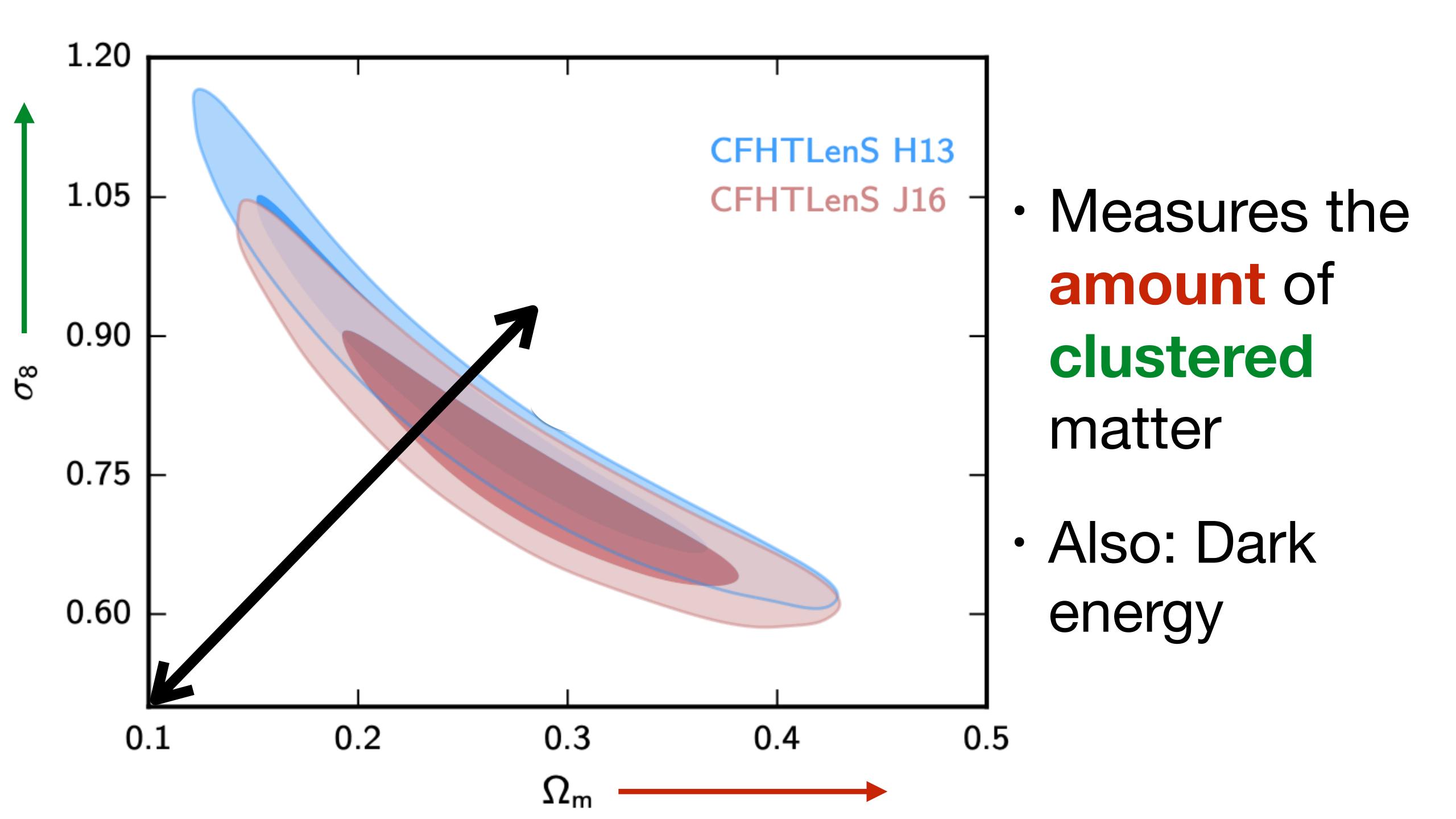
$$\xi_{\pm}(\theta) = \langle \gamma_{t} \gamma_{t} \rangle (\theta) \pm \langle \gamma_{x} \gamma_{x} \rangle (\theta)$$

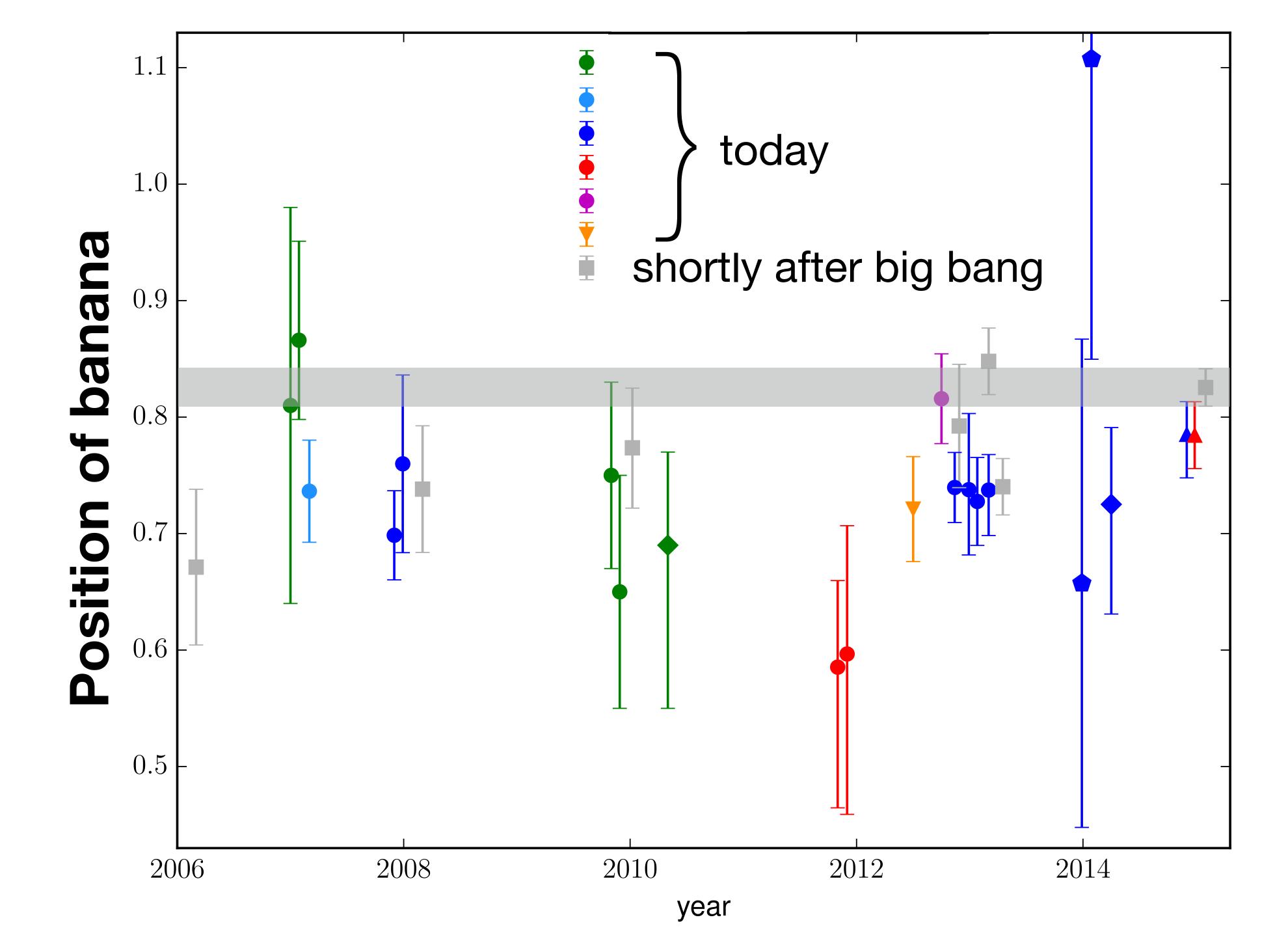
$$\xi_{+}(\theta) = \int_{0}^{\infty} \frac{\mathrm{d}\ell \,\ell}{2\pi} \, J_{0}(\ell\theta) \, P_{\kappa}(\ell) \, ; \quad \xi_{-}(\theta) = \int_{0}^{\infty} \frac{\mathrm{d}\ell \,\ell}{2\pi} \, J_{4}(\ell\theta) \, P_{\kappa}(\ell)$$

$$P_{\kappa}(\ell) = \frac{9H_{0}^{4} \Omega_{\mathrm{m}}^{2}}{4c^{4}} \, \int_{0}^{\chi_{\mathrm{h}}} \mathrm{d}\chi \, \frac{g^{2}(\chi)}{a^{2}(\chi)} \left(P_{\delta} \left(\frac{\ell}{f_{K}(\chi)}, \chi \right) \right)$$

$$g(\chi) = \int_{\chi}^{\chi_{\mathrm{h}}} \mathrm{d}\chi' \left(p_{\chi}(\chi') \right) \frac{f_{K}(\chi' - \chi)}{f_{K}(\chi')}$$

No galaxy biasing on this slide! Cosmic shear typically goes to small scales.



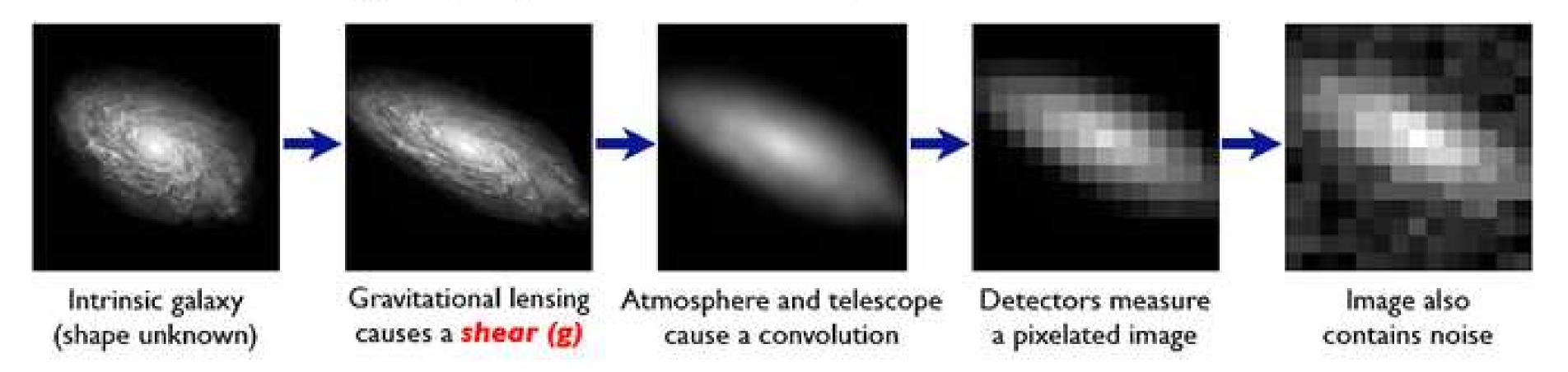


Systematic challenges

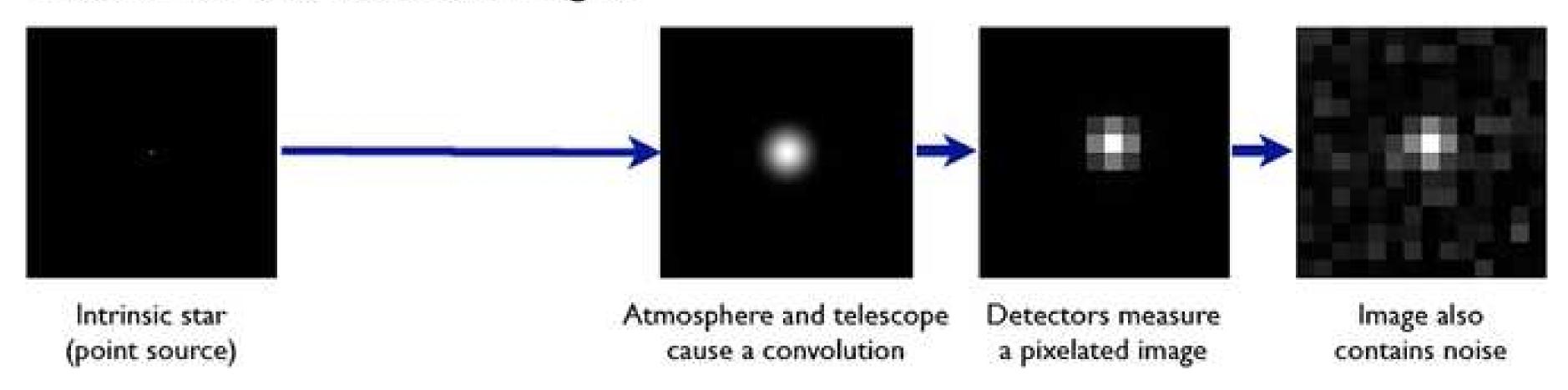
- Observational
 - Shape measurements
 - Redshift distributions
- Theoretical
 - Intrinsic alignments
 - Baryon feedback

Shape measurements

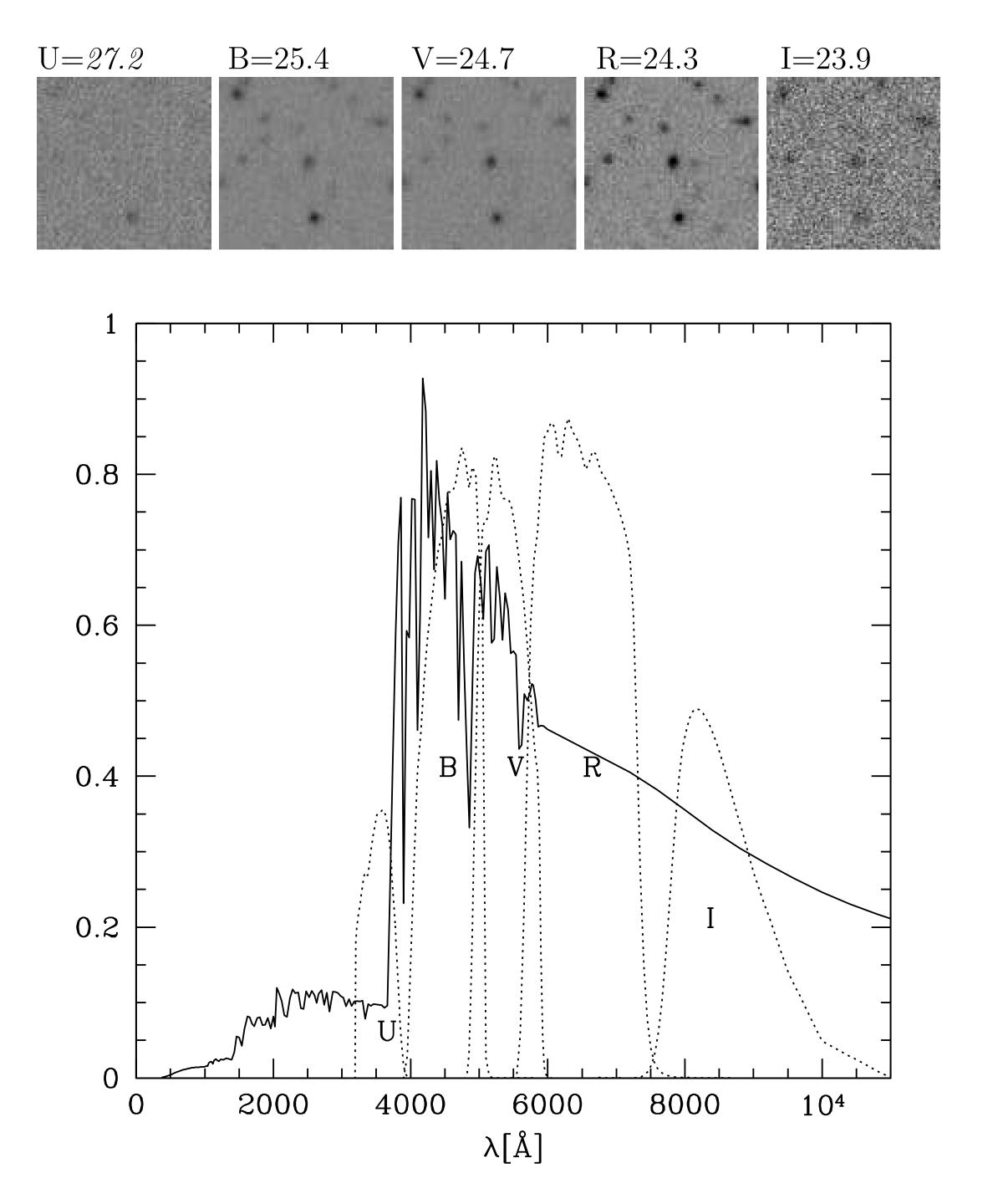
Galaxies: Intrinsic galaxy shapes to measured image:

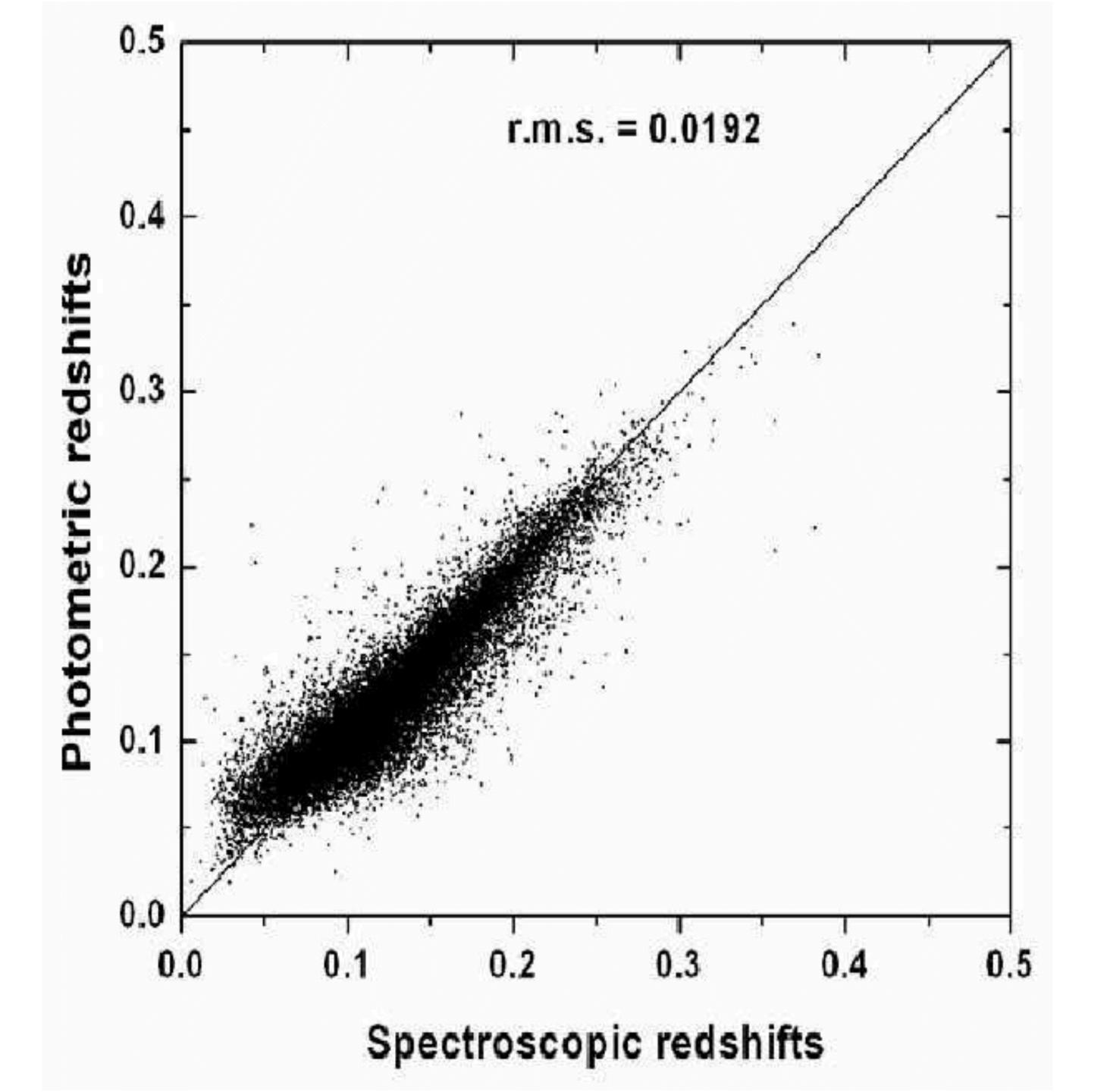


Stars: Point sources to star images:

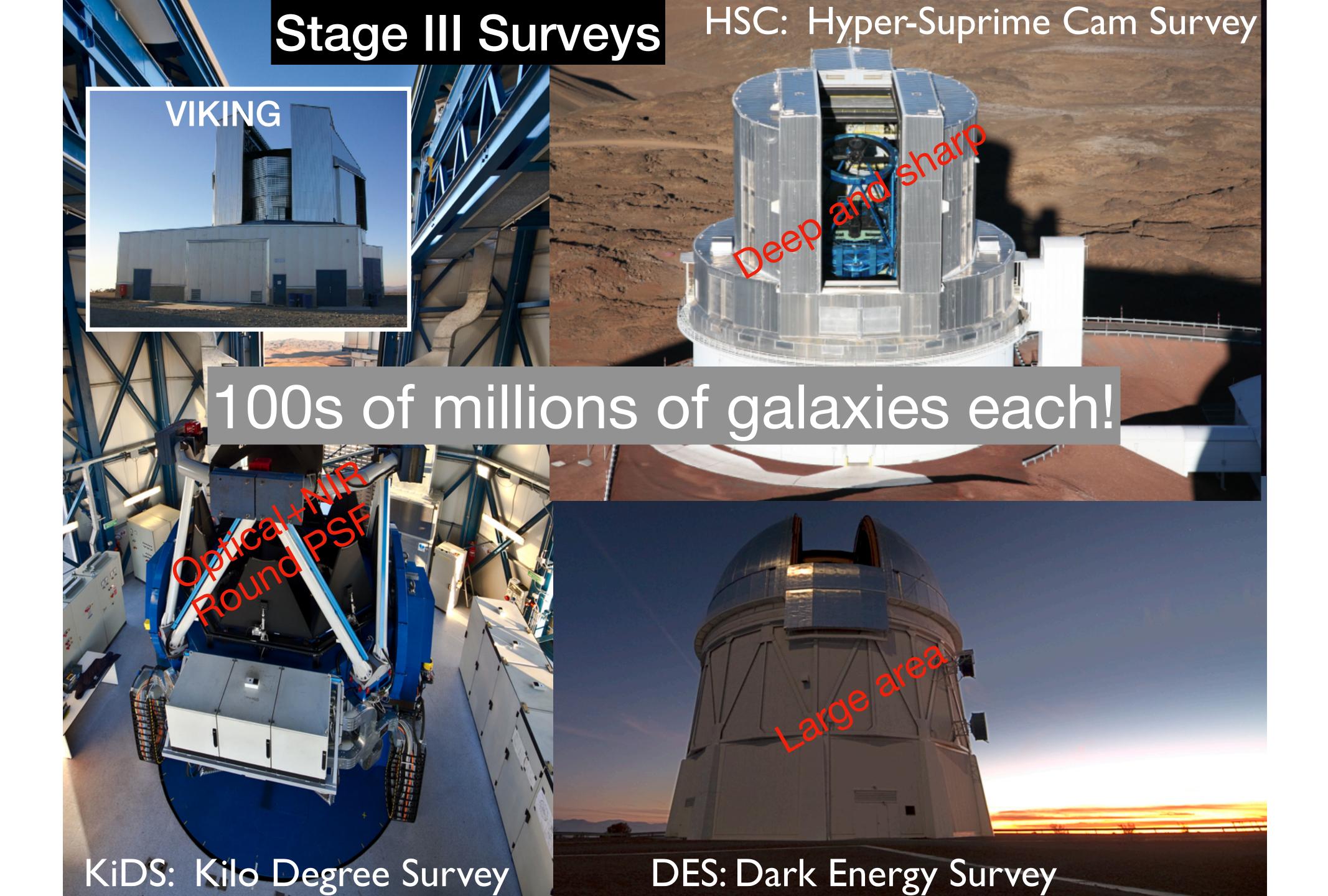


Bridle et al. (2009)

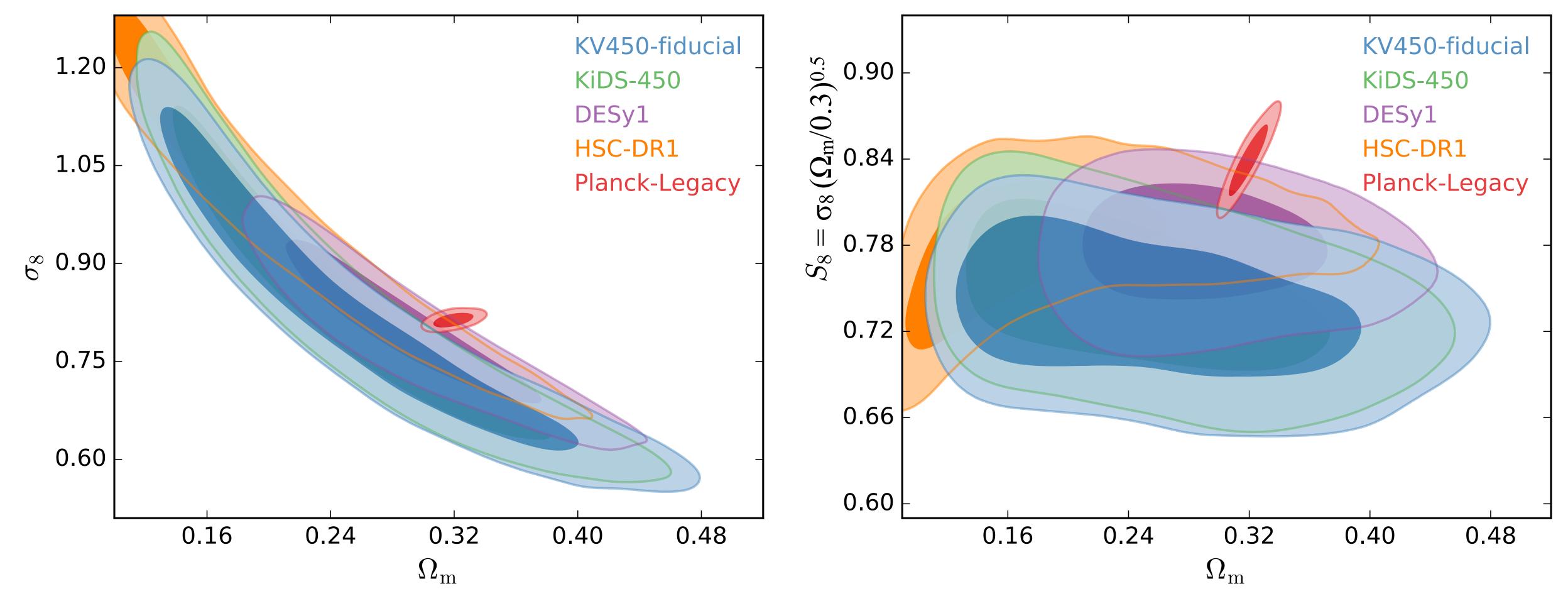








Cosmic shear before Covid-19

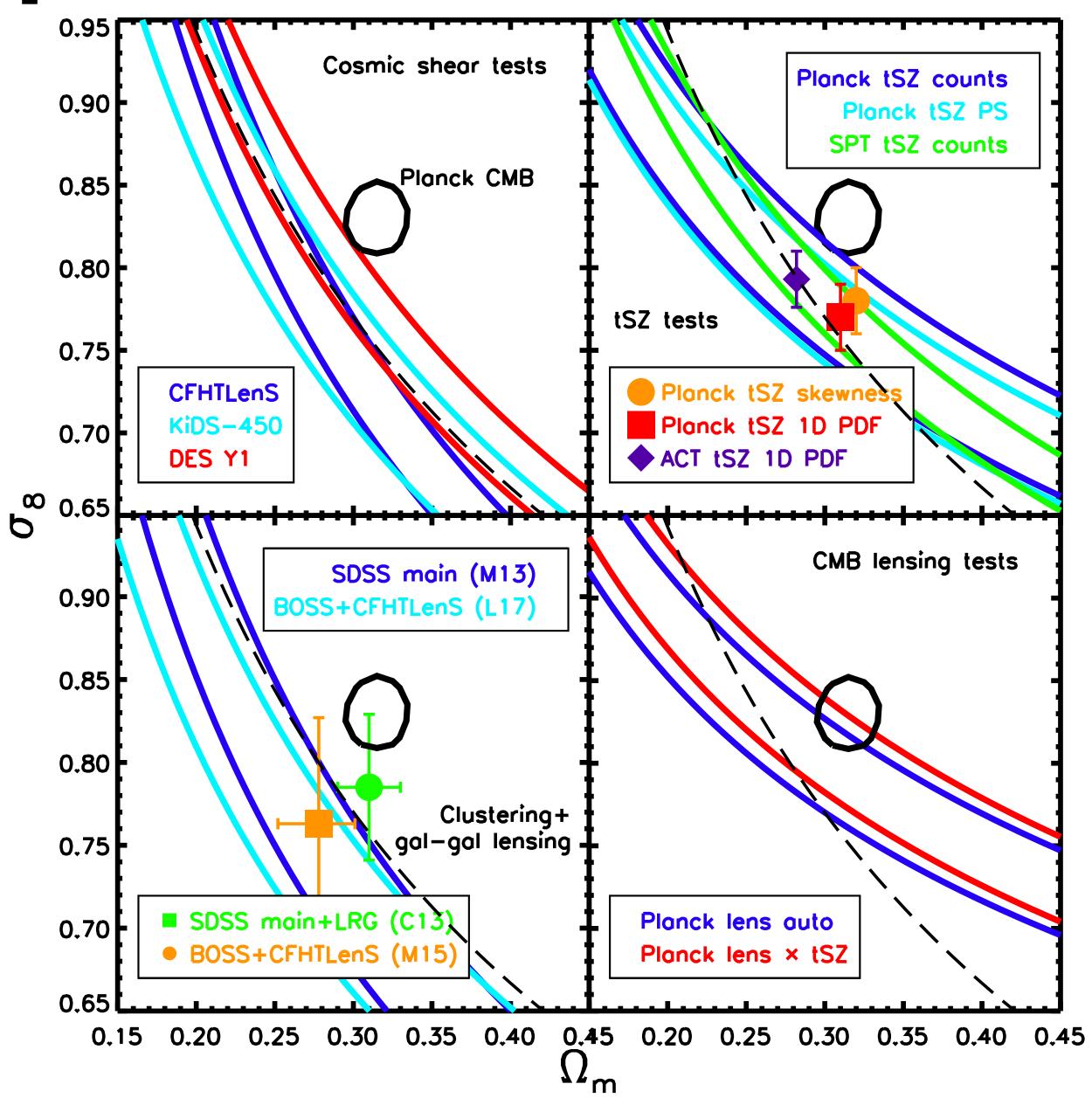


Agreement of WL measurements. All lower than Planck.

HSC-DR1: Hikage et al. (2019) DES-Y1: Troxel et al. (2018a)

KiDS-VIKING-450: Hildebrandt et al. (2020)

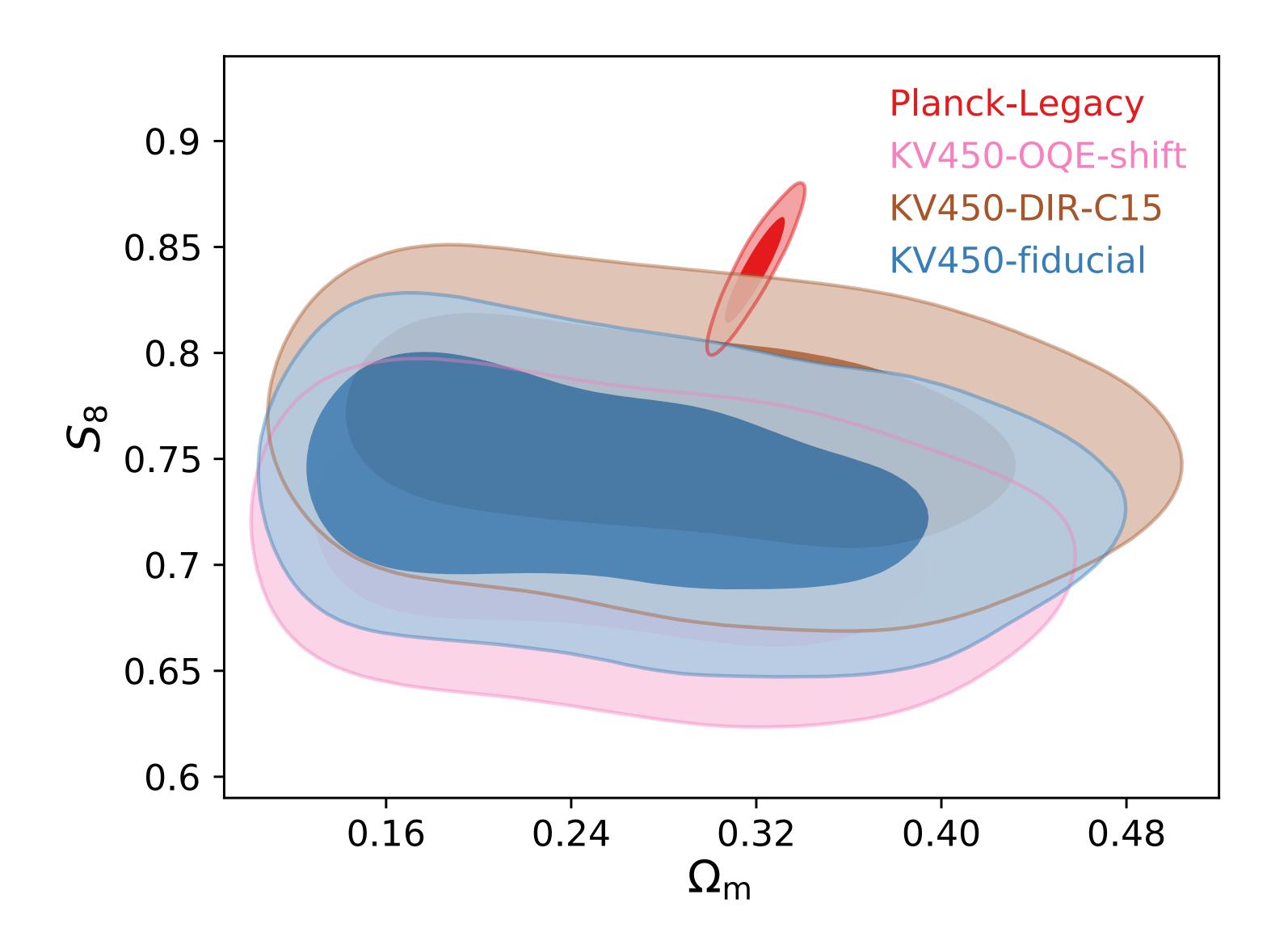
Other probes



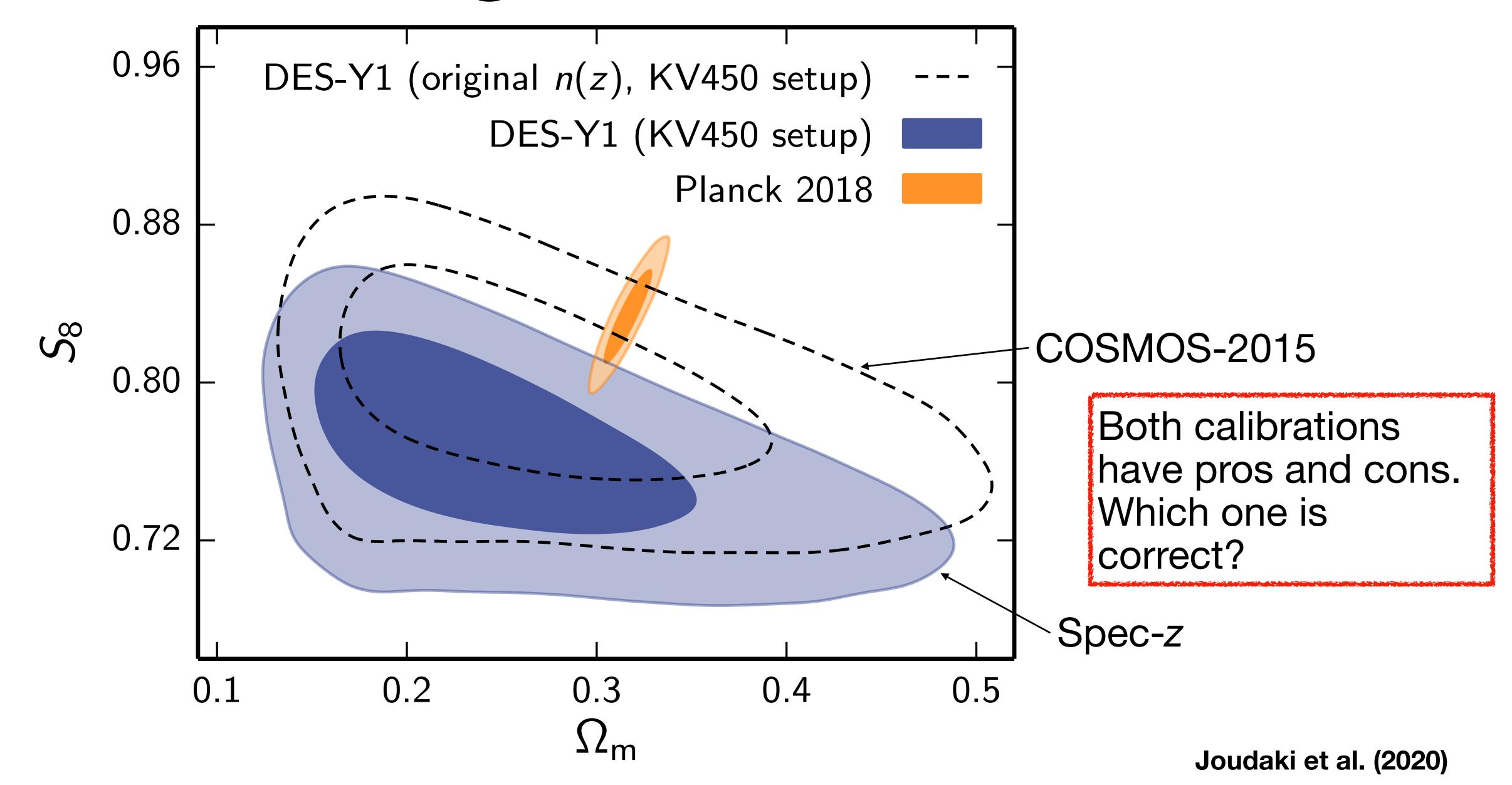
Not a single late Universe LSS measurement yields an S₈ higher than Planck.



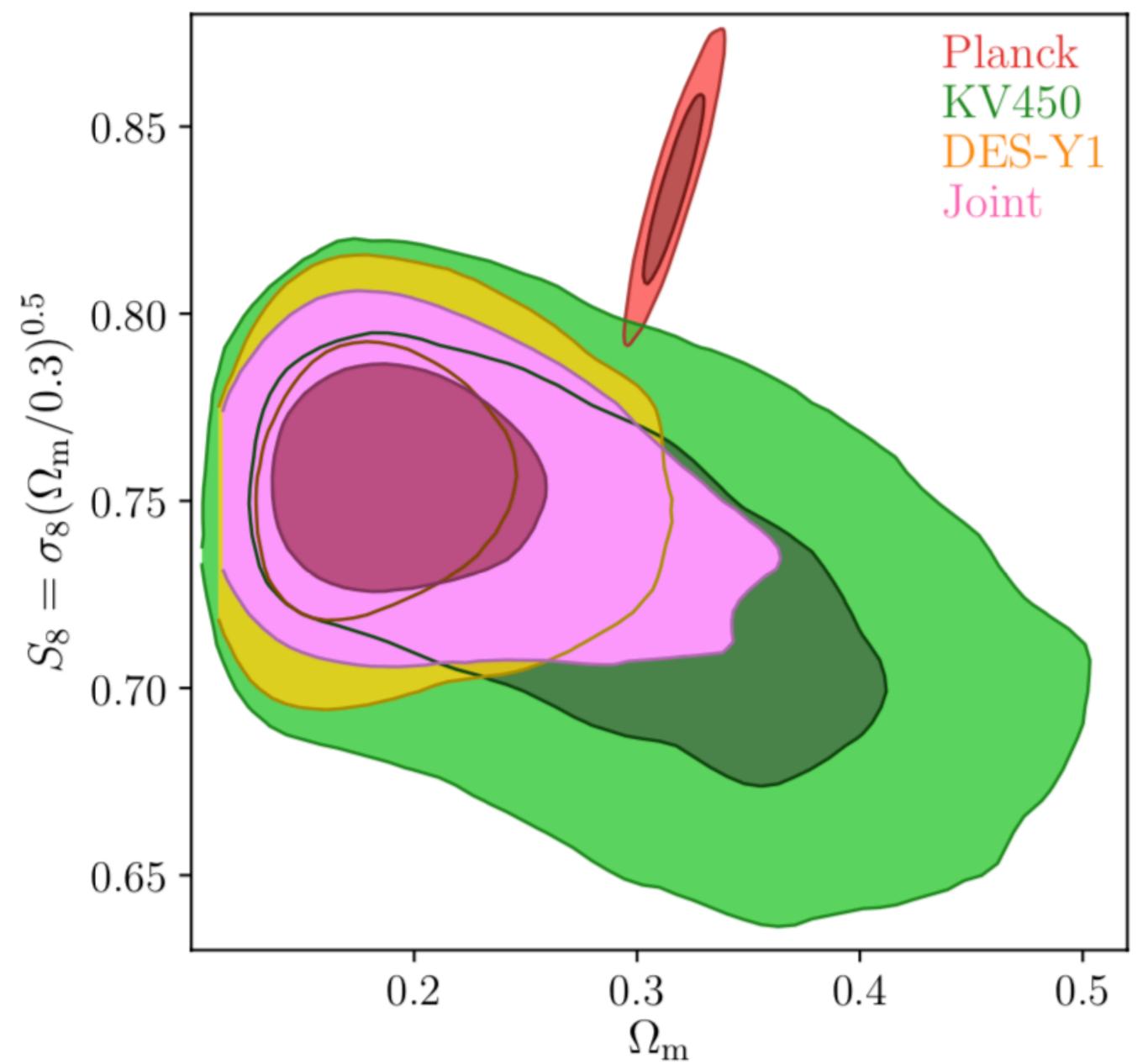
KiDS-VIKING-450 redshift calibration



Recalibrating DES redshifts

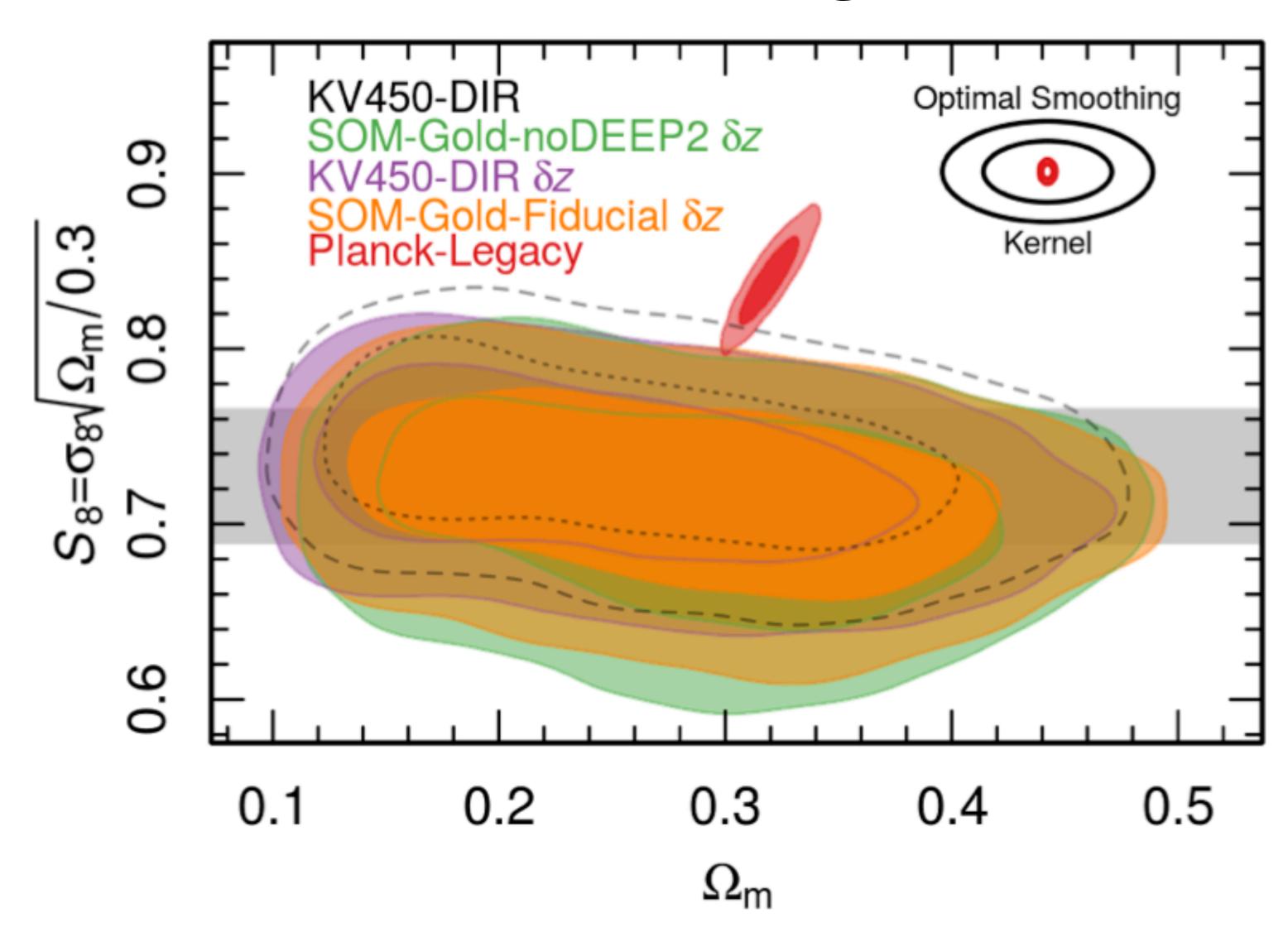


Sa constraints

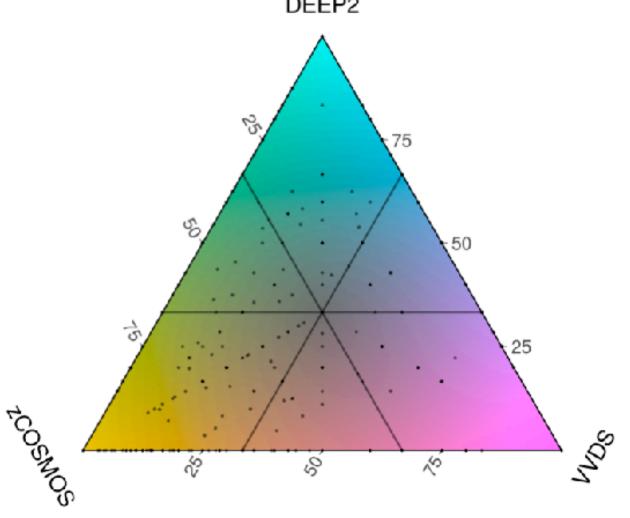


3.2σ tension between WL and Planck

KiDS-VIKING 450 "gold" sample

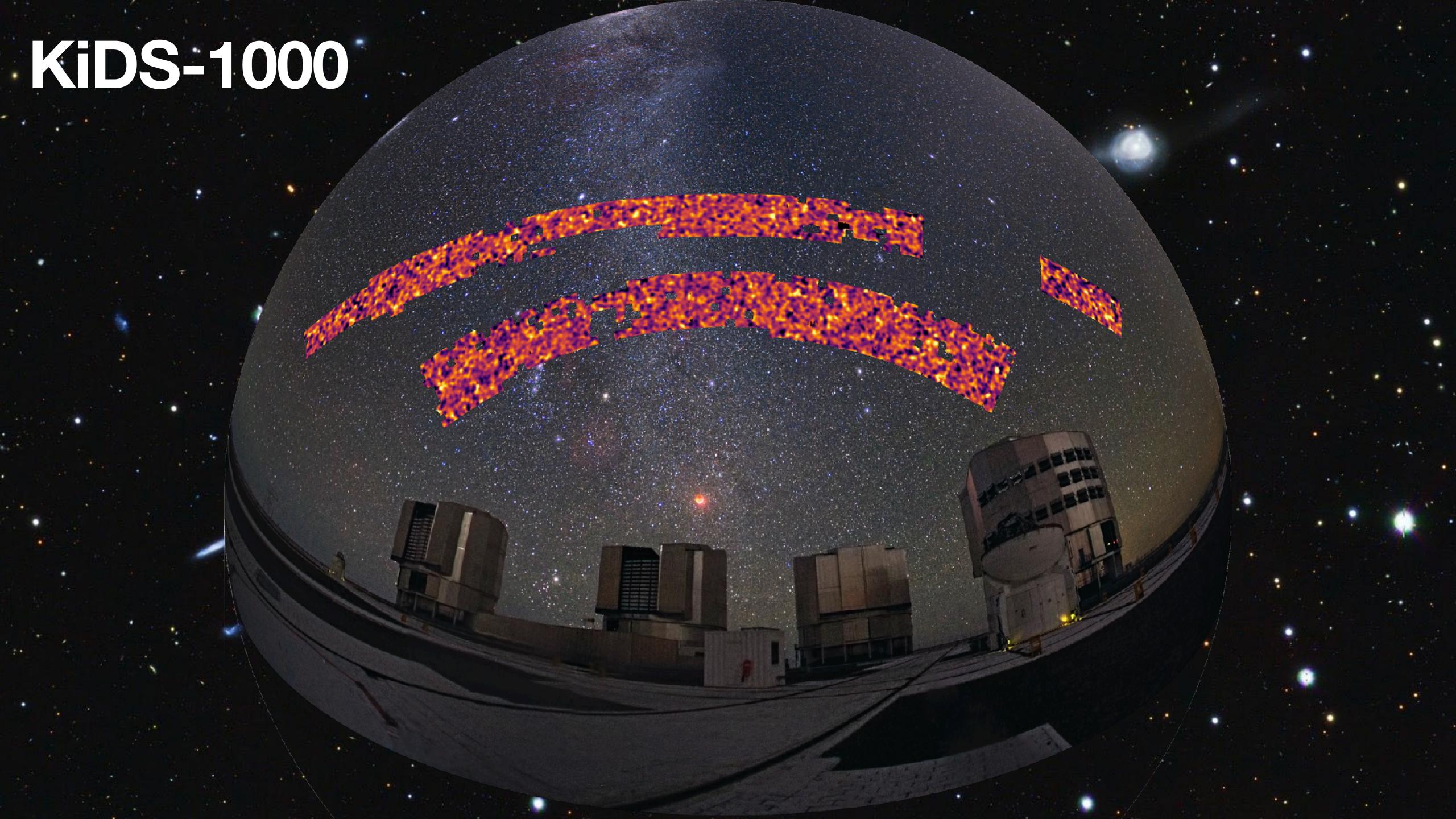


DEEP2



KiDS results robust against down-selection of sources.

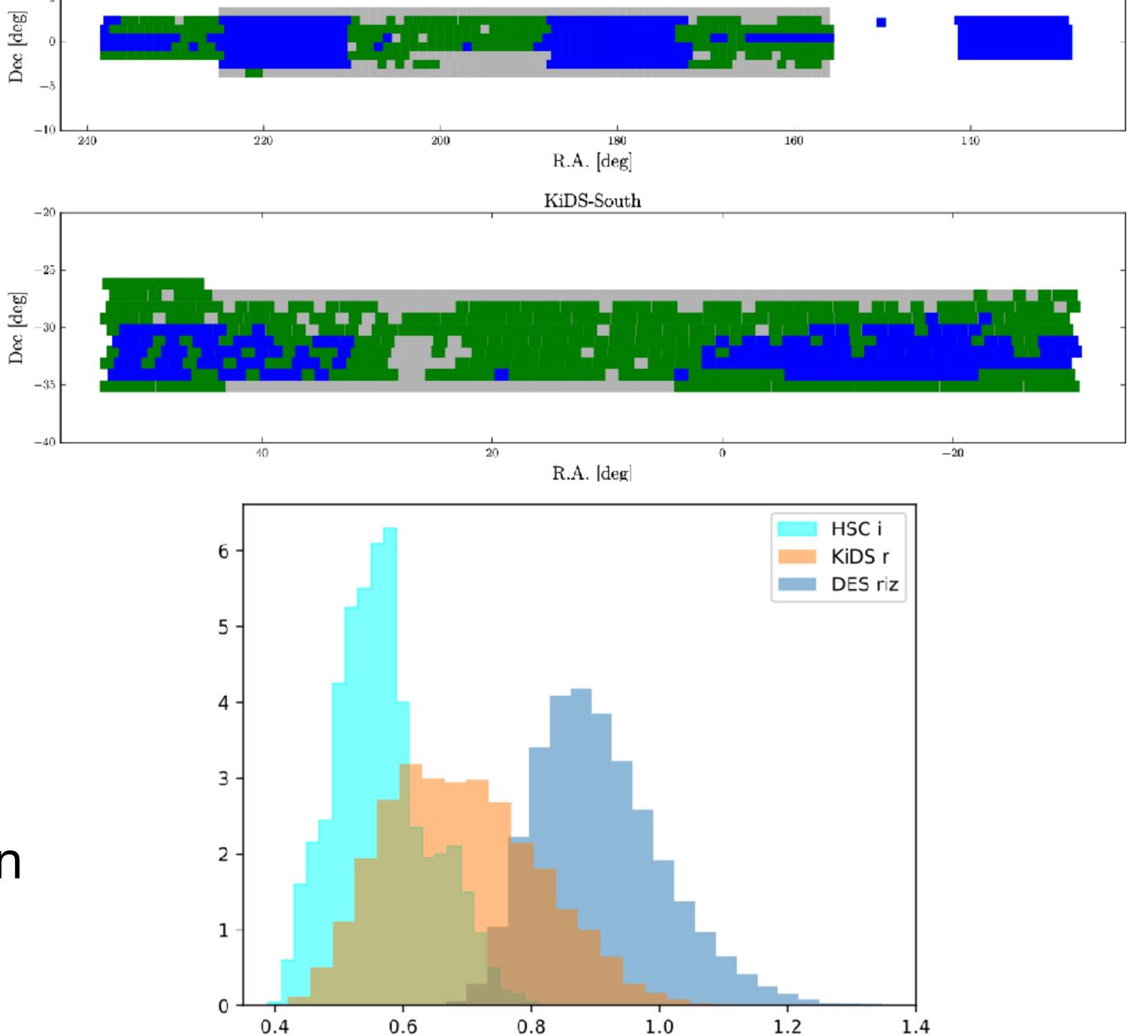
Wright et al. (2020a,2020b)



KiDS-1000 (DR4)

Key facts

- 1000deg² of *ugriZYJHK*s imaging
- 8.4gal/arcmin⁻² with shapes
- Cosmic shear improvements:
 - Extensive mock tests
 - New covariance
 - SOM-based n(z) + gold selection
 - Three different 2pt statistics

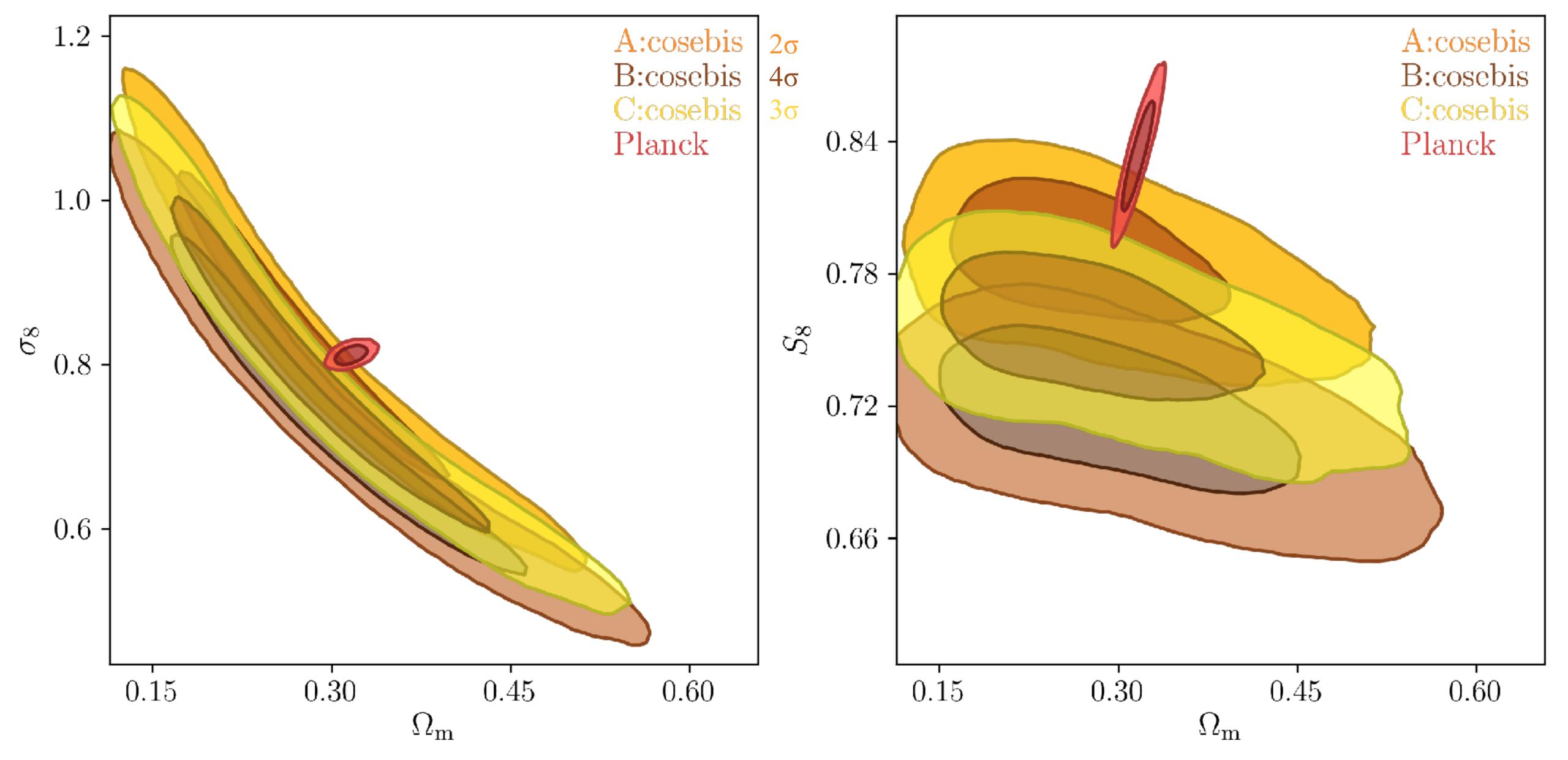


PSF FWHM [arcsec]

KiDS-North

Kuijken et al. (2019)

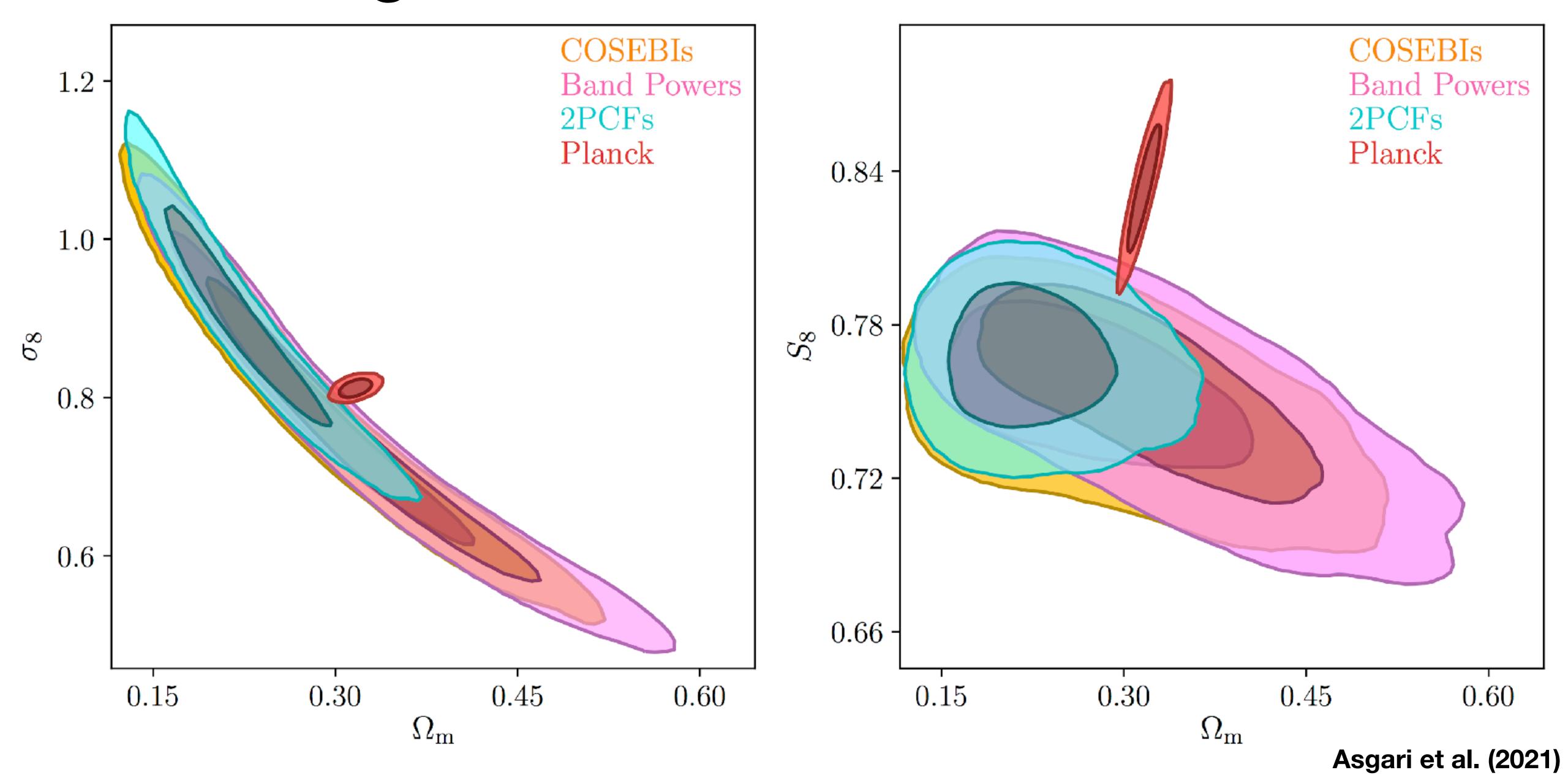
Blinding

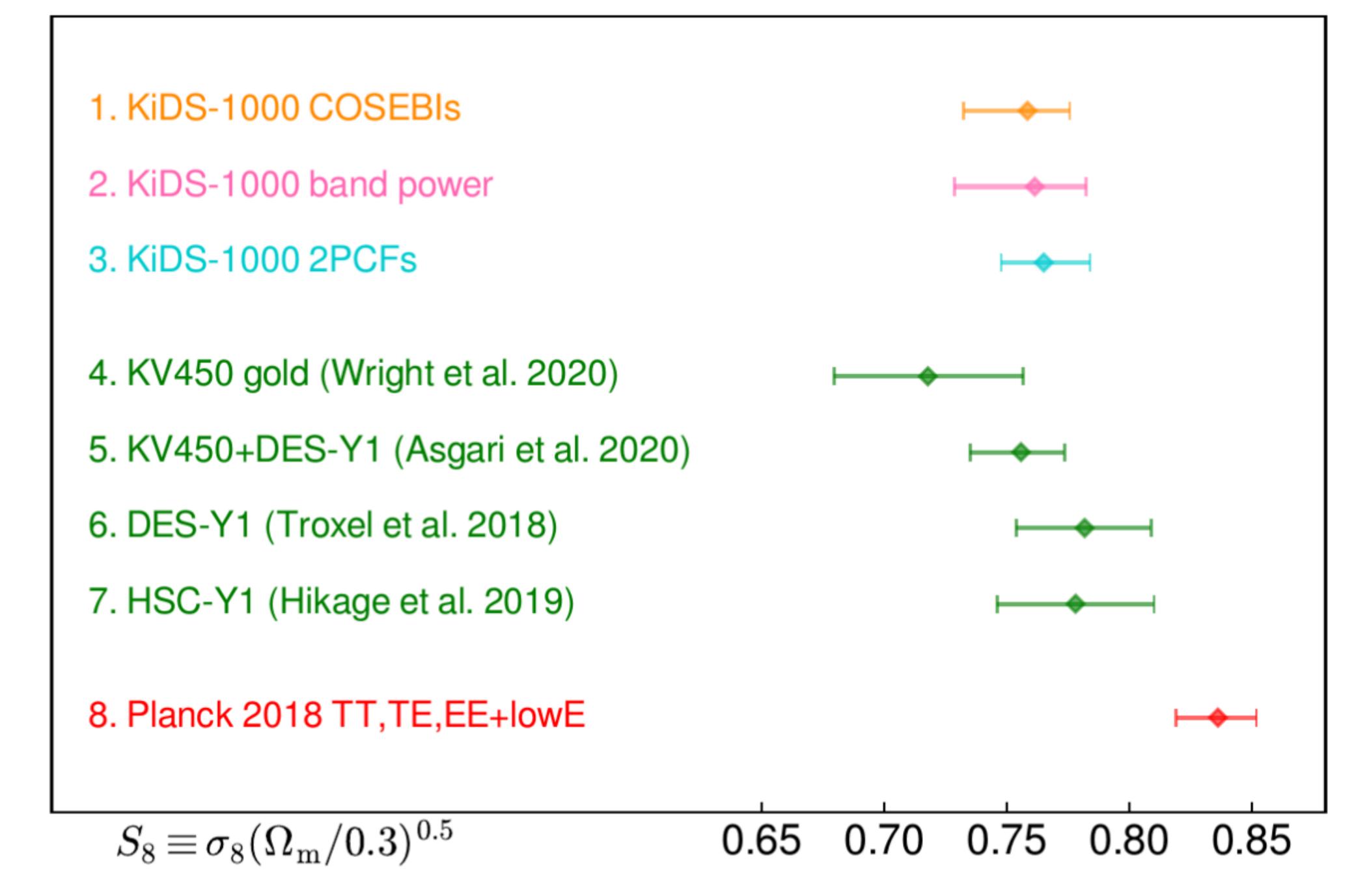


Asgari et al. (2021)

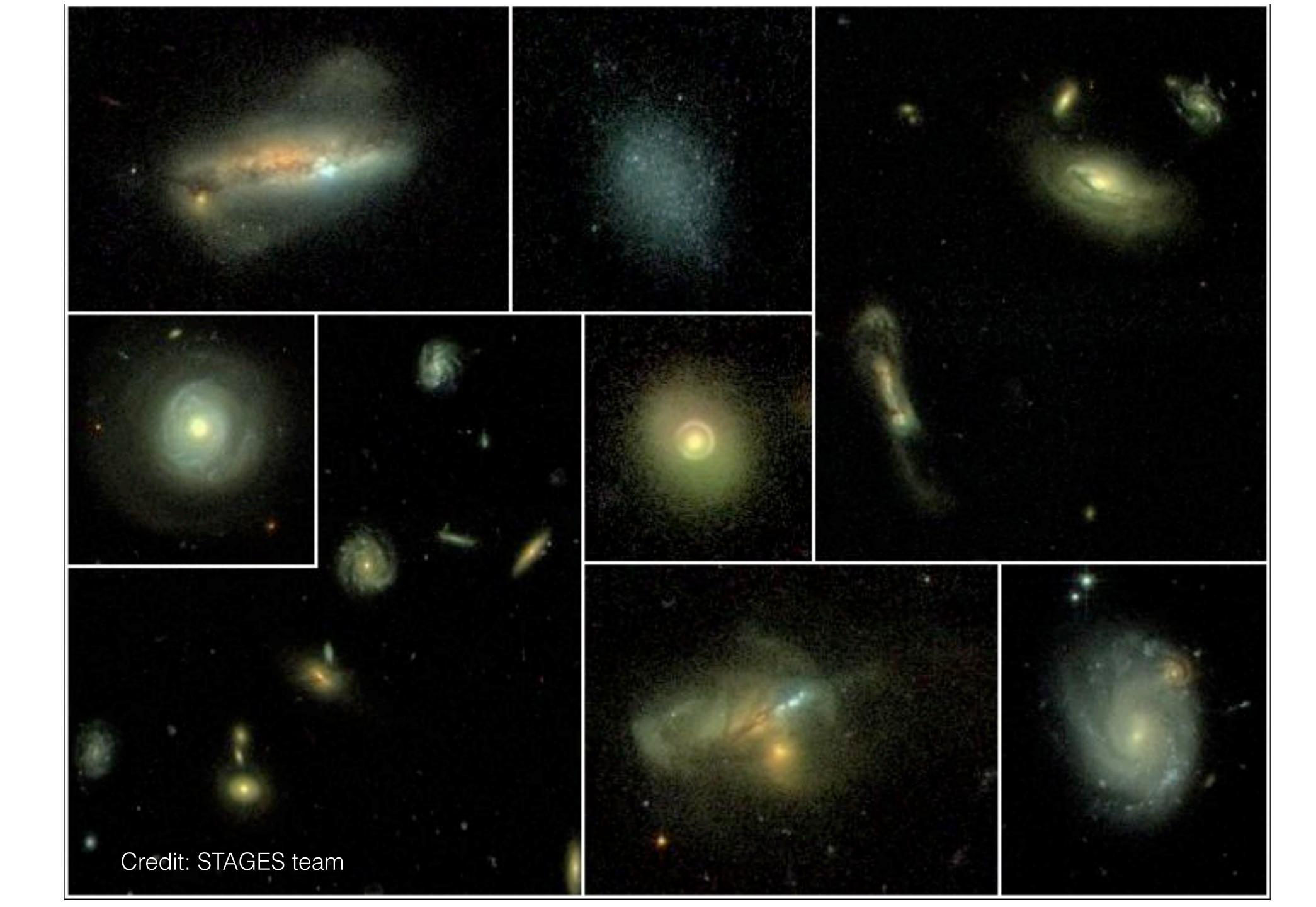


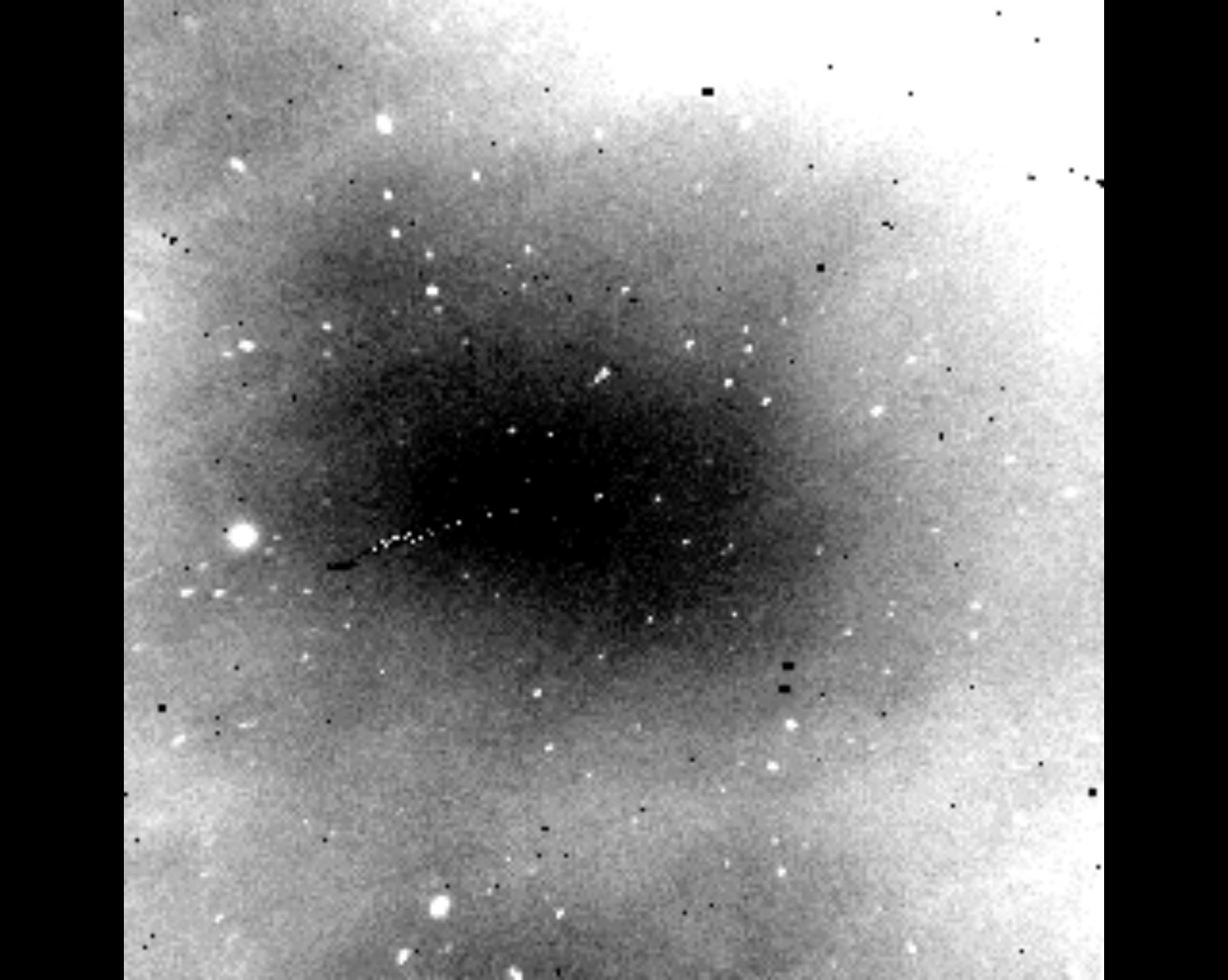
Cosmological constraints









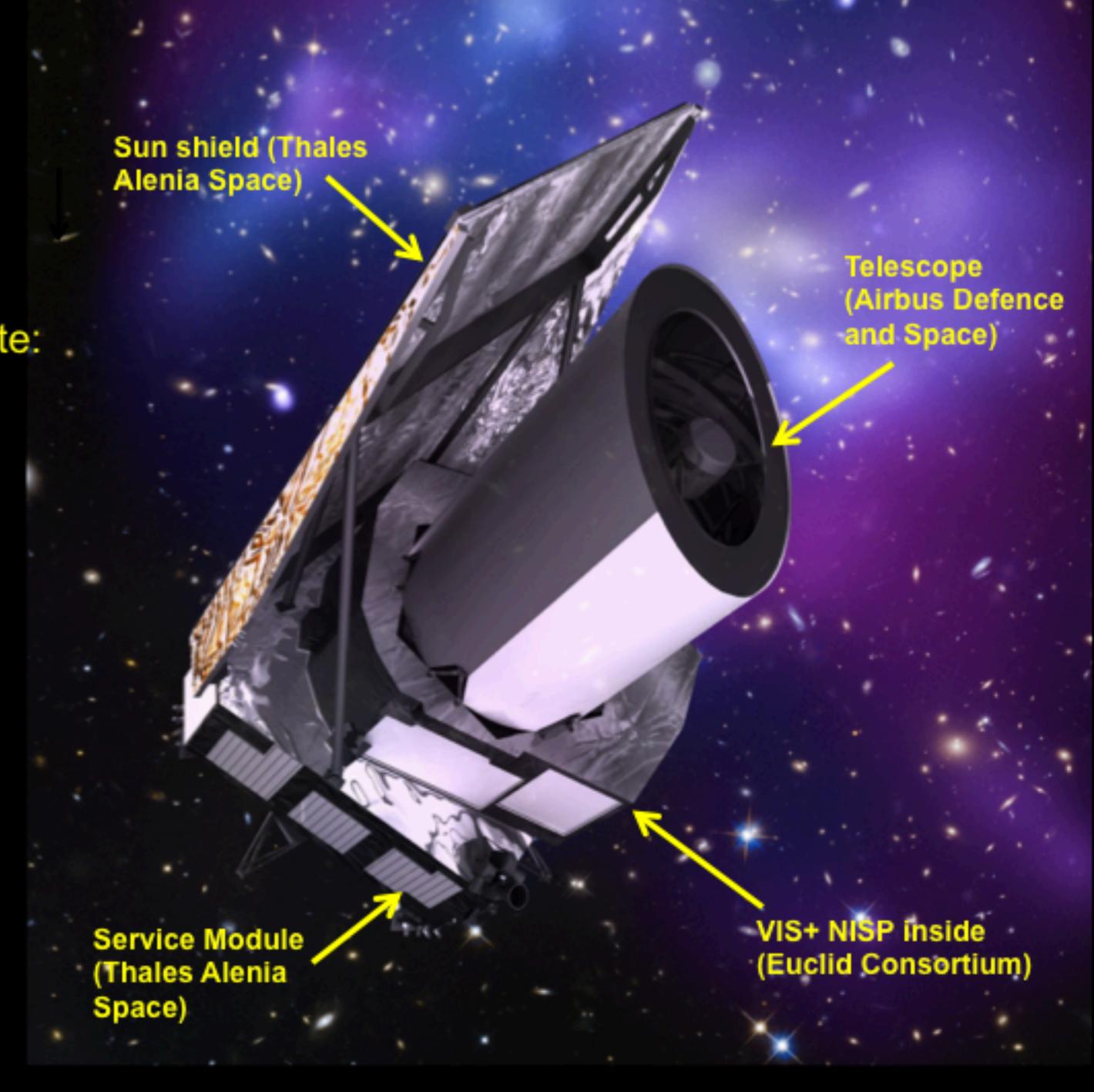


- Total mass satellite:

2200 kg

- Dimensions:

4,5 m x 3 m



Summary

- Normal matter makes up only 5% of the energy density of the Universe.
- Dark matter can be made visible with gravitational lensing.
- What is dark energy? Cosmological constant?
- Discrepancies in current data (H_0 , S_8) might be hints to a solution.
- ESA's Euclid satellite will launch in 2022 starten und solve this riddle.